Exercises for Module 2: Set Theory

1. Let $f : X \to Y$ be defined by the map $x \mapsto \sin(x)$. For what choices of X and Y is f injective, surjective, bijective, or neither?

2. Show that for sets $A, B \subseteq X$ and $f : X \to Y$, $f(A \cap B) \subseteq f(A) \cap f(B)/$

3. Let $f : X \to Y$ and $B \subseteq Y$. Prove that $f(f^{-1}(B)) \subseteq B$, with equality iff f is surjective.

4. Prove that $f(\cup_{i \in I} A_i) = \cup_{i \in I} f(A_i)$.

5. Show that $\mathbb{N}$ and $\mathbb{Z}$ have the same cardinality.

6. Show that $|(0,1)| = |(1,\infty)|$.

7. Is $\mathbb{R} \times \mathbb{R}$ with the ordering $(x_1, y_1) \preceq (x_2, y_2)$ if $x_1 \leq y_1$ a partially ordered set?

8. Let S be a non-empty set. A relation R on S is called an equivalence relation if it is

(i) Reflexive: $(x, x) \in R$ for all $x \in S$

(ii) Symmetric: if $(x, y) \in R$ then $(y, x) \in R$ for all $x, y \in S$

(iii) Transitive: if $(x, y), (y, z) \in R$ then $(x, z) \in R$ for all $x, y, z \in S$

Given $x \in S$ the equivalence class of x (with respect to a given equivalence relation R) is defined to consist of those $y \in S$ for which $(x, y) \in \mathbb{R}$. Show that two equivalence classes are either disjoint or identical.