

## Exercises for Module 2: Set Theory

1. Is  $\mathbb{R} \times \mathbb{R}$  with the ordering  $(x_1, y_1) \preceq (x_2, y_2)$  if  $x_1 \leq x_2$  a partially ordered set?

2. Let  $S$  be a non-empty set. A relation  $R$  on  $S$  is called an equivalence relation if it is

- (i) Reflexive:  $(x, x) \in R$  for all  $x \in S$
- (ii) Symmetric: if  $(x, y) \in R$  then  $(y, x) \in R$  for all  $x, y \in S$
- (iii) Transitive: if  $(x, y), (y, z) \in R$  then  $(x, z) \in R$  for all  $x, y, z \in S$

Given  $x \in S$ , the equivalence class of  $x$  (with respect to a given equivalence relation  $R$ ) is defined to consist of those  $y \in S$  for which  $(x, y) \in R$ . Show that two equivalence classes are either disjoint or identical.

3. Let  $(X, \leq)$  be a partially ordered set and  $S \subseteq X$  be bounded. Show that the infimum and supremum of  $S$  are unique (if they exist).

4. Let  $S, T \subseteq \mathbb{R}$  and suppose both are bounded above. Define  $S + T = \{s + t : s \in S, t \in T\}$ . Show that  $S + T$  is bounded above and  $\sup(S + T) = \sup S + \sup T$ .

5. Let  $f : X \rightarrow Y$ ,  $X, Y \subseteq \mathbb{R}$ , be defined by the map  $x \mapsto \sin(x)$ . For what choices of  $X$  and  $Y$  is  $f$  injective, surjective, bijective, or neither?

6. Show that for sets  $A, B \subseteq X$  and  $f : X \rightarrow Y$ ,  $f(A \cap B) \subseteq f(A) \cap f(B)$ .

7. Let  $f : X \rightarrow Y$  and  $B \subseteq Y$ . Prove that  $f(f^{-1}(B)) \subseteq B$ , with equality iff  $f$  is surjective.

8. Prove that  $f(\cup_{i \in I} A_i) = \cup_{i \in I} f(A_i)$  for  $f : X \rightarrow Y$ ,