Visual Photons

Formation and Sensation of Retinal Images

author: Jonathan D. Lettvin

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1 Introduction

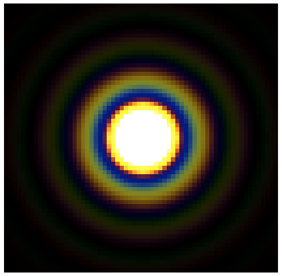


figure 1: Airy pattern

Whither and whence visual photons needs must uncertain be yet by lawful pattern ruled. Photons do not interact. As individual photons from a point source accumulate into an image they tend to accumulate as an Airy pattern. The accumulation has the character of a wave.

This pattern of intensity is ruled by equation[4]. This formula represents the idealized continuous distribution from which deviation is unmeasurable at high photon counts. Discrete photons do not form a continuous set. Nor do sensors arranged along a grid in a focal plane in an eye or camera. The zoomed "pixels" in figure[1] illustrate this grid for billions of RGB source photons seen for a centrally located point source.

Close examination reveals that different wavelengths contribute differentially to the pattern at different radii. Photographic and visual colors are typically thought to operate using RGB. The names R (red), G (green), and B (blue), as wavelengths, are typically thought to be related to color perception. In fact, color perception is independent of wavelength Lettvin et al[1]. The scientific labelling for RGB in retinal photoreceptor absorption is LMS for Long, Medium, and Short. However, since photography and displays use RGB, they will be used interchangeably.

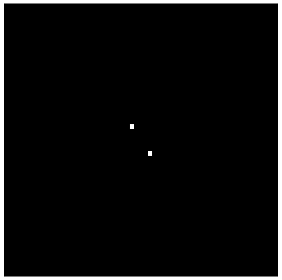


figure 2:
2 white points

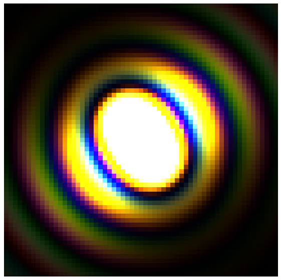


figure 3:
Retinal image with

The retinal image of two white RGB point sources incident on the human retina after traversing a 1mm pupil appears as an inchoate blur incapable of being resolved by Rayleigh criterion, Sparrow limit, or even traditional use of Fourier transform coupled with Nyquist sampling.

The wavelength dependence in figure[3] is even more apparent than for figure[1]. This is the character of image for narrow band RGB illumination. This paper will explore the problem of forming retinal equivalent incident photons using broadband illumination and methods for producing suitable sensed images for analyzing how the sensory data for vision is presented.

a 1mm pupil

Even in figure[3], enough photons are present to give the image a seemingly continuous character.

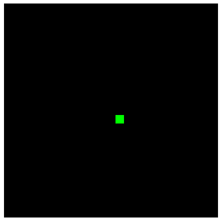


figure 4: 1 photon

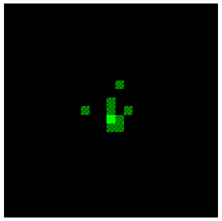


figure 5: 10 photons

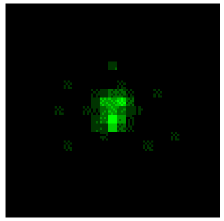


figure 6: 100 photons

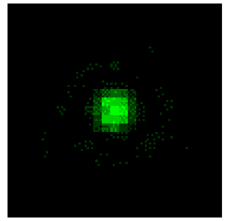


figure 7: 1000 photons

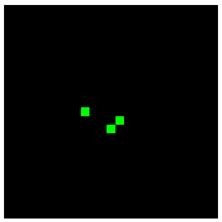


figure 8: 3 photons

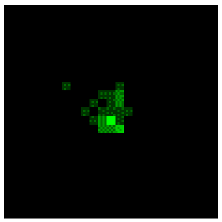


figure 9: 30 photons

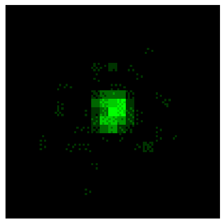


figure 10: 300 photons

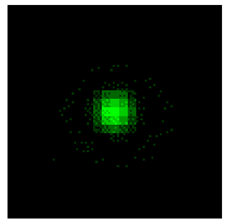


figure 11: 3000 photons

When photon flux is low, the Airy pattern becomes more difficult to discern as can be seen in the models of accumulations of single monochromatic photon events in figure[4] to figure[11].

