1. Honor Code

Signature: Andrew Liang.

2. Random Forest Motivation

(a) $E(\frac{1}{n}\sum_{i=1}^{n} Y_i) = \frac{1}{n}E(\sum_{i=1}^{n} Y_i) = \frac{1}{n}\cdot n\cdot \mu = \mu$

$Var(\frac{1}{n}\sum_{i=1}^{n} Y_i) = (\frac{1}{n})^2 \cdot n \cdot Var(Y) = \frac{1}{n}\sigma^2$

(b) $Var(\frac{1}{n}\sum_{i=1}^{n} Z_i) = \frac{1}{n^2} Var(\sum Z_i)$

$= \frac{1}{n^2}(\sum Var(Z_i) + 2\sum cov(Z_i, Z_j))$

$= \frac{1}{n^2}(n\sigma^2 + n(n-1)\sigma^2\rho)$

$= (\frac{1}{n})\sigma^2 + (\frac{n-1}{n})\sigma^2\rho$

$\underset{(X,Y)}{corr} = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$

$\therefore \rho = \frac{cov(Z_i, Z_j)}{\sigma^2}$

$2\sum cov(Z_i, Z_j)$

$= \frac{2n(n-1)}{2} \cdot cov(Z_i, Z_j)$

$n \to \infty \quad Var(mean(Z)) \to \sigma^2\rho$

$= cov(Z_i, Z_j)$

As n enlarges, $\frac{\sigma^2}{n} \to 0$, it makes averaging effect less significant

As $\rho \uparrow$, a large n will make covariance has stronger effect than variance within the data.

(c). Probability $= 1 - (1-\frac{1}{n})^n$ to be selected

ci) And $\lim_{n\to\infty}$ Probability $= 1 - \frac{1}{e} = 0.632 \approx 0.63$.

(ii). We can choose n' based on the prediction result of validation set, best using cross-val method.

3. Gaussian Kernels

(a). $K_{ij} = k(x_i, x_j)$ $\therefore$ $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(x_1, y_1) = (1,1)$ $\quad (x_1, y_2) = (1,-1)$

$(x_2, y_2) = (-1,-1)$. $\quad (x_2, y_1) = (-1,1)$

$\therefore K_{11} = K_{22} = 1, \quad K_{21} = K_{12} = 0$ $\qquad 1 \times$

$\therefore a = (1,-1)^T$ $\because$ $h(z) = \text{sign}(\sum a_i (k(x_i, z)))$

$\text{sign}(u) = \begin{cases} 1 & u \geq 0 \\ -1 & u < 0 \end{cases}$

$\therefore h(z) = \text{sign}(z)$ as it's symmetrical

(b). $K = 11^T$ $\quad 1$ is all 1's vector. $K^T = 11^T$

$K^T K a = K y$ $\qquad [n\ n \cdots n] \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

$n K a = 11^T y$ $\qquad n^2\ 1$

$\because$ count of $+1$ is the same as (-1)

$\therefore 1^T y = 0$

$\therefore a = 0$ is a must.

Now $h(z) = \text{sign}(0) = 1$

(c). $K = \begin{bmatrix} 1+\frac{1}{2\sigma^2} & 1-\frac{1}{2\sigma^2} \\ 1-\frac{1}{2\sigma^2} & 1+\frac{1}{2\sigma^2} \end{bmatrix}$ $\quad 4(1-(\frac{1}{2\sigma^2})^2)$

$\therefore K^{-1} = \frac{1}{4}\begin{bmatrix} 1+2\sigma^2 & 1-2\sigma^2 \\ 1-2\sigma^2 & 1+2\sigma^2 \end{bmatrix}$

now $a = (\sigma^2, -\sigma^2)$

$\therefore h(z) = \text{sign}(z)$ as it's symmetrical.

4.4.3. Describe Implentation details

1. I vectorize categorical values and use KNNImputer to input missing values.

2. I stop when information gain is 0.

3. I implement bootstrap (to bootstrap dataset), fit (store trees built with bagged method in a node forest). predict (predict via each tree)

