Submit on Crowdmark by Tuesday, June 22, 2021, 11:59pm

Instruction

Upload a PDF file with two parts. Part one should include your typed report (your discussions, data and figures). Part two should list your code. You will receive a Crowdmark link for uploading your results.

Perturbations in linear systems

This computing assignment is an exploration of condition numbers, perturbations, and the numerical behavior of random and not-so-random matrices. You will need to load `Data.mat` from the folder to get all the data for the assignment, including the matrices E, H, HI, $H8$, and $HI8$ referred to below. For all your computations use $\epsilon = 10^{-6}$, a variable `epsilon` with the proper value is included in the data.

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1. For $A = E$, $A = H$, compare the 1-condition number $\kappa_1(A)$ (in Matlab simply `cond(A,1)`) to the observed amplification in perturbations as well as to the Matlab estimate `rcond(A)`. Note, that `rcond(A)` estimates the reciprocal $1/\kappa_1(A)$.

a). Perturbations in the right-hand side

For each of these two matrices ($A = E$ and $A = H$) you will solve a total of 100 systems. You pair each right side `b = B(:,j)` with each perturbation direction `d = D(:,k)`; note, that all column vectors in your data have length 1 in the $||\cdot||_1$ norm. Compute (simply use the Matlab "\" backslash command) the solution of

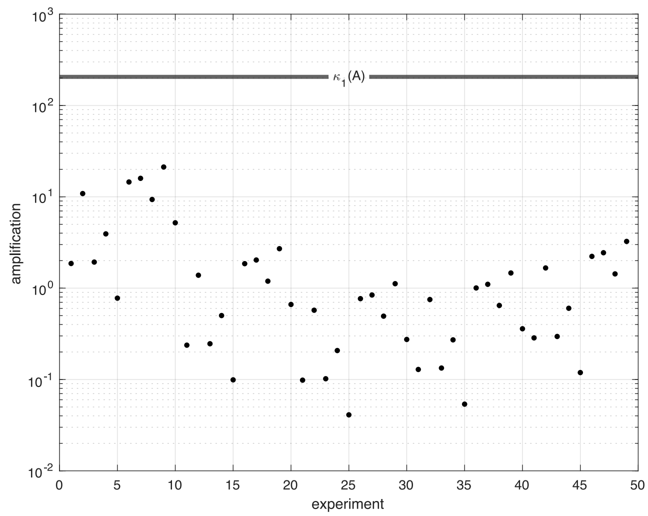
$$Ax = b, \qquad \text{and} \qquad Ay = b + \epsilon d,$$

and compare the amplification of the relative errors

$$e = \frac{\frac{||y-x||_1}{||x||_1}}{\frac{||\epsilon d||_1}{||b||_1}} = \frac{||y-x||_1}{\epsilon ||x||_1}$$

to the upper bound $\kappa_1(A)$.

Look at the average, median, and maximum of the amplification factors. Describe your observations (supported by a plot), and comment on your results. See the sample below for a possible visualization.



b). Perturbations of the matrix

For each of the two matrices E and H, solve a total of 60 linear systems to compute amplification factors. You use the same 10 right hand sides b from the first part; to get your perturbation matrices, type `C=BIGC(:,:,k)`, for $k = 1, \ldots, 6$. All the data matrices have $||C||_1 = 1$.

Compute (simply use the Matlab "\" backslash command) the solution of

$$Ax = b, \qquad \text{and} \qquad (A + \epsilon C)z = b,$$

and compare the amplification of the relative errors

$$e = \frac{\frac{||z-x||_1}{||x||_1}}{\frac{||\epsilon C||_1}{||A||_1}} = ||A||_1 \frac{||z-x||_1}{\epsilon ||x||_1}$$

to the upper bound $\kappa_1(A)$ and the Matlab estimate 1/`rcond(A)`.

Look at averages, median, and maxima of amplification factors. Plot your results, and comment on your observations.

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2. Use the Matlab command `AINV=inv(A)` to find the inverse of a matrix A, and compute the inverse of this inverse, `AC=inv(AINV)`, which mathematically equals $A = \left(A^{-1}\right)^{-1}$. The matrix I is the identity matrix.

a). For $A = E$, compute $||A * AINV - I||_1$, and $||AC - A||_1$.

b). For $A = H$, compute $||A * AINV - I||_1$, and $||AC - A||_1$. For this matrix, also compare the computed inverse to the exact inverse HI provided in the data, i.e., compute $||AINV - HI||_1$.

c). Repeat b) for the matrix $A = H8$ with exact inverse $HI8$. Compute $\kappa_1(H8)$.

Summarize your observations and highlight anything that might seem surprising.