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1 Introduction

Consider the standard 5×6 board in the popular mobile game パズドラ. If you apply a rainbow board refresh (assuming all six standard orbs are equally probable), what is the probability of i matches (including skyfalls), for $i \in \mathbb{N}$? This is a deceptively hard problem as far as I can tell. I hope there is a nice closed form solution, but there probably isn't since it seems to involve partitioning.

Consider the problem with skyfalls disabled. This limits the number of matches to $i = 0, 1, \ldots, 10$.

To consider skyfalls, we may consider the boards as states of a Markov chain (maybe). The expected

2 2×2 board with 2-matches

2.1 Initial board

Consider a simplification of the problem. The board is now 2×2, and a match occurs when at least two orbs are adjacent (non-diagonal) to each other.

It is not hard to enumerate all of the $6^4 = 1296$ possible boards, grouping them by resultant combos and board configuration.

- 0 combos
  - Each of the four orbs are distinct colors, so there are $\binom{6}{4} \cdot 4! = 360$ boards

a	b
c	d

    This probability is $360/1296 = 5/18$.
  - Two of the four orbs are the same color. In order to avoid matches, the two same-colored orbs are diagonal from each other. Therefore there are two possibilities.

a	b
c	a

b	a
a	c

This results in $2 \cdot \binom{6}{3} \cdot 3! = 240$ boards. This probability is $240/1296 = 5/27$.

- Two orbs are the same color and the remaining two orbs are both the same different color. To avoid matches, the same-color pairs must be diagonal from each other, and there is only a single case.

a	b
b	a

  This results in $\binom{6}{2} \cdot 2! = 30$ boards. This probability is $30/1296 = 5/216$.

In total, the probability of 0 combos is $(360 + 240 + 30)/1296 = 35/72$.

- 1 combo
  - With three colors, we can have the following possibilities:

a	b
a	c

a	a
b	c

b	c
a	a

b	a
c	a

  This results in $4 \cdot \binom{6}{3} \cdot 3! = 480$ boards.

  - WIth two colors, we have the following possibilities:

a	a
a	b

a	a
b	a

b	a
a	a

a	b
a	a

  This results in $4 \cdot \binom{6}{2} \cdot 2! = 120$ boards.

  - If all four orbs are the same color, there is only one configuration and 6 possible boards.

In total, the probability of 1 combo is $(480 + 120 + 6)/1296 = 101/216$.

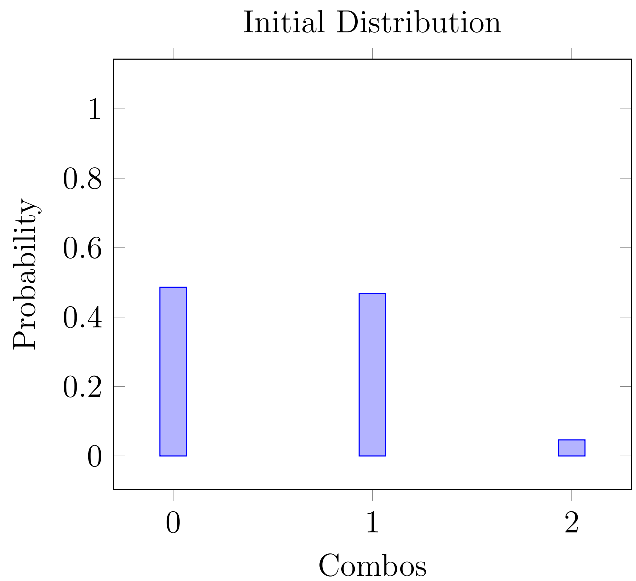
- 2 combo
  - Two adjacent orbs are the same color, and the other two adjacent orbs are another same color.

a	b
a	b

a	a
b	b

  This results in $2 \cdot \binom{6}{2} \cdot 2! = 60$ boards.

The probability of 2 combos is $60/1296 = 5/108$.



2.2 Skyfalls enabled

Now we can consider the board configurations as states of a discrete-time stochastic process. The first thing we should figure out is if this process is Markovian.

The best way to assign states is probably the distinct board configurations as listed above.

- 0 combo
  - Clearly the boards with 0 matches/combos are absorbing states as they do not give rise to any skyfalls, let alone skyfall combos. Any prior board information does not impact the future states since there cannot be any skyfalls.
- 1 combo
  - The three color case leads to three distinct possible remainder boards.

a	b

a	
b	

	a
	b

    The first board occurs with conditional probability 1/2, while the other two each occur with probability 1/4.

    What are the possible boards after skyfall? Find out on the next episode...
  - The two color case...
  - The single color case is equivalent to a complete board refresh. Therefore all of the initial boards are once again equally likely. Prior information about the board is irrelevant.
- 2 combo

  - All of the orbs are cleared, so the initial boards are equally likely. Prior information about the board is irrelevant.

2.3 Poison, mortal poison, jammer, bomb orbs enabled

What if we want to consider more than just the six standard orb colors? If we generalize to c colors, the number of boards increases to c^4 boards. However, the same analysis of board configurations as before works, replacing all instances of 6 with c, as long as $c \geq 2\times 2$. If $c < 4$, then the possible board configurations changes, along with their associated probabilities.

Keep in mind this also assumes the c different orb colors are assumed to be equally probable. Otherwise, the problem gets even more complicated.

3 3×3 board with 3-matches

We return to the situation with standard 3-matches, but still on a smaller board for simplicity.

4 $m \times n$ board with k-matches

I don't know if this is even possible to study further than running a few Monte Carlo simulations. Combinatorics is hard.