E - Reverse 2^i

Editorial (/contests/abc413/tasks/abc413_e/editorial)

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 450 points

Problem Statement

You are given a permutation $P = (P_0, P_1, \ldots, P_{2^N-1})$ of $(1, 2, 3, \ldots, 2^N)$.

You can perform the following operation any number of times (possibly zero):

- Choose non-negative integers a, b satisfying $0 \leq a \times 2^b < (a+1) \times 2^b \leq 2^N$, and reverse $P_{a\times 2^b}, P_{a\times 2^b+1}, \ldots, P_{(a+1)\times 2^b-1}$. Here, reversing $P_{a\times 2^b}, P_{a\times 2^b+1}, \ldots, P_{(a+1)\times 2^b-1}$ means simultaneously replacing $P_{a\times 2^b}, P_{a\times 2^b+1}, \ldots, P_{(a+1)\times 2^b-1}$ with $P_{(a+1)\times 2^b-1}, P_{(a+1)\times 2^b-2}, \ldots, P_{a\times 2^b}$.

Find the lexicographically smallest permutation P that can be obtained by repeating the operation.

You are given T test cases, so find the answer for each.

Constraints

- $1 \leq T \leq 10^5$
- $1 \leq N \leq 18$
- P is a permutation of $(1, 2, 3, \ldots, 2^N)$.
- For each input file, the sum of 2^N over all test cases is at most 3×10^5.
- All input values are integers.

Input

The input is given from standard input in the following format:

$$
\begin{aligned}
&T \\
&\mathrm{case}_1 \\
&\mathrm{case}_2 \\
&\vdots \\
&\mathrm{case}_T
\end{aligned}
$$

case_i represents the i-th test case and is given in the following format:

$$
\begin{aligned}
&N \\
&P_0 \quad P_1 \quad \dots \quad P_{2^N-1}
\end{aligned}
$$

Output

Output T lines. The i-th line $(1 \leq i \leq T)$ should contain the answer to the i-th test case.

Sample Input 1 Copy

Copy

```
4
1
1 2
2
1 3 4 2
2
2 3 4 1
3
8 3 4 2 1 5 7 6
```

Sample Output 1 Copy

Copy

```
1 2
1 3 2 4
1 4 2 3
1 5 6 7 2 4 3 8
```

In the first test case, when no operation is performed on P, $P = (1, 2)$. This is the lexicographically smallest permutation. Thus, the answer is $(1, 2)$.

In the second test case, when we perform the operation with $a = 1, b = 1$, P becomes $(1, 3, 2, 4)$. No matter how many operations we perform on P, we cannot obtain a permutation lexicographically smaller than $(1, 3, 2, 4)$. Thus, the answer is $(1, 3, 2, 4)$.

In the third test case, by performing operations in the following order, we can obtain $P = (1, 4, 2, 3)$:

- Perform the operation with $a = 0, b = 1$. P becomes $(3, 2, 4, 1)$.
- Perform the operation with $a = 0, b = 2$. P becomes $(1, 4, 2, 3)$.

No matter how many operations we perform on P, we cannot obtain a permutation lexicographically smaller than $(1, 4, 2, 3)$. Thus, the answer is $(1, 4, 2, 3)$.

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