CPE/EE 322
Engineering Design VI
Lesson 6: Abstraction and Modeling

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Outline

1. Abstraction
2. Importance of modeling in abstraction and design
3. Models as purposeful representations
4. Model formats and types
5. Approximations and Occam's razor
6. Developing a model
7. Sketching
8. Functional graphs and charts
9. Finite element, process simulation, and solid models
10. System and process models
11. Model enhancement

Objectives

[G. Voland, Engineering by Design, Chapter 6]

- Use abstraction to generate as many different solution categories for a given problem as possible
- Discuss a model as a purposeful representation of a process, object, or system
- Describe the use of finite element and computer simulation models
- Explain why models help organize thoughts, data, and knowledge
- Recognize that all models are approximations due to simplifying assumptions
- Justify why an engineer must be cognizant of all assumptions that were made in developing the model
- Discuss why the resolution or the minimum level of detail needed in a model must be determined if it is to properly describe a system or process
- Apply Occam's razor to ensure that a model includes only essential details
- Produce pictorial and orthographic sketches of three-dimensional objects
- Distinguish between iconic, analogic, and symbolic models
- Select the appropriate chart formats to present, compare, and analyze specific types of data
- Recognize that system models can be deterministic or stochastic, whereas process models can be prescriptive or descriptive

Lab 6 — Node.js and Pystache

- Study the GitHub repository Lesson 6
- Install Node.js and run hello-world.js, hello.js, and http.js
- Install Pystache and run say_hello.py that uses the template in say_hello.mustache

Assignment 6 — Abstraction & Modeling

- Provide applicable models for the proposed design
  - A system model
  - A process model

Program Outcome 2: (Design)
2.1 (Design assessment) Students will be able to design a system or process with considerations of economic, environmental, health and safety, manufacturability and sustainability constraints.

Abstraction

- The goal of abstraction is to obtain conceptual perspective of the problem and its possible solutions by using models to represent different possible designs
- The advantage of abstraction is that we are more likely to generate
  - Concepts that are varied in approach and value
  - As many different alternative designs as possible to maximize the likelihood of developing the best solution to the problem
- The initial steps in abstraction include
  - Parsing the problem into as many different functional parts, subproblems, or meaningful units as possible
  - Classifying the functional aspects of the problem into more general categories in accordance with their distinctive characteristics such as the general purpose, principles, approaches, contexts, operating environments, or specific subtasks that must be performed sequentially or concurrently
  - Identifying the dimensions of variation and methods to achieve variation

Importance of Modeling in Abstraction and Design

Models including computerized simulations are important in design and analysis of engineering solutions since

- Models allow us to organize data, structure our thoughts, describe relationships, and analyze proposed designs
- Models can help transform a new unfamiliar problem into a set of recognizable subproblems that may be easier to solve
- Not all proposed design can be built and tested to determine its practical value because of time or financial constraints
- Some designs may be hazardous to workers or to the environment so that one must minimize such hazards by eliminating all but the most promising designs before development

Models as Purposeful Representations

A model can be

- A working scaled (concrete) miniature used to test an engineering solution
- An abstract set of equations describing the relationships among system variables
- A computerized simulation and animation of a process
- A two-dimensional or three-dimensional graphical description of a design
- Any other purposeful representation of a process, object, or system

Model Formats

- Abstract models should describe design concepts in sufficient detail to allow evaluation and, if necessary, refinement
  - Mathematical
  - Symbolic
  - Graphical
  - Computer-based
    - Simulation
    - Finite element method (FEM)
    - Computer-aided design (CAD)
- Concrete models can be composed of modeling clay, cardboard, Play-Doh, rubber bands, wood, *etc*. for testing design concepts and discovering behavior or challenges before more investment of time and money in developing a full-scale prototype of the design

Model Types

- **Iconic models** are visually equivalent, two-dimensional (2D) or three-dimensional (3D) representations such as maps or world globes, 3D physical models, 3D models generated via CAD
- **Analogical models** are functional equivalent representations such as electrical circuit diagrams, computerized simulations of manufacturing processes that behave like the physical process or system even though they may not physically resemble the reality
- **Symbolic models** are high-level abstractions of reality that include the most important aspects of the process or system in symbolic or mathematical form, neglecting all (presumably) irrelevant details

Size of Social Network

- British anthropologist Robin Dunbar found a correlation between primate brain size and average social group size and proposed Dunbar's number in the 1990s
  - It is a suggested cognitive limit to the number of people with whom one can maintain stable social relationships in which an individual knows who each person is and how each person relates to every other person
  - By using the average human brain size and extrapolating from the results of primates, Dunbar proposed that humans can comfortably maintain only 150 stable relationships
- H. Russell Bernard and Peter Killworth 1946—2008 and associates have done a variety of field studies in the U.S. with estimated mean 290 and median 231 for the maximum likelihood of the size of a person's social network based on a number of field studies using different methods in various populations
  - The Bernard–Killworth estimate is not an average of study averages but a repeated finding

Infectious Disease

Mathematical Modeling of Infectious Disease

- Mathematical models can project how infectious diseases progress to show the likely outcome of an epidemic and help inform public health interventions
- Models use some basic assumptions and mathematics to find parameters for various infectious diseases and use those parameters to calculate the effects of different interventions such as mass vaccination programs
- Trained as a physician, Daniel Bernoulli 1700—1782 created a mathematical model to defend the practice of inoculating against smallpox in 1766
- A simple epidemic models describes the relationship between susceptible, infected, and immune individuals in a population, $P = (R^N - 1)/2$
- The basic reproduction number R of an infection is the expected number of cases directly generated by one case in a population where all P individuals are susceptible to infection, and the serial interval S is the time between successive cases in a chain of $N = \log_R(2P+1)$ generations to infect the entire population
- For the US population P = 331 449 281 , R = 3, S = 4 days

```
$ python3
>>> import math; 4*math.log(2*331449281+1,3)
73.955599255445
```

Hypertext and Hyperlinks

- Hypertext is text displayed on a computer display or other electronic devices with hyperlinks to other text that the reader can immediately access by a mouse click or keypress set, or by touching the screen
- The text that is linked from is called anchor text
- Apart from text, the term "hypertext" is also sometimes used to describe tables, images, and other presentational content formats with integrated hyperlinks
- Hypertext is one of the key underlying concepts of the World Wide Web, where Web pages are often written in the Hypertext Markup Language (HTML) with Cascading Style Sheets (CSS) and JavaScript, a triad of cornerstone technologies for the World Wide Web
- As implemented on the Web, hypertext enables the easy-to-use publication of information over the Internet
- A user following hyperlinks is said to navigate or browse the hypertext
- A program that traverses the hypertext, following each hyperlink and gathering all the retrieved documents is known as a Web crawler, spider, ant, automatic indexer, or a friend-of-a-friend (FOAF) ontology scutter

Uber

- Uber is a transportation network company (TNC) headquartered in San Francisco, California
- The name Uber is a reference to the common (and somewhat colloquial) word uber, meaning topmost or super, and having its origins in the German word über, cognate with over, meaning above
- Uber offers services including peer-to-peer ridesharing, ride service hailing, food delivery, and a bicycle-sharing system
- The company has operations in 785 metropolitan areas worldwide
- Uber is estimated to have 100 million worldwide users and a 69% market share in the United States
- Its platforms can be accessed via its websites and mobile apps
- Uber has been so prominent in the sharing economy that the changes in industries as a result of it have been referred to as Uberization and many startups have described their products as "Uber for X"

Approximations and Occam's Razor

- All models are approximations through careful evaluation of the problem
  - Why does the system need to be investigated by the model?
  - What specific contributions are to be provided by the model?
- Engineers must determine the minimum level of detail (i.e., resolution) needed in the model for it to be a valid representation of reality
- Occam's razor is the rule that only essential information should be considered when solving a problem
  - A model should never be more complex that is absolutely necessary to achieve its purpose
  - A model should never include more than the essential details needed to properly describe the process or system being modeled

Occam's Razor

- Occam's razor is a problem-solving principle attributed to William of Ockham or Occam from his name in Latin, Gulielmus Occamus, circa 1287—1347
  - Ockham is a small village in Surrey, U.K.
- His principle can be interpreted as "among competing hypotheses, the one with the fewest assumptions should be selected"
- He didn't invent this principle; its association with him may be due to the frequency and effectiveness with which he used it
- The term razor refers to distinguishing between two hypotheses either by "shaving away" unnecessary assumptions or cutting apart two similar conclusions

KISS Principle

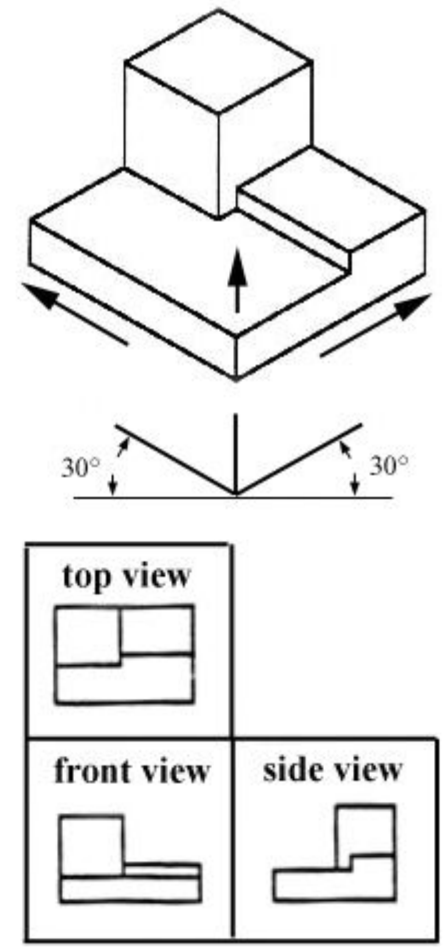
- KISS, an acronym for "keep it simple, stupid (silly)," "keep it stupid simple," "keep it short and simple," "keep it small and simple," or "keep it simple and straightforward," is a design principle noted by the U.S. Navy in 1960
- The KISS principle has been associated with Kelly Johnson 1910—1990 who contributed to aircraft designs such as Lockheed U-2 and SR-71 Blackbird
- Kelly Johnson ran Lockheed Martin Advanced Development Programs (SkunkWorks) by 14 Rules such as the 5th rule that "There must be a minimum number of reports required, but important work must be recorded thoroughly"
- An aphorism is a concise statement of a principle such as minimalism
- Leonardo da Vinci 1452—1519, "Simplicity is the ultimate sophistication"
- William Shakespeare 1564—1616, "Brevity is the soul of wit"
- Ludwig Mies van der Rohe 1886—1969, "Less is more"
- Antoine de Saint Exupéry 1900—1944, "It seems that perfection is reached not when there is nothing left to add, but when there is nothing left to take away"
- Colin Chapman 1928—1982, "Simplify, then add lightness"
- Bjarne Stroustrup, "Make Simple Tasks Simple!"

Developing a Model

- Identify the specific result that the model is expected to provide
- Consider both the types of models and the formats that could be used to represent the system or process, together with the reasons for selecting model types and formats
  - Is the model useful relative to the purpose for which it was developed? If not, modify the model accordingly
  - Does the model accurately describe the system or process under consideration? If not, identify those characteristics in the process or system that are either not included in the model or that are incorrectly embedded within the model

Sketching and Drawing

- Engineering sketches and drawings
  - Illustrate the significant elements of the design
  - Describe the shape, size, and principal features of a physical object
  - Do not need to be extremely accurate or detailed
- Sketching generally means freehand drawing
- Drawing usually means using drawing instruments
- Isometric drawing — The object's vertical lines are drawn vertically, and the horizontal lines in the width and depth planes are shown at 30 degrees to the horizontal
- Orthographic or multiview drawing — Typically, only three views (front, top, and right side) are needed to convey the needed geometric data of a part
- University College London (UCL) Drawing Gym
- MIT OpenCourseWare: Ernesto Blanco 1922—2017, *et al.*, *Design Handbook: Engineering Drawing and Sketching*

