In [15]:
```
import pandas as pd
from fredapi import Fred
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
import yfinance as yf
import math
from sklearn.decomposition import PCA
from numpy import linalg as LA
sns.set()
```

In [3]:
```
fred = Fred(api_key='5079f41d061a4037d81f3da69e018803')
series_ids = ['DGS6MO', 'DGS1', 'DGS3', 'DGS5', \
              'DGS10', 'DGS20', 'DGS30']
```

In [4]:
```
def get_yield_data(series_id):
    data = fred.get_series(series_id, observation_start="2020-01-01", obs
    return data

yields_d = {series_id: get_yield_data(series_id) for series_id in series_
yields = pd.DataFrame(yields_d)
yields.columns = ['6 Month', '1 Year', '3 Year', '5 Year', \
                  '10 Year', '20 Year', '30 Year']
yields.index = pd.to_datetime(yields.index)
yields = yields.dropna()
yields
```

Out[4]:

	6 Month	1 Year	3 Year	5 Year	10 Year	20 Year	30 Year
2020-01-02	1.57	1.56	1.59	1.67	1.88	2.19	2.33
2020-01-03	1.55	1.55	1.54	1.59	1.80	2.11	2.26
2020-01-06	1.56	1.54	1.56	1.61	1.81	2.13	2.28
2020-01-07	1.56	1.53	1.55	1.62	1.83	2.16	2.31
2020-01-08	1.56	1.55	1.61	1.67	1.87	2.21	2.35
...	...	...	...	...	...	...	...
2024-04-29	5.43	5.20	4.80	4.65	4.63	4.86	4.75
2024-04-30	5.44	5.25	4.87	4.72	4.69	4.90	4.79
2024-05-01	5.43	5.21	4.79	4.64	4.63	4.85	4.74
2024-05-02	5.42	5.16	4.71	4.57	4.58	4.82	4.72
2024-05-03	5.41	5.12	4.63	4.48	4.50	4.75	4.66

1087 rows × 7 columns

In [5]:
```
pip install nelson_siegel_svensson
```

```
Requirement already satisfied: nelson_siegel_svensson in /usr/local/lib/py
thon3.11/site-packages (0.5.0)
Requirement already satisfied: Click>=8.0 in /usr/local/lib/python3.11/sit
e-packages (from nelson_siegel_svensson) (8.1.8)
Requirement already satisfied: numpy>=1.22 in /usr/local/lib/python3.11/si
te-packages (from nelson_siegel_svensson) (1.26.2)
Requirement already satisfied: scipy>=1.7 in /usr/local/lib/python3.11/sit
e-packages (from nelson_siegel_svensson) (1.11.4)
Requirement already satisfied: matplotlib>=3.5 in /usr/local/lib/python3.1
1/site-packages (from nelson_siegel_svensson) (3.8.2)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.
11/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (1.2.0)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.11/s
ite-packages (from matplotlib>=3.5->nelson_siegel_svensson) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python
3.11/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (4.46.0)
Requirement already satisfied: kiwisolver>=1.3.1 in /usr/local/lib/python
3.11/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (1.4.5)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.1
1/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (23.1)
Requirement already satisfied: pillow>=8 in /usr/local/lib/python3.11/site
-packages (from matplotlib>=3.5->nelson_siegel_svensson) (9.3.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.
11/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (3.1.1)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/pyth
on3.11/site-packages (from matplotlib>=3.5->nelson_siegel_svensson) (2.8.
2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/site-
packages (from python-dateutil>=2.7->matplotlib>=3.5->nelson_siegel_svenss
on) (1.16.0)
WARNING: Running pip as the 'root' user can result in broken permissions a
nd conflicting behaviour with the system package manager. It is recommende
d to use a virtual environment instead: https://pip.pypa.io/warnings/venv

[notice] A new release of pip available: 22.3.1 -> 24.3.1
[notice] To update, run: pip install --upgrade pip
Note: you may need to restart the kernel to use updated packages.
```

In [6]:

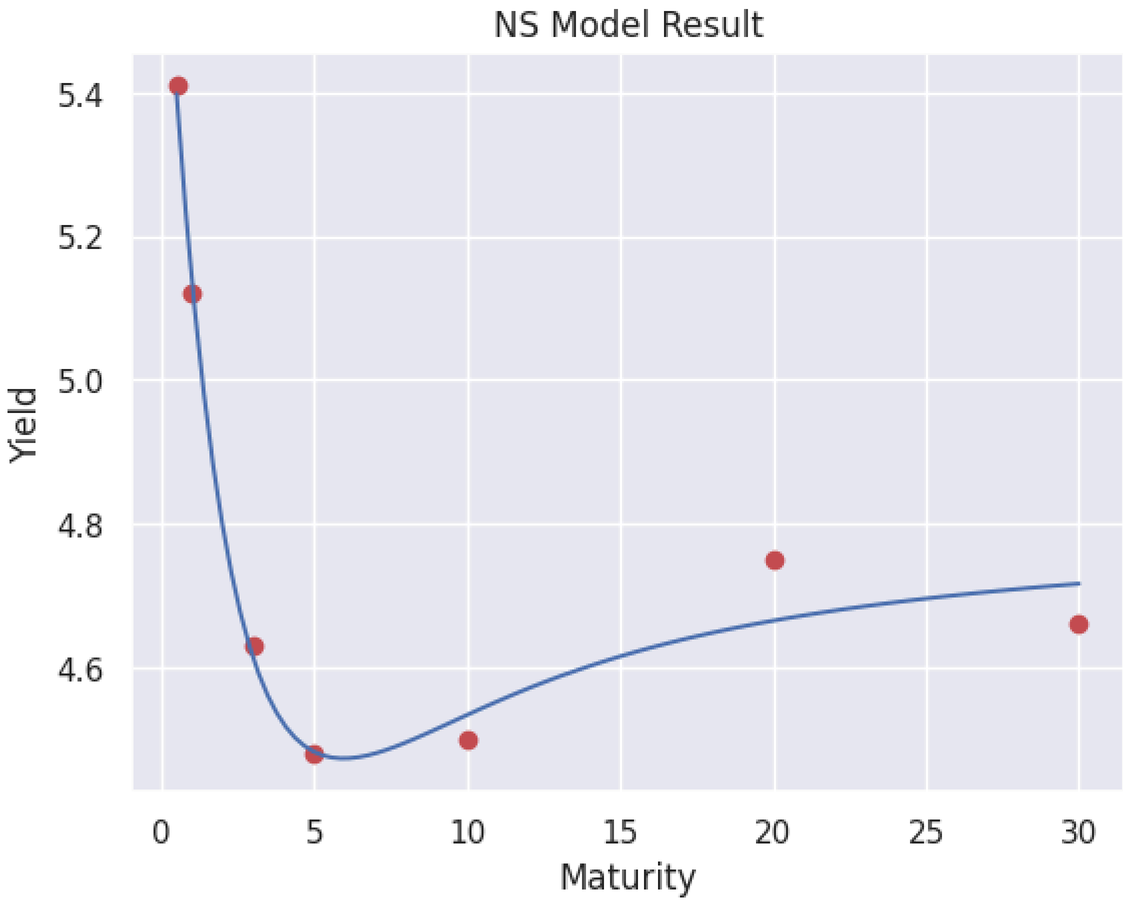
```python
from nelson_siegel_svensson.calibrate import calibrate_ns_ols
t = np.array([0.5,1,3,5,10,20,30])
y = np.array(yields.loc["2024-05-03"])
```

