APPROXIMATE EXPONENTIAL FUNCTION WITH TAYLOR EXPANSION

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots,$$

where $n!$ denotes the factorial of n and $f^{(n)}(a)$ denotes the nth derivative of f evaluated at the point a.

In the more compact sigma notation, this can be written as

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n.$$

The derivative of order zero of f is defined to be f itself and $(x-a)^0$ and $0!$ are both defined to be 1. When $a = 0$, the series is also called a Maclaurin series.

The Taylor series for the exponential function e^x at $a = 0$ is

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{x^n}{n!}&=\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots\\&=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\cdots.\end{aligned}$$

The above expansion holds because the derivative of e^x with respect to x is also e^x and e^0 equals 1. This leaves the terms $(x-0)^n$ in the numerator and $n!$ in the denominator for each term in the infinite sum.

Example 1:

$$f(x)=e^x\approx 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}$$

Try to approximate f(3.14).

Output:

```
Please enter a number as the input x of original exponential function: 3.14
Please enter a positive integer as the number of terms in taylor expansion: 5
Please enter the parameter in taylor expansion with default value of 0:
The approximate value of the original exponential function is 18.2801.
```

Example 2:

$$f(x) = e^x \approx \sum_{i=1}^{n} \frac{e^a}{(i-1)!} \times (x-a)^{i-1}$$

Given a=1 and n=3, try to approximate f(2).

```
Please enter a number as the input x of original exponential function: 2
Please enter a positive integer as the number of terms in taylor expansion: 3
Please enter the parameter in taylor expansion with default value of 0: 1
The approximate value of the original exponential function is 6.7957.
```