Future Value

- One-period:

$$FV = C_0 \times (1 + r)$$

Where C_0 is cash flow today (time zero), and
r is the appropriate interest rate.

Present Value

$$\$9{,}523.81 = \frac{\$10{,}000}{1.05}$$

Note that $10,000 = $9,523.81 × (1.05).

Present Value

- In the one-period case:

$$PV = \frac{C_1}{1+r}$$

Where C_1 is cash flow at date 1, and

r is the appropriate interest rate.

Net Present Value

- The Net Present Value (*NPV*) of an investment is the present value of the expected cash flows, less the cost of the investment.
- Suppose an investment that promises to pay $10,000 in one year is offered for sale for $9,500. Your interest rate is 5%. Should you buy?
- ONLINE POLL – Q10

Net Present Value

$$NPV = -\$9,500 + \frac{\$10,000}{1.05}$$

$$NPV = -\$9,500 + \$9,523.81$$

$$NPV = \$23.81$$

The present value of the cash inflow is greater than the cost. In other words, the Net Present Value is positive, so the investment should be purchased.

Net Present Value

In the one-period case, the formula for *NPV* can be written as:

$$NPV = -Cost + PV$$

If we had *not* undertaken the positive *NPV* project considered on the last slide, and instead invested our \$9,500 elsewhere at 5 percent, our *FV* would be less than the \$10,000 the investment promised, and we would be worse off in *FV* terms :

$$\$9{,}500 \times (1.05) = \$9{,}975 < \$10{,}000$$

4.2 The Multiperiod Case

$$FV = C_0 \times (1 + r)^T$$

Future Value

- Suppose a stock currently pays a dividend of \$1.10, which is expected to grow at 40% per year for the next five years.
- What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