In [7]:

```python
curve, status = calibrate_ns_ols(t, y, tau0=1.0)
assert status.success
y_est = curve
t_est = np.linspace(0.5,30,100)
plt.plot(t, y, 'ro')
plt.plot(t_est, y_est(t_est))
plt.xlabel("Maturity")
plt.ylabel("Yield")
plt.title("NS Model Result")
curve
```

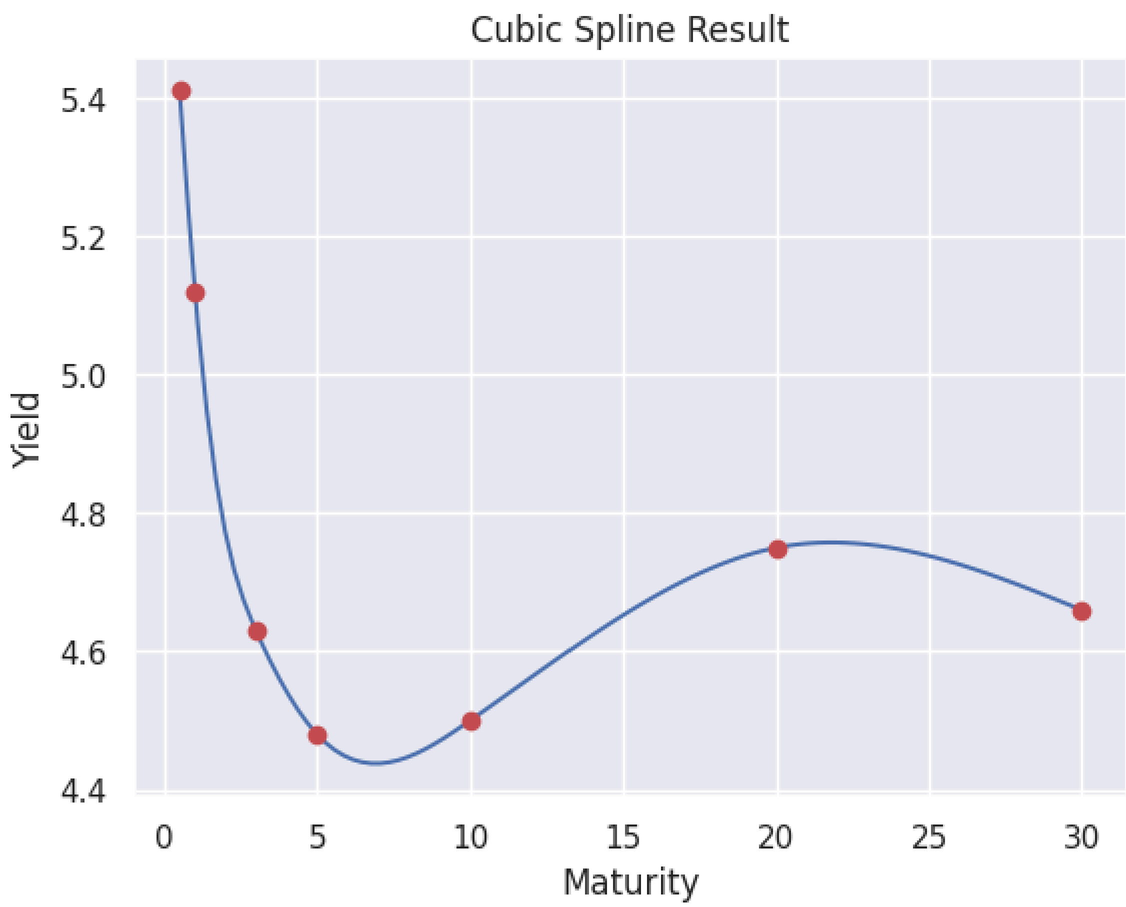
Out[7]:

```
NelsonSiegelCurve(beta0=4.820017888453549, beta1=0.9216139083815179, bet
a2=-2.389113874880242, tau=2.1135354375132294)
```



In [8]:
```
from scipy.interpolate import CubicSpline
t = np.array([0.5,1,3,5,10,20,30])
y = np.array(yields.loc["2024-05-03"])
f = CubicSpline(t, y, bc_type='natural')
t_new = np.linspace(0.5,30,100)
y_new = f(t_new)
plt.plot(t_new, y_new)
plt.plot(t, y, 'ro')
plt.xlabel("Maturity")
plt.ylabel("Yield")
plt.title("Cubic Spline Result")
```

Out[8]:
```
Text(0.5, 1.0, 'Cubic Spline Result')
```



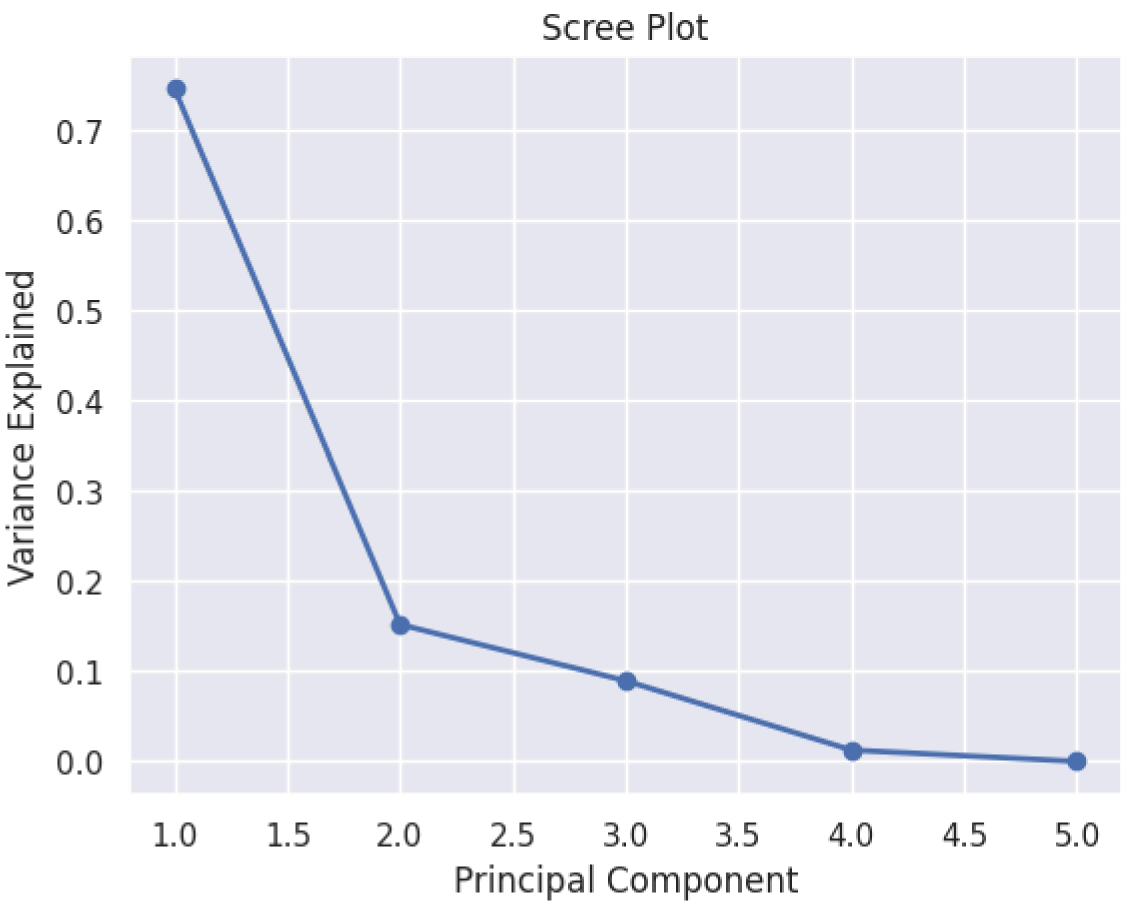
In [9]:
```
# The following is for Task 3
mean = 0
std_dev = 0.01
num_variables = 5  # Number of random variables
num_samples = 1  # Single sample of each

np.random.seed(42)
random_variables = np.random.normal(mean, std_dev, (num_samples, num_vari

print(random_variables)
```

```
[[ 0.00496714 -0.00138264  0.00647689  0.0152303  -0.00234153]]
```