As can be seen, even at 3000 photons from a point source the Airy pattern is yet to assume the continuous character achieved at much higher photon fluxes experienced in daylight or even low incandescent lighting.

Vision still works fairly well at low light levels so whatever explanation is made for vision must account for the ability to convert patterns such as these to good enough quality internal reconstructions of the scene.

It is important to note that, when point sources are eccentric from the center, different wavelength photons refract and diffract to different centers aligned radially from the center; yet the perception of a polychromatic point is of a single point source.

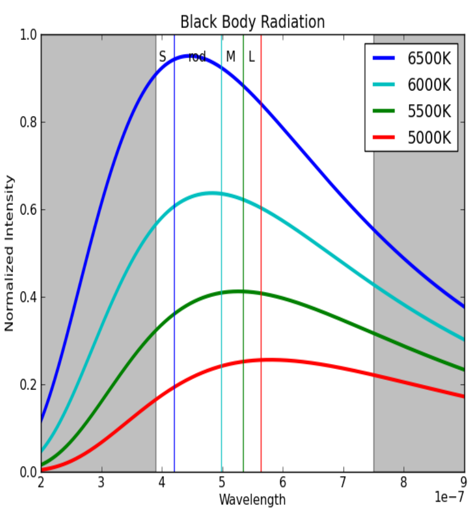


figure 12: Blackbody radiation

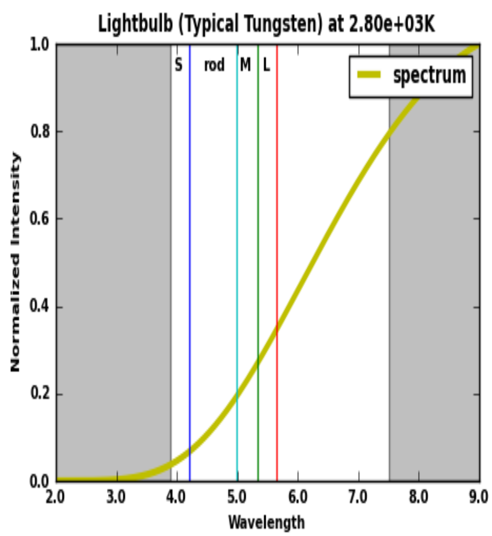


figure 13: Light bulb

Natural lighting does not divide photons into the 3 or 4 energy bands discussed in the literature. Spectra are considered continuous in that no energies or wavelengths are forbidden. Typically, ambient light is broad spectrum as in figure[12]. Visible wavelengths are within the white region of the figure. The peak absorption of typical human photoreceptors are shown as colored vertical lines. Sunlight resembles the 6500K curve with atmospheric absorption. An incandescent lightbulb peak temperature is 3695K which yields a slope that appears to rise monotonically from the blue end to the red end. The spectrum of figure[13] is typical for a light bulb at 2800K. When walking from sunlight into incandescent light one does not perceive a change in colors. The change of spectrum does not change the colors seen.

The two spectra figure[12] and figure[13] were produced with equation[2]. The theory of this curve is well explored. Blackbody radiation may be specified with two parameters, T (degrees Kelvin).

$$I(\lambda, T) = \frac{2hc^3}{\lambda^5} \frac{1}{exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Blackbody equation in λ [1]

$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{exp\left(\frac{h\nu}{kT}\right) - 1}$$

Blackbody equation in ν [2]

where:
h = *Planck constant*
c = *speed of light in a vacuum*
k = *Boltzmann constant*
ν = *photon frequency*
T = *body temperature in Kelvin*

Other spectra like figure[14] and figure[15] are generated using short lists of coefficient triples (a,b,c). The curves in both of these figures took one or two triples each. This equation is used for convenient curve-fitting.

$$Gauss(x) = a\ exp\left(-\frac{(x-b)^2}{2c^2}\right)$$

Gaussian function [3]

The typical trichromat human has three cone photoreceptors L, M, and S. A fourth, rod photoreceptor, is labelled X in figure[14]. As can be seen, all human photoreceptors absorb photons having short wavelengths. Absorption of photons at the longer end have sharp relative cutoffs such that the S cone absorbs almost no photons at wavelengths above ~550nm. The M cone cuts off at ~625nm, and L cone cuts off at