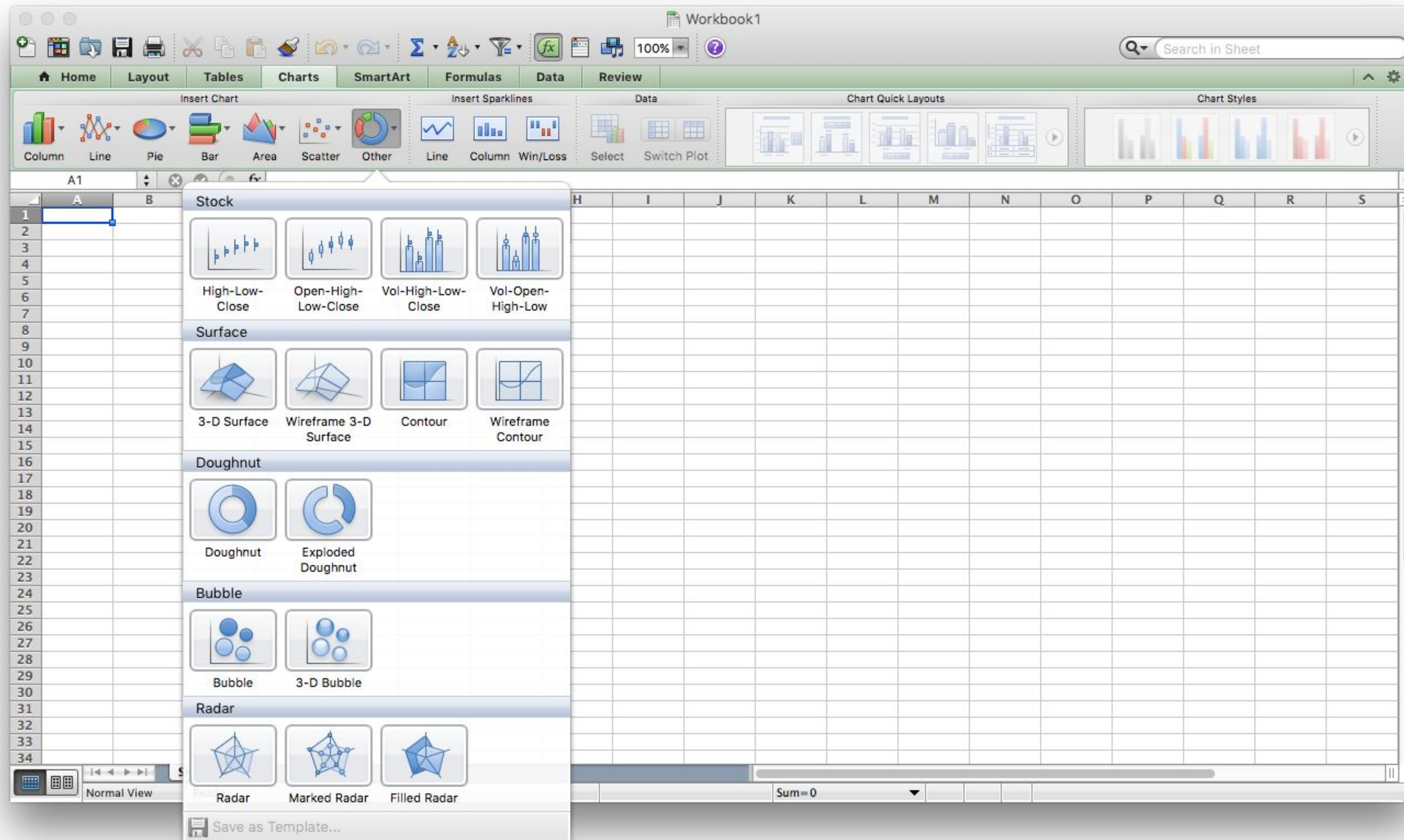


Functional Graphs and Charts

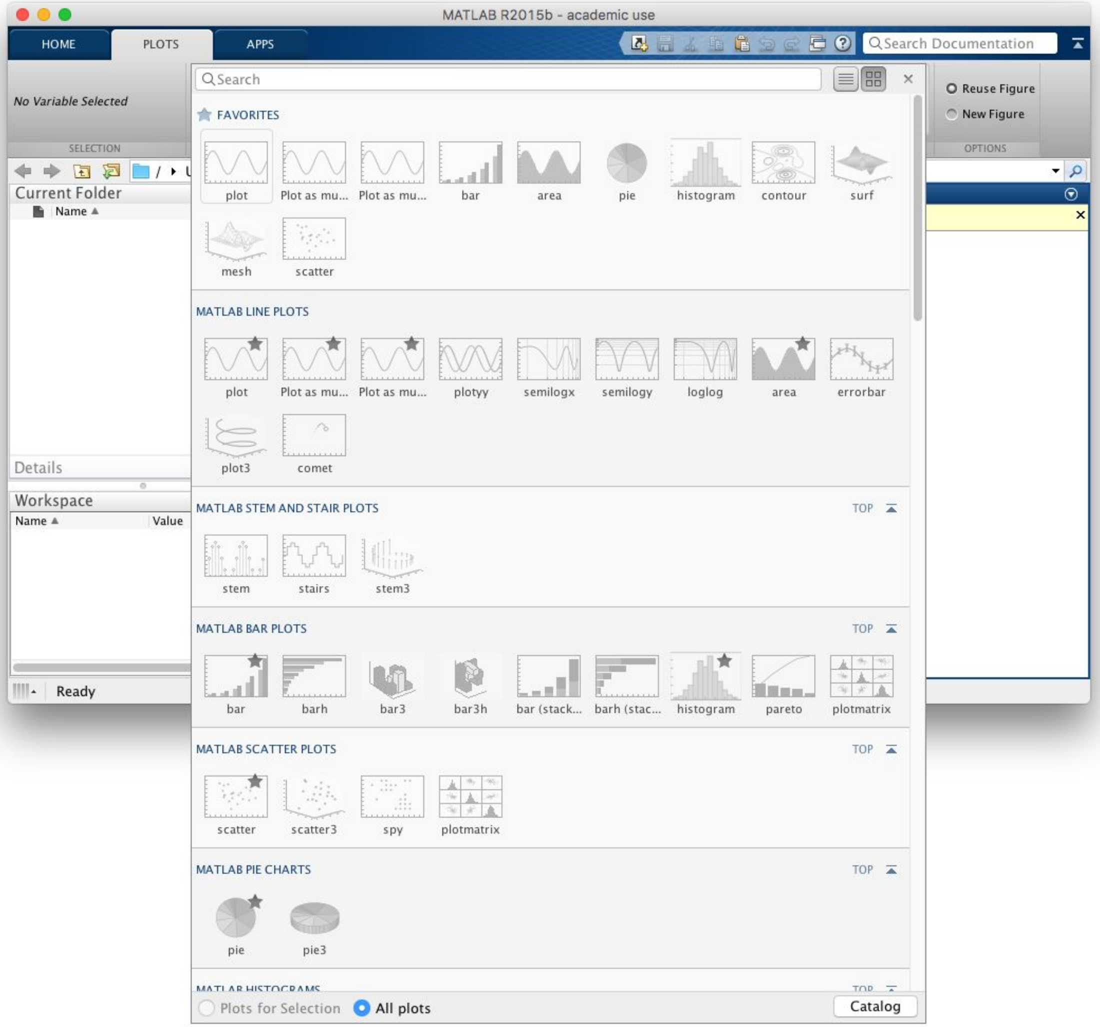
Chart Format	Use
Simple line	Continuous variations over time
Multiple-curve	Variations in two or more variables
Column and bar	Discrete variations
Subdivided-column or bar	Proportional variations over time
Sliding-bar	Proportional variations over time w.r.t. a baseline
Paired-bar	Variations in two distinct, linked entities
Pie	Proportional components of a whole