In [10]:
```
num_samples = 10  # Expand to 10 samples
random_variables_multiple = random_variables + np.random.normal(0, 0.0001
cov_matrix = np.cov(random_variables_multiple, rowvar=False)
pca = PCA()
pca.fit(cov_matrix)
principal_components = pca.components_
explained_variance = pca.explained_variance_ratio_

print("\nPrincipal Components (Eigenvectors):")
print(principal_components)
print("\nExplained Variance Ratios:")
print(explained_variance)
```

```
Principal Components (Eigenvectors):
[[-0.20194479  0.52911366 -0.41790326 -0.08770455  0.70492681]
 [-0.34440756  0.63113918  0.44985046 -0.39355854 -0.3546731 ]
 [ 0.69421065  0.05722716 -0.18585788 -0.6918283  -0.0403369 ]
 [-0.22622202 -0.4614553   0.51389309 -0.43425683  0.53218183]
 [ 0.55452403  0.32478627  0.56952802  0.41257023  0.30404048]]

Explained Variance Ratios:
[7.46026249e-01 1.51998062e-01 8.93912327e-02 1.25844556e-02
 5.16402205e-34]
```

In [11]:
```
PC_values = np.arange(pca.n_components_) + 1
plt.plot(PC_values, pca.explained_variance_ratio_, 'o-', linewidth=2)
plt.title('Scree Plot')
plt.xlabel('Principal Component')
plt.ylabel('Variance Explained')
plt.show()
```



In [12]:
```
yields = yields[["6 Month", "1 Year", "3 Year", "5 Year", "10 Year"]]
yields = yields.dropna()
daily_yields = yields.pct_change()
daily_yields_filtered = daily_yields[daily_yields.index >= '2023-12-03']
daily_yields_filtered
```

Out[12]:

	6 Month	1 Year	3 Year	5 Year	10 Year
2023-12-04	0.015009	0.009901	0.020882	0.021739	0.014218
2023-12-05	-0.007394	-0.007843	-0.015909	-0.021277	-0.023364
2023-12-06	0.001862	0.001976	0.000000	-0.004831	-0.014354
2023-12-07	-0.003717	-0.003945	-0.004619	-0.002427	0.004854
2023-12-08	0.005597	0.015842	0.032483	0.031630	0.021739
...	...	...	...	...	...
2024-04-29	0.005556	-0.001919	-0.008264	-0.006410	-0.008565
2024-04-30	0.001842	0.009615	0.014583	0.015054	0.012959
2024-05-01	-0.001838	-0.007619	-0.016427	-0.016949	-0.012793
2024-05-02	-0.001842	-0.009597	-0.016701	-0.015086	-0.010799
2024-05-03	-0.001845	-0.007752	-0.016985	-0.019694	-0.017467

105 rows × 5 columns

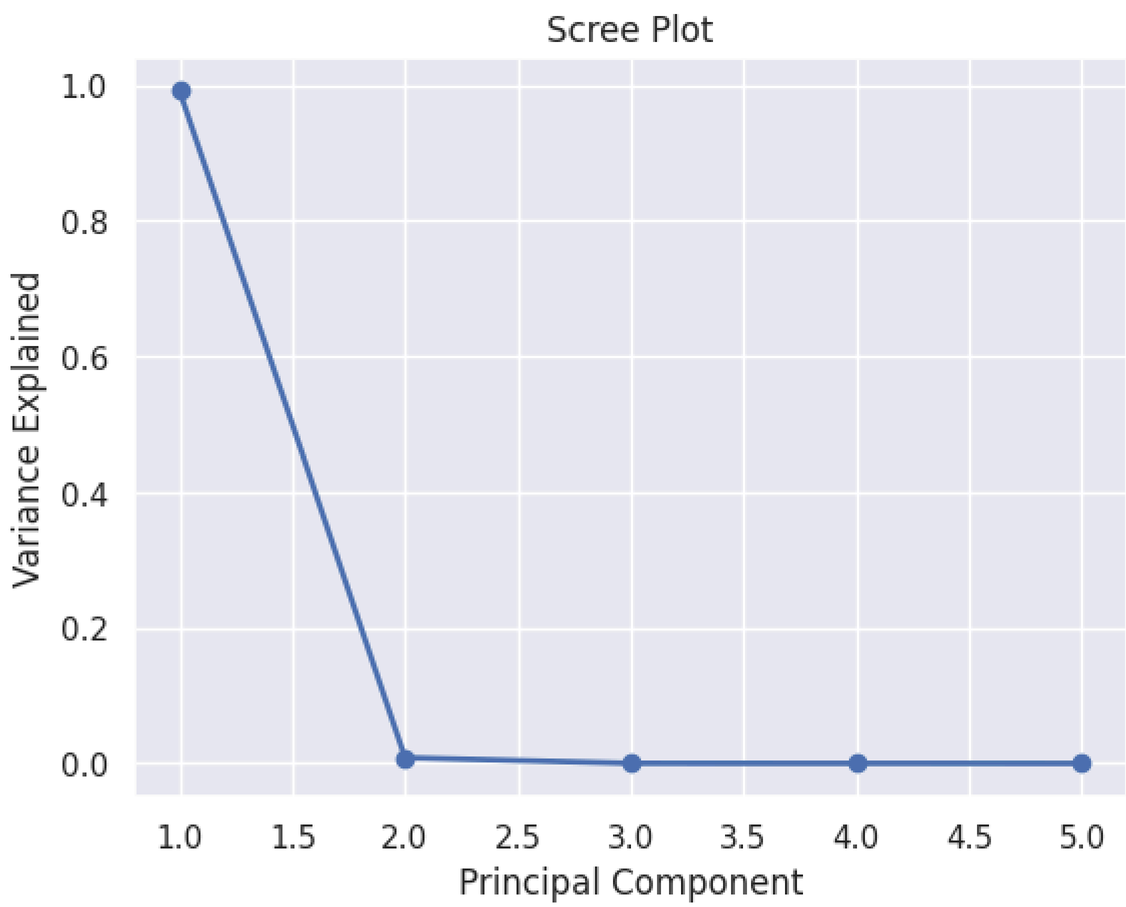
In [13]:

```
covariance_matrix = daily_yields_filtered.cov()
covariance_matrix
pca = PCA()
pca.fit(covariance_matrix)
principal_components = pca.components_
explained_variance = pca.explained_variance_ratio_

print("\nPrincipal Components (Eigenvectors):")
print(principal_components)
print("\nExplained Variance Ratios:")
print(explained_variance)
```

```
Principal Components (Eigenvectors):
[[-0.07930553 -0.26028494 -0.55415933 -0.59211031 -0.5179529 ]
 [-0.20249222 -0.57657443 -0.39882405  0.05788     0.68128417]
 [-0.48530221  0.72552298 -0.43448251 -0.03861165  0.21870586]
 [ 0.08136083 -0.07087925 -0.46972889  0.79048864 -0.37794051]
 [ 0.84295297  0.26154582 -0.35274146 -0.14032871  0.2773182 ]]

Explained Variance Ratios:
[9.91303253e-01 8.47130057e-03 1.55187392e-04 7.02589226e-05
 7.68298739e-35]
```

