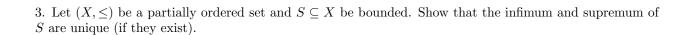
## Exercises for Module 2: Set Theory

1. Is  $\mathbb{R} \times \mathbb{R}$  with the ordering  $(x_1, y_1) \leq (x_2, y_2)$  if  $x_1 \leq x_2$  a partially ordered set?

- 2. Let S be a non-empty set. A relation R on S is called an equivalence relation if it is
  - (i) Reflexive:  $(x, x) \in R$  for all  $x \in S$
  - (ii) Symmetric: if  $(x, y) \in R$  then  $(y, x) \in R$  for all  $x, y \in S$
- (iii) Transitive: if  $(x, y), (y, z) \in R$  then  $(x, z) \in R$  for all  $x, y, z \in S$

Given  $x \in S$ , the equivalence class of x (with respect to a given equivalence relation R) is defined to consist of those  $y \in S$  for which  $(x, y) \in R$ . Show that two equivalence classes are either disjoint or identical.



4. Let  $S,T\subseteq\mathbb{R}$  and suppose both are bounded above. Define  $S+T=\{s+t:\ s\in S,t\in T\}$ . Show that S+T is bounded above and  $\sup(S+T)=\sup S+\sup T$ .

5. Let  $f: X \to Y, X, Y \subseteq \mathbb{R}$ , be defined by the map  $x \mapsto \sin(x)$ . For what choices of X and Y is f injective, surjective, bijective, or neither?

6. Show that for sets  $A, B \subseteq X$  and  $f: X \to Y, f(A \cap B) \subseteq f(A) \cap f(B)$ .

7. Let  $f: X \to Y$  and  $B \subseteq Y$ . Prove that  $f(f^{-1}(B)) \subseteq B$ , with equality iff f is surjective.

8. Prove that  $f(\bigcup_{i\in I} A_i) = \bigcup_{i\in I} f(A_i)$  for  $f: X \to Y$ ,