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MATH 3113
Quiz 1

INSTRUCTIONS: Please show your work. This quiz is closed-book and notes are not allowed.

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1. **[4 points]** Recall the u-substitution from Calculus:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

with $u = g(x)$ and $du = g'(x)dx$. Also recall that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$.
Find the general solution for the differential equation:

$$y' = \tan(x)$$

We have

$$y' = \frac{\sin x}{\cos x}$$

Since the RHS is a function depends only on x and the LHS is the derivative of y, we can integrate on both sides and get

$$\begin{aligned} y = \int y'dx &= \int \frac{\sin x}{\cos x}dx \\ &= \int \frac{d(-\cos x)}{\cos x} \\ &= -\ln(|\cos x|) + C \end{aligned}$$

2. **[6 points]** The relations between acceleration a, velocity v and displacement s with independent variable time t are given by

$$v(t) = s'(t)$$
$$a(t) = v'(t)$$

Sherlock Holmes noticed a robbery occurred 50 meters away from him. The robber is running away from Holmes at a constant acceleration 9 m/s^2, Sherlock Holmes is chasing the robber at a constant acceleration 10 m/s^2. Assume that both of them were standing still at the beginning, and are running in the same direction, how long does it take Holmes to catch the robber?
Let a_h, v_h, s_h be the acceleration, velocity and displacement of Holmes, respectively. Since $a_h = 10$, we have

$$v_h = \int 10dt = 10t + C_1$$

Since at $t = 0$, $v_h = 0$, we have $C_1 = 0$, so $v_h = 10t$. Then

$$s_h = \int v_h dt = \int 10t = 5t^2 + C_2.$$

Since $s_h(0) = 0$, we have $C_2 = 0$. So

$$s_h = 5t^2.$$

Similarly, let a_r, v_r, s_r be the acceleration, velocity and displacement of the robber, respectively. Then

$$v_r = \int 9dt = 9t + D_1$$

and since $v_r(0) = 0$, $D_1 = 0$. Then

$$s_r = \int v_r dt = \int 9tdt = \frac{9}{2}t^2 + D_2.$$

Since the robber was $50m$ away from Holmes at the beginning, $s_r(0) = 50$, therefore $D_2 = 50$, and so $s_r = \frac{9}{2}t^2 + 50$

Finally, when Holmes catches the robber we have $s_h = s_r$, this means

$$\begin{aligned} 5t^2 &= \frac{9}{2}t^2 + 50 \\ \implies \frac{1}{2}t^2 &= 50 \\ \implies t^2 &= 100 \\ \implies t &= 10 \end{aligned}$$

So Holmes catches the robber after 10 seconds.