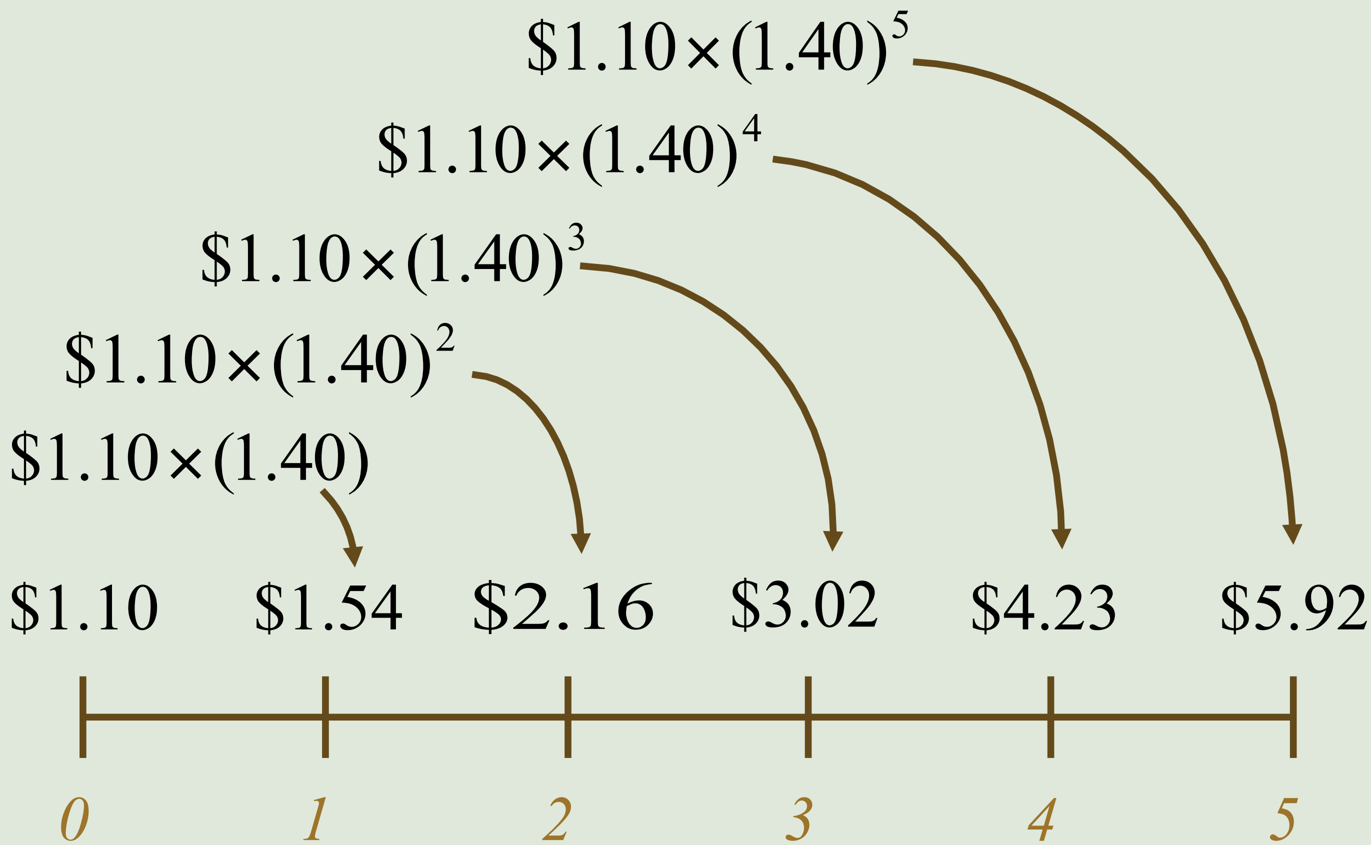
Future Value and Compounding

- Notice that the dividend in year five, \$5.92, is considerably higher than the sum of the original dividend plus five increases of 40-percent on the original \$1.10 dividend:

$$\$5.92 > \$1.10 + 5 \times [\$1.10 \times .40] = \$3.30$$

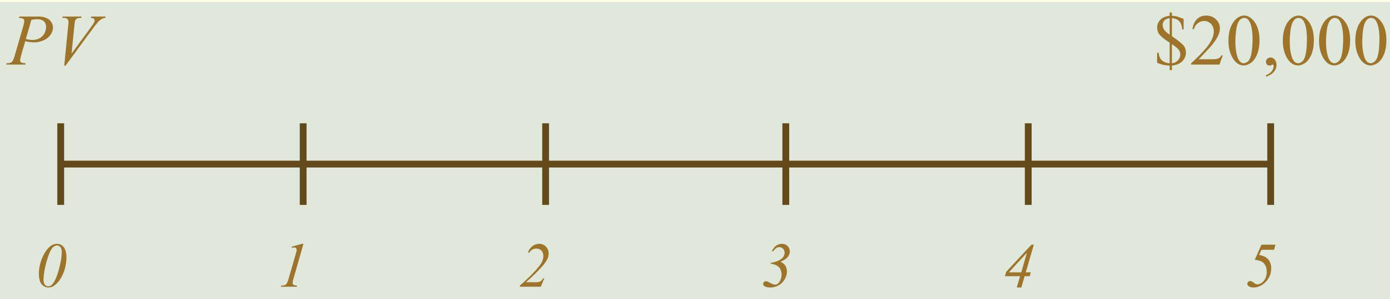
This is due to *compounding*.

