# *APPROXIMATE EXPONENTIAL FUNCTION WITH TAYLOR EXPANSION*

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number $a$ is the power series

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots,$$

where $n!$ denotes the factorial of $n$ and $f^{(n)}(a)$ denotes the $n$th derivative of $f$ evaluated at the point $a$.

In the more compact sigma notation, this can be written as

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

The derivative of order zero of $f$ is defined to be $f$ itself and $(x-a)^0$ and $0!$ are both defined to be 1. When $a = 0$, the series is also called a Maclaurin series.

The Taylor series for the exponential function $e^x$ at $a = 0$ is

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{x^n}{n!} &= \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots.\end{aligned}$$

The above expansion holds because the derivative of $e^x$ with respect to $x$ is also $e^x$ and $e^0$ equals 1. This leaves the terms $(x-0)^n$ in the numerator and $n!$ in the denominator for each term in the infinite sum.