In [14]:

```
PC_values = np.arange(pca.n_components_) + 1
plt.plot(PC_values, pca.explained_variance_ratio_, 'o-', linewidth=2)
plt.title('Scree Plot')
plt.xlabel('Principal Component')
plt.ylabel('Variance Explained')
plt.show()
```



In [5]:
```
# The following is for Task 4
tickers = [
    "AAPL", "MSFT", "NVDA", "AVGO", "ORCL", "TSM", "ADBE", "CSCO", "CRM",
    "TXN", "QCOM", "ACN", "AMD", "IBM", "AMAT", "NOW", "ADI", "NXPI", "LR
    "PAYC", "SNPS", "FTNT", "PANW", "CDNS", "ANSS", "KEYS", "MU", "MPWR",
]
end_date = pd.Timestamp.today().strftime('%Y-%m-%d')
start_date = (pd.Timestamp.today() - pd.DateOffset(months=6)).strftime('%
data = yf.download(tickers, start=start_date, end=end_date, interval="1d"
closing_prices = {ticker: data[ticker]['Close'] for ticker in tickers}
closing_prices_df = pd.DataFrame(closing_prices)
closing_prices_df
```

```
[*********************100%***********************]  30 of 30 completed
```

Out[5]:

Date	AAPL	MSFT	NVDA	AVGO	ORCL	TSM
2024-07-08	227.306534	465.401154	128.180191	173.548660	144.297211	185.355087
2024-07-09	228.164581	458.713226	131.359726	172.301117	139.969177	183.259521
2024-07-10	232.454895	465.411133	134.889175	173.432343	141.352173	189.744904
2024-07-11	227.057098	453.881927	127.380325	169.581375	142.439713	183.239655
2024-07-12	230.020386	452.733978	129.220047	169.056519	144.445206	186.070190
...	...	...	...	...	...	...
2024-12-30	252.199997	424.829987	137.490005	235.580002	166.910004	200.389999
2024-12-31	250.419998	421.500000	134.289993	231.839996	166.639999	197.490005
2025-01-02	243.850006	418.579987	138.309998	231.979996	166.029999	201.580002
2025-01-03	243.360001	423.350006	144.470001	232.550003	166.320007	208.610001
2025-01-06	245.000000	427.850006	149.429993	236.410004	165.690002	220.009995

127 rows × 30 columns

In [20]:

```
daily_returns = closing_prices_df.pct_change().dropna()
daily_returns
```

Out[20]:

Date	AAPL	MSFT	NVDA	AVGO	ORCL	TSM	AD
2024-07-09	0.003775	-0.014370	0.024805	-0.007188	-0.029994	-0.011306	-0.016
2024-07-10	0.018804	0.014602	0.026869	0.006565	0.009881	0.035389	-0.002
2024-07-11	-0.023221	-0.024772	-0.055667	-0.022204	0.007694	-0.034284	-0.012
2024-07-12	0.013051	-0.002529	0.014443	-0.003095	0.014080	0.015447	0.002
2024-07-15	0.016743	0.000904	-0.006190	0.007956	-0.011743	-0.011369	0.011
...	...	...	...	...	...	...	
2024-12-30	-0.013263	-0.013240	0.003503	-0.025522	-0.012133	-0.006150	-0.001
2024-12-31	-0.007058	-0.007838	-0.023275	-0.015876	-0.001618	-0.014472	-0.002
2025-01-02	-0.026236	-0.006928	0.029935	0.000604	-0.003661	0.020710	-0.008
2025-01-03	-0.002009	0.011396	0.044538	0.002457	0.001747	0.034874	-0.023
2025-01-06	0.006739	0.010630	0.034332	0.016599	-0.003788	0.054647	0.001

126 rows × 30 columns

In [21]:

```
daily_returns_means = daily_returns.mean()
daily_returns_stds = daily_returns.std()
standardized_returns = (daily_returns - daily_returns_means) / daily_retu

standardized_returns_dvd_sqrt_n=(standardized_returns/math.sqrt(len(stand
standardized_returns_cov = standardized_returns_dvd_sqrt_n.T@standardized
standardized_returns_cov
```

Out[21]:

