Written Homework 1

Exercise 1

Solve the following IVP:

1. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x+2}}, \quad y(2) = -1$
2. $\dfrac{dy}{dx} = x\sqrt{x^2+9}, \quad y(-4) = 0$
3. $\dfrac{dy}{dx} = xe^{-x}, \ \ y(0) = 1$

Exercise 2

Draw the solution curves of

$$y' = \frac{-x}{\sqrt{1-x^2}}$$

for the initial conditions $y(0) = 1, y(0) = 2, y(0) = 3$. Draw the curves in the same $xy-$plane.

Exercise 3

Verify that the function $y(x) = x + Cx^{-2}$ is a solution of the differential equation $xy' + 2y = 3x$. In addition, find the value of $C \in \mathbb{R}$ so that we have $y(1) = 5$.

Exercise 4

Find the function $y = f(x)$ that satisfies the IVP

$$y' = (2 + 5x)e^{\frac{1}{3}x}, \ \ y(0) = 5.$$

Exercise 5

A ball of mass 1 kg is thrown upward from the ground with a velocity of $v = 30\text{m/s}$ on Mars, where the acceleration of the gravity is $g = -3.7\text{m/s}^2$. How much longer is the ball in the air on Mars than on Earth, where $g = -9.8\text{m/s}^2$?

Exercise 6

Use separation of variables to find the general solutions (implicit if necessary, explicit if it can be done).

1. $y' = \dfrac{1+\sqrt{x}}{1+\sqrt{y}}$

2. $y' = \dfrac{(x-1)y^5}{x^2(2y^3 - y)}$

3. $y' = 1 + x + y + xy$

Exercise 7

Determine if the following IVP has a unique solution:

1. $y' = \sqrt[3]{y}, y(0) = 1$

2. $y' = \sqrt[3]{y}, y(0) = 0$

3. $y' = \ln(1 + y^2), y(0) = 0$