Excel charts and MATLAB plots

Types of Excel Charts

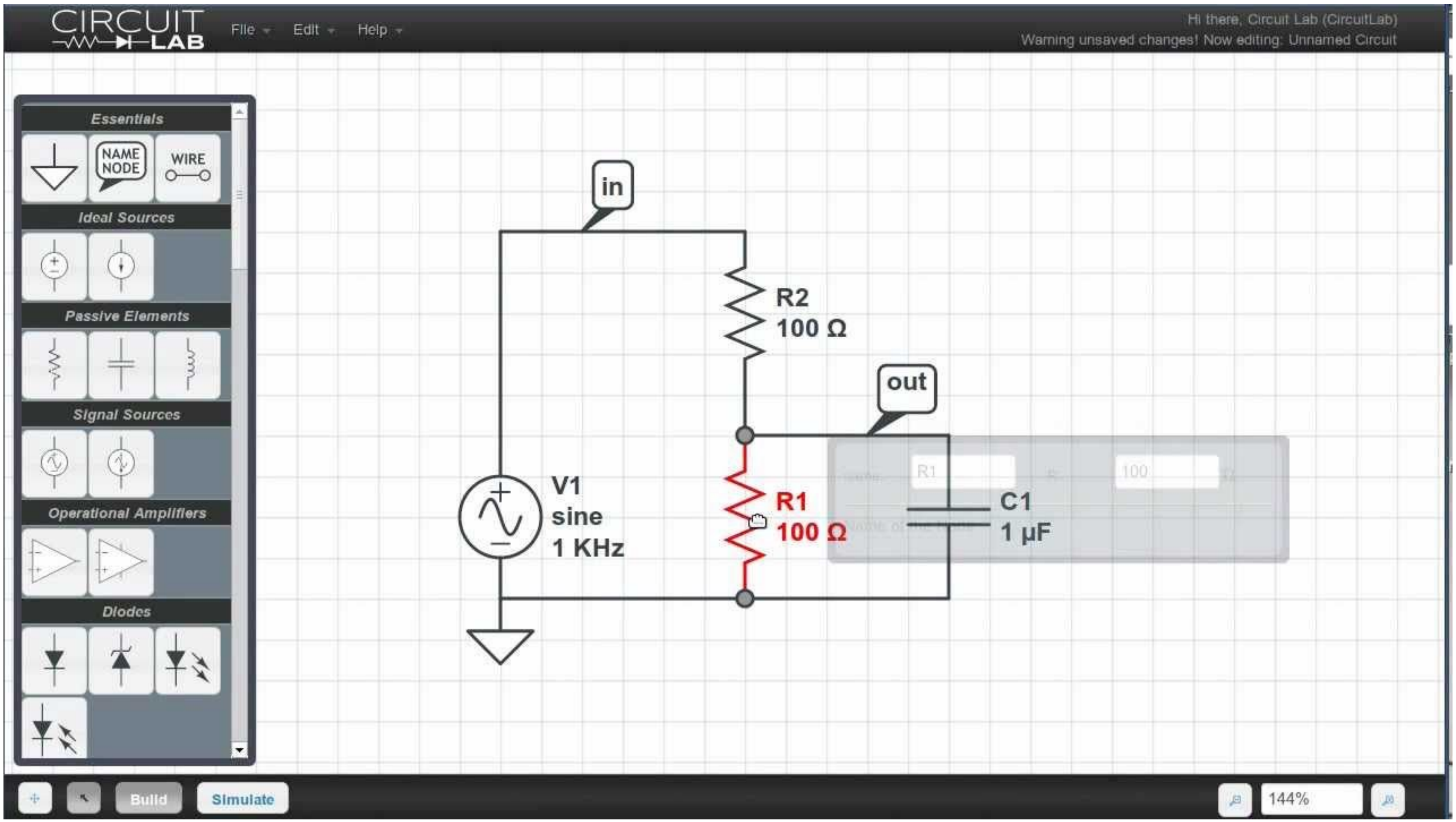


Types of MATLAB Plots

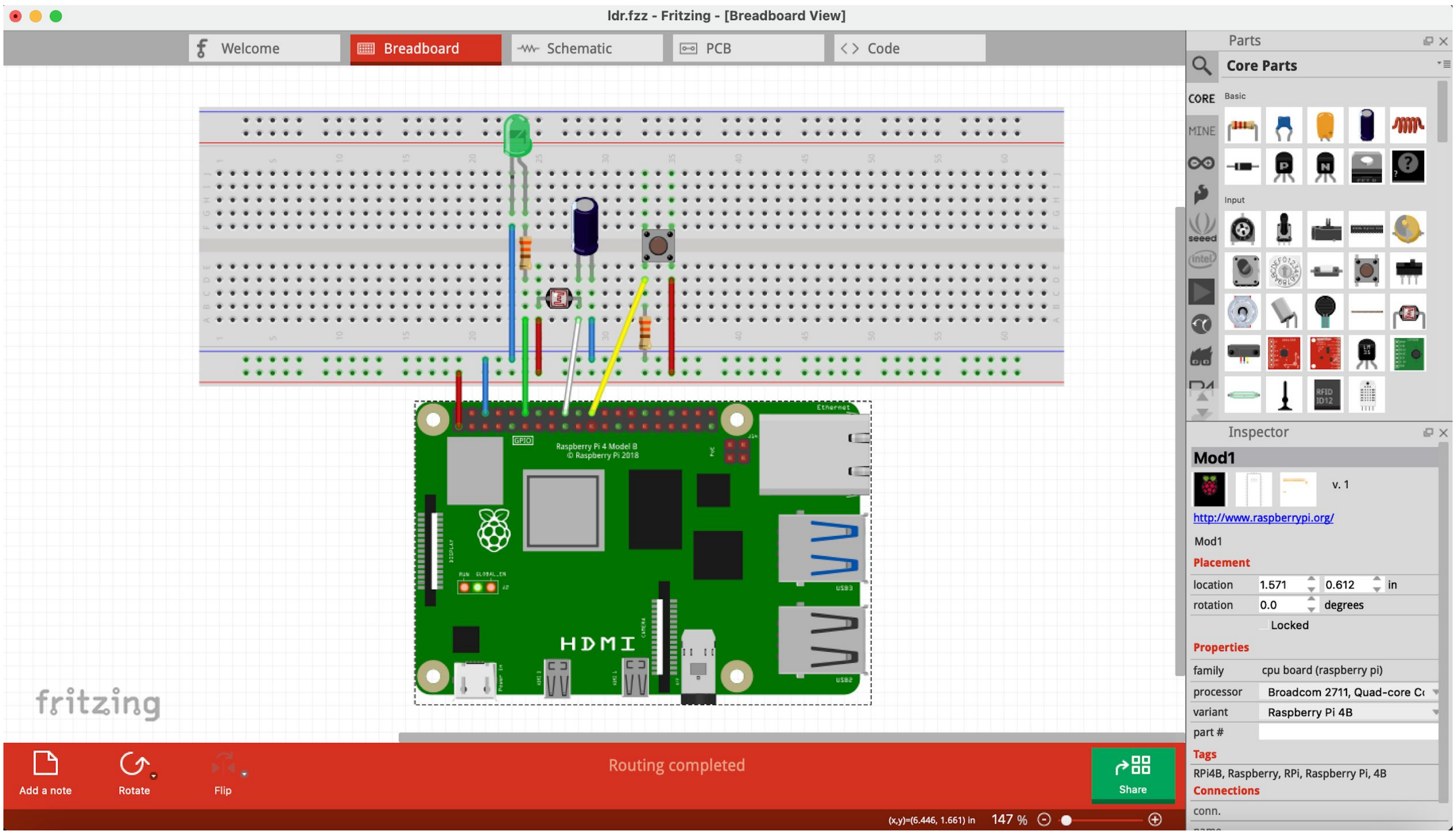


CircuitLab

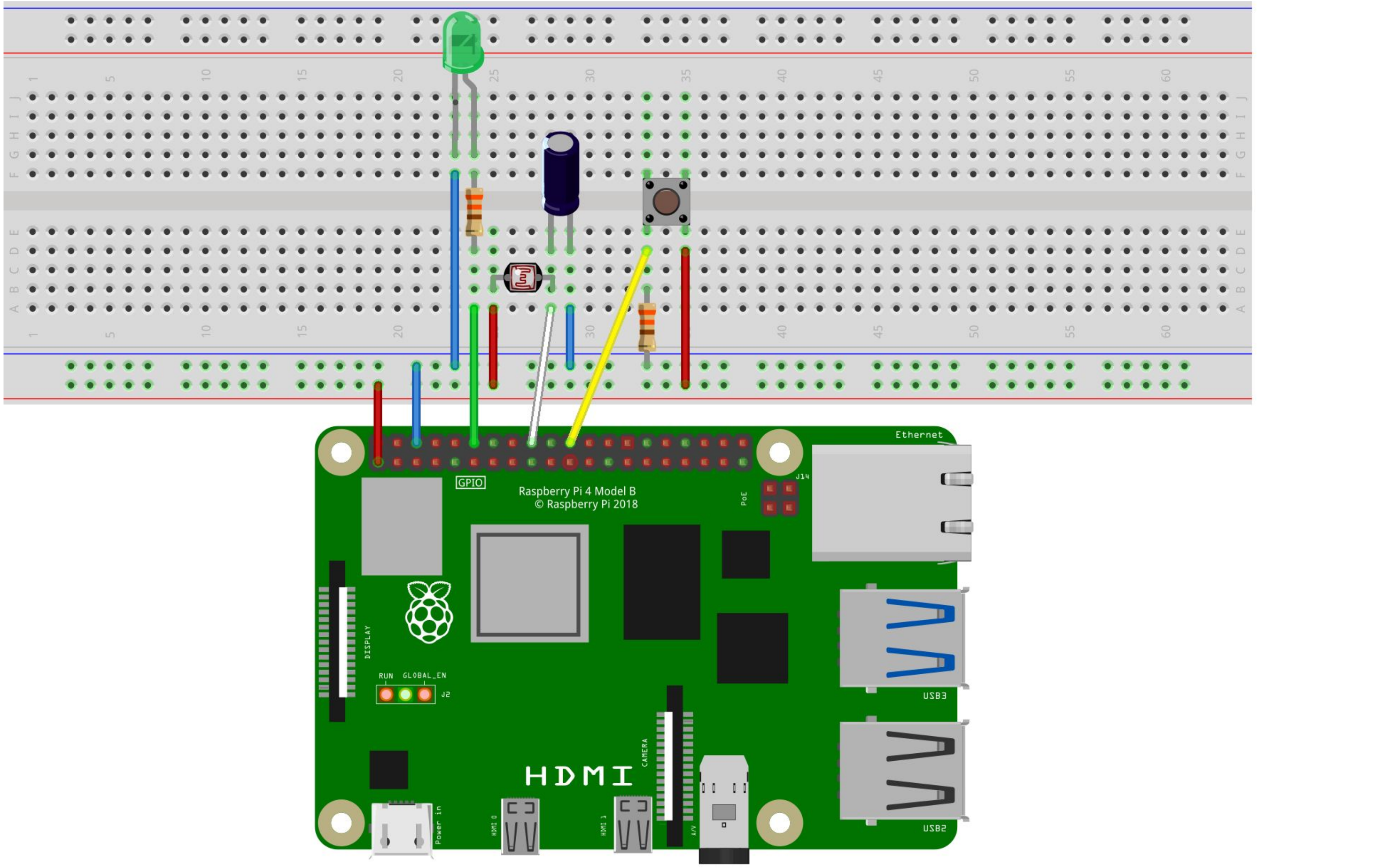
https://www.circuitlab.com



Fritzing Breadboard View

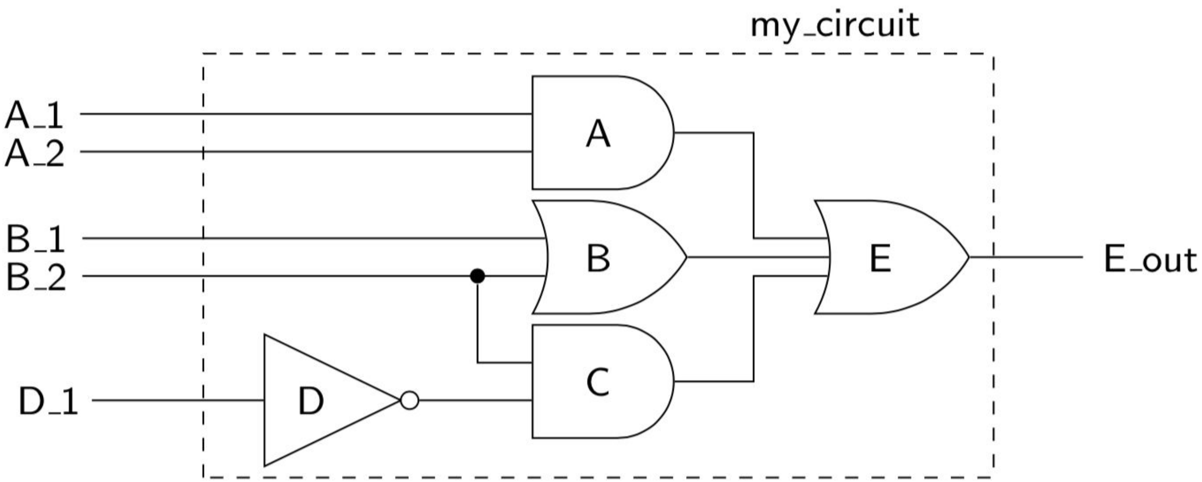


File Export as Image



Hardware Description Language

VHDL



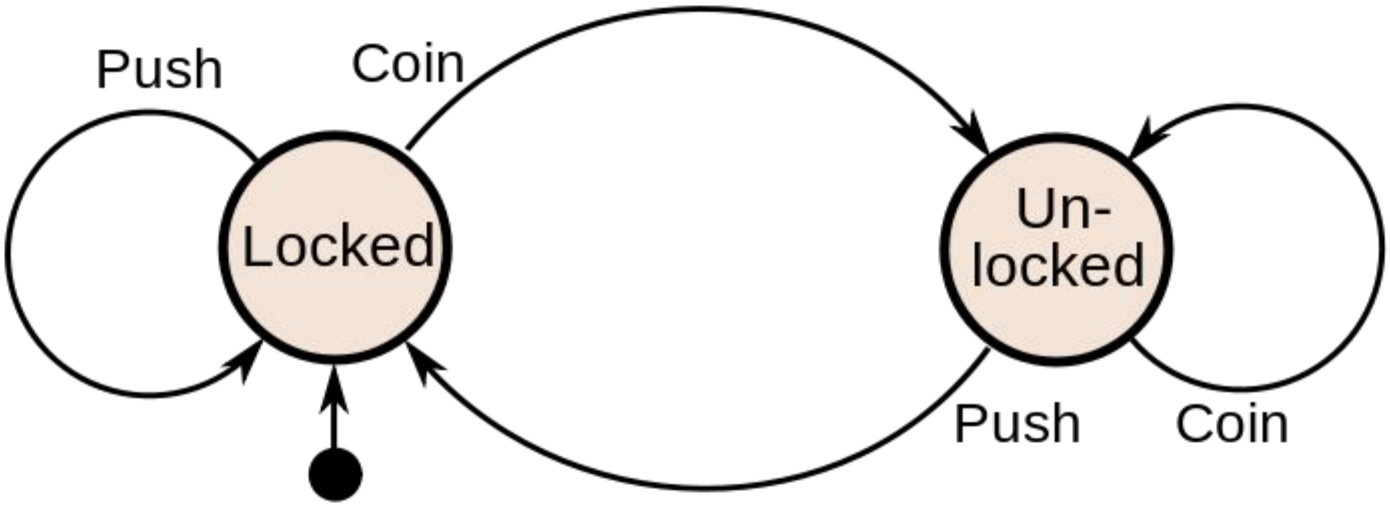

```
-- library declaration
library IEEE;
use IEEE.std_logic_1164.all;

-- entity
entity my_circuit is
    port ( A_1,A_2,B_1,B_2,D_1 : in  std_logic;
           E_out               : out std_logic);
end my_circuit;

-- architecture
architecture my_circuit_arc of my_circuit is
    signal A_out, B_out, C_out : std_logic;
begin
    A_out <= A_1 and A_2;
    B_out <= B_1 or B_2;
    C_out <= (not D_1) and B_2;
    E_out <= A_out or B_out or C_out;
end my_circuit_arc;
```

Finite-State Machine (FSM)

FSM

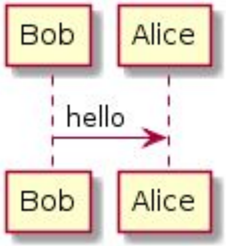


Current State	Input	Next State	Output
Locked	Coin	Unlocked	Unlock the turnstile for pushing through
	Push	Locked	None
Unlocked	Coin	Unlocked	None
	Push	Locked	Lock the turnstile after pushing through

Unified Modeling Language (UML)

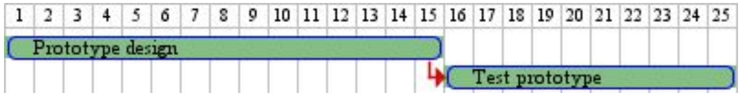
[UML](), [PlantUML]()

```
@startuml
Bob -> Alice : hello
@enduml
```




```
@startuml
(*) --> "First Activity"
"First Activity" --> (*)
@enduml
```

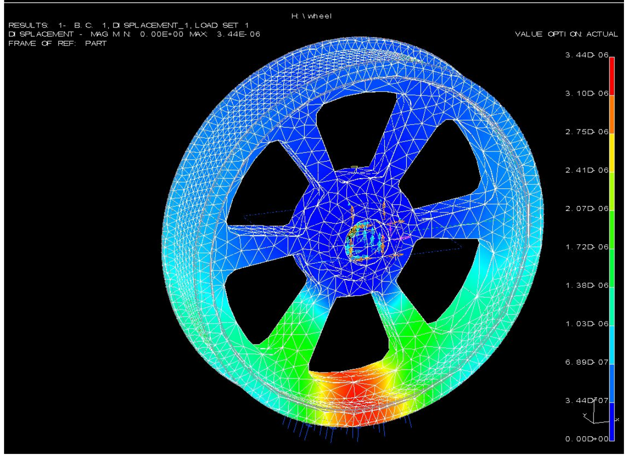

```
@startgantt
[Prototype design] lasts 15 days
[Test prototype] lasts 10 days
[Test prototype] starts at [Prototype design]'s end
@endgantt
```



Finite Element, Process Simulation, and Solid Modeling Software

Representation of engineering designs for further analysis and refinement

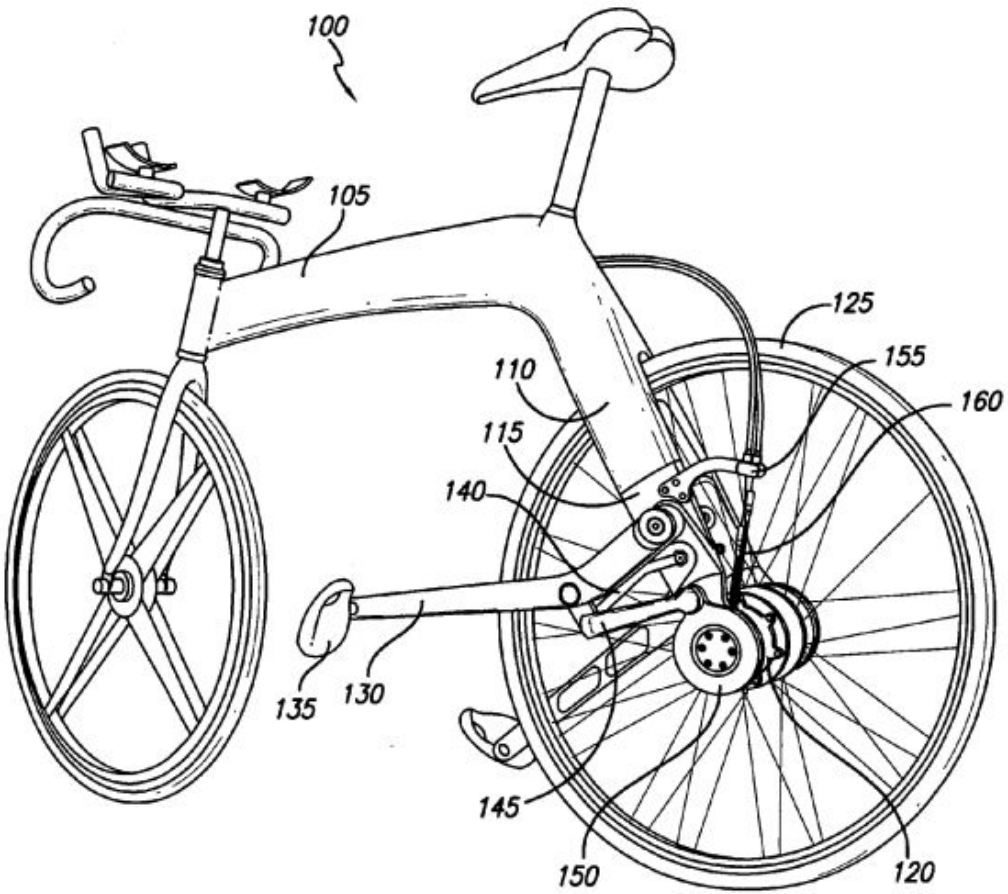
- Finite element method (FEM) or finite element analysis (FEA) describes a system as a collection of independent discrete parts, each represented by a set of equations
- Process simulation uses a computerized model to analyze a set of subordinate activities from an entire process
- Solid modeling software allows engineers to develop detailed 3D descriptions of a design, perform analysis upon the solid model, and create concrete prototype



