Problem Set 5

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October 21, 2023

Contents

Problem 1 1

Problem 2 3

Problem 1

(a) The graph of the integrand $x^{a-1}e^{-x}$ for $a = 2, 3, 4$ is below

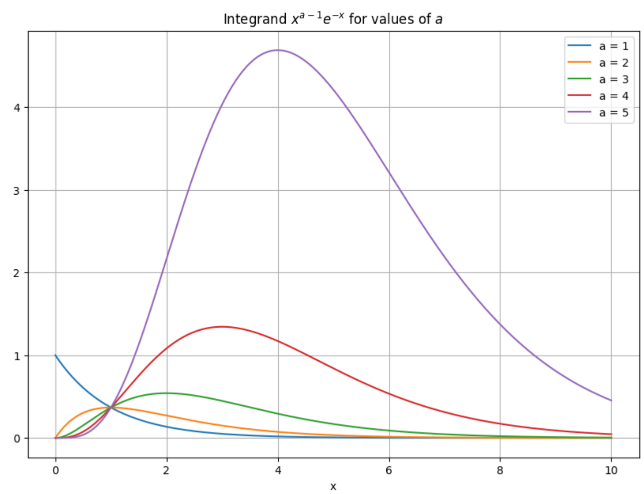

Figure 1: Integrand $x^{a-1}e^{-x}$ for $a = 2, 3, 4$

(b) Using product rule, we can evaluate $\frac{d}{dx}(x^{a-1}e^{x})$. It follows

$$\frac{d}{dx}(x^{a-1}e^{x}) = x^{a-1}e^{x} + (a-1)x^{a-2}e^{x}$$

To find the values of a for which $\frac{df}{dx} = 0$, we factor out the common term $x^{a-2}e^x$

$$x^{a-2}e^x[(a-1)+x] = 0$$

Since $e^x > 0$ for all x, $x^{a-2} = 0$, $x^{a-2} = 0$ only if $x = 0$. It follows, if $x \neq 0$,

$$a - 1 + x = 0 \implies x = 1 - a$$

(c) Given the change of variables $z = \frac{x}{c+x}$, it follows

$$\frac{1}{2} = z = \frac{x}{c+x} \implies c = x = 1 - a$$

(d) The integrand $x^{a-1}e^{-x}$ is numerically unstable for large x and a, as e^{-x} reaches 0 exponentially while x^a becomes very large. To address this, we can write the x^{a-1} as

$$x^{a-1} = e^{(a-1)\ln x}$$

Substituting, we obtain the integrand as

$$e^{(a-1)\ln x}e^{-x} = e^{(a-1)\ln x - x}$$

It is more stable numerically because the factor grows exponentially, and will avoid issues with a large factors like x^{a-1}.

(e) From **newman_5.17.py**: gamma value for 3/2 is : 0.8862269254527004

(f) From **newman_5.17.py**:
gamma values for 3, 6, 10 are : [2.0, 119.99999999999997, 362879.9999999999]
factorial values for 3, 6, 10 are [2, 120, 362880]

Problem 2

(a)

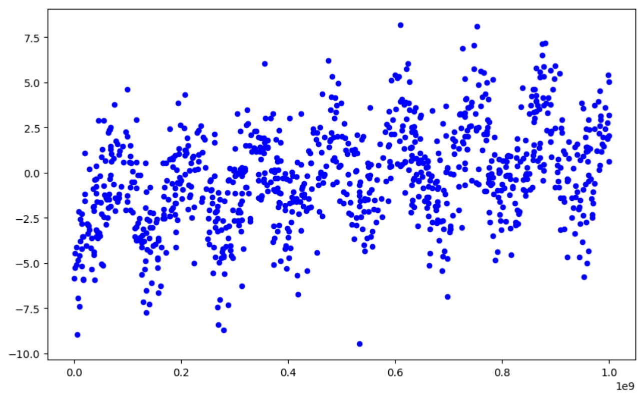

Figure 2: Plot of signal

(b)

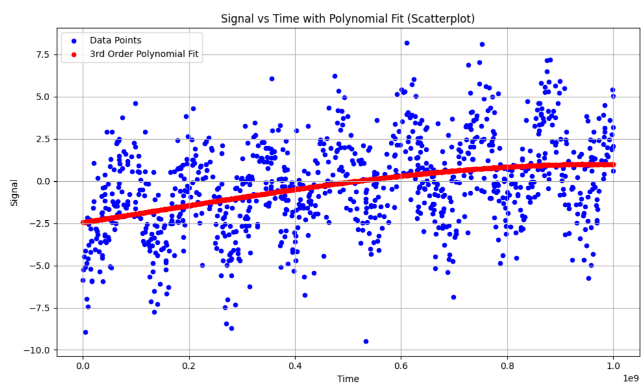

Figure 3: Fitted polynomial degree 3

(c) From the residuals plot, we can observe significant fluctuations. The deviations of the data from the model are of the same order of magnitude as the measurement uncertainties and such, it is not an ideal fit for the data. Residuals plot is shown in the next page.

Figure 4: Residuals

(d)

(e) As the polynomial order rises, the condition numbers escalate quickly. Higher order polynomials will lead to matrices that are ill-conditioned and, as a result, numerically unstable. Therefore, the model will be unreliable for large order polynomial fits. Below a polynomial of degree 7 is fitted.

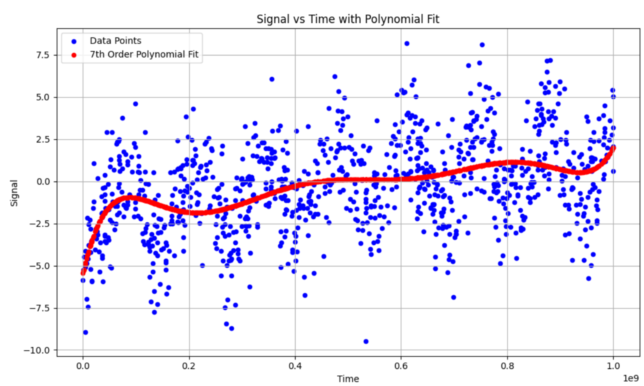

Figure 5: Fitted polynomial degree 7

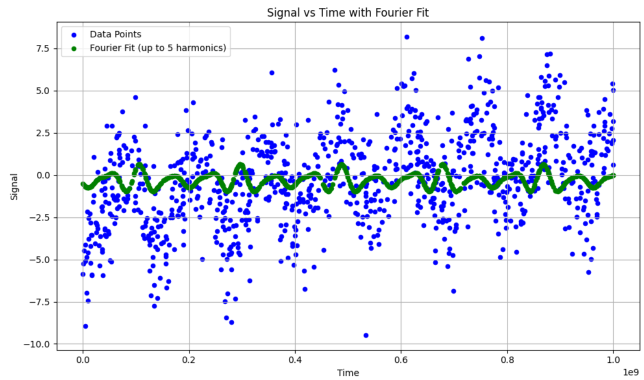

Figure 6: Fourier Fit

(f) The scatterplot above depicts the original signal data and the Fourier series fit using up to the 5th harmonic. The Fourier series expansion is expressed as a combination of sine and cosine terms, starting with a period equal to half of the time span covered. From the fit, it seems like the fourier transform did capture some of the periodicity, but it did not do as ideal of a job capturing the larger oscillations.