	AAPL	MSFT	NVDA	AVGO	ORCL	TSM	ADBE
AAPL	1.000000	0.560328	0.398547	0.367579	0.243876	0.359072	0.324499
MSFT	0.560328	1.000000	0.496958	0.439703	0.540636	0.418683	0.390648
NVDA	0.398547	0.496958	1.000000	0.551871	0.427426	0.702929	0.329807
AVGO	0.367579	0.439703	0.551871	1.000000	0.379655	0.646363	0.249623
ORCL	0.243876	0.540636	0.427426	0.379655	1.000000	0.300212	0.352013
TSM	0.359072	0.418683	0.702929	0.646363	0.300212	1.000000	0.204235
ADBE	0.324499	0.390648	0.329807	0.249623	0.352013	0.204235	1.000000
CSCO	0.246310	0.355489	0.296197	0.223837	0.362673	0.186361	0.303947
CRM	0.282489	0.386166	0.412004	0.309744	0.454833	0.320130	0.422898
INTC	0.229316	0.430174	0.426441	0.355079	0.379559	0.403216	0.206875
TXN	0.333014	0.418606	0.494471	0.401914	0.293677	0.493971	0.256041
QCOM	0.491205	0.519853	0.712643	0.612914	0.353227	0.657477	0.285559
ACN	0.214552	0.224685	0.179476	0.123968	0.349084	0.095089	0.288978
AMD	0.406846	0.438485	0.631988	0.511143	0.358100	0.613231	0.188798
IBM	0.194722	0.226739	0.242450	0.208581	0.331137	0.240670	0.203223
AMAT	0.433198	0.486339	0.692181	0.579698	0.338287	0.707713	0.300493
NOW	0.309432	0.360331	0.358643	0.282424	0.412972	0.299388	0.401788
ADI	0.455914	0.524606	0.608762	0.505576	0.349738	0.603887	0.326727
NXPI	0.419637	0.458670	0.565787	0.492073	0.246930	0.556857	0.256912
LRCX	0.385135	0.482235	0.650199	0.605076	0.307810	0.683399	0.306996
PAYC	0.126635	-0.024213	0.047402	0.093969	0.102359	0.071797	0.210988
SNPS	0.411668	0.558834	0.611778	0.525543	0.513622	0.536392	0.359240
FTNT	0.192342	0.143533	0.013953	-0.011697	0.152683	0.026247	0.143837
PANW	0.402794	0.502343	0.405113	0.376971	0.466895	0.300622	0.386327
CDNS	0.381076	0.573280	0.551784	0.534155	0.484256	0.537173	0.378000
ANSS	0.475376	0.524563	0.593905	0.489901	0.482232	0.560070	0.424442
KEYS	0.365399	0.361604	0.424112	0.374049	0.411386	0.391983	0.287321
MU	0.306204	0.396046	0.528773	0.572495	0.286053	0.587497	0.241545
MPWR	0.422209	0.519900	0.649121	0.611252	0.340308	0.637320	0.260137
KLAC	0.403887	0.454596	0.687008	0.605885	0.321045	0.701669	0.276163

