

Module 5: Topology

Operational math bootcamp



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Outline



Equivalent metrics

Definition (Equivalent metrics)

Two metrics d_1 and d_2 on a set X are *equivalent* if the identity maps from (X, d_1) to (X, d_2) and from (X, d_2) to (X, d_1) are continuous.

Proposition

Two metrics d_1, d_2 on a set X are equivalent if and only if they have the same open sets or the same closed sets.

Definition

Two metrics d_1 and d_2 on a set X are *strongly equivalent* if for every $x, y \in X$, there exists constants $\alpha > 0$ and $\beta > 0$ such

$$\alpha d_1(x, y) \leq d_2(x, y) \leq \beta d_1(x, y).$$

If two metrics are strongly equivalent then they are equivalent. The proof of this is one of the exercises.

Example

We show that the Euclidean distance (induced by 2-norm) and the metric induced by the ∞ -norm are equivalent on \mathbb{R}^n .

References

Runde ,Volker (2005). *A Taste of Topology*. Universitext. url:
<https://link.springer.com/book/10.1007/0-387-28387-0>