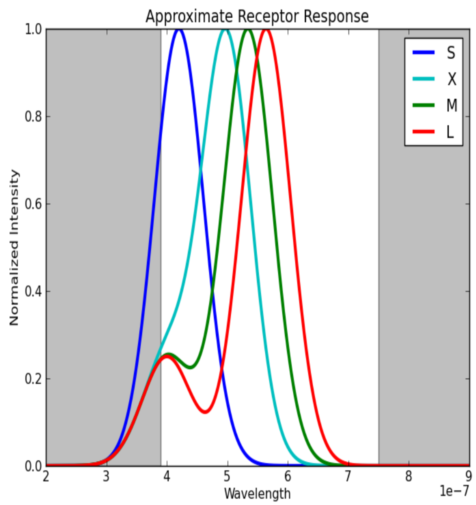


figure 14: Retinal receptor absorption

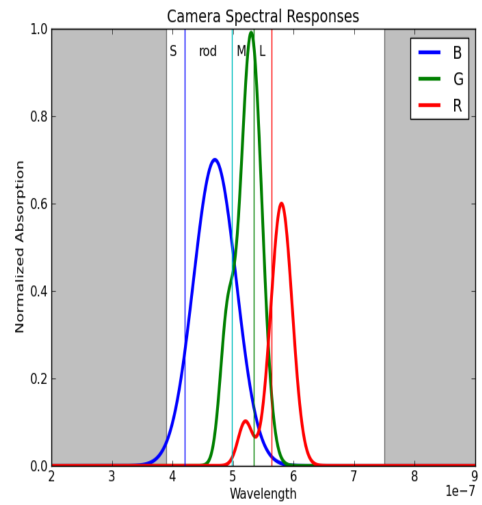


figure 15: Camera pixel absorption

~710nm. Note the curious increase of L and M cone absorption at shorter wavelengths. All this indicates that the wavelengths in a scene are not as sharply divided into color planes as is assumed in standard image processing.

A critical function of vision is to identify readiness of a resource by its color. Regardless of the ambient light, a ripe apple should look red. In fact, it does when viewed under a blue sky, a green forest canopy, a red sunset, or by sodium light having only two yellow and one blue spectral line. The redness of the apple does not depend on the presence of long wavelength photons. This is called color constancy and is what was described for perception under figure[12] and figure[13].

$$I = I_0 \left(\frac{2J_1(u)}{u} \right)^2 \qquad \textit{Airy equation} \qquad [4]$$

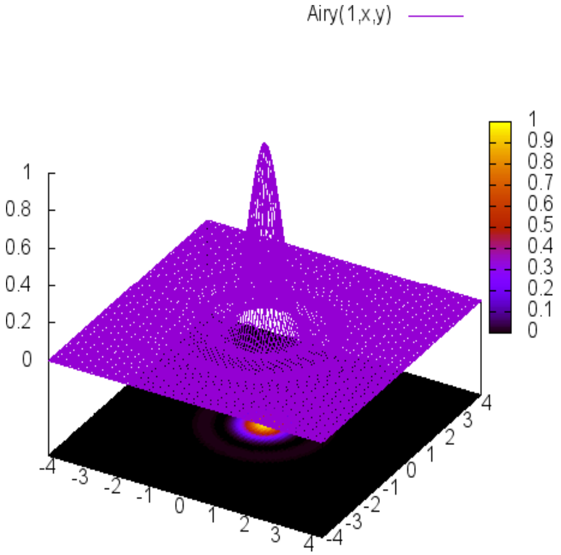
$$u = \frac{2\pi a q}{\lambda R} \qquad \textit{Airy parameter} \qquad [5]$$

where:
$J_1 = Bessel\ function\ of\ the\ first\ kind$
$a = aperture\ radius$
$q = focal\ plane\ offset$
$\lambda = photon\ wavelength$
$R = distance\ from\ aperture\ to\ focal\ plane$

Wave superposition rules when multiple points are being projected onto an image plane. The waves from all the points are summed and the result is squared to yield the intensity map (image).

$$A = A_0 \frac{2J_1(u)}{u} \qquad \textit{Airy wave equation} \qquad [6]$$

where:
$A_0 = \sqrt{I_0}$



gnuplot 1: Airy(x,y)

The conditions under which this occurs are special. An achromatic lens is required.