Future Value and Compounding



Present Value and Discounting

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

Using the tables:

TABLE 1 Future Value of $1

$FV = \$1\,(1 + i)^n$

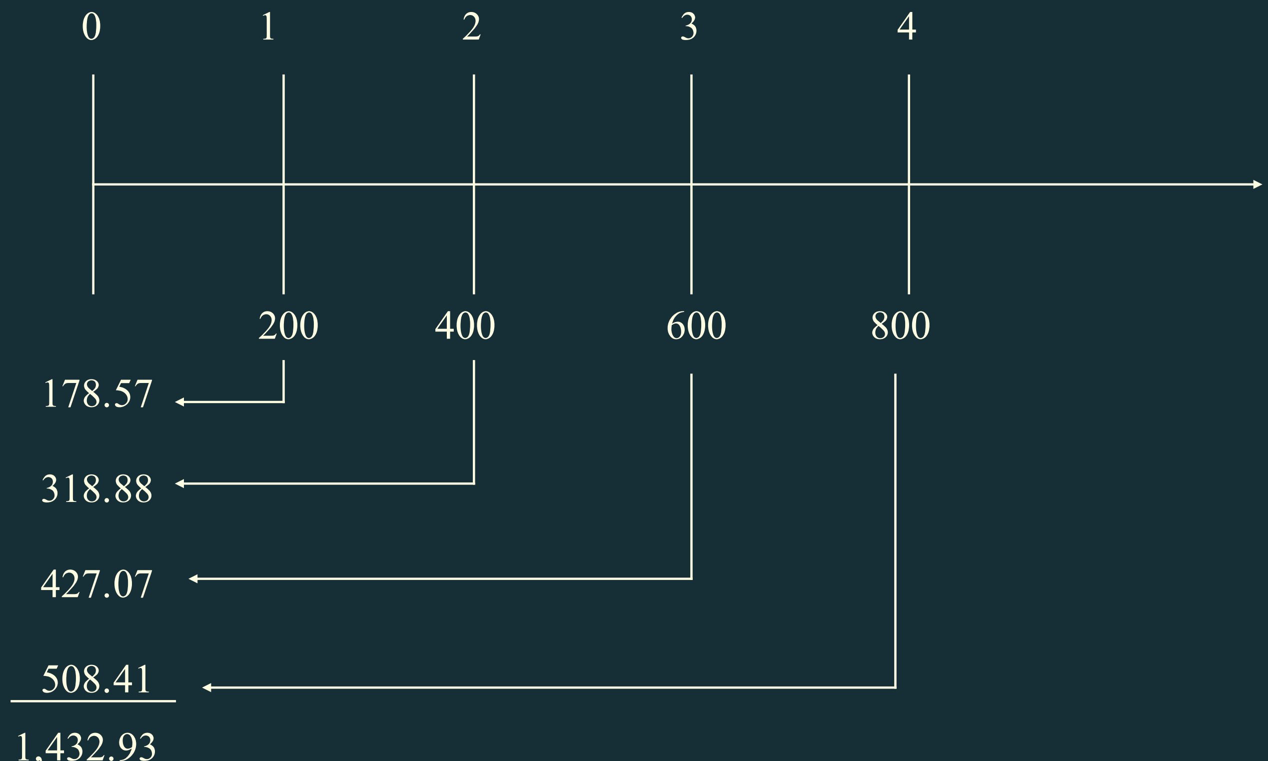
n/i	1.0%	1.5%	2.0%	2.5%
1	1.01000	1.01500	1.02000	1.02500
2	1.02010	1.03022	1.04040	1.05063
3	1.03030	1.04568	1.06121	1.07689
4	1.04060	1.06136	1.08243	1.10381
5	1.05101	1.07728	1.10408	1.13141

TABLE 2 Present Value of $1

$$PV = \frac{\$1}{(1 + i)^n}$$

n/i	1.0%	1.5%	2.0%	2.5%
1	0.99010	0.98522	0.98039	0.97561
2	0.98030	0.97066	0.96117	0.95181
3	0.97059	0.95632	0.94232	0.92860
4	0.96098	0.94218	0.92385	0.90595
5	0.95147	0.92826	0.90573	0.88385

Multiple Cash Flows – Price of the following cash flow is $1,5000, would you buy?



