Written Homework 1

Exercise 1

Verify that the function $y(x) = x + Cx^{-2}$ is a solution of the differential equation $xy' + 2y = 3x$. In addition, find the value of $C \in \mathbb{R}$ so that we have $y(1) = 5$.

Exercise 2

Solve the following IVP:

1. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x+2}}, \quad y(2) = -1$
2. $\dfrac{dy}{dx} = x\sqrt{x^2+9}, \quad y(-4) = 0$
3. $\dfrac{dy}{dx} = xe^{-x}, \; y(0) = 1$

Exercise 3

Draw the solution curves of

$$y' = \frac{-x}{\sqrt{1-x^2}}$$

for the initial conditions $y(0) = 1, y(0) = 2, y(0) = 3$. Draw the curves in the same $xy-$plane.

Exercise 4

Find the function $y = f(x)$ that satisfies the IVP

$$y' = (2+5x)e^{\frac{1}{3}x}, \; y(0) = 5.$$

Exercise 5

Right before Sherlock Holmes and Watson catches the robber, the robber throws away the loot with a launch speed $v_0 = 30m/s$ and a launch angle $\theta = \pi/3$. The position of the loot at time t is given by the Newtonian dynamics

$$\frac{d^2x}{dt} = 0, \qquad \frac{d^2y}{dt} = -g$$

where g is the gravitational acceleration and x, y are the horizontal and vertical displacement of the loot, respectively. Also recall that the relation between the initial velocities in the horizontal and vertical directions are given by

$$x_0 = v_0\cos(\theta), \qquad y_0 = v_0\sin(\theta).$$

1. At what time does the loot reach its maximum height? What is the maximum height?

2. Watson, who is at the same position as the robber, is trying to catch the loot on a bike. What is the acceleration of Watson's bike if he wants to catch the loot when it falls to the ground, assuming the relation between the distance s and the acceleration a is given by

$$\frac{d^2s}{dt} = a.$$

Exercise 6

Use separation of variables to find the general solutions (implicit if necessary, explicit if it can be done).

1. $y' = \dfrac{1+\sqrt{x}}{1+\sqrt{y}}$

2. $y' = \dfrac{(x-1)y^5}{x^2(2y^3-y)}$

3. $y' = 1 + x + y + xy$

Exercise 7

Determine if the following IVP has a unique solution:

1. $y' = \sqrt[3]{y}, \;\; y(0) = 1$

2. $y' = \sqrt[3]{y}, \;\; y(0) = 0$

3. $y' = \ln(1+y^2), \;\; y(0) = 0$