Problem Set 3

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Contents

Problem 1

(a) From newman4.3.py, we have:
Numerical Derivative: 1.010000000000001
Analytical Derivative: 1
Evidently, $\frac{d}{dx}(x(x-1)) = 2x-1$, and when $x=1$, $\frac{df(x)}{dx} = 1$. The two are different due to rounding error (with limitations due to floating point arithmetic) and approximation error (due to δ being finite, and therefore cannot achieve perfect accuracy).

(b) From newman4.3.py, we have:
Numerical Derivative for delta = 0.0001: 1.0000999999998899
Numerical Derivative for delta = 1e-06: 1.0000009999177333
Numerical Derivative for delta = 1e-08: 1.0000000039225287
Numerical Derivative for delta = 1e-10: 1.000000082840371
Numerical Derivative for delta = 1e-12: 1.0000889005833413
Numerical Derivative for delta = 1e-14: 0.9992007221626509
The error becomes worse with small delta due to the increasing prevalence of round of error, when δ is small, $f(x+\delta) - f(x)$ becomes increasingly smaller, leading to loss of precision.
Attached is a plot of the log-log plot of error and delta.

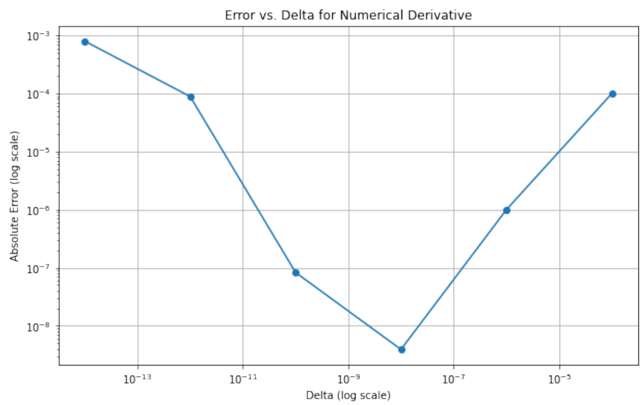

Figure 1: Error for Numerical Derivative

Problem 2

The results below are obtained from newman4.2ex.py.
The graphs below indicate the empirical scaling of the naive method and the dot method, as well as a plot of N (the size of the matrix is $N \times N$) and $\frac{t}{N^3}$, the time that it takes to execute the program divided by N^3 as to determine the empirical scaling of the algorithm. The matrix sizes selected were 10, 30, 50, and 70.
As displayed in Figure 3 below, $\frac{t}{N^3}$ is roughly constant, hovering around 3×10^{-7}, and therefore behaves roughly as predicted.

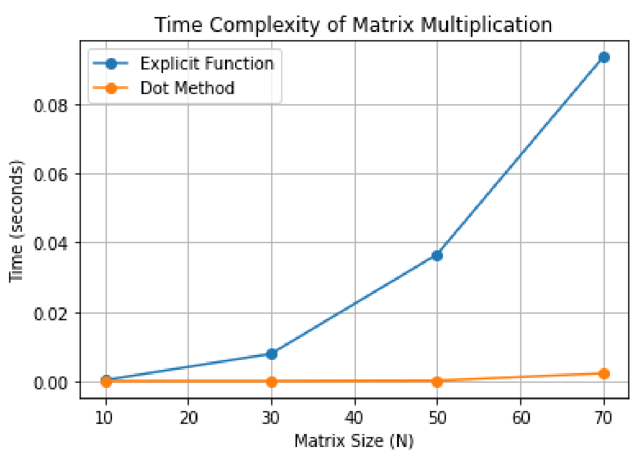

Figure 2: Time Complexity of Matrix Multiplication

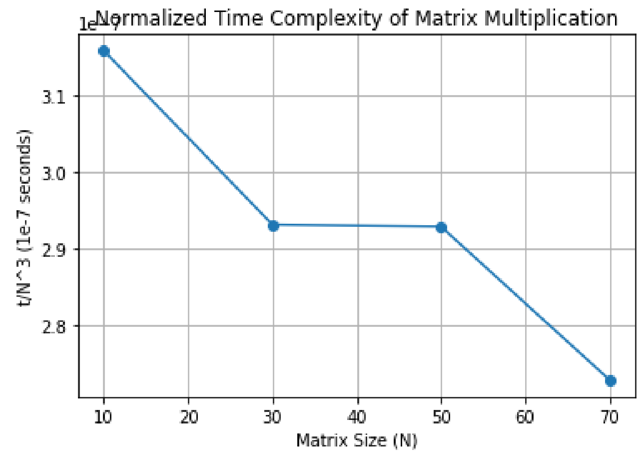

Figure 3: Normalized Time Complexity for Naive Method

Problem 3

The graph below was obtained from newman10.2.py.

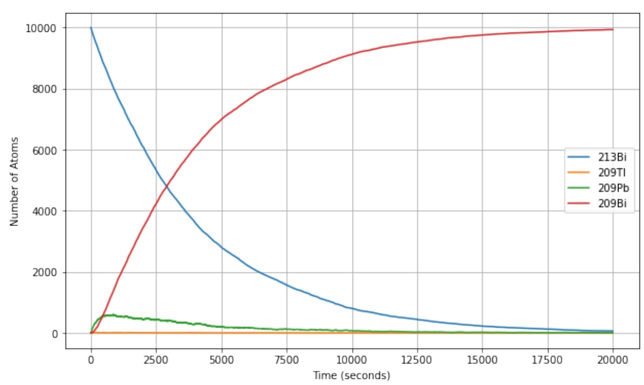

Figure 4: Number of atoms in radioactive decay

As seen above, the number of atoms are plotted for each time t across the 20,000 second duration, and the totals for each respective element across the 20,000 seconds are displayed. Initially, 213Bi (as indicated by the blue line) starts off with 10,000 atoms, and gradually decays into the other elements until there is none remaining.

Problem 4

Attached is the decay of 208TI. Uniform random numbers were generated to obtain and plot the exponentially distributed x values. In this problem, $\mu = \ln(\frac{2}{\tau})$, and the x values plotted (on the y axis) are generated by plugging in random values of z from 0 to 1 into $x = -\frac{1}{\mu}\ln(1-z)$.

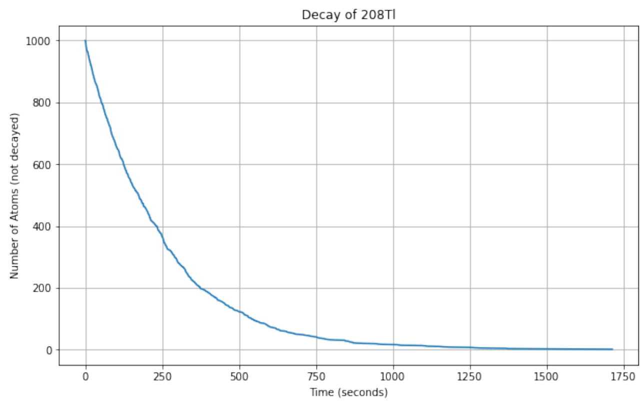

Figure 5: 208TI decay