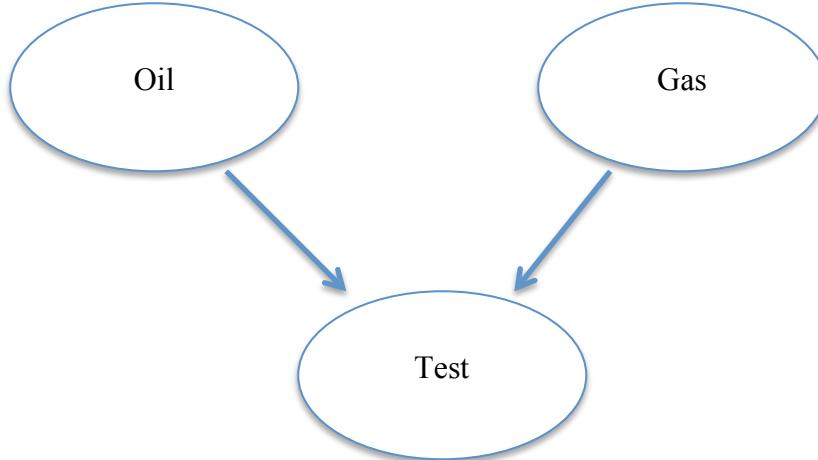


CS161 Homework 6

1.

(a)



Oil	P(Oil)
T	0.5
F	0.5

Gas	P(Gas)
T	0.2
F	0.8

Test	P(Oil)	P(Gas)	P(Test Oil, Gas)
T	F	F	0.1
T	F	T	0.3
T	T	F	0.9
T	T	T	0.0

(b)

$$\text{Using Bayes' Rule, } P(\text{Oil} \mid \text{Test}) = \frac{P(\text{Test} \mid \text{Oil})P(\text{Oil})}{P(\text{Test})}$$

$$P(\text{Test}) = \sum_{\text{Oil}, \text{Gas}} P(\text{Test} \mid \text{Oil}, \text{Gas})P(\text{Oil}, \text{Gas})$$

$$P(\text{Test}) = P(\text{Test} \mid \text{Oil})P(\text{Oil}) + P(\text{Test} \mid \text{Gas})P(\text{Gas}) + P(\text{Test} \mid \neg \text{Oil}, \neg \text{Gas})P(\neg \text{Oil}, \neg \text{Gas})$$

$$P(\text{Test}) = (0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3) = 0.54$$

$$P(\text{Oil} \mid \text{Test}) = \frac{(0.9)(0.5)}{0.54} = \mathbf{0.83}$$

2.

(a)

$$P(A, B, C, D, E, F, G, H) = P(A) * P(B) * P(C | A) * P(D | A, B) * P(E | B) * P(F | C, D) * P(G | H) * P(H | E, F)$$

(b)

$$\begin{aligned} P(A, B, C, D, E, F, G, H) &= f_1(F, G) * f_2(E, F, H) * f_3(C, D, F) * f_4(B, E) * f_5(A, B, D) * \\ &f_6(A, C) * f_7(B) * f_8(A) \end{aligned}$$

$$f_9(A, B, C, D, E, F, G, H) = \sum_A \sum_B \sum_C \sum_D [f_1(F, G) * f_2(E, F, H) * f_3(C, D, F) * f_4(B, E) * f_5(A, B, D) * f_6(A, C) * f_7(B) * f_8(A)]$$

- Factor out D
 - $f_{10}(A, B, C, E, F, G, H) = \sum_A \sum_B \sum_C f_9(A, B, C, D, E, F, G, H)$
- Factor out C
 - $f_{11}(A, B, E, F, G, H) = \sum_A \sum_B f_{10}(A, B, C, E, F, G, H)$
- Factor out B
 - $f_{12}(A, E, F, G, H) = \sum_A f_{11}(A, B, E, F, G, H)$
- Finally, factoring out A will get the final result
 - $P(E, F, G, H) = f_{13}(E, F, G, H)$