Present Value < Cost → Do Not Purchase

Effect of Interest Rate on Tail Heavy Cash Flow

Year	0	1	2	3	4	5	6	7	8	9	10
Cash Flow I	-10,000	3,000	3,000	3,000	3,000	3,000	4,000	4,000	4,000	4,000	4,000
Cash Flow II	-10,000	4,000	4,000	4,000	4,000	4,000	3,000	3,000	3,000	3,000	3,000

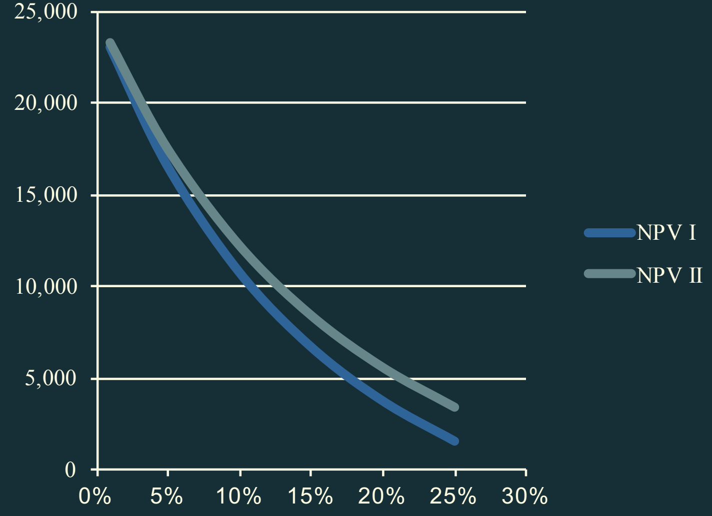
Online Poll: Q11

As the interest rate goes up, which one would lose value faster?

Effect of Interest Rate on Tail Heavy Cash Flow

Year	0	1	2	3	4	5	6	7	8	9	10
Cash Flow I	-10,000	3,000	3,000	3,000	3,000	3,000	4,000	4,000	4,000	4,000	4,000
Cash Flow II	-10,000	4,000	4,000	4,000	4,000	4,000	3,000	3,000	3,000	3,000	3,000

Rate	NPV I	NPV II
1%	23,032	23,267
5%	16,557	17,495
10%	10,787	12,224
15%	6,723	8,408
20%	3,779	5,568
25%	1,593	3,401



4.3 Compounding Periods

Compounding an investment *m* times a year for *T* years provides for future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

Effective Annual Rates of Interest

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times3} = \$50 \times (1.06)^6 = \$70.93$$

