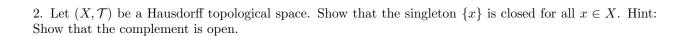
Exercises for Module 6: Metric Spaces IV $\,$

4. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in \mathbb{R} . Show that $\lim_{n\to\infty}x_n=0$ if and only if $\limsup_{n\to\infty}|x_n|=0$.

1. Let (X, \mathcal{T}) be a topological space and $A \subseteq X$ be dense. Show that if $A \subseteq B \subseteq X$, then B is dense as well.



3. Let (X, \mathcal{T}_X) , (Y, \mathcal{T}_Y) and (Z, \mathcal{T}_Z) be topological spaces and let $f: X \to Y$, $g: Y \to Z$ be continuous. Show that $g \circ f: X \to Z$ is continuous as well.

4. Let (X,d) be a metric space and $K \subset X$ compact. Show that for all $\epsilon > 0$ there exists $\{x_1, x_2, \dots, x_n\} \subseteq K$ such that for all $y \in K$ we have $d(y, x_i) < \epsilon$ for some $i = 1, \dots, n$.