

Practical Class 04

Convex Functions and Logistic Regression

1. Consider the sigmoid function $\sigma(x) = \frac{1}{1+\exp(-x)}$. Is this a convex function? Show your working.
2. Consider again the sigmoid function $\sigma(x) = \frac{1}{1+\exp(-x)}$. Is $-\log(\sigma(x))$ a convex function? Show your working.
3. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ in the form

$$f(\mathbf{x}) = f(x_1, \dots, x_n) = a_0 + a_1 x_1 + \dots + a_n x_n = \mathbf{a}^T \mathbf{x} + a_0$$

is called an affine function. An affine function is convex, convince yourself that this is the case. (In fact, affine functions are the only functions that are both convex and concave as $f''(\mathbf{x}) = 0$.)

Now, let $g : \mathbb{R} \rightarrow \mathbb{R}$ be a convex function. Is that $g \circ f = g(f(\mathbf{x}))$ also convex? Show your working.

4. Use all the results above, show that the loss of a logistic model, which is written in the form of negative log-likelihood, is convex.