The straight lines predicted by optics between this origin identify the center of a probability distribution at the destination. The relative probability of a photon of specific energy from a specific source being absorbed within a specific tile in a grid of a tiled surface is calculated by integrating the infinitesimal probabilities over the tile.

Retinal Photoreceptor Species Mosaic

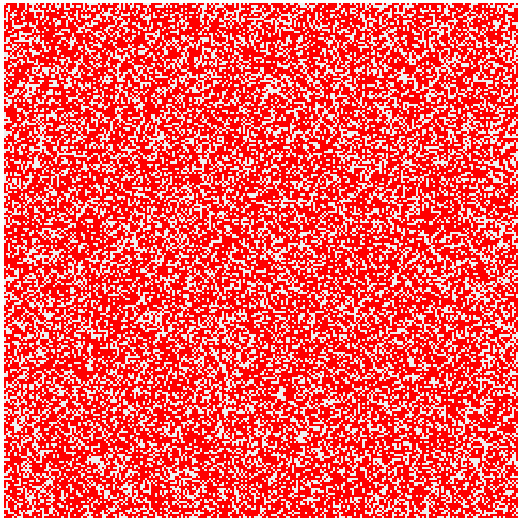


figure 16: Mosaic L cones

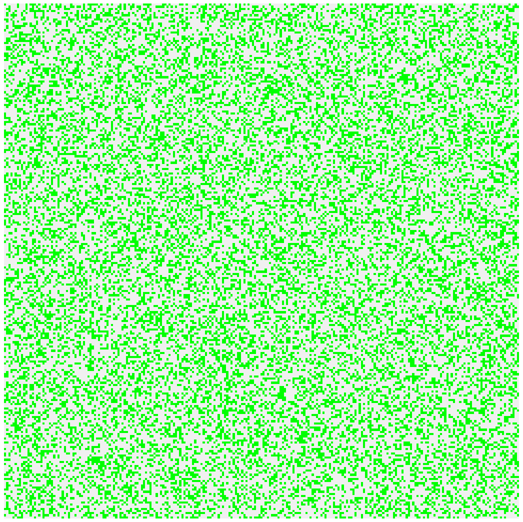


figure 17: Mosaic M cones

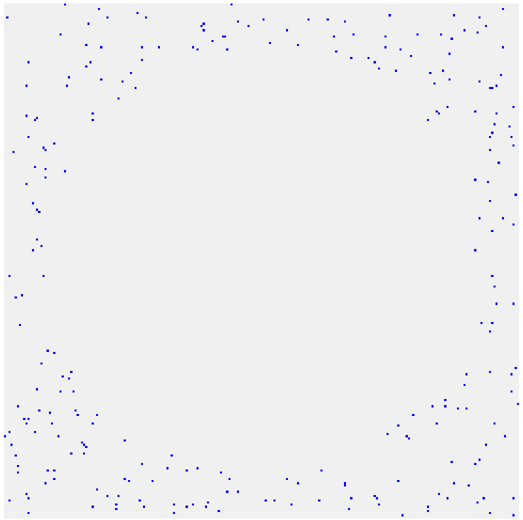


figure 18: Mosaic S cones

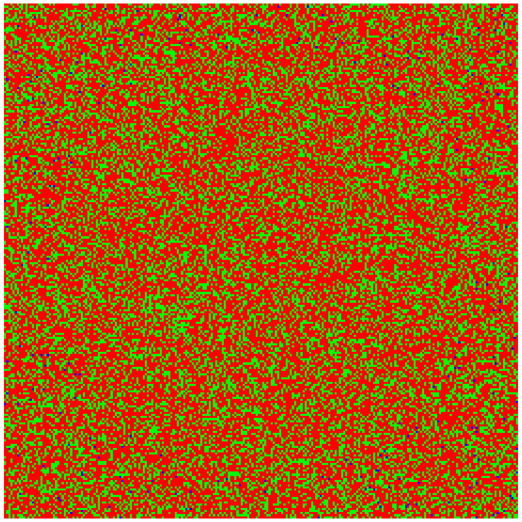


figure 19: Species Mosaic

There are four photoreceptor species in the typical trichromat human eye. Variations in species presence and distribution are found in individuals. These images are fictional characterizations of what is found in vivo. The images are produced on a rectangular grid but, in vivo, the grid is more similar to a hexagonal grid, but nowhere near reliably. These distributions are largely consistent with published data: such as the site:
http://www.cvrl.org/database/text/intros/introdens.htm