SOLIDWORKS

- SolidWorks Corporation was founded in December 1993 by Massachusetts Institute of Technology graduate Jon Hirschtick
- Hirschtick used $1 million he had made while a member of the MIT Blackjack Team (featured in the movie 21) to set up the company and recruited a team of engineers with the goal of building 3D CAD software that was easy-to-use, affordable, and available on the Windows desktop
- SolidWorks released its first product SolidWorks 95, in November 1995
- In 1997, Dassault Systèmes (abbreviated 3DS), best known for its CATIA CAD software, acquired SolidWorks for $310 million in stock
- 3DEXPERIENCE is the 3DS response to the rise of today’s experience economy where consumers put greater value on the usage of a product than on the product itself
- Comparison of CAD software including Autodesk , FreeCAD, LibreCAD, and OpenSCAD

Chainless Bicycles



- There are several [types](#) of [chainless bicycles](#)
- [NuBike](#) features vertical pedaling, lightweight frame, strong materials, and reduced joint stress
- Although Rodger Parker abandoned his patent application on "bicycle propulsion mechanism" [US20070228687A1](#), his team eventually found a strong metal and hardening process used in military applications to solve the problem of failing axles

System and Process Models

System Models	Process Models
Deterministic models perform according to an expected pattern and produce the expected answer	Descriptive models provide the actual procedure that is followed to achieve a desired goal
Stochastic models based on various sample sets of empirical data predict the behavior of a system with a certain degree of uncertainty in the results	Prescriptive models provide general guidelines and rules about how a process should be performed to achieve a desired objective

Model Enhancement

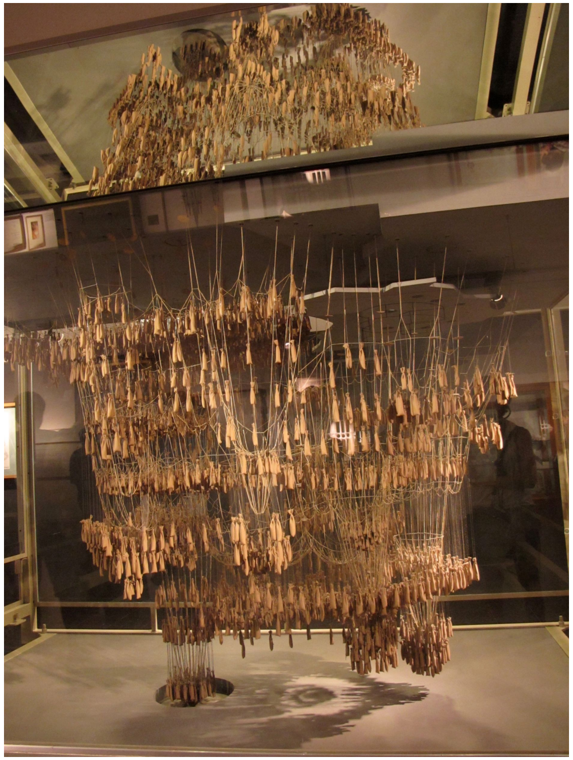
- Explain the model to another person (not someone on the design team); this may lead to a deeper understanding of the model's relative strengths and weaknesses
- Identify boundaries or constraints on the model: Are these realistic and consistent with the problem to be solved?
- The model reflects various simplifying assumptions: Are these assumptions reasonable? If not, adjust the model, i.e., discard any unreasonable assumptions and make the model more complex
- Develop additional simplifying assumptions and modify the model accordingly
- Are all known facts and data properly embedded within the model? If not, why?
- Obtain additional information if necessary
- Are there similar problems/models available? If so, how does the model differ (if at all) from these other models? Why?
- Use another (different) format to model the process or system and compare the two formats: Which is the more accurate representation and why?

Lesson 6 Summary

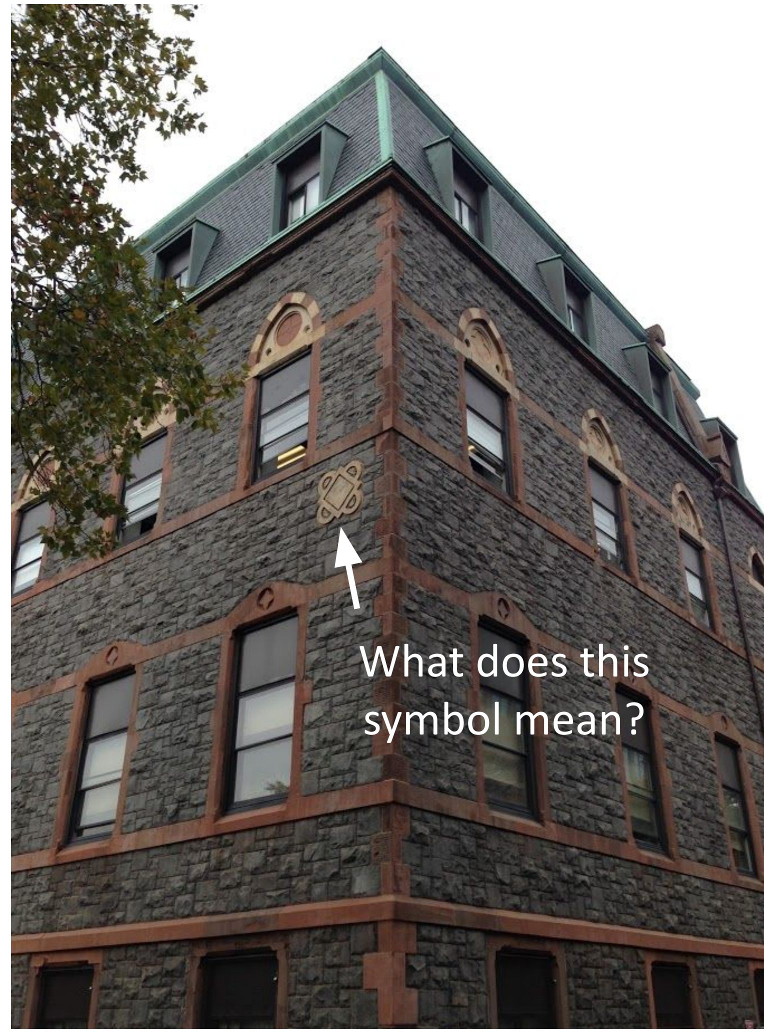
- In abstraction, one tries to generate as many different solution categories as possible
- A model is a purposeful representation of a process, object, or system
- Models help organize thoughts, data, and knowledge
- All models are approximations because of simplifying assumptions
- Engineers must be cognizant of all assumptions made in developing the model
- Engineers must determine the resolution (the minimum level of detail) needed in the model for it to properly describe the system under consideration
- A model should never include more than the essential details needed to properly describe a process or system (Occam's razor)
- Models can be abstract or concrete and may be iconic, analogical, or symbolic
- System models can be deterministic or stochastic, whereas process models can be descriptive or prescriptive
- Freehand pictorial, orthographic, and other sketches can be used to describe and develop design concepts
- A wide variety of charts can be used to compare and analyze different kinds of data

Modeling in Architecture

Antoni Gaudí 1852—1926 made a hanging chain model for the Church of Colònia Güell

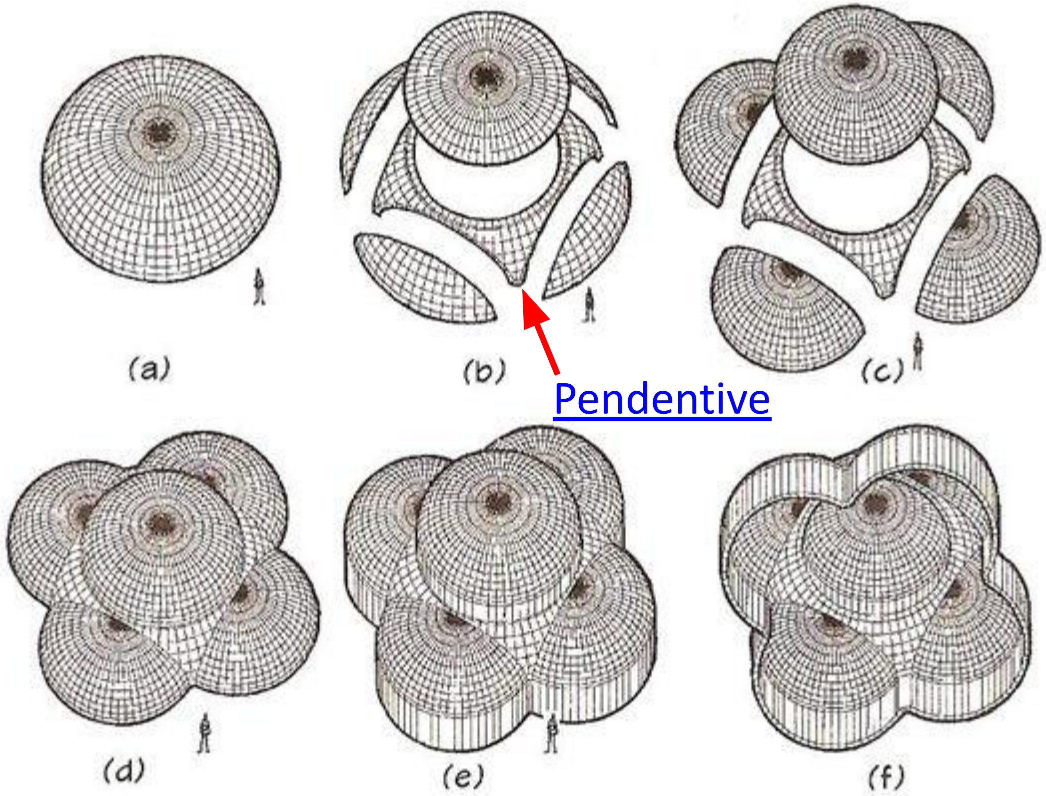


Edwin A. Stevens Hall Est. 1870



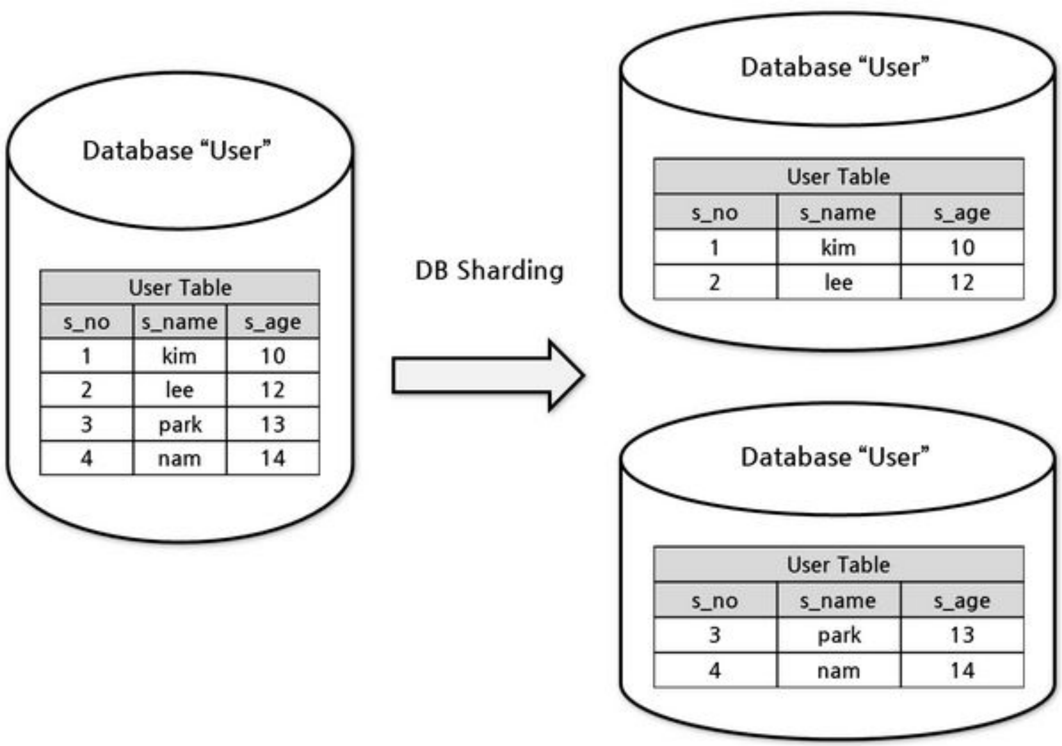
Pendentive Dome

D. Vaccari, "The Barbed Quatrefoil Is a 1500-Year-Old Symbol of Architectural Advance," 2018. [Online]. Available.



Sharding

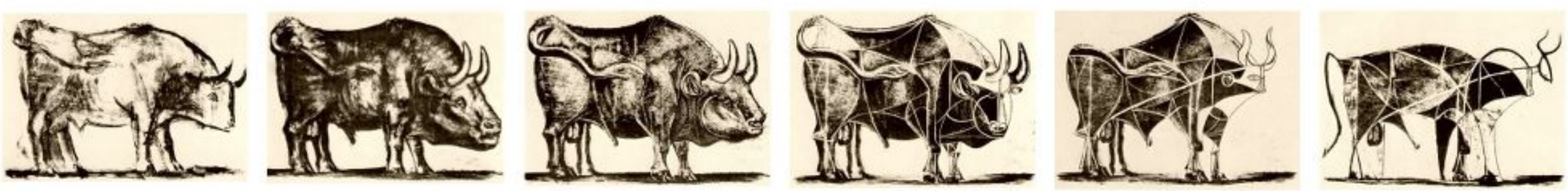
https://en.wikipedia.org/wiki/Shard_(database_architecture)



