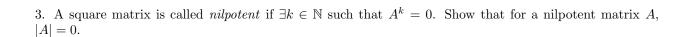
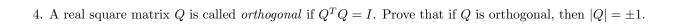
Exercises for Module 8: Linear Algebra II

1. Let $D \in \mathcal{L}(\mathbb{P}_4(\mathbb{R}), \mathbb{P}_3(\mathbb{R}))$ be the differentiation map, Dp = p'. Find bases of $\mathbb{P}_4(\mathbb{R})$ and $\mathbb{P}_3(\mathbb{R})$ such that the matrix representation of $\mathcal{M}(D)$ with respect to these basis is given by

$$\mathcal{M}(D) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

2. Show that matrix multiplication of square matrices is not commutative, i.e find matrices $A, B \in M_2$ such that $AB \neq BA$.





5. An
$$n \times n$$
 matrix is called *antisymmetric* if $A^T = -A$. Prove that if A is antisymmetric and n is odd, then $|A| = 0$.

6. Let V be an inner product space, U a vector space and $S \colon U \to V$, $T \colon U \to V$ be linear maps. Show that $\langle S\mathbf{u}, \mathbf{v} \rangle = \langle T\mathbf{u}, \mathbf{v} \rangle$ for all $\mathbf{u} \in U$ and $\mathbf{v} \in V$ implies S = T.