# Binary Conversions

In the modern world, we use decimal, or base 10, notation to represent integers. We can represent numbers using any base b, where b is a positive integer greater than 1.

Base 10:

- When we write 965, this can be translated as: $9 \cdot 10^2 + 6 \cdot 10^1 + 5 + 10^0$
- $n = a_k b^k + a_{k-1} b^{k-1} + \ldots a_1 b + a_0$

Base b:

- Let b be a positive integer greater than 1. Then if n is a positive integer, it can be expressed uniquely in the form, where k is a nonnegative integer and $a_0, a_1 \ldots a_k$ are nonnegative integers less than b.
- this representation of n is called the base b expansion of n and can be denoted by $(a_k a_{k-1} \ldots a_1 a_0)_b$

Binary expansions:

- Computers represent integers and do arithmetic with binary (base 2) expansions of integers. In these expansions, the only digits used are 0 and 1.
- Example: What is the decimal expansion of the integer that has $(11011)_2$ as its binary expansion?

  Solution: $(11011)_2$

  $= 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

  $= 16 + 8 + 0 + 2 + 1$

  $= 27$

Base Conversion:

- To construct the base b expansion of the integer n, divide n by b to obtain a quotient and remainder: $n = bq_0 + a_0,\ 0 \leq a_0 < b$
- The remainder, $a_0$, is the rightmost digit in the base b expansion of n.
- Next, divide $q_0$ by b: $q_0 = bq_1 + a_1,\ 0 \leq a_1 < b$
- The remainder, $a_1$, is the 2nd digit from the right in the base b expansion of n.
- Continue by successively dividing the quotients by b, obtaining the additional base b digits as the remainder. The process terminates when the quotient is 0.