30 rows × 30 columns

In [13]:

```
pca = PCA()
pca.fit(standardized_returns_cov)
```

```
principal_components = pca.components_
explained_variance = pca.explained_variance_ratio_

print("\nPrincipal Components (Eigenvectors):")
print(principal_components)

print("\nExplained Variance Ratios:")
print(explained_variance)
```

```
Principal Components (Eigenvectors):
[[-7.54532350e-02 -1.09479628e-01 -2.09009390e-01 -1.88298810e-01
  -1.34183275e-02 -2.31012212e-01  1.70613508e-02  3.78098113e-02
   7.40922630e-03 -1.35096783e-01 -2.15093951e-01 -2.63182962e-01
   1.08486559e-01 -2.25816523e-01  8.74494630e-02 -2.79084789e-01
   2.36829663e-02 -2.44013101e-01 -2.71488208e-01 -2.95084329e-01
   9.59831545e-02 -1.96404025e-01  1.39996509e-01  3.35002295e-02
  -2.02994753e-01 -1.60585335e-01 -1.07922170e-01 -2.14961349e-01
  -2.67097925e-01 -2.83184623e-01]
 [-9.92030568e-02 -2.90914767e-01 -1.09056452e-01 -4.80525395e-02
  -3.66891556e-01  2.62997992e-02 -2.57301817e-01 -3.66377141e-02
  -3.32049374e-01 -5.69824669e-03  1.72724978e-01  6.41033336e-02
  -7.45289955e-02  2.65052320e-02 -7.24724337e-03  3.65018367e-02
  -3.45521852e-01  1.19866398e-01  1.59042314e-01  4.85706001e-02
   1.23405922e-01 -2.82808956e-01 -2.85969921e-02 -3.23669842e-01
  -2.90233239e-01 -2.81643755e-01  1.34970626e-02  7.40838951e-02
   4.86493214e-02  5.30122914e-02]
 [ 1.62328845e-01  6.04800543e-02  9.10916648e-02  1.87816108e-01
  -1.03467534e-01  1.76493986e-01  7.37708877e-03 -3.97075571e-01
  -5.57521712e-02 -2.92275115e-01 -2.78866454e-01  7.42088658e-02
  -3.66421481e-01  2.88941757e-02 -3.54565142e-01  3.42254837e-02
  -5.28032441e-02 -2.21340890e-01 -1.72691380e-01  8.59080637e-02
  -1.68585030e-01 -6.25443642e-02  2.40082790e-01  2.26859589e-02
  -2.13193063e-02 -1.00588139e-01 -3.08074347e-01  8.41298412e-02
  -1.09474358e-02  2.99396483e-02]
 [-2.41669248e-01 -3.62284663e-01 -2.32321115e-02  3.86708986e-02
  -1.31948053e-01  7.37420728e-02  3.09651990e-02 -2.87194439e-01
   1.11471703e-01 -3.54998400e-02 -9.58759816e-02 -1.73329179e-02
  -1.22211926e-01 -3.99094938e-02  6.48820702e-02  1.45655173e-01
   2.69605285e-01 -1.13824771e-01 -1.07926994e-01  1.37056878e-01
   6.04066582e-01  1.11885392e-01 -2.49004499e-01  2.56836853e-02
   1.16859179e-01  1.20509330e-01  2.72538741e-02  4.12367575e-02
  -1.76895279e-01  1.37197587e-01]
 [-5.48455289e-02 -6.03610015e-02 -2.29742303e-01 -3.61720777e-01
  -1.63305124e-01 -2.52858318e-01  5.07455726e-02  2.27282466e-02
   2.51513611e-01  2.07899875e-01  1.93061591e-01 -5.61644648e-02
  -2.78845787e-01 -4.72661847e-02 -2.93457957e-01  9.36418347e-02
  -3.10743599e-02  9.98025604e-02  1.11310598e-01  1.31668610e-01
  -1.63603115e-02  1.57830056e-01  5.04258632e-01  8.09607992e-03
   9.81377893e-02  9.06203625e-02  6.06134113e-02  2.51856852e-02
  -2.16251876e-01  7.17912776e-02]
 [ 4.80595368e-01  1.79764433e-01 -1.21584162e-01 -1.32098857e-01
  -1.93988475e-01 -2.17642763e-01  4.60656306e-01 -1.02465216e-01
  -2.04658442e-01 -1.62828992e-01  8.48102498e-02  6.69461729e-02
   1.57723024e-02 -1.59199680e-01 -2.31511634e-01 -2.88185980e-02
   2.45821324e-02  1.20907215e-01  1.37070643e-01 -2.01144963e-02
   2.43886866e-01 -2.47814144e-02 -2.80211294e-01 -5.96608936e-02
   7.25156217e-02  3.62655648e-02  1.00034200e-02 -2.09929047e-01
   8.74132089e-02 -4.13381141e-02]
 [-2.95245640e-01  1.09552843e-01 -5.41065408e-02  1.95006475e-01
   2.44768606e-01 -5.20197009e-02  1.87944761e-01  2.49009739e-01
  -1.34270096e-01  3.00405791e-01 -3.60121268e-02 -1.00509178e-01
  -3.41633692e-01 -2.32400885e-01 -3.51870562e-01 -1.12132878e-01
  -2.15556060e-01 -7.18694534e-02 -1.01984809e-01  6.18572881e-03
   6.04992386e-02  1.37901062e-02 -3.13096179e-01 -8.49904435e-02
   8.44413061e-02 -1.09732091e-01  1.33659149e-02  2.52546230e-01
  -2.90353602e-02 -1.34165450e-01]
 [-2.34942288e-01 -1.97967674e-01  1.54257003e-01 -2.45265748e-02
  -1.13566665e-01  8.45786761e-02  4.55050265e-01  2.26637427e-01
   3.61125249e-01 -2.85873668e-01  8.73091512e-02 -2.74566817e-02
```

```
 -9.70051490e-02 -2.14773456e-03 -2.33004535e-02 -6.50159058e-02
  3.15329318e-01  5.98301493e-02  4.91834889e-02 -2.18193267e-02
 -2.15266788e-01 -2.09010296e-01 -1.84378863e-02 -3.23615594e-01
 -1.21794178e-01 -7.74797722e-02 -3.21161683e-02  6.18939948e-02
  1.97782441e-01  4.70618674e-02]
[-3.71956111e-01 -1.34161482e-01 -5.37584352e-02 -9.76988973e-02
  2.26974871e-01 -6.83307537e-02  2.25769434e-01 -3.99647803e-01
 -2.04858046e-01 -5.56532202e-02 -6.58716912e-02 -4.24906310e-02
  4.31963745e-01 -8.15976045e-02 -1.26026183e-01  1.09847675e-01
 -2.13368385e-01 -2.63840576e-02  2.57050685e-02  7.45198247e-02
 -2.49719903e-01  1.56115521e-01  4.04261646e-02 -2.97093512e-01
  1.50802153e-01  1.11308559e-01 -2.09588758e-02 -1.45219140e-01
  2.21217044e-02  7.55886487e-02]
[ 2.55810272e-01 -1.48252787e-01 -1.69467118e-01  1.26518307e-01
 -3.00953023e-03 -3.69253454e-02 -7.81762856e-02  6.79507969e-02
  8.54724022e-02 -3.47580201e-01 -1.70604541e-01 -1.75813147e-01
  3.97445439e-03  4.46294506e-02 -1.99090688e-02 -2.57464475e-02
 -1.89926606e-01 -9.73282039e-02 -8.03270097e-02 -5.07138249e-02
 -3.16399759e-02  1.22833642e-01  3.78878475e-02 -2.26598364e-01
  8.58391205e-02  2.05184425e-01  6.14672448e-01  3.25995459e-01
  2.87885637e-02 -8.19790078e-02]
[-1.99587778e-02  9.15873477e-02 -1.22277826e-01 -6.84541744e-02
  3.66083818e-01 -7.58598618e-02 -4.68409587e-01 -1.37531662e-01
  2.02431054e-02 -9.85900671e-02  1.07516762e-01  1.41014533e-02
 -2.61274787e-01 -1.12885682e-02 -1.73887202e-01 -1.12586614e-01
  3.54624026e-01  5.42901913e-02  2.26946000e-02 -1.13554867e-01
  3.85440755e-02 -2.04042697e-02 -6.24498560e-02 -3.45734186e-01
  1.06530773e-01 -8.72405980e-02  1.12644773e-01 -3.35837111e-01
  2.16659780e-01  3.82358097e-02]
[-1.22288983e-01 -2.91639153e-02  1.67329284e-01 -1.23254396e-01
  3.96480554e-01 -1.84849943e-01  1.35570589e-01 -3.20914740e-02
  1.50840093e-01 -3.02782140e-01  9.54790969e-02  2.36648694e-01
 -2.03802302e-02  4.49207120e-01 -1.15438983e-01 -1.13863004e-02
 -3.04936683e-01  3.61374243e-02  3.42719638e-02  3.96586593e-03
  2.86237318e-01 -9.93226951e-02  3.74111388e-02  2.45970330e-01
 -6.99641861e-02 -2.37114509e-01  1.25911709e-01 -2.66709543e-02
 -4.19591049e-02 -7.99838177e-02]
[-1.87369030e-01 -9.10898921e-02 -2.14751562e-01  3.06927468e-01
 -1.90014573e-01 -5.52798866e-02 -1.18793293e-01  2.34046876e-01
  5.89787310e-02 -3.73111566e-01  1.36589478e-01 -2.89910592e-02
 -2.30191465e-02 -2.35880480e-02  4.18221668e-02 -1.29954040e-01
 -2.29212262e-01  8.03915159e-02  1.33814845e-01 -1.09543533e-01
  6.52963626e-02  2.24747218e-01  4.13653709e-02  1.32241679e-01
  4.22201456e-01  5.41057786e-02 -3.55679715e-01 -1.60562085e-01
  1.62951569e-01 -5.83623363e-02]
[ 1.43065262e-01 -1.37482978e-01  4.13755467e-01 -2.91010638e-01
 -1.08991080e-01  1.99846723e-01 -1.32339677e-01  2.96775843e-01
  9.39048556e-02  9.09021319e-02 -1.91289254e-01  1.48457074e-02
  2.29815369e-02  1.94923589e-01 -2.38809334e-01  1.73091584e-02
 -2.11804035e-01 -8.87724684e-02 -5.56819986e-02 -1.36280406e-01
  4.91088472e-02  1.51099662e-01 -1.61555170e-01 -3.04794474e-01
  1.25667981e-01  1.69406844e-01 -1.49190970e-01 -2.66939010e-01
 -1.91211309e-01 -6.61229045e-02]
[-3.55716962e-02  1.89404219e-01  1.50309489e-01 -2.46077824e-01
  1.70376127e-01  2.01028522e-01  1.72622439e-01  7.83967284e-02
 -2.58822113e-01 -2.03513570e-01  4.41622624e-02 -1.18979715e-01
 -4.12381404e-01 -2.03620964e-01  5.13935215e-01  1.19038447e-01
 -1.99979196e-01  4.27512594e-02 -1.06702276e-01  3.91528028e-02
  1.08609995e-01  6.70032645e-02  1.89204059e-01 -1.18906059e-01
  6.87302473e-02  6.19362165e-02  3.52290674e-02 -1.13037505e-01
```

```
 -1.18023094e-02  1.27741676e-01]