(c)

$$P(A, \neg B, C, D, \neg E, F, \neg G, H) = P(H | \neg E, F) * P(\neg G | F) * P(F | C, D) * P(\neg E | \neg B) * P(D | A, \neg B) * P(C | A) * P(\neg B) * P(A)$$

$$P(A, \neg B, C, D, \neg E, F, \neg G, H) = P(H | \neg E, F) * P(\neg G | F) * P(F | C, D) * 0.1 * 0.6 * P(C | A) * 0.3 * 0.2$$

$$P(A, \neg B, C, D, \neg E, F, \neg G, H) = \mathbf{0.0036} * P(H | \neg E, F) * P(\neg G | F) * P(F | C, D) * P(C | A)$$

(d)

$$P(\neg A, B) = P(\neg A)P(B) = (0.8)(0.7) = \mathbf{0.56}$$

- A and B are independent, so can just apply the definition of independence

$$P(\neg E | A) = P(\neg E) = P(\neg E | B)P(B) + P(\neg E | \neg B)P(\neg B) = (0.9)(0.7) + (0.1)(0.3) = \mathbf{0.66}$$

- A and E are independent (i.e. E is only dependent on its parent A), so E is conditionally independent of A. We can then apply the Law of Total Probabilities

(e)

Markovian assumptions states that a node X is conditionally independent of its non-descendants given its parents

- A is conditionally independent of all nodes
- B is conditionally independent of all nodes
- C is conditionally independent of all nodes except for its parent A
- D is conditionally independent of all nodes except for its parents A and B
- E is conditionally independent of all nodes except for its parent B
- F is conditionally independent of all nodes except for its parents C and D
- G is conditionally independent of all nodes except for its parent F
- H is conditionally independent of all nodes except for its parents E and F

(f)

Markov blanket states that a node X is conditionally independent of all other nodes in the network given its parents, children, and children's parents

Market blanket for D = {A, B, C, F}

(g)

A	B	D	P(D A, B)	B	E	P(E B)	A	B	D	E	P(D A, B) * P(E B)
F	F	F	0.2	F	F	0.1	F	F	F	F	0.2 * 0.1 = 0.02
F	F	T	0.8	F	T	0.9	F	F	F	T	0.2 * 0.9 = 0.18
F	T	F	0.9	T	F	0.9	F	F	T	F	0.8 * 0.1 = 0.08
F	T	T	0.4	T	T	0.1	F	F	T	T	0.8 * 0.9 = 0.72
T	F	F	0.1				F	T	F	F	0.9 * 0.9 = 0.81
T	F	T	0.6				F	T	F	T	0.9 * 0.1 = 0.09
T	T	F	0.5				F	T	T	F	0.1 * 0.9 = 0.09
T	T	T	0.5				F	T	T	T	0.1 * 0.1 = 0.01
							T	F	F	F	0.4 * 0.1 = 0.04
							T	F	F	T	0.4 * 0.9 = 0.36
							T	F	T	F	0.6 * 0.1 = 0.06
							T	F	T	T	0.6 * 0.9 = 0.54
							T	T	F	F	0.5 * 0.9 = 0.45
							T	T	F	T	0.5 * 0.1 = 0.05
							T	T	T	F	0.5 * 0.9 = 0.45
							T	T	T	T	0.5 * 0.1 = 0.05

(h)

$$f_{15}(A, B, E) = \sum_D f_{14}(A, B, D, E) = f_{14}(A, B, D, E) + f_{14}(A, B, \neg D, E)$$

A	B	E	$f_{15}(A, B, E)$
F	F	F	$0.08 + 0.02 = 0.1$
F	F	T	$0.72 + 0.18 = 0.9$
F	T	F	$0.09 + 0.81 = 0.9$
F	T	T	$0.01 + 0.09 = 0.1$
T	F	F	$0.06 + 0.04 = 0.1$
T	F	T	$0.54 + 0.36 = 0.9$
T	T	F	$0.45 + 0.45 = 0.9$
T	T	T	$0.05 + 0.05 = 0.1$