4. I make sure max_depth and bootstrap_times aren't too large to avoid overfitting and training time.

5. Not that I'm aware of.

4.5.3

hw5

April 4, 2024

[465]:
```
from collections import Counter
import random
import numpy as np
import pandas as pd
from numpy import genfromtxt
from scipy import io
from sklearn.tree import DecisionTreeClassifier, export_graphviz
from sklearn.base import BaseEstimator, ClassifierMixin
from sklearn.model_selection import cross_val_score
#from pydot import graph_from_dot_data
#import io

eps = 1e-5  # a small number




if __name__ == "__main__":
    data = io.loadmat('../hw5/datasets/spam_data/spam_data.mat')
    #print("\nloaded %s data!" % data)
    fields = "training_data", "test_data", "training_labels"
    for field in fields:
        print(field, data[field].shape)
```

```
training_data (5629, 32)
test_data (5400, 32)
training_labels (1, 5629)
```

[466]:
```
X_test = data['test_data']
```

[467]:
```
class Node():
    def __init__(self, idx=None, thresh=None, left=None, right=None,␣
 ↪info_gain=None, value=None):
        ''' constructor '''

        # for decision nodeidx
        self.idx = idx
        self.thresh = thresh
```

```
        self.left = left
        self.right = right
        self.info_gain = info_gain

        # for leaf node
        self.value = value
```

[468]:
```
class DecisionTree():
    def __init__(self, min_split=2, max_depth=2, feature_labels = None):
        ''' constructor '''

        # initialize the root of the tree
        self.root = None

        # stopping conditions
        self.min_split = min_split
        self.max_depth = max_depth
        self.pred = None
        self.features = feature_labels

    def build_tree(self, dataset, curr_depth=0, random_subspace=None):
        ''' recursive function to build the tree '''

        X, Y = dataset[:,:-1], dataset[:,-1]
        num_samples, num_features = np.shape(X)

        # split until stopping conditions are met
        if (num_samples >= self.min_split) and (curr_depth<=self.max_depth):
            # find the best split
            best_split = self.get_best_split(dataset, num_samples,␣
↪num_features, random_subspace)
            # check if information gain is positive
            if best_split and best_split.get("info_gain", 0) > 0:
                # recur left
                left_subtree = self.build_tree(best_split["dataset_left"],␣
↪curr_depth+1, random_subspace)
                # recur right
                right_subtree = self.build_tree(best_split["dataset_right"],␣
↪curr_depth+1, random_subspace)
                # return decision node
                return Node(best_split["idx"], best_split["thresh"],
                            left_subtree, right_subtree,␣
↪best_split["info_gain"])

        # compute leaf node
        leaf_value = self.leaf_value(Y)
        # return leaf node
```

```
        return Node(value=leaf_value)

    def get_best_split(self, dataset, num_samples, num_features, 
↪random_subspace=None):
        ''' function to find the best split '''
        # dictionary to store the best split
        best_split = {}
        max_info_gain = -float("inf")
        if random_subspace and random_subspace <= num_features:
            feature_indices = random.
↪sample(population=list(range(num_features)), k=random_subspace)
        else:
            feature_indices = list(range(num_features))


        # loop over all the features
        for idx in feature_indices:
            feature_values = dataset[:, idx]
            possible_thresholds = np.unique(feature_values)
        # loop over all the feature values present in the data
            for thresh in possible_thresholds:
            # get current split
                dataset_left, dataset_right = self.split(dataset, idx, thresh)
            # check if childs are not null
                if len(dataset_left)>0 and len(dataset_right)>0:
                    y, left_y, right_y = dataset[:, -1], dataset_left[:, -1], 
↪dataset_right[:, -1]
                # compute information gain
                    curr_info_gain = self.information_gain(y, left_y, right_y, 
↪"entropy")
                # update the best split if needed
                    if curr_info_gain>max_info_gain:
                        best_split["idx"] = idx
                        best_split["thresh"] = thresh
                        best_split["dataset_left"] = dataset_left
                        best_split["dataset_right"] = dataset_right
                        best_split["info_gain"] = curr_info_gain
                        max_info_gain = curr_info_gain



        # return best split
        return best_split




    def split(self, dataset, idx, thresh):
        ''' function to split the data '''
```

```
        dataset_left = np.array([row for row in dataset if row[idx]<=thresh])
        dataset_right = np.array([row for row in dataset if row[idx]>thresh])
        return dataset_left, dataset_right




    def information_gain(self, parent, left_child, right_child, mode="gini"):
        ''' function to compute information gain '''

        weight_left = len(left_child) / len(parent)
        weight_right = len(right_child) / len(parent)
        if mode=="gini":
            gain = self.gini(parent) - (weight_left*self.gini(left_child) +␣
↪weight_right*self.gini(right_child))
        else:
            gain = self.entropy(parent) - (weight_left*self.entropy(left_child)␣
↪+ weight_right*self.entropy(right_child))
        return gain

    def entropy(self, y):
        ''' function to compute entropy '''

        labels = np.unique(y)
        entropy = 0
        for ls in labels:
            p_ls = len(y[y == ls]) / len(y)
            entropy += -p_ls * np.log2(p_ls)
        return entropy

    def gini(self, y):
        ''' function to compute gini index '''

        labels = np.unique(y)
        gini = 0
        for ls in labels:
            p_ls = len(y[y == ls]) / len(y)
            gini += p_ls**2
        return 1 - gini

    def leaf_value(self, Y):
        ''' function to compute leaf node '''

        Y = list(Y)
        return max(Y, key=Y.count)
```

```
    def fit(self, X, Y):

        dataset = np.concatenate((X, Y), axis=1)
        self.root = self.build_tree(dataset)

    def predict(self, X):
        preditions = [self.one_prediction(x, self.root) for x in X]
        return preditions

    def one_prediction(self, x, tree):

        if tree.value!=None:
            return tree.value
        feature_val = x[tree.idx]
        if feature_val<=tree.thresh:
            return self.one_prediction(x, tree.left)
        else:
            return self.one_prediction(x, tree.right)

    def print_predict(self, x):
        '''
        Function to print the splits made by the decision tree for a given data␣
↪point.
        '''
        # Ensure that the tree has been built
        if self.root is None:
            return

        # Get the prediction path for the given data point
        prediction_path = self.get_prediction_path(x, self.root, [])

        # Print the splits and the final prediction
        for idx, node in enumerate(prediction_path[:-1]):
            if node.left == prediction_path[idx+1]:
                direction = "<="
            else:
                direction = ">"
            print(f"({self.features[node.idx]} {direction} {node.thresh})")
        pred = ''
        if prediction_path[-1].value == 0:
            pred = 'ham email'
        else:
            pred = 'spam email'
        print("Therefore this email was", pred)
```

```
    def get_prediction_path(self, x, tree, path):
        '''
        Helper function to get the prediction path for a given data point.
        '''
        path.append(tree)
        if tree.value is not None:
            return path
        feature_val = x[tree.idx]
        if feature_val <= tree.thresh:
            return self.get_prediction_path(x, tree.left, path)
        else:
            return self.get_prediction_path(x, tree.right, path)
```