- The word shard means a small part of a whole
- Sharding is a type of database partitioning that separates very large databases into smaller, faster, more easily managed parts called data shards
- Each shard is held on a separate, independent, self-sufficient database server instance to spread load via a shared nothing (SN) distributed computing architecture

Abstraction in Steps

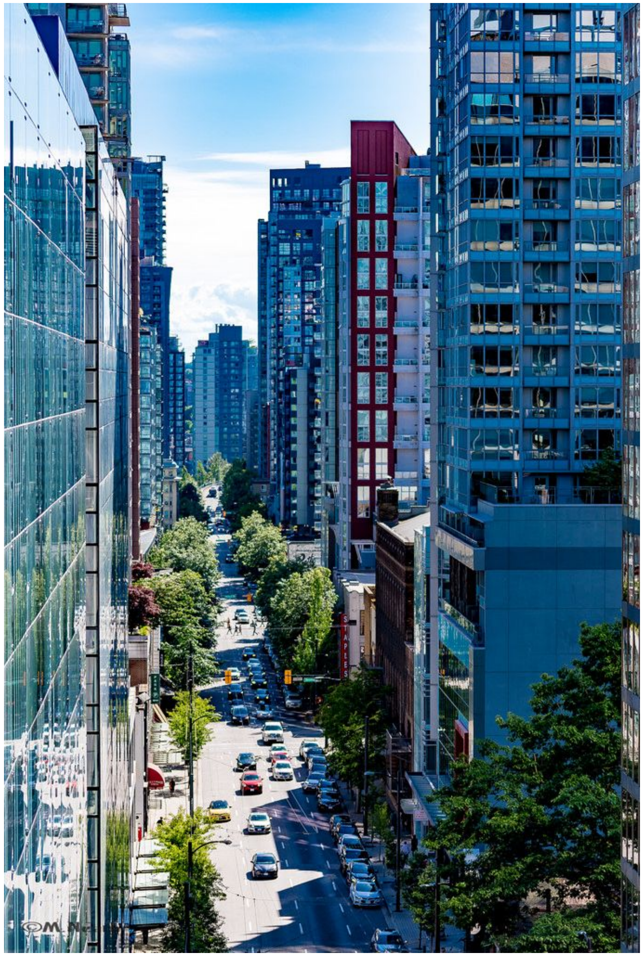
Pablo Picasso 1881—1973 dissected the image of a bull to discover its essential presence through a progressive analysis of its form



1945-12-05 1945-12-12 1945-12-18 1945-12-22 1945-12-24 1945-12-26

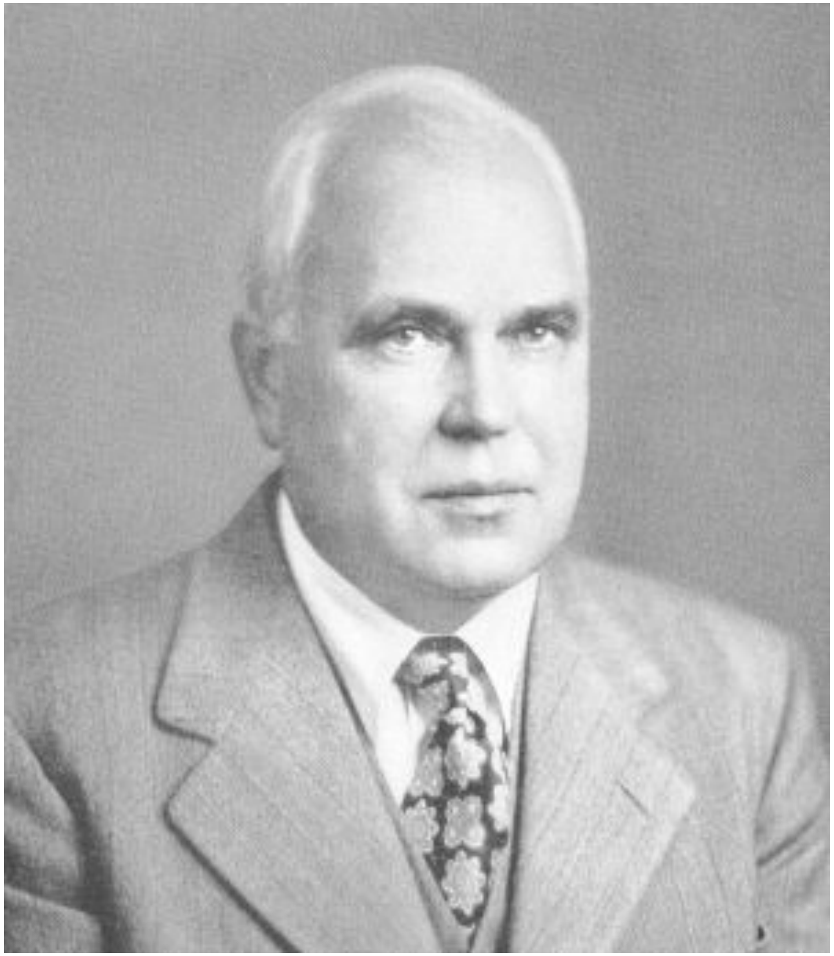
1945-12-28 1946-01-02 1946-01-05 1946-01-10 1946-01-17

Urban Canyons



- An urban canyon or a street canyon is a place where the street is flanked by buildings on both sides creating a canyon-like environment
- Urban canyons affect various local conditions, including temperature, wind, air quality, and radio reception, including satellite navigation signals
- When using Global Positioning System (GPS) receivers in street canyons with tall buildings, the fading (shadowing) and multipath effects may contribute to poor GPS signal reception

Friis Transmission Equation

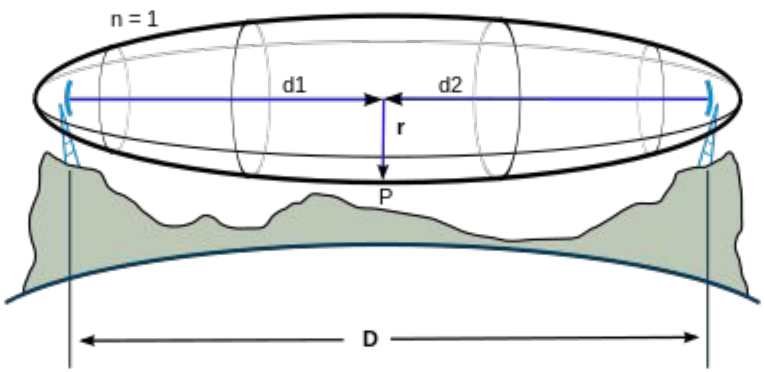


- The Friis transmission equation was derived in 1945 by Danish-American radio engineer Harald T. Friis 1893—1976 at Bell Labs
- Given two antennas with gains *G*, wavelength λ, and distance *R*, the ratio of input power at the receiving antenna to output power at the transmitting antenna is

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$$

- The ideal conditions are without obstructions or reflections from buildings or the ground, e.g., satellite communications or anechoic chambers

Fresnel Zones and Radii



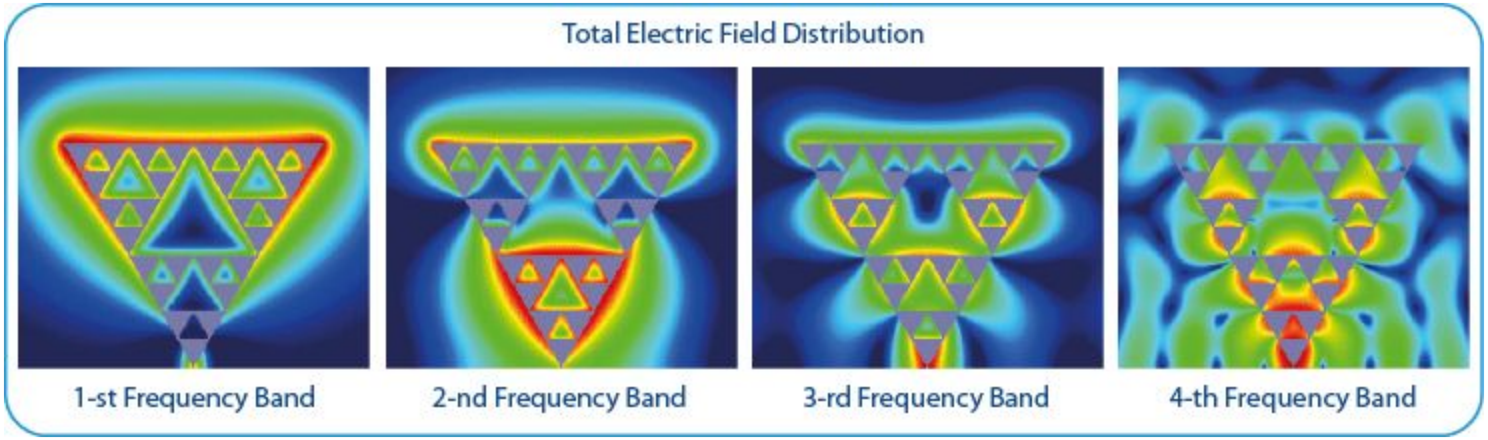
$$r = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}, \qquad d_1,\ d_2 \gg n\lambda$$

- A Fresnel zone named after French engineer Augustin-Jean Fresnel 1788—1827 is one of a series of concentric prolate ellipsoidal regions surrounding the radio frequency (RF) line of sight (LoS) between and around a transmitting antenna and a receiving antenna system with wavelength λ
- The first zone (n=1) must be kept largely free from obstructions to avoid interfering with the radio reception

- Ray tracing for electromagnetic radiation often relies on approximate solutions to Maxwell's equations that are valid as long as the waves propagate through and around objects whose dimensions are much greater than the wavelength
- Ray theory does not describe phenomena such as interference and diffraction, which require wave theory (involving the phase of the wave)

Fractal Antennas

- Normally standard antennas have to be "cut" for the frequency for which they are to be used—and thus the standard antennas only work well at that frequency
- Fractal antennas that uses a fractal self-similar design (e.g., Sierpiński triangle) are compact, multiband or wideband, and have useful applications in cellular telephone and microwave communications
- The response of a fractal antenna (e.g., Sierpiński gasket monopole) differs markedly from traditional antenna designs, in that it is capable of operating with good-to-excellent performance at many different frequencies simultaneously



Physical Theory of Diffraction

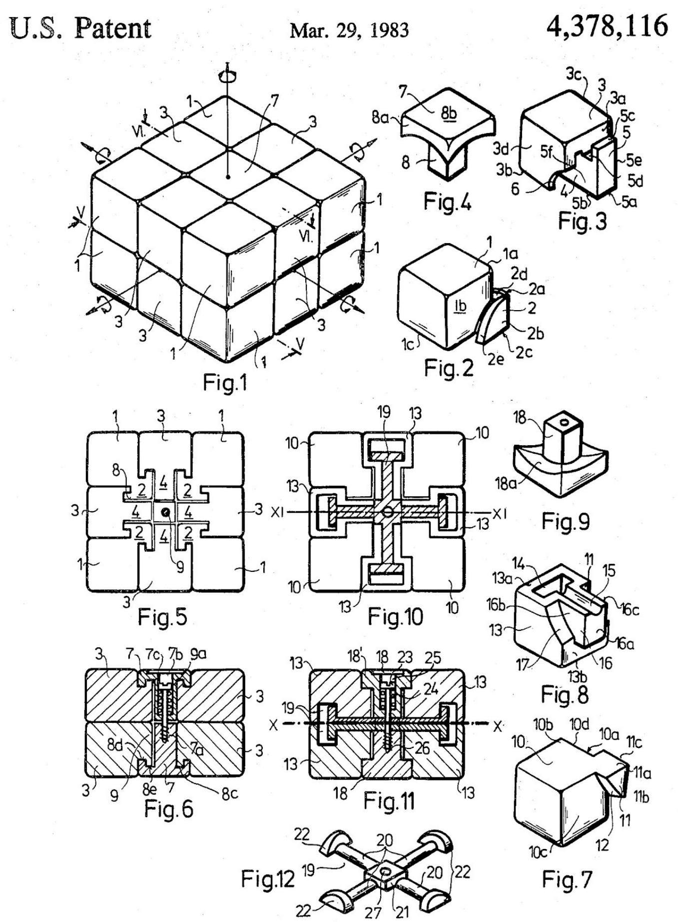
- Stealth technology makes personnel, aircraft, ships, submarines, missiles, satellites, and ground vehicles less visible (ideally invisible) to radar, infrared, sonar and other detection methods
- In the 1960s, Petr Ufimtsev began developing a high-frequency asymptotic theory for predicting the scattering of electromagnetic waves from two-dimensional and three-dimensional objects
- This theory played a critical role in the design of stealth aircraft Lockheed F-117 Nighthawk and Northrop Grumman B-2 Spirit

