Computational Physics Problem Set 2

Jason Li

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Contents

Problem 1

$121 = 2^6 + 2^5 + 2^4 + 2^3 + 0 \times 2^2 + 0 \times 2^1 + 2^0 = 1111001_2$

Problem 2

Here is the result from newman2.9.py.

The Madelung constant for L = 100 using madelung_constant_0 is approximately:	1.7418198158
The Madelung constant for L = 100 using madelung_constant_1 is approximately:	1.7418198158
Execution time of madelung_constant_0:	6.574964 seconds
Execution time of madelung_constant_1:	0.259492 seconds

Problem 3

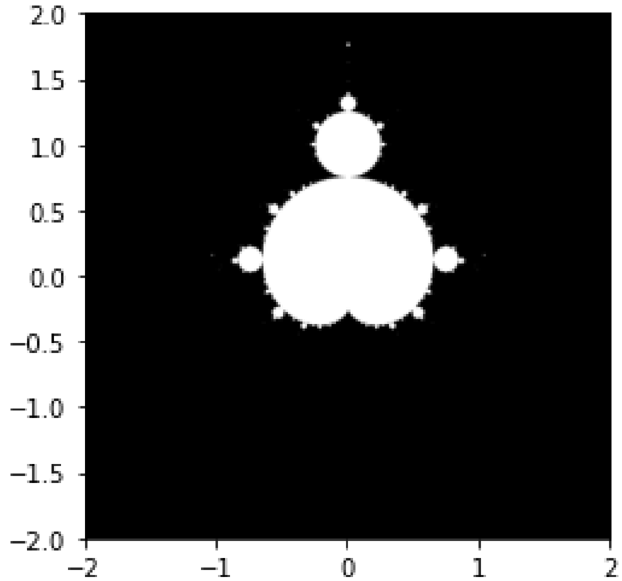

Figure 1: Mandelbrot

Problem 4

(a) Solutions to the equation $0.001x^2 + 1000x + 0.001 = 0$ are:
Root 1: -9.999894245993346e-07
Root 2: -999999.999999
This is obtained using the standard formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ with the function, quadraticstandard in newman4.2py

(b) The standard solution to the quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

.

Multiplying the top and bottom fractional components yields:

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} \\
x &= \frac{b^2 - (b^2 - 4ac)}{2a(-b \mp \sqrt{b^2 - 4ac})} \\
x &= \frac{2a(2c)}{2a(-b \mp \sqrt{b^2 - 4ac})} \\
x &= \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}
\end{aligned}$$

Here is the result from newman2.9.py using the function quadraticalternative.
Root 1: -1000010.5755125057
Root 2: -1.000000000001e-06
We know that the first root should be slightly less than 10^{-6} because

$$-b - \sqrt{b^2 - 4ac} > -b - b$$

Therefore,

$$\frac{2c}{-b - \sqrt{b^2 - 4ac}} < \frac{c}{-b} = -10^{-6}$$

Additionally, we know that the second roots should be slightly more than -10^6 because

$$-b - \sqrt{b^2 - 4ac} > -b - b$$

Therefore,

$$\frac{-b - \sqrt{b^2 - 4ac}}{2a} > -\frac{b}{a} = -10^6$$

This would leave one to conclude that Root 1 of both programs are inaccurate.

(c) The crux of the problem (in this specific scenario) results when the computer computes $-b + \sqrt{b^2 - 4ac}$. Since $-b + \sqrt{b^2 - 4ac}$ is close to 0 in this case, the roots computed using this formula lead to numerical instability. Therefore, one should avoid computations involving values close to 0. See newman4.2.py for the function named quadraticroots.