$$\$50 \times (1 + EAR)^3 = \$70.93$$

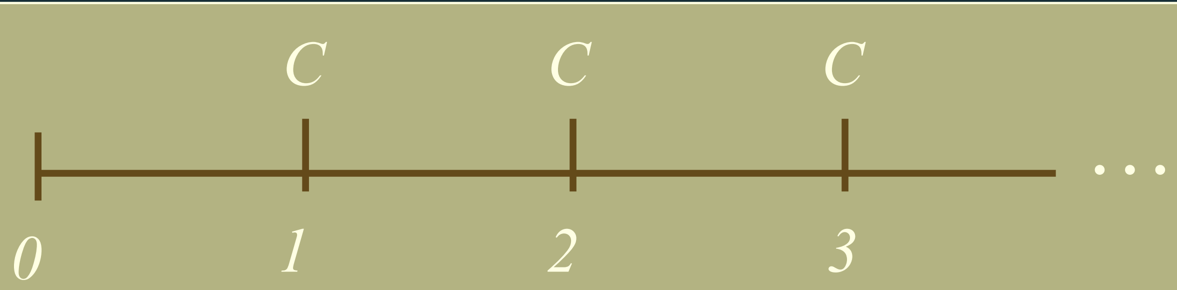
EAR = 12.36%

4.4 Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity

A constant stream of cash flows that lasts forever



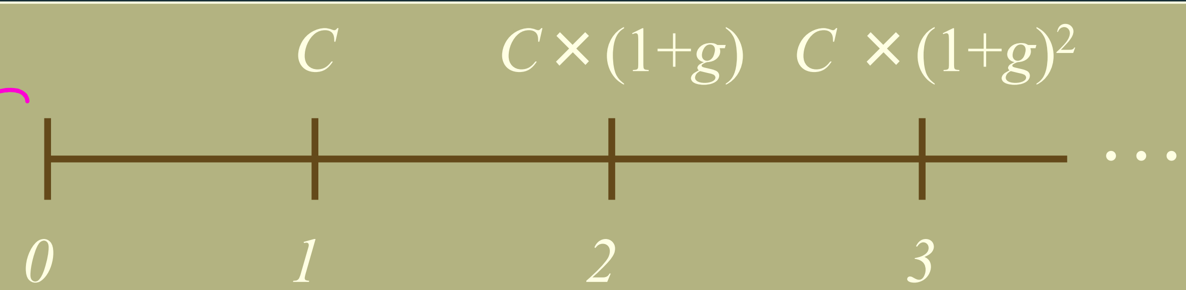
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

$$\frac{1}{1-\left(\frac{1}{1+r}\right)} \qquad \frac{C}{(1+r)}\underbrace{\left(1+\frac{1}{1+r}+\frac{1}{(1+r)^2}+\cdots\right)}$$

Growing Perpetuity

A growing stream of cash flows that lasts forever



no cash here

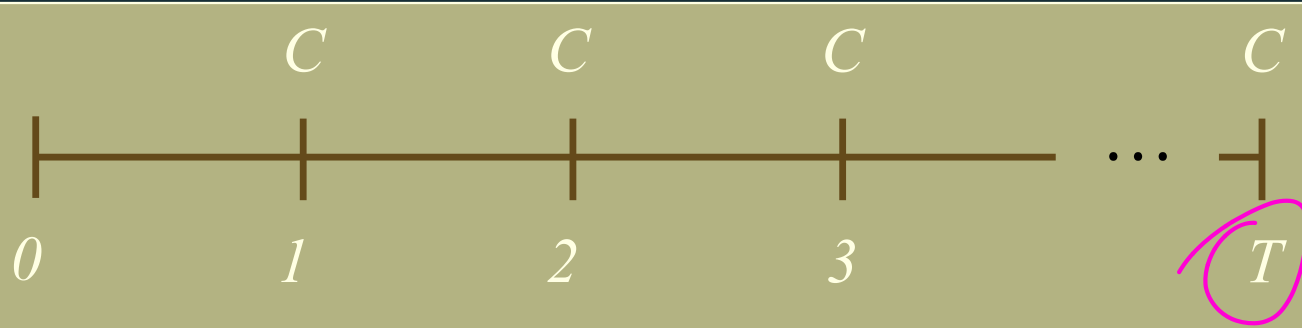
$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \cdots$$

$$PV_0 = \frac{C}{r-g}$$

→ value at t=0!