<u>table 1:</u> Photoreceptor Species

Photoreceptor Species		
L	photoreceptors (cones shown as red)	~566nm peak absorption
M	photoreceptors (cones shown as green)	~533nm peak absorption
I	photoreceptors (rods shown as cyan)	~498nm peak absorption
S	photoreceptors (cones shown as blue)	~433nm peak absorption

L cones typically dominate in the macula but M cone dominance is not rare. S cones do not appear within 100μM of center and are far fewer in number. Rods are not present in these figures and only begin to populate the retina in the outer part of the macula where they increase to 100% outside the macula. table[1] represents the species in the typical trichromat human. Note that the distribution has no reliable pattern and is effectively random.

As is shown in figure[16] to figure[18], were the species to be treated as image planes, the effective image for each plane would have lacunae. This will be treated in a later paper focused on processing based on these effective images and the shapes of neurons. Here, it will be shown what information is present when full-spectrum scenes cause absorption events in the retina.

Here is PHP code which generates the receptor mosaic images shown here.

```
<?php
error_reporting(E_ALL | E_STRICT);

function radius($x,$y) { return sqrt($x*$x + $y*$y); }

function species() {
    list($r0, $x0, $y0) = array_fill(0, 3, 120);
    list(     $X , $Y ) = array_fill(0, 2, range(-$r0, +$r0));
    list($P, $M, $D, $rs, $Rmax) = [[], 255, count($X), 100, radius($x0, $y0)];
    $image = ['L' => ''       , 'M' => ''       , 'S' => ''       , 'LMS' => '' ];
    $count = ['L' => 650      , 'M' => 340      , 'S' => 10       , 'I'   => 0  ];
    $cones = ['L' => [$M,0,0], 'M' => [0,$M,0], 'S' => [0,0,$M]               ];
    foreach (array_keys($image) as $k)
```

```
        ${'im'.$k} = $image[$k] = @imagecreatetruecolor($D, $D)
        or die("{$k} mosaic fail");
    foreach ($image as $im)
        imagefill($im, 0, 0, imagecolorallocate($im, 240, 240, 240));
    foreach ($cones as $cone => $channel) {
        list($r, $g, $b)    = $channel;
        $$cone              = imagecolorallocate($imLMS, $r, $g, $b);
        ${$cone.'L'}        = imagecolorallocate($imL  , $r, $g, $b);
        ${$cone.'M'}        = imagecolorallocate($imM  , $r, $g, $b);
        ${$cone.'S'}        = imagecolorallocate($imS  , $r, $g, $b);
        if (($N=$count[$cone])) $P = array_merge($P, array_fill(0, $N, $cone));
    }
    foreach ($X as $x1) {
        $x = $x1+$x0;
        foreach ($Y as $y1) {
            $y   = $y1+$y0;
            $r   = radius($x1, $y1);
            $LMS = ($LM  = $count['L'] + $count['M']) + $count['S'];
            $c   = $P[rand(0, (($r < $rs) ? $LM : $LMS) - 1)];
            imagesetpixel($imLMS    , $x, $y, $$c); // Combined mosaic
            imagesetpixel($image[$c], $x, $y, $$c); // Species  mosaic
        }
    }
    foreach ($image as $key => $im) imagepng(    $im, "species{$key}.png");
    foreach ($image as         $im) imagedestroy($im                     );
}

species();
?>
```

Listing 1: Generating the mosaic

2 Methods

Full spectrum image file representation.

