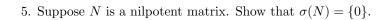
Exercises for Module 9: Linear Algebra III

1. Let V be an inner product space and $\mathbf{x}_1, \dots, \mathbf{x}_n$ be an orthonormal basis and $\mathbf{y} \in V$. Then, \mathbf{x} has a unique representation $\mathbf{y} = \sum_{i=1}^n \alpha_i \mathbf{x}_i$. Show that $\alpha_i = \langle \mathbf{y}, \mathbf{x}_i \rangle$ for all $i = 1, \dots, n$.

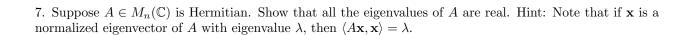
2. Let V be an inner product space and $U \subseteq V$ a subset. Show that U^{\perp} is a subspace of V.

3. Let $A, B \in M_n(\mathbb{F})$ be similar matrices. Show that their characteristic polynomials coincide.

4. Show that $A \in M_n(\mathbb{C})$ is invertible if and only if $0 \notin \sigma(A)$.



6. Let $A \in M_n(\mathbb{C})$ be an invertible matrix. Show that λ is an eigenvalue of A if and only if λ^{-1} is an eigenvalue of A^{-1} .



8. Let $A \in M_n(\mathbb{R})$. Show that the eigenvalues of A^TA are non-negative.