CBOE VIX Prediction

1. Methodology and Implementation

Inspired by Liu, Guo and Qiao (2015), I decided to apply GARCH (1,1) Model to forecast VIX.

- Using historical data to estimate parameters

$$r_t = \beta^* + \varepsilon_t$$
$$\varepsilon_t = \sqrt{h_t} u_t$$
$$h_t = \mu^* + \delta^* h_{t-1} + \emptyset^* {\varepsilon_{t-1}}^2$$

r_t is measured by daily log return of VIX from Apr. 29, 2015 to Apr. 30, 2020 (Data Source: Yahoo Finance). There are four parameters and β is shown in the equation of r_t. I used MLE to maximize the likelihood of data of $r_t's$. Assuming $u_t's$ are i.i.d. standard normal, $r_t \sim N(E_{t-1}(r_t), V_{t-1}(r_t))$. Specifically, $r_t \sim N(\beta^*, h_t)$. The conditional density function of r_t can be written as follows:

$$p(r_t|r_{t-1}) = \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{1}{2}\left(\frac{\varepsilon_t}{\sqrt{h_t}}\right)^2} = \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{1}{2}\left(\frac{r_t - \beta^*}{\sqrt{h_t}}\right)^2}.$$

The likelihood function is:

$$L(r, \theta) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{1}{2}\left(\frac{r_t - \beta^*}{\sqrt{h_t}}\right)^2}.$$

The log-likelihood function is:

$$l(r, \theta) = -T\log(\sqrt{2\pi}) - \sum_{t=1}^{T} \log(\sqrt{h_t}) - \sum_{t=1}^{T} \frac{1}{2h_t}(r_t - \beta^*)^2,$$

where $h_t = \mu^* + \delta^* h_{t-1} + \emptyset^* {\varepsilon_{t-1}}^2,\ t > 1$.

Suppose $h_1 = \widehat{var(r)}$. So that $\widehat{\theta_T} = max_\theta l(r, \theta) = max_\theta \frac{l(r,\theta)}{T}$, where $\theta = (\mu, \delta, \emptyset, \beta)$.

- Calculating standard errors and t-statistics

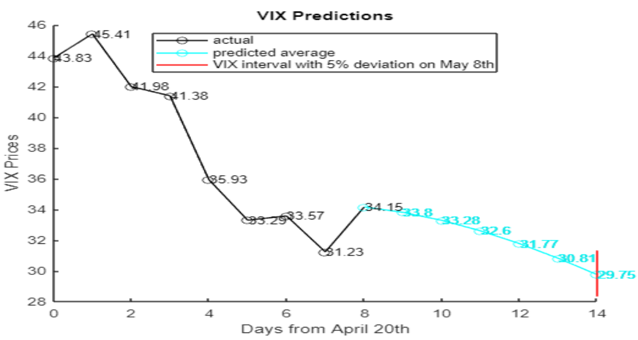
$$se(\widehat{\theta_T}) = \sqrt{var(\widehat{\theta_T})} = \frac{1}{T}\widehat{\Omega_T}^{-1} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial l_t(r,\theta)}{\partial \theta} \times \frac{\partial l_t(r,\theta)}{\partial \theta^{\mathrm{T}}}\right),$$

where $\frac{\partial l_t(r,\theta)}{\partial \theta} = \frac{\partial l_t(r,\theta)}{\partial h_t} \times \frac{\partial h_t}{\partial \theta} = (-\frac{1}{2h_t} + \frac{1}{2{h_t}^2}(r_t - \beta)^2) \times (1, h_{t-1}, {\varepsilon_{t-1}}^2)$ for $\theta = (\mu, \delta, \emptyset)$; $\frac{\partial l_t(r,\theta)}{\partial \beta} = \frac{r_t - \beta}{h_t}$.

- Performing Monte Carlo simulations to predict future returns

I did 1 million monte carlo simulations for u_t, ε_t, r_t and VIX_t, where $r_{t+1} = \ln\left(\frac{VIX_{t+1}}{VIX_t}\right)$, $VIX_{t+1} = VIX_t e^{r_{t+1}} = VIX_t e^{\beta^* + \varepsilon_{t+1}} = VIX_t e^{\beta^* + \sqrt{h_{t+1}} u_{t+1}} = VIX_t e^{\beta^* + \sqrt{\mu^* + \delta^* h_t + \emptyset^* {\varepsilon_t}^2} u_{t+1}}$, and averaged future VIX_t.

2. Results

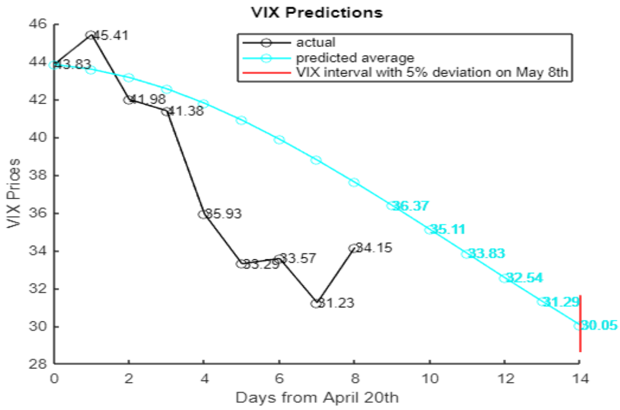


parameters	Estimates_MLE	Std Errors	t_statistics
"mu"	-0.0101	7.8602e-05	-128.5
"delta"	0.63	0.0048941	128.73
"phi"	0.4	0.0031276	127.89
"beta"	0	0.0013543	0

Predicted final VIX price on May 8th is: $29.75.

Predicted interval within 5% deviation is: [$28.34, $31.32].

3. Robustness Check



parameters	Estimates_MLE	Std Errors	t_statistics
"mu"	-0.0084	0	-94.74
"delta"	0.6	0.01	94.44
"phi"	0.4	0	97.94
"beta"	0	0	0

Trial final VIX price on May 8th is: $30.05.

Trial interval with 5% deviation is:[$28.62, $31.63].

To verify the accuracy of GARCH (1,1) model, I deliberately ignored available data of the last 8 trading days (4.20-4.30) and assumed it as unknown. I applied the same GARCH model and predicted future VIX in the next 8+6=14 days. Results showed that all predictions in robustness test were closer to initial ones.

Liu, Q., Guo, S., & Qiao, G. (2015). VIX forecasting and variance risk premium: A new GARCH approach. The North American Journal of Economics and Finance, 34, 314-322.