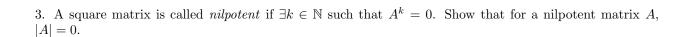
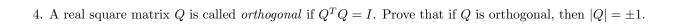
Exercises for Module 8: Linear Algebra II

1. Let $D \in \mathcal{L}(\mathbb{P}_4(\mathbb{R}), \mathbb{P}_3(\mathbb{R}))$ be the differentiation map, Dp = p'. Find bases of $\mathbb{P}_4(\mathbb{R})$ and $\mathbb{P}_3(\mathbb{R})$ such that the matrix representation of $\mathcal{M}(D)$ with respect to these basis is given by

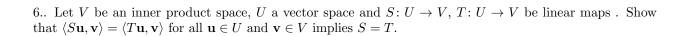
$$\mathcal{M}(D) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

2. Show that matrix multiplication of square matrices is not commutative, i.e find matrices $A, B \in M_2$ such that $AB \neq BA$.





5. An
$$n \times n$$
 matrix is called *antisymmetric* if $A^T = -A$. Prove that if A is antisymmetric and n is odd, then $|A| = 0$.



1. Let V be an inner product space and $\mathbf{x}_1, \dots, \mathbf{x}_n$ be an orthonormal basis and $\mathbf{y} \in V$. Then, \mathbf{x} has a unique representation $\mathbf{y} = \sum_{i=1}^n \alpha_i \mathbf{x}_i$. Show that $\alpha_i = \langle \mathbf{y}, \mathbf{x}_i \rangle$ for all $i = 1, \dots, n$.

2. Let V be an inner product space and $U \subseteq V$ a subset. Show that U^{\perp} is a subspace of V.

- 3. Let U, V, W be inner product spaces and $S, T \in \mathcal{L}(U, V)$ and $R \in \mathcal{L}(V, W)$. Show that the following holds
 - 1. $(S + \alpha T)^* = S^* + \overline{\alpha} T^*$ for all $\alpha \in \mathbb{F}$
 - 2. $(S^*)^* = S$
 - 3. $(RS)^* = S^*R^*$
 - 4. $I^* = I$, where $I \colon U \to U$ is the identity operator on U