[ ]:

[469]:
```
class RandomForest(DecisionTree):

    def __init__(self, n_trees, n_bootstrap, n_features, max_depth): #n_tree␣
↪larger meaning better avoid overfitting
        self.rf_root = None
        self.n_trees = n_trees
        self.n_bootstrap = n_bootstrap
        self.n_features = n_features
        self.max_depth = max_depth
        #self.min_splits = min_splits
        #self.bags = [DecisionTree(self.max_depth, self.min_splits) for i in␣
↪range(self.n_bootstrap)]
        super().__init__()#max_depth=max_depth)

    def bootstrap(self, dataset):
        np.random.seed(420)

        bootstrap_indices = np.random.randint(low=0, high=dataset.shape[0],␣
↪size=self.n_bootstrap)
        dataset_bootstrapped = dataset[bootstrap_indices] #dataset being␣
↪multidimensional array

        return dataset_bootstrapped



    def fit(self, X, Y):
        forest = []
        dataset = np.concatenate((X, Y), axis=1)

        for i in range(self.n_bootstrap):
```

```
            dataset_bootstrapped = self.bootstrap(dataset)
            tree = self.build_tree(dataset_bootstrapped, curr_depth=0,␣
↪random_subspace=self.n_features)
            forest.append(tree)
        self.rf_root = forest


    def predict(self, X):
        rf_predictions = {}
        for i in range(len(self.rf_root)):
            column_name = "tree()".format(i)
            predictions = [DecisionTree().one_prediction(x, self.rf_root[i])␣
↪for x in X]
            rf_predictions[column_name] = predictions

        rf_predictions = pd.DataFrame(rf_predictions)
        random_forest_predictions = rf_predictions.mode(axis=1)[0]

        return random_forest_predictions
```

[470]:

```
#spam/ham
```

[471]:

```
X = data['training_data']
Y = data['training_labels'].reshape(-1,1)
from sklearn.model_selection import train_test_split
X_train, X_val, Y_train, Y_val = train_test_split(X, Y, test_size=.2,␣
↪random_state=7)
```