Polybius Square

	1	2	3	4	5
1	Α	Β	Γ	Δ	Ε
2	Ζ	Η	Θ	Ι	Κ
3	Λ	Μ	Ν	Ξ	Ο
4	Π	Ρ	Σ	Τ	Υ
5	Φ	Χ	Ψ	Ω	

- The Polybius square is a device invented by Cleoxenus and Democleitus, and further developed by Polybius 200—118 BC
- The device is used for fractionating plaintext characters so that they can be represented by a smaller set of symbols, useful for telegraphy steganography, and cryptography
- The device was originally used for fire signaling, allowing for the coded transmission of any message, not just a finite amount of predetermined options

Rubik's Cube

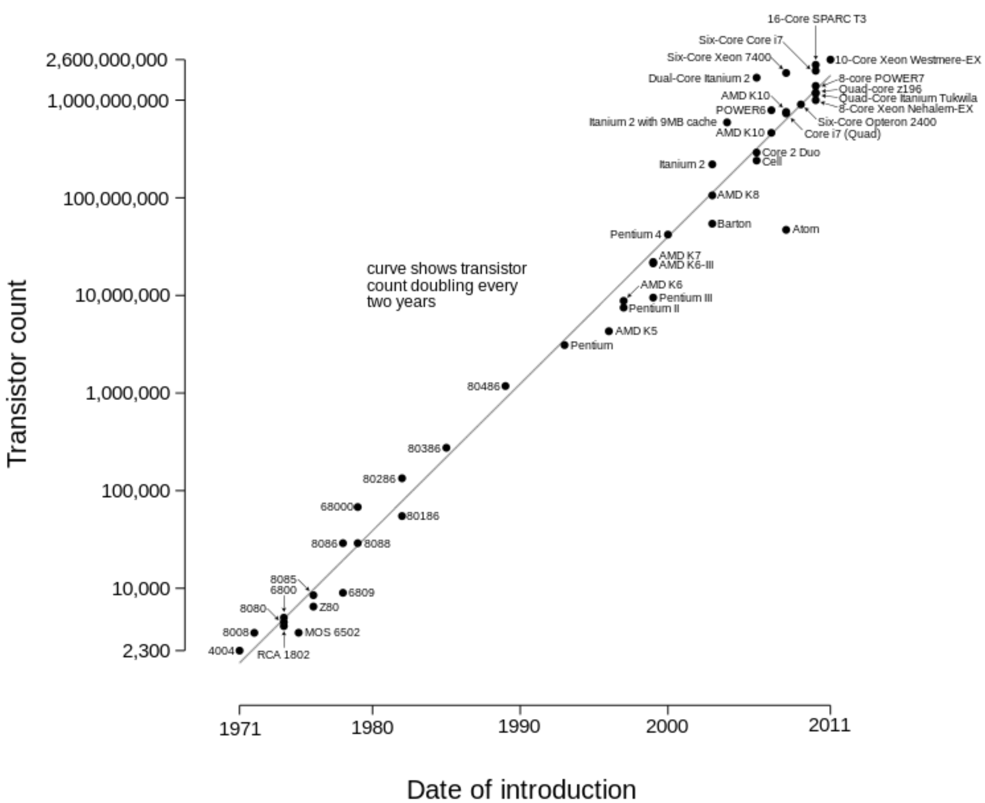


- Ernő Rubik invented Rubik's Cube (originally called the Magic Cube) in 1974 and applied for a patent (HU170062) at the Hungarian Intellectual Property Office on 1975-01-30
- Rubik was granted U.S. patent 4,378,116 on 1983-03-29 that expired in 2000
- On 2016-11-10, the European Union's Court of Justice, ruled that the puzzle's shape was not sufficient to grant it trademark protection
- There are $8! \times 3^7 \times (12!/2) \times 2^{11}$ = 43,252,003,274,489,856,000 permutations
- J. Rokicki, H. Kociemba, M. Davidson, and J. Dethridge, "The Diameter of the Rubik's Cube Group is Twenty," *SIAM J. Discrete Math.*, Vol. 27, No. 2, pp. 1082-1105, 2013 [PDF]
- God's algorithm, God's number

Moore's Law

https://en.wikipedia.org/wiki/Moore%27s_law

Microprocessor Transistor Counts 1971-2011 & Moore's Law

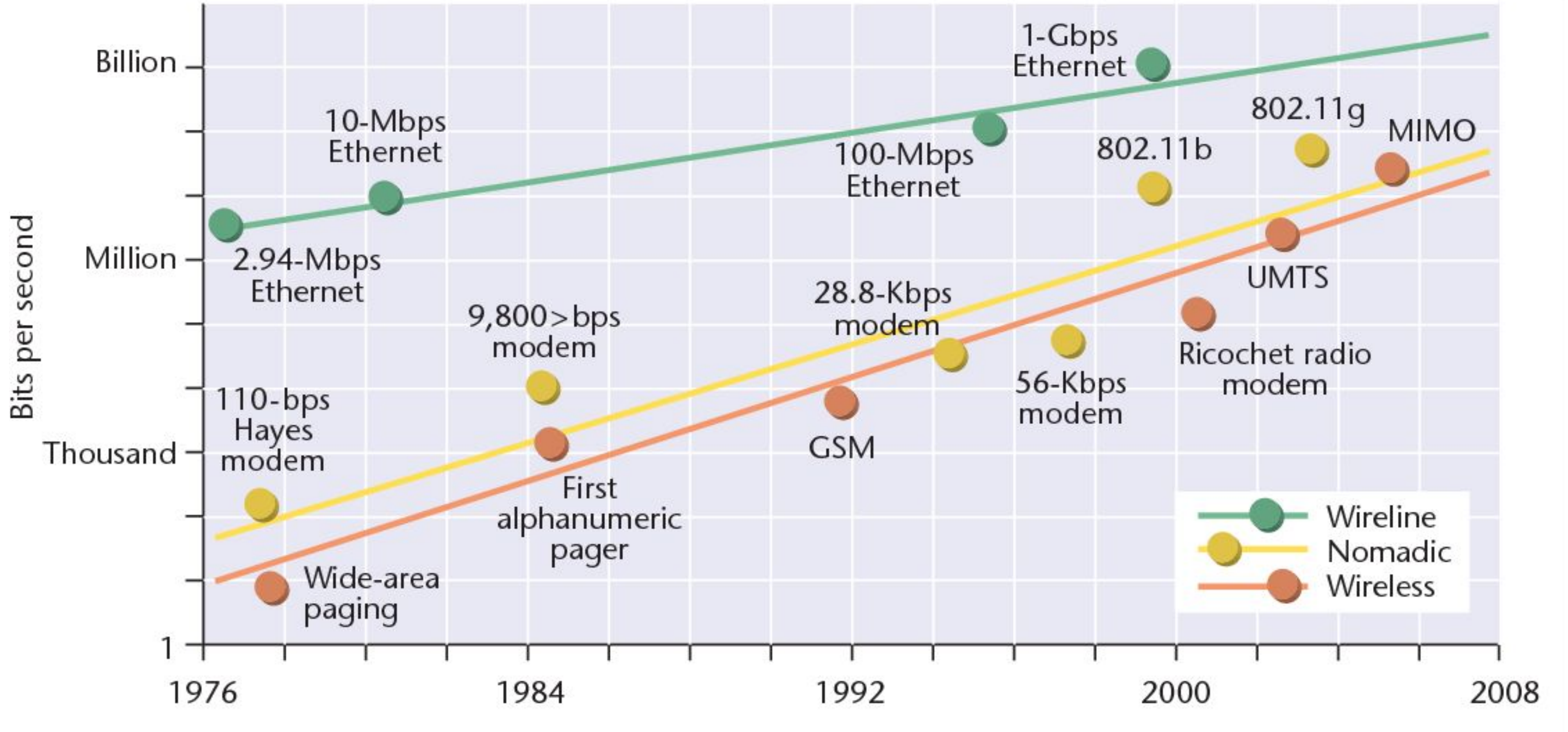


- [Gordon Moore](), co-founder and chairman emeritus of Intel Corporation
- His 1965 paper described a doubling every year in the number of components per integrated circuit, and projected this rate of growth would continue for at least another decade
- In 1975, looking forward to the next decade, he revised the forecast to doubling every two years

Edholm's Law

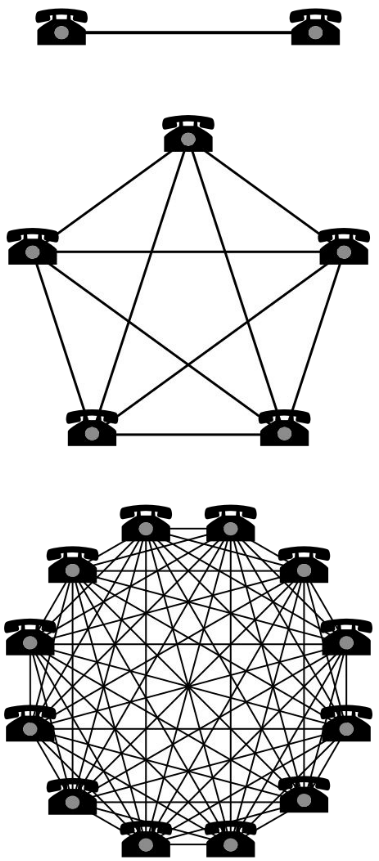
https://en.wikipedia.org/wiki/Edholm%27s_law

Phil Edholm of Nortel Networks observed that telecommunication bandwidth (including Internet access bandwidth) was doubling every 18 months from the late 1970s through to the early 2000s



Metcalfe's Law

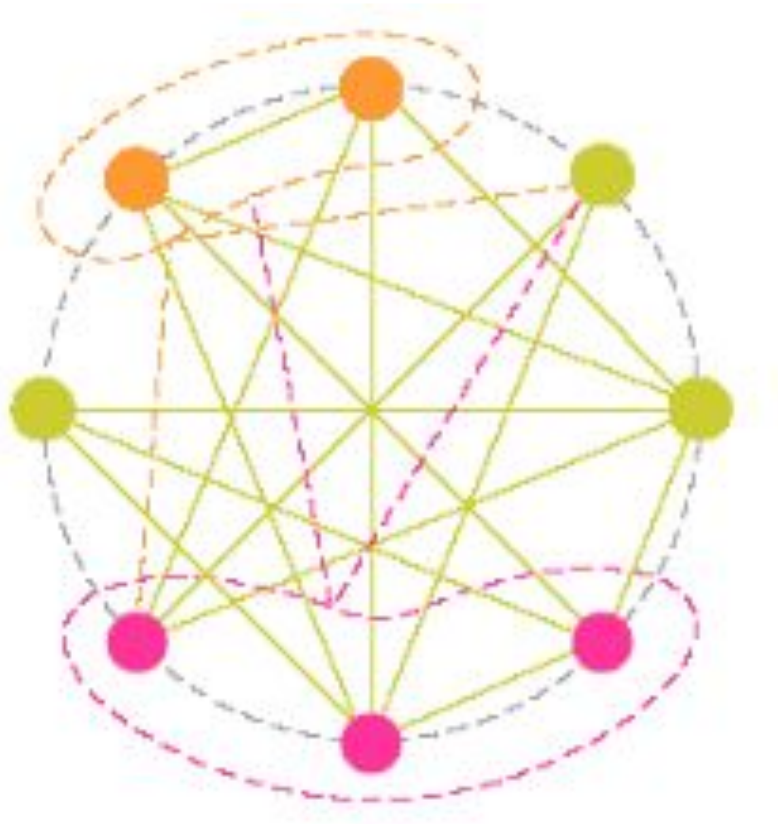
https://en.wikipedia.org/wiki/Metcalfe%27s_law



- The value of a telecommunications network of n nodes is proportional to $n(n - 1)/2$, which is proportional to n^2 asymptotically
- Attributed to Robert (Bob) Metcalfe, co-inventor of Ethernet in 1973
- In 2013, Bob Metcalfe used the netoid function to model Facebook user growth from 2004 to 2013, and fits his law to the associated revenue — validated in 2015 by Zhang Xingzhou, Liu Jingjie, and Xu Zhiwei (Chinese Academy of Sciences in Beijing) with both Facebook and Tencent data