Annuity

A constant stream of cash flows with a fixed maturity



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

$$PV_0 = \frac{100}{r} - \frac{\frac{100}{r}}{(1+r)^T}$$

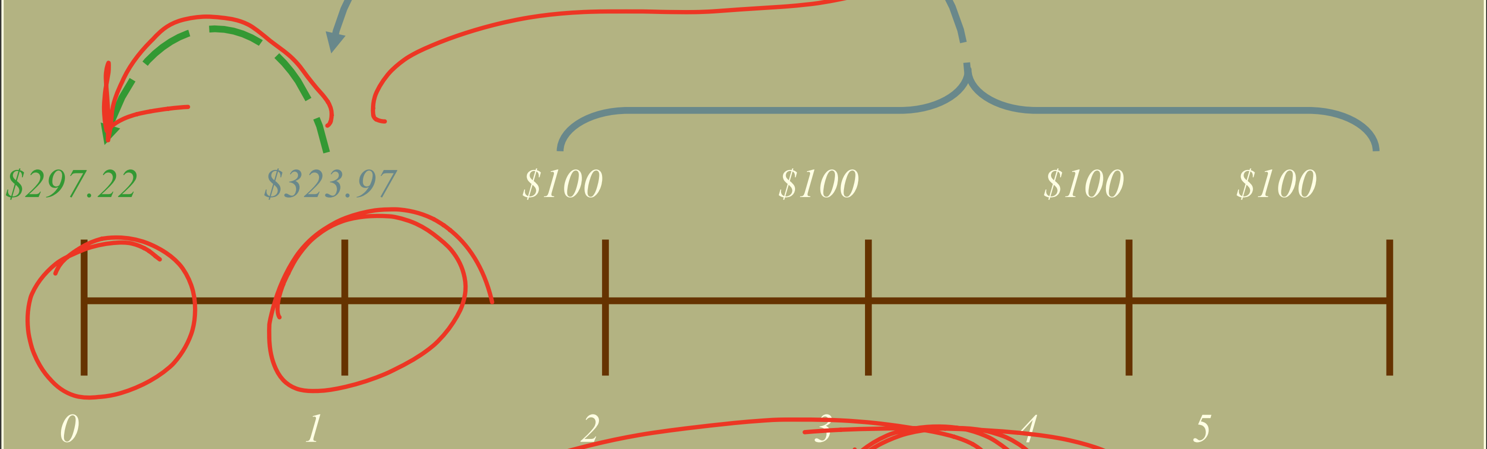
	100	100	...	100	100	100 ...	
0	1	2	...	T	T+1	...	∞

$$PV = \frac{100}{1.10} + \frac{100}{1.10^2} + \frac{100}{1.10^3} + \cdots + \frac{100}{1.10^{10}}$$

What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$

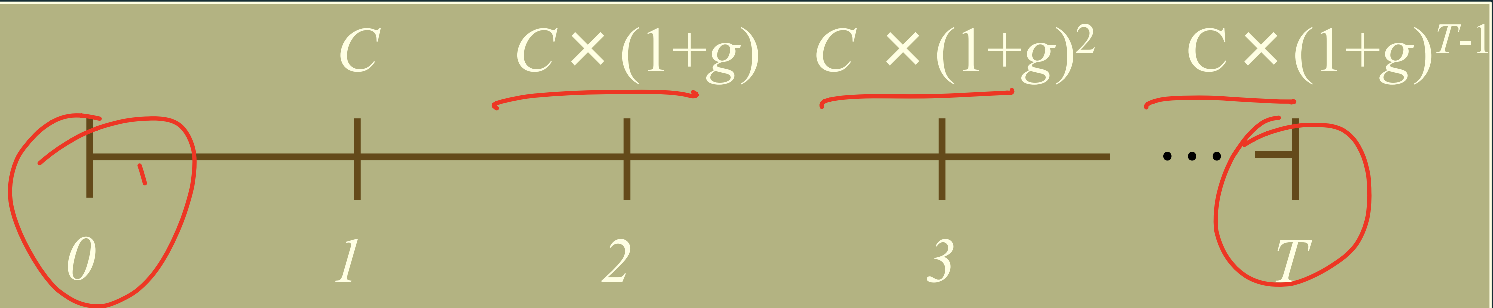
$$PV_1 = \frac{100}{0.09}\left(1 - \frac{1}{(1+9\%)^4}\right)$$



$$PV_o = \frac{\$323.97}{1.09} = \$297.22$$

Growing Annuity

A growing stream of cash flows with a fixed maturity



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \cdots + \frac{C\times(1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{(1+r)}\right)^T\right]$$

Chapter 5&6

Net Present Value and Other Project Evaluation Methods

Making Capital Investment Decisions

5.1 Why Use Net Present Value?

- Accepting positive NPV projects benefits shareholders.
  - ✓ NPV uses cash flows
  - ✓ NPV uses all the cash flows of the project
  - ✓ NPV discounts the cash flows properly