Currently, there is no popular stored image format capable of storing a full spectrum image of the sort needed for this analysis. It is possible to specify a format for use in this paper. The format for a given pixel may be given as a list of specifications. Each specification has 1 bit to identify the coefficients as either blackbody figure[12] equation[2] or gaussian equation[7] (a variation on equation[3]). If identified as blackbody, a pair of (I_0,T) coefficients or (a,b) follows. If identified as gaussian, an $(I_0, \lambda_{peak}, RMSwidth)$ or (a,b,c) coefficient triple follows.

$$I(\lambda) = I_0 \; exp\left(-\frac{(\lambda-\lambda_{peak})^2}{2RMSwidth^2}\right) \qquad \textit{Gaussian function in } \lambda \qquad [7]$$

The spectrum of a pixel may be fully described as a list of components: (B:((a1,b1),(a2,b2)...), G:((a3,b3,c3),(a4,b4,c4)...)). Each coefficient is a floating point number limited to a small range. Using the "half" float (16 bits), it is possible for blackbody specifications to occupy 32 bits and gaussian to occupy 48 bits. I_0 Cannot be negative, so the sign bit of coefficient a may be used to indicate whether it is a G (gaussian) or B (blackbody). For shorthand, uppercase letters will indicate Blackbody coefficients when the first coefficient is negative. Here is a fanciful pixel spectrum: (A1,B1,a2,b2,c2,a3,b3,c3,A4,B4). In most cases, only one or two components will be used. In no case shown in this paper will blackbody and gaussian components be mixed.

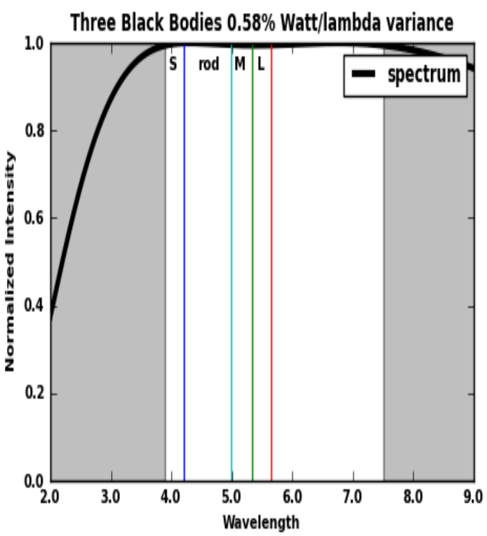


figure 20: White Body Spectrum

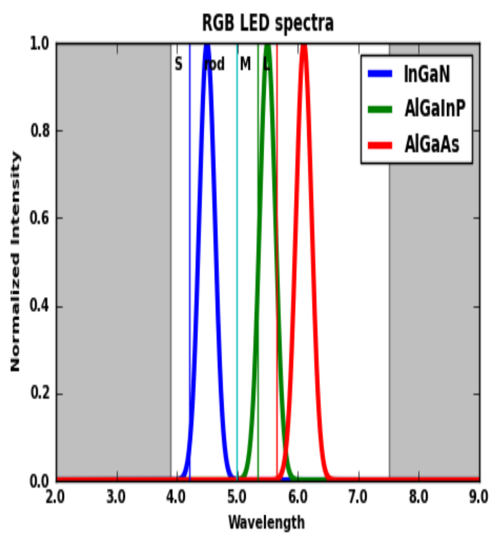


figure 21: RGB LED Spectrum

An example is the white body spectrum figure[20] where intensity variation over the entire visible spectrum is less than 1%. This is expressable as the 96 bit or 12 byte (A1,B1,A2,B2,A3,B3) with values (-1e-2,8.7e+3,-3.226e-2,5.6e+3,-1.732e+0,3e+3). Note the negative values for A1, A2, and A3 indicating black body coefficients.

Another example is the spectral composition of a white LED pixel when using LEDs of types R:AlGaAs, G:AlGaInP, B:InGaN. The b coefficient is given in nanometers. figure[21]. In this figure, each component is given by a single gaussian. (I_R,610,0.18e-7,I_G,550,0.18e-7,I_B,450,1.7e-7,0.18e-7)