Reed's Law

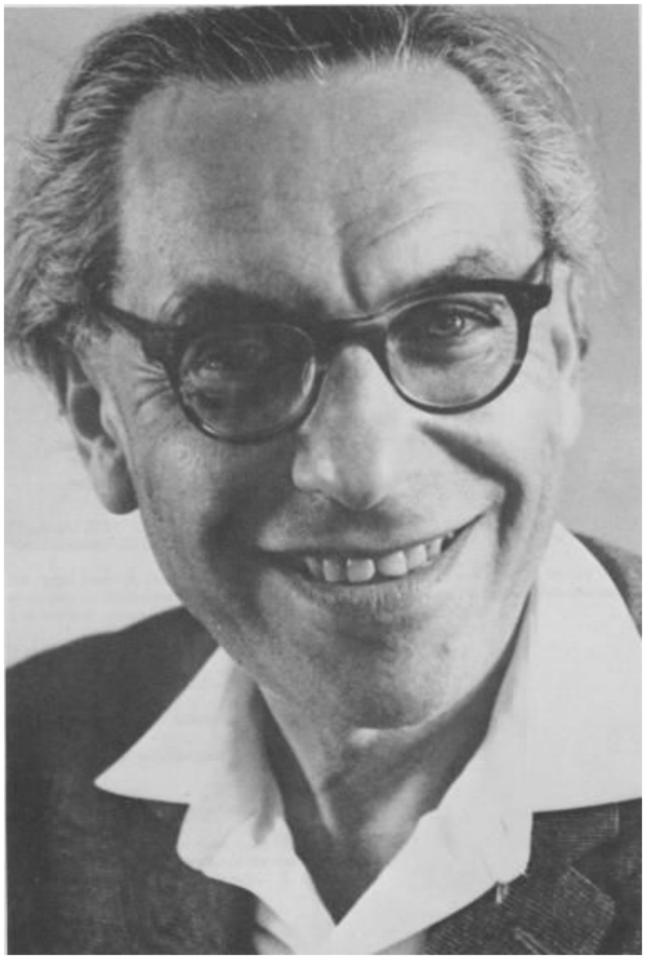
https://en.wikipedia.org/wiki/Reed%27s_law



- David P. Reed asserted that the utility of large networks, particularly social networks, can scale exponentially with the size of the network in "The Law of the Pack," *Harvard Business Review*, pp. 23–24, February 2001
- A set of *n* people has $2^n - n - 1$ possible subsets with at least two people

$$\sum_{k=2}^{n} \binom{n}{k} = 2^n - n - 1$$

Erdős Pál 1913—1996



- Paul Erdős was a Hungarian mathematician who published at least 1,525 mathematical papers with 511 collaborators
- The order of Hungarian names is the family name first, followed by the given name
- The Erdős number defined in 1969 by Prof. Casper Goffman 1913—2006 of Purdue University describes the "collaborative distance" between Erdős and another person in not only mathematics but also biology, chemistry, economics, law, medicine, physics, and sociology
- Related concepts include six degrees of separation, LinkedIn connections, and ever shrinking average distance of Facebook or Twitter users

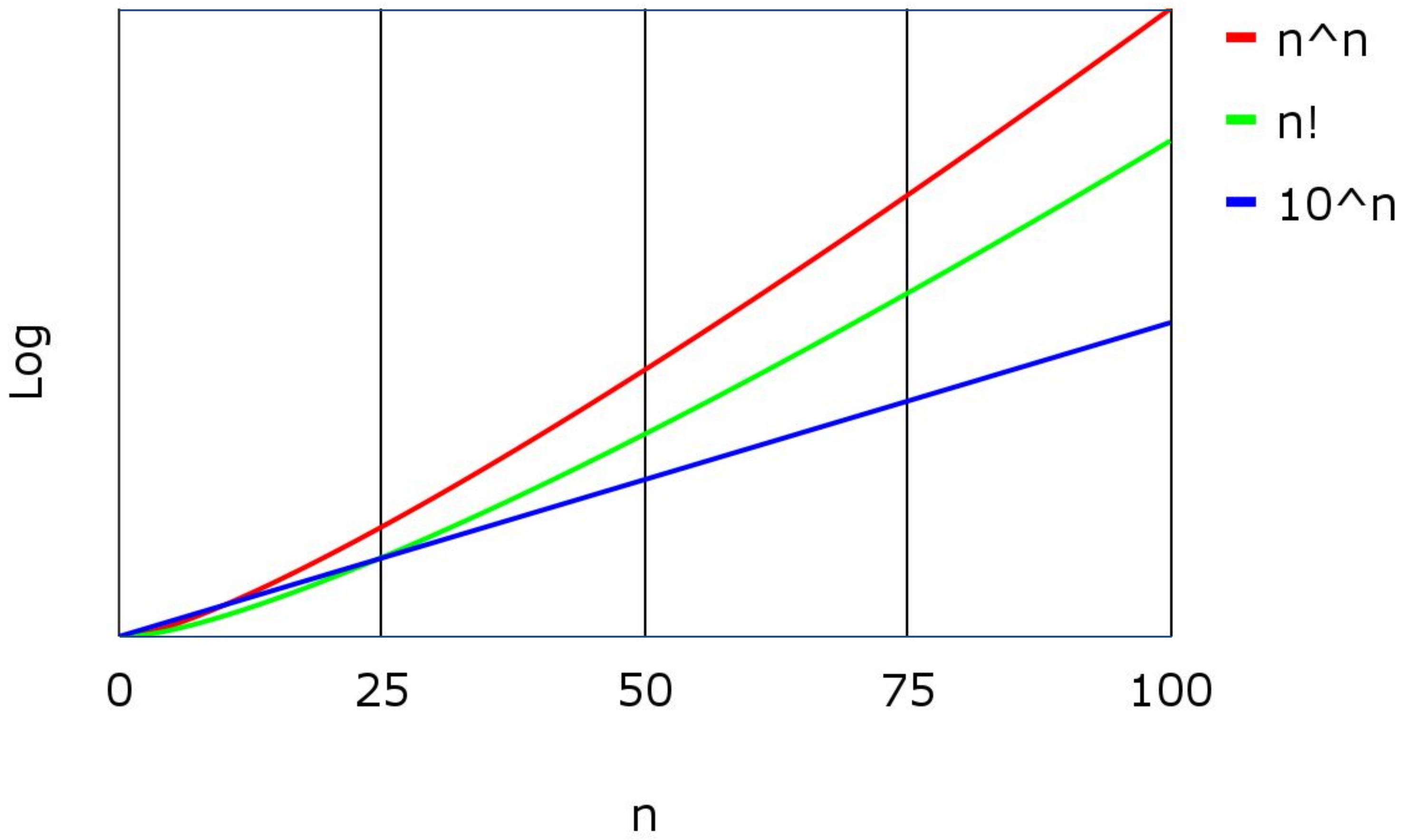
Erdős Number

https://files.oakland.edu/users/grossman/enp/Erdos1.html
https://files.oakland.edu/users/grossman/enp/Erdos2.html

	Erdős Number	Number of People
Paul Erdős	0	1
Peter E. Ney	1	511
John S. Sadowsky	2	11,009
Kevin W. Lu	3	?

Exponential vs. Factorial

https://en.wikipedia.org/wiki/Large_numbers



Hyperoperation

https://en.wikipedia.org/wiki/Hyperoperation

- The hyperoperation sequence is an infinite sequence of arithmetic operations
- Here are the first seven hyperoperations

Hyper0	successor	$1 + b$	
Hyper1	addition	$a + b$	
Hyper2	multiplication	$a \times b$	$a \cdot b$
Hyper3	exponentiation	a^b	$a \uparrow b$
Hyper4	tetration	${}^{b}a$	$a \uparrow\uparrow b$
Hyper5	pentation	$a \uparrow^3 b$	$a \uparrow\uparrow\uparrow b$
Hyper6	hexation	$a \uparrow^4 b$	$a \uparrow\uparrow\uparrow\uparrow b$

Other Large Numbers

- Avogadro constant named after Amedeo Avogadro 1776—1856
  $6.02214076 \times 10^{23}$
- Skewes' number named after Stanley Skewes 1899—1988
- Steinhaus-Moser notation named after Hugo Steinhaus 1887—1972 and Leo Moser 1921—1970
- Graham's number named after Ronald Graham 1935—2020

$$G = \left.\begin{matrix} 3\underbrace{\uparrow\uparrow\cdots\cdots\cdots\cdots\cdots\cdots\cdots\uparrow}3 \\ 3\underbrace{\uparrow\uparrow\cdots\cdots\cdots\cdots\cdots\uparrow}3 \\ \underbrace{\qquad\qquad\vdots\qquad\qquad} \\ 3\underbrace{\uparrow\uparrow\cdots\cdots\uparrow}3 \\ 3\uparrow\uparrow\uparrow\uparrow 3 \end{matrix}\right\}\text{64 layers}$$

  where $3\uparrow\uparrow\uparrow\uparrow3 = 3\uparrow\uparrow\uparrow(3\uparrow\uparrow\uparrow3)$ and $3\uparrow\uparrow\uparrow3 = 7{,}625{,}597{,}484{,}987$
- TREE function by Joseph Kruskal 1928—2010
- SSCG function by Harvey Friedman (SSCG: simple subcubic graph)

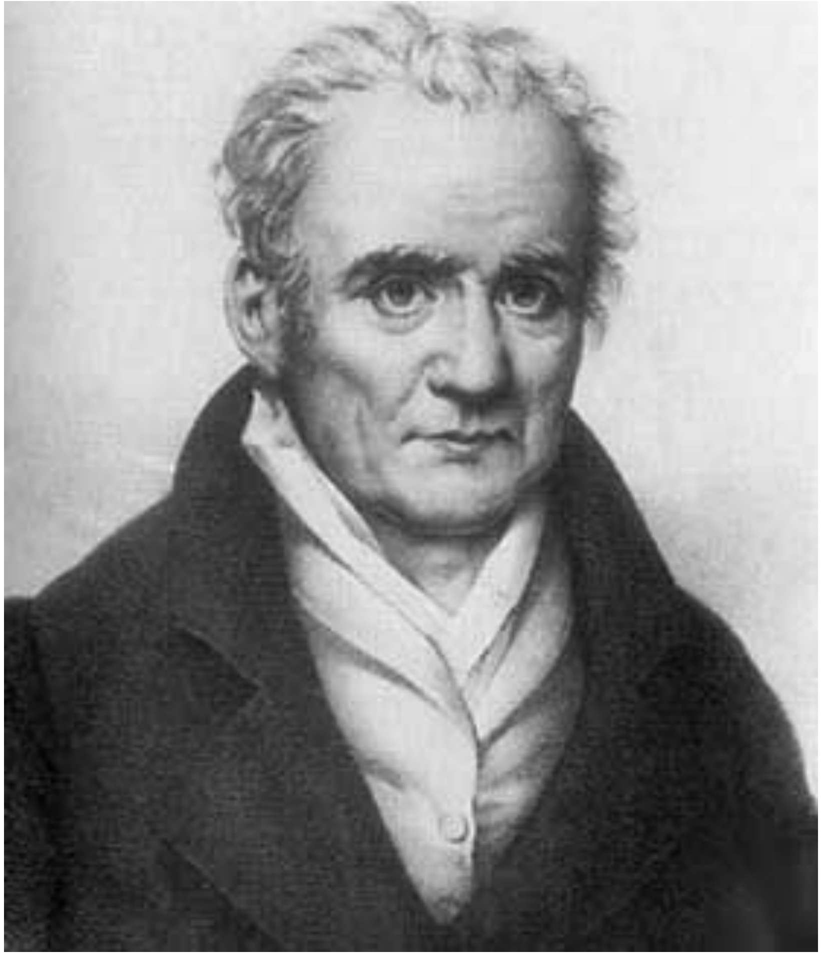
Find the Next Number in the Series

1, 11, 21, 1211, …

Find the Next Number in the Series

1, 11, 21, 1211, 111221, …

Gaspard Monge 1746—1818



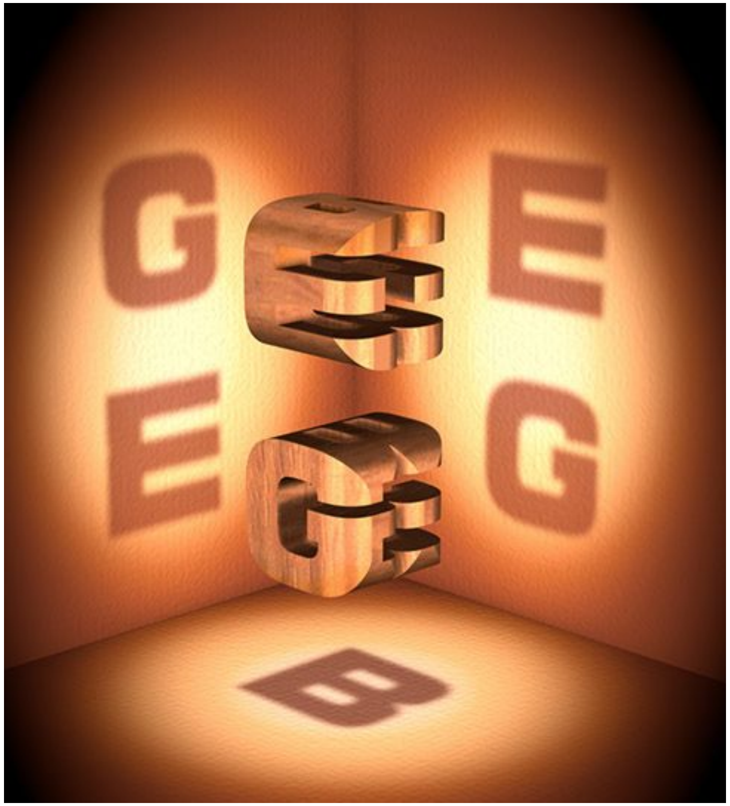
- Gaspard Monge was a French mathematician, the inventor of descriptive geometry (the mathematical basis of technical drawing), and the father of differential geometry
- He proposed that images of a three-dimensional object, when projected onto a set of coordinated two-dimensional planes, could be used to describe that object