[472]:

```
import matplotlib.pyplot as plt
from sklearn.metrics import accuracy_score

features = np.array([
"pain", "private", "bank", "money", "drug", "spam", "prescription", "creative",␣
↪"height", "featured", "differ", "width", "other", "energy", "business",␣
↪"message", "volumes", "revision", "path", "meter", "memo", "planning",␣
↪"pleased", "record", "out", "semicolon", "dollar", "sharp", "exclamation",␣
↪"parenthesis", "square_bracket", "ampersand"
])
assert len(features) == 32

class_label = ['ham','spam']

classifier = DecisionTree(min_split=2, max_depth=10, feature_labels = features)
classifier.fit(X_train, Y_train)
```

```
Y_pred_train = classifier.predict(X_train)
Y_pred_val = classifier.predict(X_val)

#4.4 decision tree, spam/ham
spam_val_accuracy = accuracy_score(Y_val, Y_pred_val)
spam_train_accuracy = accuracy_score(Y_train, Y_pred_train)
print('DecisionTree training accuracy of spam data is:', spam_train_accuracy)
print('DecisionTree validation accuracy of spam data is:', spam_val_accuracy)

#4.5.2 tree split for one single data
classifier.print_predict(X_train[10])#, class_label)
classifier.print_predict(X_train[0])
```

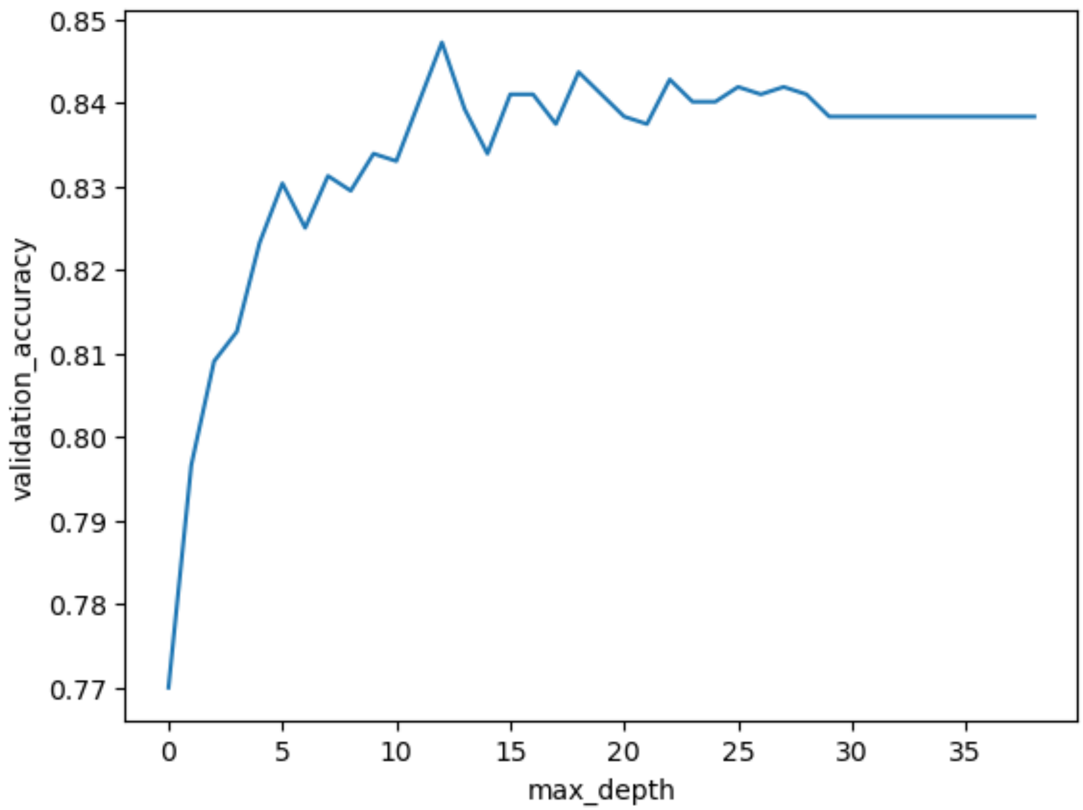
```
DecisionTree training accuracy of spam data is: 0.8525427492782589
DecisionTree validation accuracy of spam data is: 0.8339253996447602
(exclamation <= 0.0)
(parenthesis > 0.0)
(money <= 0.0)
(dollar <= 1.0)
(energy <= 0.0)
(meter <= 0.0)
(pain <= 0.0)
(featured <= 0.0)
(message <= 0.0)
(bank <= 0.0)
(spam <= 0.0)
Therefore this email was ham email
(exclamation > 0.0)
(parenthesis > 0.0)
(dollar > 1.0)
(exclamation <= 3.0)
(ampersand <= 0.0)
(parenthesis <= 16.0)
(energy <= 0.0)
(semicolon <= 0.0)
(other <= 0.0)
(volumes <= 0.0)
(record <= 1.0)
Therefore this email was spam email
```

