Problem Set 7

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Contents

Problem 1

The results outputted from **p1.py** are as follows

Method	**Result**	**Function Value**
Manual Brent's Method	0.300048828125	3.22×10^{-9}
Scipy Brent's Method	0.300000000023735	7.60×10^{-22}

The manual implementation of Brent's method was obtained from some Python implementation from the link https://mathsfromnothing.au/brents-method/?i=1. The currrent program code adjusted the logic to look for a minimum value rather than a sign change. It involves checking for a change in the direction of the function's slope (derivative) rather than its value crossing zero.
As seen in the table, the Manual Brent method and the Scipy's Brent method both accurately determined the minima (using the interval (0,1)) to be 0.3.

Problem 2

The maximum likelihood estimates for the parameters are

$$\beta_0 \text{ (intercept)} = -5.620$$
$$\beta_1 \text{ (slope)} = 0.110$$

The covariance matrix for the parameters are

$$\begin{bmatrix} -1.945 \times 10^{-3} & 1.945 \times 10^{-3} \\ 1.945 \times 10^{-3} & -4.002 \times 10^{-5} \end{bmatrix}$$

The negative log-likelihood at the estimated parameters is approximately 34.726. The log-likelihood is derived as follows. We first arrive from the logistic function, which is defined as

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

The likelihood for a single observation, x_i, y_i, is $p(x_i)$ for a success ($y_i = 1$), and $1 - p(x_i)$ for a failure ($y_i = 0$). It follows that the likelihood function is the product of these individual events, or

$$L(\beta_0, \beta_1) = \prod_{i=1}^{n} p(x_i)^{y_i} (1 - p(x_i))^{(1-y_i)}$$

Taking the log, we obtain

$$\ell(\beta_0, \beta_1) = \sum_{i=1}^{n} [y_i \log(p(x_i)) + (1 - y_i) \log(1 - p(x_i))]$$

The graph plotted in **p2.py** is below

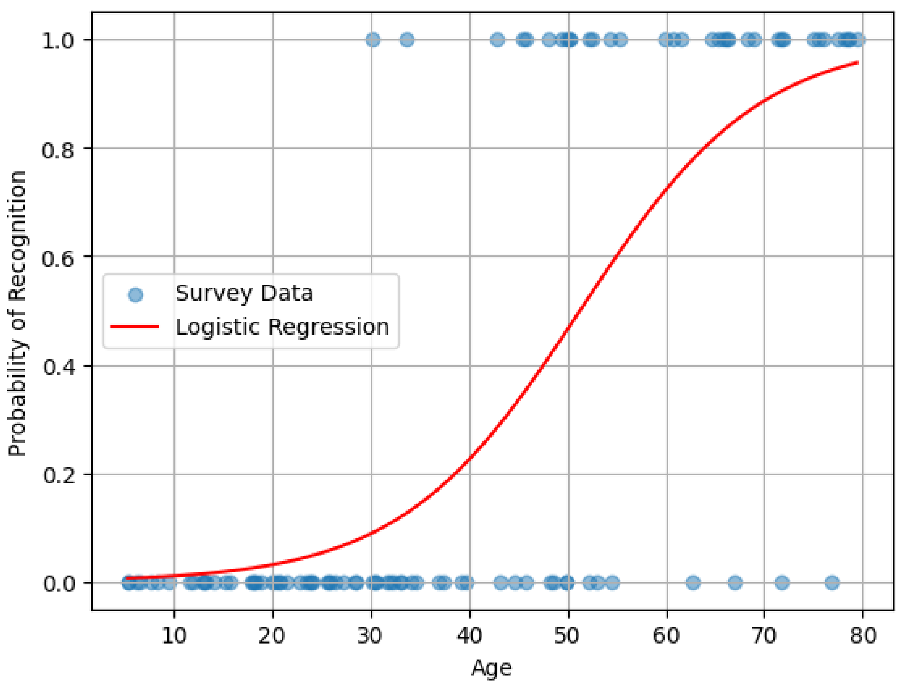

Figure 1: Plot of logistic regrtession