[-2.37832835e-02 -3.09169688e-01  3.04182159e-01  2.21160987e-01
 -3.43295625e-02  2.48145297e-01  6.83416690e-02 -3.50121135e-02
 -1.19185703e-01  9.37360744e-02  1.55363159e-01  2.52020141e-03
 -1.37376900e-02 -2.97115547e-01 -2.35394028e-01 -1.35462856e-01
 -2.28462806e-02  9.85802053e-02 -4.39100804e-02 -1.44238061e-01
  3.13374822e-02  1.53564306e-02  2.40845903e-01  3.04952962e-01
 -7.88214215e-02  4.74363714e-02  3.81012082e-01 -3.53823512e-01
  1.37396155e-01 -3.87317345e-02]
[ 2.30149911e-01 -4.58059547e-02 -1.98777009e-01  5.17768963e-01
  2.09619289e-01  3.30678003e-02  1.54571215e-01  7.95137150e-03
  3.13978698e-01  1.88610263e-01 -2.09712704e-02  1.03545646e-01
 -1.65103424e-02 -2.22649627e-02  1.47590605e-01  1.01016489e-01
 -1.90788481e-01  5.09611169e-02  5.96444761e-02  4.08708045e-02
  8.02713898e-02 -8.42731514e-02  1.08834755e-01 -3.05223810e-01
 -1.38335235e-01 -2.43463358e-02 -5.92727187e-02 -3.19687503e-01
 -2.93408546e-01  1.26350789e-01]
[-7.07078660e-02  2.48602431e-01 -7.96706747e-02 -1.35307967e-01
 -4.27812951e-02  4.28590364e-01 -1.20706661e-02 -4.17046157e-01
  3.70361735e-01  4.91491735e-02  1.91206046e-01 -8.95426947e-02
 -1.93441700e-03 -6.57414045e-02  2.69349057e-02 -2.43459648e-01
 -2.24811754e-01  1.54092654e-01  1.30693894e-01 -2.22253102e-01
  8.62231392e-02 -4.37312699e-02 -1.03317638e-01 -4.07508902e-02
 -6.93168747e-02  1.72240175e-01 -3.76526700e-02  1.40482606e-01
 -3.41124213e-03 -2.87024571e-01]
[-8.65149682e-02 -8.87183192e-02 -2.75052200e-01  2.76389013e-02
 -1.45590237e-01  1.22578446e-01  2.34424012e-01 -1.02155493e-01
 -2.14666086e-01  2.00181198e-01 -8.37372573e-02 -1.57386139e-01
 -2.85566458e-01  6.02471656e-01  1.27063357e-01 -1.26061996e-01
  6.99614605e-02 -1.11241323e-02  2.41396632e-02 -1.06210571e-01
 -2.33168651e-01 -1.55978540e-02 -1.08606431e-01  9.04721890e-02
  1.18201452e-01  1.60534169e-02  1.64823285e-01 -2.62149274e-01
 -4.38062708e-02 -9.27920632e-02]
[-2.56654830e-01  5.22777297e-01 -2.39849960e-02  4.21945101e-02
 -3.45876343e-01  1.44254821e-01 -3.85623372e-02  7.38918221e-02
  1.67221538e-01 -5.56861541e-02 -1.46587590e-01 -7.13258892e-02
  1.84500626e-01 -8.83656174e-03 -1.07457475e-01  1.54463792e-01
 -5.10892323e-02 -5.67792302e-02 -1.13828056e-01  1.46104028e-01
  8.39683735e-02 -9.34701911e-02  1.08887315e-02 -8.80336960e-03
  1.59015994e-01 -3.40884965e-01  3.29614393e-01 -2.44344383e-01
 -3.07946768e-02  1.30453586e-01]
[ 7.88522555e-02 -3.08047209e-02 -3.10516744e-01 -5.41922027e-02
  2.38948336e-01  4.81312029e-01  1.01553538e-03  2.48245505e-01
 -1.98519049e-01 -2.02546843e-01  2.44323994e-01 -2.54366175e-01
  2.02863341e-01  9.80347362e-02 -2.89623536e-01  1.55593097e-01
  1.52747639e-01  8.55058599e-02 -8.52741582e-02  1.24767072e-01
  7.45577529e-02 -1.74516400e-01  4.47601304e-02  8.90321219e-02
 -2.52488913e-02  7.42400656e-02 -1.14023342e-01  4.74959024e-02
 -2.34720713e-01  5.89750036e-02]
[-2.79951916e-01  3.36786974e-01  1.73386221e-01  2.64068077e-01
 -1.28641253e-01 -1.69437858e-01 -1.55090568e-02  5.31771852e-02
 -2.67153792e-01 -1.38069897e-01  7.64379871e-02  1.19559726e-01
  1.36073793e-04  1.54156658e-01 -7.72861015e-02 -5.17525365e-02
  1.88155865e-01  4.07673959e-03  1.23044378e-01 -8.73047870e-02
  1.37642757e-01  8.52602561e-03  1.82862131e-01 -1.77851057e-01
 -2.56687883e-01  4.54687852e-01  3.59371097e-02  1.38644149e-02
 -2.75399486e-01 -1.57688820e-01]
[-6.29848185e-02 -4.24063690e-02 -3.06915313e-01 -1.49484258e-01
  3.06738111e-02  2.90257017e-01  7.61861801e-02  1.57219230e-01
 -9.16460383e-02  5.19886374e-02 -2.42758492e-01  7.25396222e-01
```

```
  4.55599380e-02 -1.07797058e-01  4.25424812e-02 -7.12457118e-02
  1.00079545e-01 -1.05601227e-01  5.78332048e-02 -2.62278227e-02
  6.90750859e-02 -2.49934323e-02  1.79478110e-01 -5.86288810e-02
  1.44195078e-01  1.12474239e-03  7.54666767e-02  5.34315657e-02
  5.54866530e-02 -2.15630746e-01]
[ 3.96093170e-02 -8.01213513e-02  2.12633999e-01  3.59517839e-02
  4.97362864e-02  3.16667185e-02 -5.40417070e-02 -7.33057316e-02
 -7.31877828e-02 -1.35870250e-01 -6.44467548e-02 -4.96228953e-02
 -4.25782243e-02 -1.71822959e-01  6.57356990e-02 -2.22058664e-03
  1.48373356e-01  9.89520315e-02  5.25254598e-01  8.77445765e-03
 -1.80822306e-01 -6.12350853e-02 -6.95773179e-02  3.35924209e-02
  3.09767130e-01 -3.02409509e-01  1.02084174e-01  7.73644038e-02
 -5.28634402e-01 -1.84880186e-01]
[ 7.24668498e-02 -1.93754658e-02  7.74542234e-02  4.99670035e-02
 -1.10383586e-01 -6.24860045e-02  1.75794491e-03 -9.27905551e-02
 -3.95966475e-03 -6.75658581e-03  6.27300644e-01  2.55024923e-01
  2.20971858e-02  4.73613361e-03  9.15792937e-02 -2.31803637e-02
  6.47031885e-03 -2.48831913e-01 -4.73638106e-01  2.78314608e-03
 -1.85092792e-01 -4.85664313e-02 -9.48823960e-02 -1.06027101e-01
  2.66647952e-01 -6.58579270e-02  5.89721970e-02  3.37139746e-02
 -2.40238736e-01 -1.10477509e-01]
[ 8.25289961e-02  2.70095939e-02  9.34108629e-02  9.26903961e-02
 -1.46594746e-02  4.43722718e-02  1.21243210e-01 -5.42770739e-02
 -1.30710858e-01  1.24705074e-01 -4.35894111e-02 -6.10743551e-02
  1.55923471e-01  1.65598670e-01 -3.25341973e-03 -1.59278061e-01
  1.52779038e-01  3.92082229e-01 -2.30338029e-01 -3.24204148e-01
  2.26871870e-01  2.91236379e-01  2.83279481e-01 -2.21901555e-01
  1.44850761e-01 -3.68845478e-01 -7.47745603e-02  2.61791245e-01
 -1.77793118e-02  8.63081268e-02]
[-1.92847961e-03  1.37805905e-02 -1.05065877e-01 -1.48136255e-02
 -2.30766201e-02  1.23748244e-01  1.76756445e-02  3.75991309e-02
  1.03137824e-02 -1.29380587e-01  1.28111595e-01  3.13537360e-02
 -3.33797341e-02 -2.09499199e-02 -9.74850826e-06  5.91019307e-02
  6.68987145e-02 -1.33631716e-01  5.80994228e-02  1.37055082e-01
 -9.17148042e-02  7.20169392e-01 -1.32274749e-01 -6.12392525e-03
 -4.57369374e-01 -2.84443160e-01  1.98974104e-02 -9.77379740e-02
 -5.19561189e-02 -1.99658274e-01]
[ 3.99624254e-02 -5.70644432e-02  9.20065123e-02  4.67805452e-02
 -2.75432226e-03 -4.80192813e-02 -3.82752513e-02 -1.30203765e-02
  9.61828646e-03  4.17220485e-02 -9.46511856e-02 -1.03233349e-01
  2.42048763e-02  4.13213253e-02  6.53228233e-02 -7.12777414e-02
  4.01079373e-02  3.40166329e-01 -2.08286055e-01  6.30116220e-01
  5.70968819e-02 -6.51367941e-02  8.78807727e-02 -1.01436775e-01
  7.53570987e-02  2.21858052e-02 -4.69252570e-02 -9.03746255e-02
  1.16397517e-01 -5.78013403e-01]
[ 1.76040570e-02 -1.35231442e-02  2.50486228e-02  7.12299228e-02
  1.37890275e-02 -2.12882594e-02  4.35348344e-02 -2.17156819e-02
  6.41942747e-03  1.05605622e-01  7.80592308e-02 -1.34513858e-01
 -1.13219545e-02  3.27362782e-02  4.49264853e-03  7.08781484e-01
  5.66081092e-02 -1.79433555e-01  9.17338403e-02 -3.26636138e-01
  1.01302825e-01 -6.95830594e-02  1.27726248e-01 -4.86110208e-02
  7.49065148e-02 -7.53440033e-02 -4.03666311e-02  1.88742205e-02
  2.38649366e-01 -4.44194263e-01]
[-1.06719777e-01 -3.24457352e-02 -1.07085050e-01 -3.06844393e-02
 -4.76129193e-02 -8.40349755e-02 -8.08166458e-02 -1.19803289e-02
  5.45148965e-02 -1.50657872e-01 -1.64642098e-01  2.08391986e-01
 -1.23365047e-01 -1.11258779e-01 -3.56515567e-02  3.44531464e-01
 -3.12072531e-02  5.92565217e-01 -3.22519293e-01 -2.38568650e-01
 -2.47644145e-01  1.41902996e-03 -2.47139653e-01  1.40776526e-01
 -1.14997417e-01  9.35634430e-02  3.32214913e-02 -4.41274602e-02
```

```
 -1.86251752e-01 -1.96324657e-02]]

Explained Variance Ratios:
[6.61561661e-01 7.88690210e-02 5.73069803e-02 3.74154448e-02
 2.66998151e-02 2.40502321e-02 1.94563473e-02 1.59664947e-02
 1.31386968e-02 1.02402552e-02 9.11738029e-03 7.75253384e-03
 6.77863479e-03 6.23984044e-03 5.07614808e-03 4.46558192e-03
 3.33196235e-03 2.59584192e-03 2.40459967e-03 2.29067149e-03
 1.89966378e-03 1.00084217e-03 8.62959689e-04 5.70130150e-04
 4.16751353e-04 2.39598500e-04 1.65326675e-04 6.56532645e-05
 2.09316831e-05 2.58280607e-32]
```