[473]:
```
#4.5.3 visualization of spam data
plot_y = []
for i in range(1, 40):
    one_clf = DecisionTree(min_split=1, max_depth=i)
    one_clf.fit(X_train, Y_train)
    one_pred = one_clf.predict(X_val)
    one_val_acc = accuracy_score(Y_val, one_pred)
```

```
    plot_y.append(one_val_acc)

plt.plot(plot_y)
plt.xlabel('max_depth')
plt.ylabel('validation_accuracy')
# From the plot, it seems that the validation accuracy stays stable after about␣
↪10 for max_depth with an accuracy of between 0.84.
```

[473]: Text(0, 0.5, 'validation_accuracy')



[474]:
```
#4.4 randomforest spam train/val data

rf_spam_class = RandomForest(n_trees = 10, n_bootstrap = 50, n_features = 20,␣
↪max_depth = 10)
rf_spam_class.fit(X_train, Y_train)
rf_spam_Y_pred_val = rf_spam_class.predict(X_val)
rf_spam_Y_pred_train = rf_spam_class.predict(X_train)
val_accuracy_rf = accuracy_score(Y_val, rf_spam_Y_pred_val)
train_accuracy_rf = accuracy_score(Y_train, rf_spam_Y_pred_train)

print('RainForest training accuracy of spam data is:', train_accuracy_rf)
```

```
print('RainForest validation accuracy of spam data is:', val_accuracy_rf)
```

```
RainForest training accuracy of spam data is: 0.7181878747501665
RainForest validation accuracy of spam data is: 0.7255772646536413
```

[475]:
```
titanic_whole_data = pd.read_csv("../hw5/datasets/titanic/titanic_training.
 ↪csv")#, skiprows=1, header=None, names=col_names)
titanic_test = pd.read_csv("../hw5/datasets/titanic/titanic_testing_data.csv")
titanic_data = titanic_whole_data.drop(columns = ['survived'])
titanic_label = titanic_whole_data['survived']

# replace NAN to mode via pandas trick
#use one_hot_encode pclass to first,second,third, sex to male 0, female 1,
#for missing value in sex, use mode of sex; for age missing value, use mode of␣
 ↪age in (pclass, sex); for cabin, mode of cabin in pclass
```

[477]:
```
from sklearn.impute import KNNImputer
from sklearn.preprocessing import OneHotEncoder
import pandas as pd

def impute_encode(data):

    categorical_cols = [col for col in data.columns if col in␣
 ↪['sex','embarked', 'pclass']]
    numerical_cols = [col for col in data.columns if col in␣
 ↪['age','sibsp','parch','fare']]

    # Step 2: Impute missing values in numerical columns using KNN
    imputer = KNNImputer(n_neighbors=2)
    titanic_data_num = imputer.fit_transform(data[numerical_cols])
    titanic_data_num = pd.DataFrame(data = titanic_data_num, columns=␣
 ↪['age','sibsp','parch','fare'])
    #experiment_titanic = pd.concat([titanic_data[categorical_cols],␣
 ↪titanic_data_num],axis=0)
    #experiment_titanic
    titanic_data_num

    #step 3: impute missing value of categorical column
    intermediate = pd.concat([titanic_data_num, data[['sex','embarked']]], axis␣
 ↪= 1)
    intermediate

    intermediate.sex = intermediate.sex.map({'male':0, 'female':1})
    intermediate.embarked = intermediate.embarked.map({'S':0, 'C':1, 'Q':2})

    new_intermediate = imputer.fit_transform(intermediate)
```

```
    new_intermediate = pd.DataFrame(data = new_intermediate, columns=␣
↪['age','sibsp','parch','fare','sex','embarked'])
    new_intermediate['pclass'] = data['pclass']
    new_intermediate
    new_intermediate.sex = new_intermediate.sex.replace({1:10})
    new_intermediate.pclass = new_intermediate.pclass.replace({1: 10, 3: 0})


    #step 4 one hot encode
    ohe = OneHotEncoder()
    one_hot_encoded = ohe.
↪fit_transform(new_intermediate[['sex','embarked','pclass']]).toarray()
    one_hot_encoded_df = pd.DataFrame(one_hot_encoded, columns=ohe.
↪get_feature_names_out())
    revised_titanic = pd.concat([new_intermediate, one_hot_encoded_df], axis=1)
    revised_titanic.drop(columns=['sex','embarked','pclass'], inplace=True)
    return new_intermediate
```

