Exercises for Module 1: Proofs

1. Prove De Morgan's Laws for propositions: $\neg(P \land Q) = \neg P \lor \neg Q$ and $\neg(P \lor Q) = \neg P \land \neg Q$ (Hint: use truth tables).

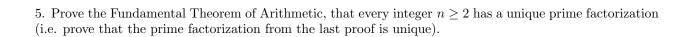
- 2. Write the following statements and their negations using logical quantifier notation and then prove or disprove them:
- (i) Every odd integer is divisible by three.

(ii) For any real number, twice its square plus twice itself plus six is greater than or equal to five. (You may assume knowledge of calculus.)
(iii) Every integer can be written as a unique difference of two natural numbers.
3. Prove the following statements: (i) If $a b$ and $a, n \in \mathbb{Z}_{>0}$ (positive integers), then $a \leq b$.

(ii)	If $a b$ and	a c, t	hen $a (a$	xb + uc).	where x	$u \in \mathbb{Z}$

(iii) Let
$$a, b, n \in \mathbb{Z}$$
. If n does not divide the product ab , then n does not divide a and n does not divide b .

4. Prove that for all integers $n \ge 1$, $3|(2^{2n} - 1)$.



6. Let $A = \{x \in \mathbb{R} : x < 100\}$, $B = \{x \in \mathbb{Z} : |x| \ge 20\}$, and $C = \{y \in \mathbb{N} : y \text{ is prime}\}$. Find $A \cap B$, $B^c \cap C$, $B \cup C$, and $(A \cup B)^c$.