

1. The worst case occurs when the largest or smallest element is always chosen for the pivot
  - a. Results in a sub-array of size 0, and another of size  $n - 1$
  - b. Is  $O(n^2)$
2. Let's consider a scenario where we always pick the last element as the pivot. A vector that incurs the worst-case complexity in this situation would be one that is already sorted in ascending order, as the pivot (being the last element) will always be the largest in the sub-array.

Vector = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]

Now, let's manually show the working of quicksort on this vector:

**1. First Pass (Pivot = 16):**

- i. Elements less than 16: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
- ii. Elements greater than 16: []
- iii. New Vector: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, |16|]

**2. Second Pass (Pivot = 15):**

- i. Elements less than 15: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]
- ii. Elements greater than 15: []
- iii. New Vector: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, |15|, 16]

**3. Third Pass (Pivot = 14):**

- i. And so on, until each element has been "sorted".

In each pass, the pivot is placed in its correct position (indicated by the vertical bar | |), but since it's always the largest element in the sub-array, we end up having to sort the rest of the elements in a very inefficient manner. Each step reduces the problem size by only 1, leading to  $n-1$  recursive calls, which is the worst-case scenario.

This process continues, with each step sorting a sub-array that is one element smaller than the previous sub-array, until each element is in its correct place. This results in  $O(n^2)$  time complexity due to the  $n+(n-1)+(n-2)+\dots+1$  sequence of operations, where  $n$  is the number of elements in the vector.

4.