Typically, images are laid out as either raster sequences of pixels or as vector graphics. Another alternative, used here, is to give the coordinates of a pixel at which a spectrum is expressed. With (x,y) expressed as 12 bits for each coordinate (24 bits) a pixel is expressed as (x,y,a1,b1,c1,a2,b2,c2...). 3 bytes for coordinates and 6 bytes for each gaussian or 4 bytes for each blackbody component. This pixel spectrum may be used for spectral calculations such as multiplying the probabilities of capture by a photoreceptor by the probabilities of photons arriving from a scene. An S photoreceptor figure[14] takes 9 bytes. An M and L photoreceptor each take 15 bytes for complete spectral representation. For instance with positive a1, and a2 for gaussian coefficients:

list 1: Spectra for human cones

- L cone spectrum (x1,y1,I0,564,0.58e-7,0.25*I_0,400,0.58e-7)
- M cone spectrum (x2,y2,I0,534,0.58e-7,0.25*I_0,400,0.58e-7)
- I rods spectrum (x3,y3,I0,498,0.58e-7,0.25*I_0,400,0.58e-7)
- S cone spectrum (x4,y4,I0,420,0.58e-7)

The spectra of figure[14] were generated using these coefficients and they may be compared favorably with published absorption spectra for human photoreceptors, notably Bowmaker J.K. and Dartnall H.J.A.[2].

While the stored form of a full photograph would be expanded by about a factor of 4 with use of this representation, an image composed of photons from point sources may actually be far smaller than with traditional RGB digital images yet capable of expressing the full spectra required for analyzing vision in natural light settings.

Photon probability chain

Probabilities for photon capture are products of probabilities for every step between emission and absorption. The spectrum of blackbody emission E_λ is a probability function of wavelength. Objects have a surface reflectance spectrum S_λ. The cornea and lens have a refraction spectrum L_λ at each retinal photoreceptor. The pupil has a diffraction spectrum P_λ at each retinal photoreceptor where it is absorbed. Finally, each species of photoreceptor has an absorption spectrum A_λ. The relative probability of a photon from the point source being absorbed by the particular photoreceptor is shown in equation[8].

$$E_\lambda * S_\lambda * L_\lambda * P_\lambda * A_\lambda \qquad \textit{Probability chain} \qquad [8]$$

graph 1: Probability chain

In effect, each spectrum between emission and absorption modifies the image as absorbed. No single step in the chain stands alone as dominating the result. Isotropic emission from a blackbody doesn't alter the course of a photon to the photoreceptor. Isotropic reflection, also, doesn't differentially change photon pathways. Refraction performs an Affine transform with the pattern seen for short wavelength photons displaced further to the outside. Diffraction performs two function: convolution with a spatially limited discrete Airy kernel followed by an affine transform with long wavelength photons displaced further to the outside.

The resulting function of wavelength forms a probability pattern which may be converted to a wave equation for superposition. Since the final step in the chain is multiplication by the absorption spectrum of the photoreceptor, the apparent image incident on each species of receptor differs substantially from that for other species.

RGB test pattern

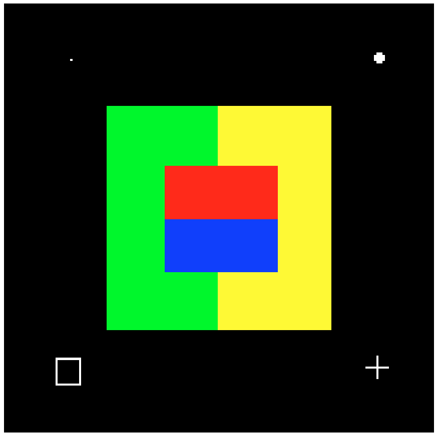


figure 22: RGB test pattern

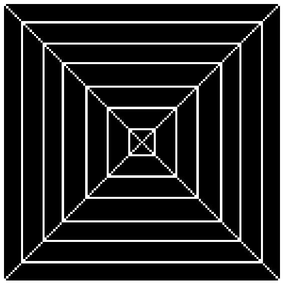


figure 23: Another RGB pattern

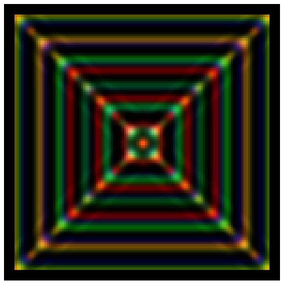


figure 24: As incident on retina

3 Results

4 Discussion

5 Acknowledgments

6 Literature

1. The Colors Of Things, Jerome Y. Lettvin et al, Lettvin et al, Scientific American, 1986, September, 255.3, 84-92
2. Visual pigments of rods and cones in a human retina, Bowmaker J.K. and Dartnall H.J.A., J. Physiol., 1980, 298, 501-511

7 Appendices

8 Missing