[478]:
```
new_intermediate.sex = new_intermediate.sex.replace(1,6)
new_intermediate.pclass = new_intermediate.pclass.replace({1: 6, 3: 1})
```

[479]:
```
#4.4 decision tree titanic data

X_titanic = impute_encode(titanic_data).values
Y_titanic = titanic_label.values.reshape(-1,1)

X_train_titanic, X_val_titanic, Y_train_titanic, Y_val_titanic =␣
↪train_test_split(X_titanic, Y_titanic, test_size=.2, random_state=420)

classifier_titanic = DecisionTree(min_split=2, max_depth=4, feature_labels =␣
↪impute_encode(titanic_data).columns.values)
classifier_titanic.fit(X_train_titanic, Y_train_titanic)
Y_pred_train_titanic = classifier_titanic.predict(X_train_titanic)
Y_pred_val_titanic = classifier_titanic.predict(X_val_titanic)

dt_train_acc_titanic = accuracy_score(Y_pred_train_titanic, Y_train_titanic)
dt_val_acc_titanic = accuracy_score(Y_pred_val_titanic, Y_val_titanic)

print('DecisionTree titanic data training accuracy is:', dt_train_acc_titanic)
print('DecisionTree titanic data validation accuracy is:', dt_val_acc_titanic)
```

```
DecisionTree titanic data training accuracy is: 0.838909541511772
DecisionTree titanic data validation accuracy is: 0.7920792079207921
```

[480]:
```
#4.4 randomforest titanic train/val data
```

```
rf_titanic_class = RandomForest(n_trees = 5, n_bootstrap = 50, n_features = 8,␣
 ↪max_depth = 8)
rf_titanic_class.fit(X_train_titanic, Y_train_titanic)
rf_titanic_Y_pred_val = rf_titanic_class.predict(X_val_titanic)
rf_titanic_Y_pred_train = rf_titanic_class.predict(X_train_titanic)
val_accuracy_rf = accuracy_score(Y_val_titanic, rf_titanic_Y_pred_val)
train_accuracy_rf = accuracy_score(Y_train_titanic, rf_titanic_Y_pred_train)

print('RainForest training accuracy of spam data is:', train_accuracy_rf)
print('RainForest validation accuracy of spam data is:', val_accuracy_rf)
```

```
RainForest training accuracy of spam data is: 0.7608426270136307
RainForest validation accuracy of spam data is: 0.7475247524752475
```

[ ]:

[481]:
```
import pandas as pd
def results_to_csv(y_test):
    #y_test = y_test.astype(int)
    df = pd.DataFrame({'Category': y_test})
    df.index += 1 # Ensures that the index starts at 1
    if y_test == y_spam:
        df.to_csv('submission_spam.csv', index_label='Id')
    else:
        df.to_csv('submission_titanic.csv', index_label='Id')
```

[ ]:

[482]:
```
y_spam = classifier.predict(X_test)
results_to_csv(y_spam)
```

[483]:
```
y_titanic = classifier_titanic.predict(impute_encode(titanic_test).values)
results_to_csv(y_titanic)
```

[ ]:

[ ]:

Text

```
(base) m.k.leung@MKs-MacBook-Pro code % python -m unittest -v tests.test_layers
TestConv2D

test_backward (tests.test_layers.TestConv2D.test_backward) ... /Users/m.k.leung
anaconda3/lib/python3.11/unittest/case.py:678: DeprecationWarning: It is deprec
ted to return a value that is not None from a test case (<bound method TestConv
D.test_backward of <tests.test_layers.TestConv2D testMethod=test_backward>>)
  return self.run(*args, **kwds)
ok
test_forward (tests.test_layers.TestConv2D.test_forward) ... /Users/m.k.leung/a
aconda3/lib/python3.11/unittest/case.py:678: DeprecationWarning: It is deprecat
d to return a value that is not None from a test case (<bound method TestConv2D
test_forward of <tests.test_layers.TestConv2D testMethod=test_forward>>)
  return self.run(*args, **kwds)
ok

----------------------------------------------------------------------
Ran 2 tests in 4.668s

OK
```