

# Exercise 1: Basic programming in R

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## Prat 1: Matrix and vector operations.

1. Solve the following system:

$$A \begin{bmatrix} a_1 & b_1 & & & 0 \\ c_1 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & & a_{99} & b_{99} \\ 0 & & & c_{99} & a_{100} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{100} \end{bmatrix}$$

where

$$a_j = j, \quad b_j = 1/j, \quad c_j = 1, \quad d_j = \sin(j\pi/200)$$

and print  $x_1, x_2, \dots, x_5$ .

## Part 2: For loops.

1. Using a **for** loop, calculate the following with a sequence of  $m$ , and generate a plot for  $m$  verses  $E_m$ . Avoid using a **for** loop, can you complete the same task?

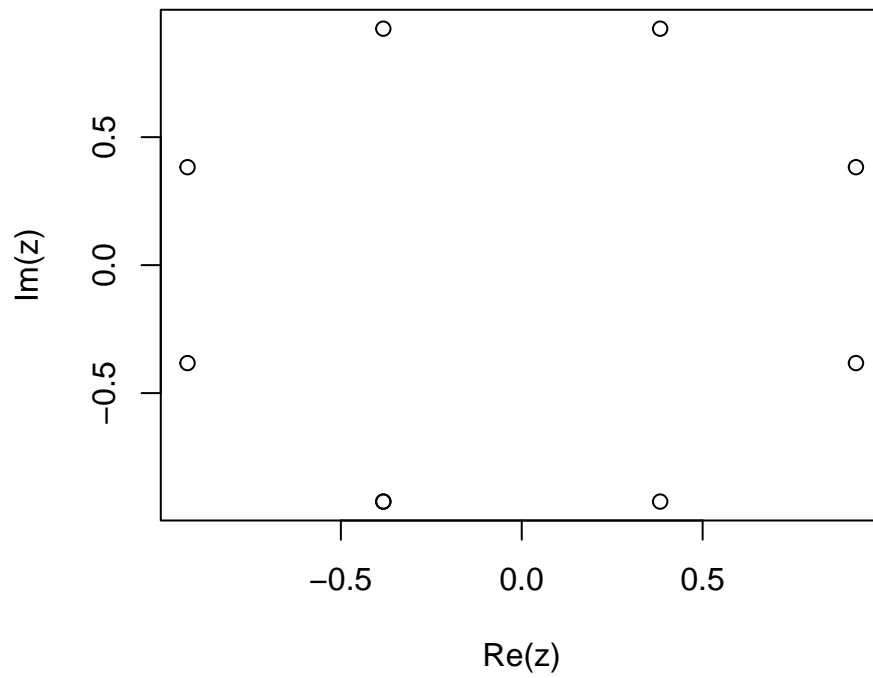
$$E_m = 1 + \frac{1}{2} + \dots + \frac{1}{2^m} - \log(2^m)$$

2. Let's draw a regular polygon of  $n$  sides, with a horizontal bottom side, and the corners of the polygon staying on the unit circle. For given  $n = 5$  and  $r = 0.6$ , start the first point  $p_1 = (x_1, y_1)$  as a random number in  $(-0.5, 0.5) \times (-0.5, 0.5)$ , generate  $10^4$  points interactively. In the  $j$ th iteration, we choose one corner  $z_*$  of the polygon randomly and let  $p_{j+1} = (x_{j+1}, y_{j+1})$  be the point on the line segment between  $p_j$  and  $z_*$ , with the distance from  $p_{j+1}$  to  $p_j$  being  $r$  times the distance from  $z_j$  to  $p_j$  and then draw all these points as dots in the  $xy$  panel.

Hint: Complex numbers can be used to represent points in the  $xy$  plane. The following script works for even or odd  $n$ , and the polygon always has a flat bottom.

```
n <- 8
t <- c(0:n) - 0.5
z <- exp(2i * pi * (t/n - 0.25))

plot(Re(z), Im(z))
```



You will have a plot like this.

