

Exercise 5: Statistical inference (III)

Siyue Yang

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Part 1: MLE and optim

The likelihood for a linear model where we assume $\epsilon_i \sim N(0, \sigma^2)$ and observe X_1, \dots, X_n is,

$$\mathcal{L}(\beta_0, \beta_1, \beta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma^2} (Y_i - (\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}))^2\right)$$

The log-likelihood

$$\ell(\beta_0, \beta_1, \beta_2) = \sum_{i=1}^n -\log(\sqrt{2\pi}\sigma) - \frac{1}{2\sigma^2} (Y_i - (\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}))^2$$

1. Write a function that calculates the negative log-likelihood.
2. Use the `optim()` function to find the the MLE of β when the outcome Y is birthweight, X_1 is smoking, and X_2 mother's weight.

Part 2: EM and Newton-Raphson implementation

The ABO-gene or ABO-locus is on chromosome 9. It has 3 alleles (antigens) (A, B, O) and it determines 4 blood type (A, B, AB, O).

genotype	phenotype
AA AO	A
BB BO	B
AB	AB
OO	O

A, B are dominant to O.
O is recessive to A, B.
A, B are co-dominant.

We have a large random sample obtained from Berlin (Bernstein 1925, Sham's book page 44):

- $n_A = 9123$ blood type A
- $n_B = 2987$ blood type B
- $n_{AB} = 1269$ blood type AB
- $n_O = 7725$ blood type O

For instance, $n_A = 9123 = n_{AA} + n_{AO}$: Among 9123 blood type A individuals, some have genotype AA and the others have genotype AO.

Our interest is to estimate the allele frequencies of alleles A, B, and O. i.e. $p = \text{freq}(\text{allele } A)$, $q = \text{freq}(\text{allele } B)$, $1 - p - q = \text{freq}(\text{allele } O)$.

1. Write out the log-likelihood $L(p, q)$.
2. Is there a closed-form solution of this log-likelihood function?
3. Formulate the problem as a missing data problem and use the Newton-Raphson algorithm to find the MLEs, \hat{p} and \hat{q} , that maximize the log-likelihood, $\ln L(p, q)$.
4. (Advanced) Use the EM algorithm to find the Maximum Likelihood Estimates (MLEs) of parameters, \hat{p} and \hat{q} .

Hint: Lei Sun's STA2080 Modern genetic statistics notes ([link](#)).