Isometric/Orthographic Drawing

https://www.tensorflow.org

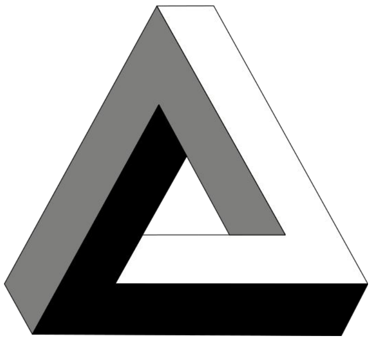
TensorFlow

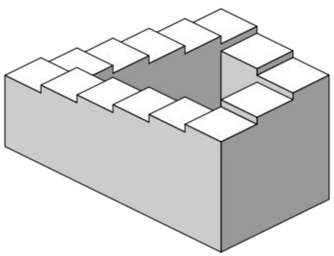
Gödel, Escher, Bach (*GEB*)

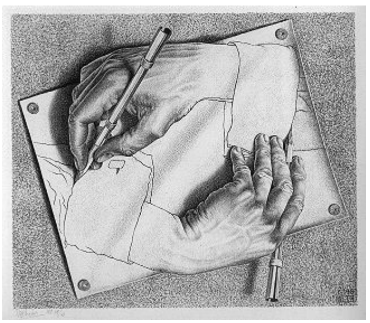


Douglas Hofstadter designed and made two 4-inch cubic redwood triplets suspended in space, casting shadows on three planes that meet at the corner of a room to symbolize the unity of Gödel, Escher, and Bach by fusing their names for the cover of his 1979 book titled *Gödel, Escher, Bach: An Eternal Golden Braid* — taglined "A metaphorical fugue on minds and machines in the spirit of Lewis Carroll"

Strange Loop

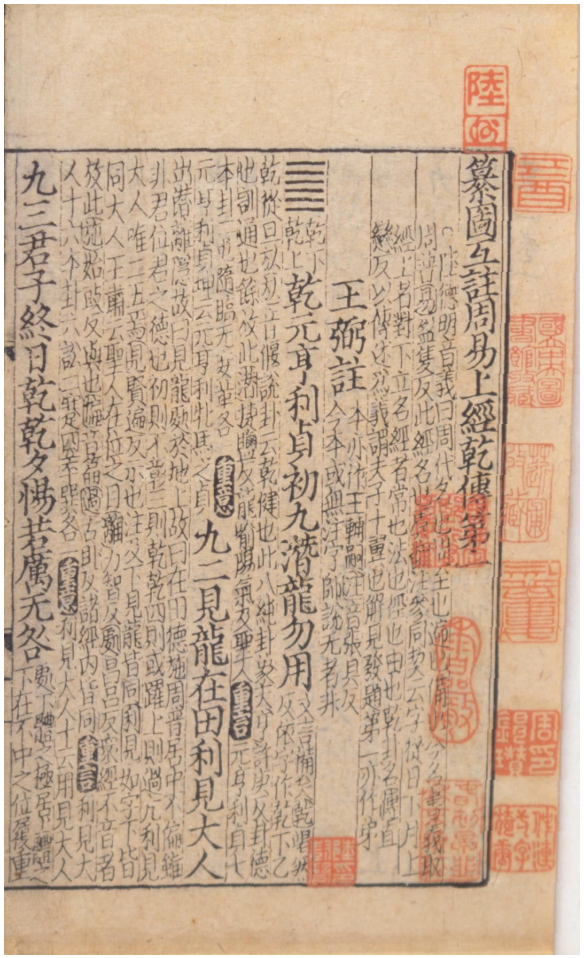






- A strange loop is a cyclic structure that may involve self-reference (ouroboros) and paradox, e.g.,
  - *Impossible Triangle* (1934) and *Impossible Stairs* (1937) by Oscar Reutersvärd 1915—2002, and independently by Lionel Penrose 1898—1972 and his son Roger Penrose in 1958 and 1959, respectively
  - *Drawing Hands* (1948) by M. C. Escher 1898—1972
- Douglas Hofstadter proposed and discussed the concept of a strange loop to demonstrate how the properties of self-referential systems such as Gödel's incompleteness theorems can be used to describe the unique properties of minds that animate beings can come out of inanimate matter
  - *Gödel, Escher, Bach: An Eternal Golden Braid* (1979) PDF
  - *I Am a Strange Loop* (2007) in PDF

Yìjīng and Sigmoid Function



纂圖互註周易上經乾傳第一

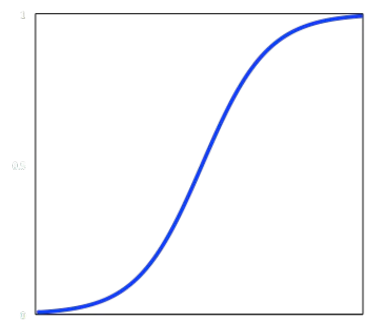
王弼註

乾元亨利貞初九潛龍勿用

九二見龍在田利見大人

九三君子終日乾乾夕惕若厲无咎

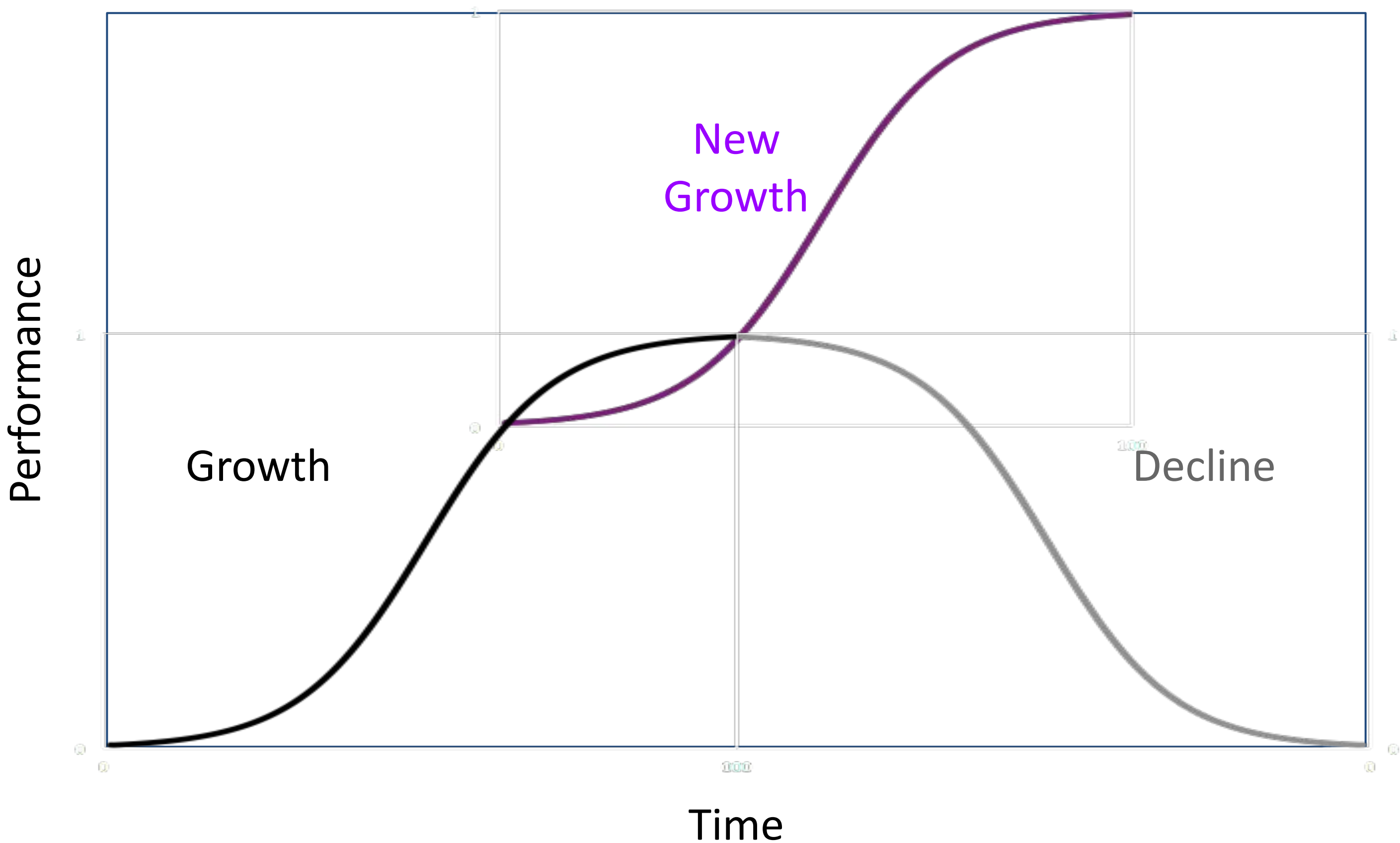
- *Yìjīng*, *I Ching*, or *Book of Changes* was an ancient Chinese divination manual circa 1000 BC
- Contains 64 hexagrams, each with the hexagram name, a short hexagram statement, and six line statements that were used to determine the results of divination
- The statements of the first hexagram describe the pattern and characteristics of growth similar to a sigmoid function of an S-shaped curve such as the following special case of the logistic function



$$S(x)=\frac{1}{1+e^{-x}}$$

The Second Curve

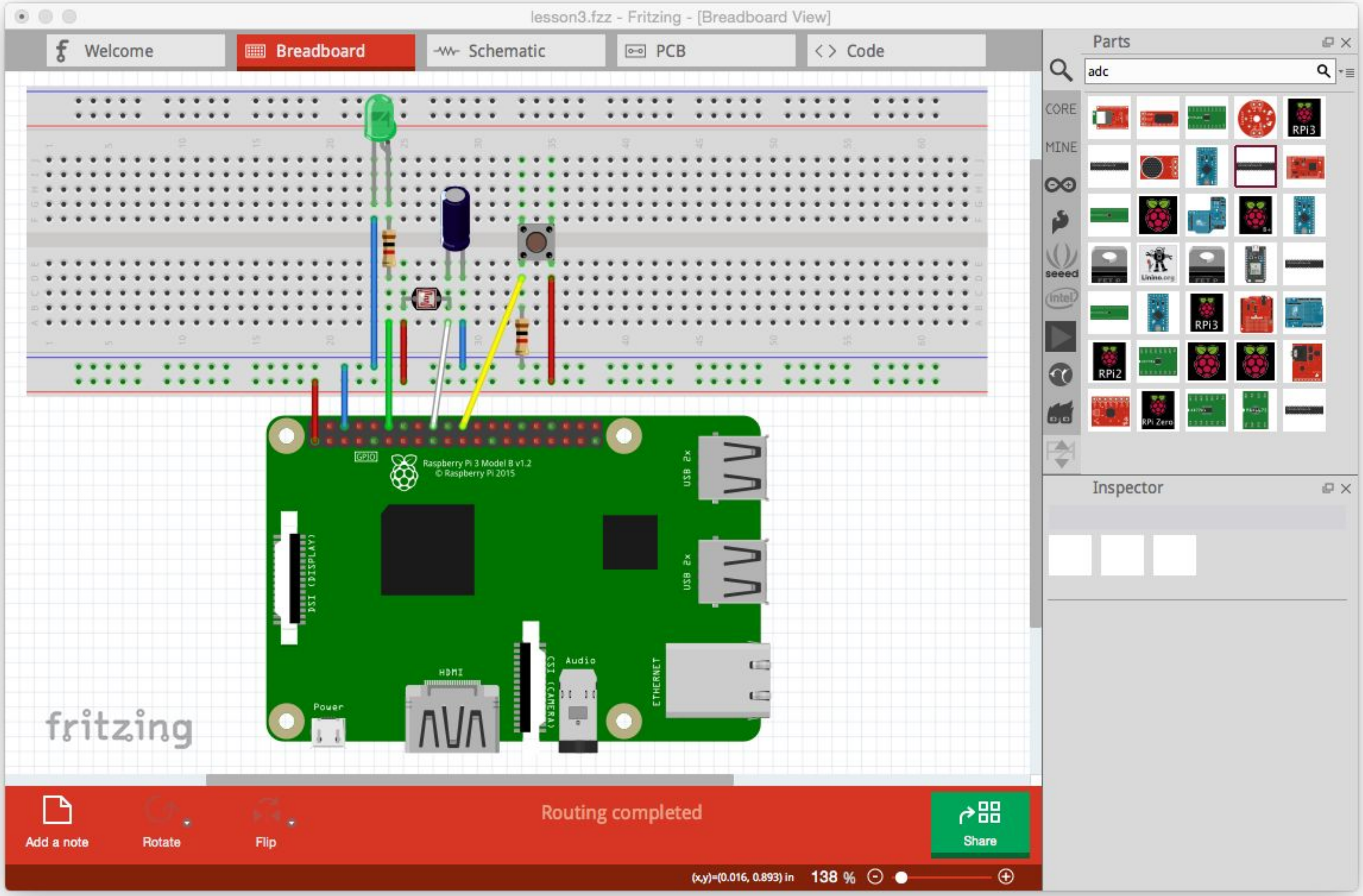
Ian Morrison 1996 and Charles Handy 2015



Crown Cork

- William Painter 1838—1906 patented the crown cork with 24 teeth on 1892-02-02 (U.S. Patent 468,258)
- The current version of 21 teeth since 1930s has the best tightness, rigidity, and ease of opening
- In the 1960s, the German standard DIN 6099 reduced the height of the crown cap and specified the "twist-off" crown cap, eliminating the need for an opener
- The crown cork inspired King C. Gillette 1855—1932 to invent the disposable razor when he was a salesman for Crown Cork & Seal Co.

Fritzing Breadboard View



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