The Net Present Value (NPV) Rule

- Net Present Value (NPV) =

  Total PV of future CF's + Initial Investment
- Estimating NPV:
  1. Estimate future cash flows: how much? and when?
  2. Estimate discount rate
  3. Estimate initial costs
- Minimum Acceptance Criteria: Accept if NPV > 0
- Ranking Criteria: Choose the highest NPV

Year	Project I	Project II
0	-1000	-2000
1	600	1000
2	800	400
3	400	680
4	700	800
NPV @10%	985.24	296.97

5.2 The Payback Period Method

- How long does it take the project to "pay back" its initial investment?
- Payback Period = number of years to recover initial costs
- Minimum Acceptance Criteria:
  - Set by management
- Ranking Criteria:
  - Set by management

Payback Period:

$r = 10\%$

	A	B	Years
	−1000	−1000	0
182.	+200	+400	1
165.	200	600	2
376	500	100	3
342	500	0	4
310	500	0	5

A: 3 years $\frac{100}{500}$ → 3.2 years

B: 2 years

3.8 years ⟶ $\underbrace{182+165+376}_{723}$ → $\frac{277}{342}$ → 3 years 0.8

The Payback Period Method

- Disadvantages:
  - Ignores the time value of money
  - Ignores cash flows after the payback period
  - Biased against long-term projects
  - Requires an arbitrary acceptance criteria
  - A project accepted based on the payback criteria may not have a positive NPV
- Advantages:
  - Easy to understand
  - Biased toward liquidity
  - Helpful in assessing lower management

5.3 The Discounted Payback Period

- How long does it take the project to “pay back” its initial investment, taking the time value of money into account?
- Decision rule: Accept the project if it pays back on a discounted basis within the specified time.
- By the time you have discounted the cash flows, you might as well calculate the NPV.

5.4 The Internal Rate of Return

- IRR: the discount rate that sets NPV to zero

- Advantages:
  - Easy to understand and communicate
  - No need to use a market rate to calculate IRR

  What is IRR?

  I invest $10 today and get $12 in one year

  ⟹ IRR 20%

  I borrow $10 today and pay $12 in one year

  ⟹ IRR 20%

  What is the interpretation?

5.6 The Profitability Index (PI)

$$PI = \frac{\text{Total PV of Future Cash Flows}}{\text{Initial Investment}}$$

- Minimum Acceptance Criteria:
  - Accept if PI > 1
- Ranking Criteria:
  - Select alternative with highest PI

Using PI on Mutually Exclusive Projects

Project	**A**	**B**	**A-B**
Investment	-100	-70	-30
Year 1	120	20	100
Year 2	80	140	-60
NPV (@10%)	75.21	63.88	11.32
PI(@10%)	1.75	1.91	1.38

The Profitability Index

- Disadvantages:
  - Problems with mutually exclusive investments
- Advantages:
  - May be useful when available investment funds are limited
  - Easy to understand and communicate
  - Correct decision when evaluating independent projects

5.7 The Practice of Capital Budgeting

- Varies by industry
- The most frequently used technique for large corporations is either IRR or NPV.
- Military uses PI

Summary – Discounted Cash Flow

- Net present value
  - Difference between market value and cost
  - Accept the project if the NPV is positive
  - Has no serious problems
  - Preferred decision criterion
- Profitability Index
  - Benefit-cost ratio
  - Take investment if PI > 1
  - Cannot be used to rank mutually exclusive projects
  - May be used to rank projects in the presence of capital rationing

Summary – Payback Criteria

- Payback period
  - Length of time until initial investment is recovered
  - Take the project if it pays back in some specified period
  - Does not account for time value of money, and there is an arbitrary cutoff period
- Discounted payback period
  - Length of time until initial investment is recovered on a discounted basis
  - Take the project if it pays back in some specified period
  - There is an arbitrary cutoff period