

## Exercise 4: Statistical inference (I)

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05/10/2022

### Part 1: Probability distributions

1. A contestant on a game show needs to answer 10 questions correctly to win the jackpot. However, if they get 4 incorrect answers, they are kicked off the show. Suppose one contestant consistently has a 80% chance of correctly responding to any question.
  - (a) What is the probability distribution?
  - (b) What is the probability that she will correctly answer 10 questions before 4 incorrect responses?
  - (c) Write out the R code to calculate (b).
2. A small town's police department issues 5 speeding tickets per month on average.
  - (a) Using a Poisson random variable, what is the likelihood that the police department issues 3 or fewer tickets in one month?
  - (b) What is the probability that 10 days or fewer elapse between two tickets being issued?
  - (c) Write out the R code to calculate (a), (b).

### Part 2: Statistical inference

1. (AoS 6.6.2) Let  $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$  and let  $\hat{\theta} = \max \{X_1, \dots, X_n\}$ . Find the bias, se and MSE of this estimator.
2. (AoS 6.6.3) Let  $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$  and let  $\hat{\theta} = 2\bar{X}_n$ . Find the bias, se and MSE of this estimator.
3. Let  $X_1, \dots, X_n \sim \text{Uniform}(0, 1)$ . Let  $Y_n = \bar{X}_n^2$ . Find the limiting distribution of  $Y_n$ . (Hint: CLT)