Solution 1: Basic programming in R

Jianhui Gao

07/07/2025

Part 1: Matrix and vector operations.

1. Solve the following system:

$$\begin{bmatrix} a_1 & b_1 & & & 0 \\ c_1 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & & a_{99} & b_{99} \\ 0 & & & c_{99} & a_{100} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{100} \end{bmatrix}$$

where

$$a_j = j, \quad b_j = 1/j, \quad c_j = 1, \quad d_j = \sin(j\pi/200)$$

and print $x_1, x_2, \ldots, x_5$.

Solution:

```
# Define A.
A <- matrix(rep(0, 100*100), nrow = 100, ncol = 100, byrow = TRUE)

for (i in c(1:100)) {
    A[i, i] <- i

    if (i + 1 < 101)  A[i, i + 1] <- 1/i
    if (i - 1 > 0)    A[i, i - 1] <- 1
}

# Define D.
d <- c(1:100)
d <- sin(d*pi/200)

# Solve Ax = d.
x <- solve(A, d)
x[1:5]
```

```
## [1] 0.005473329 0.010233988 0.010938907 0.012167224 0.012730871
```

Part 2: For loops.

1. Write a function that uses a `for` loop to calculate the following with a sequence of m, and generate a plot for m verses E_m. Avoid using a `for` loop, can you complete the same task?

$$E_m = 1 + \frac{1}{2} + \cdots + \frac{1}{2^m} - \log(2^m)$$

Solution:

```
# Using for loop.
E_m <- function(m) {
  res <- 1
  for (i in c(1:m)) {
    res <- res + 1/(2^i)
  }
  res <- res - log(2^m)

  return(res)
}

# Avoid using for loop.
E_m2 <- function(m) {
  res <- 1

  index <- c(1:m)
  denom <- 2^index
  res <- res + sum(1/denom)

  res <- res - log(2^m)

  return(res)
}

E_m(100)
```

```
## [1] -67.31472
```

```
E_m2(100)
```

```
## [1] -67.31472
```