In [23]:

```
U_st_return, s_st_return, VT_st_return = np.linalg.svd(standardized_retur
print("U matrix:")
print(U_st_return)
print("\nSingular values (S):")
print(s_st_return)
print("\nVt matrix:")
print(VT_st_return.T)
```

```
U matrix:
[[ 0.02770263  0.10654319 -0.02116893 ...  0.13669214  0.06796821
   0.0305015 ]
 [-0.07870497  0.05296426 -0.00977978 ... -0.02624099 -0.09543401
  -0.30576   ]
 [ 0.10474048 -0.09546045 -0.12562549 ...  0.14859127  0.05186406
   0.0651288 ]
 ...
 [ 0.0082343   0.08497417 -0.00656594 ...  0.89317108 -0.03719237
  -0.06885962]
 [-0.09441086  0.02404942 -0.03429764 ... -0.02847863  0.88706707
  -0.0469891 ]
 [-0.07674075  0.13161157  0.02574944 ... -0.05277643 -0.0222516
   0.69251012]]

Singular values (S):
[3.71995943 1.70637896 1.2306415  1.07043207 1.01858079 0.98274151
 0.95724865 0.87404264 0.86504717 0.79301445 0.77400563 0.73227637
 0.70558309 0.69589064 0.65730011 0.64015118 0.61314037 0.56105951
 0.54828291 0.53275577 0.51894334 0.47170411 0.43122274 0.39580107
 0.36958009 0.34385779 0.27958136 0.23886386 0.21416184 0.15608039]

Vt matrix:
[[-1.48310310e-01 -7.49072060e-02  1.45795230e-01 -2.28376847e-01
   2.72802626e-01 -9.50397224e-02  5.44412145e-01 -2.45363524e-02
  -2.37922403e-01  2.70845810e-01 -2.69316177e-01 -1.52037533e-01
  -5.65399470e-02  2.17234924e-01  4.50495780e-02  7.48937144e-02
   1.77465507e-01 -8.22425862e-02 -6.69547395e-02 -2.74913936e-01
  -1.18427885e-02  1.61206409e-01  2.43954996e-01 -8.50909001e-02
  -7.48481499e-02 -1.16443136e-01  9.57334702e-03  5.89181637e-02
   4.99576244e-04 -3.12670405e-02]
 [-1.78257710e-01 -9.74435360e-02  2.34714659e-01 -3.67420905e-01
  -1.03606197e-01 -3.59333381e-02  1.64621336e-01 -1.39334693e-01
  -2.01596213e-01  1.18293484e-01  1.38647185e-01  6.08141184e-02
  -1.54166779e-01 -5.01732346e-03  1.06740701e-02 -3.39273229e-01
   1.40667380e-01  8.40684170e-02  2.96787901e-01  4.95399529e-01
   1.04474473e-01 -6.97464766e-02 -3.26800293e-01 -2.66879945e-02
   6.59788867e-02  2.26731092e-02  2.62812864e-02 -2.93006765e-02
  -5.21705506e-02 -2.72844140e-02]
 [-2.06758318e-01  5.91166283e-02  1.36396592e-01 -2.13106618e-02
   2.93086120e-02 -2.43613662e-01 -1.01775820e-01 -2.91165476e-03
   1.53802936e-01  5.28943644e-03  1.75685720e-01 -5.38443297e-03
   2.13744371e-01  4.14689240e-01  2.84543974e-01  1.32974432e-01
  -2.89714068e-01  1.95765749e-01  6.02149238e-03 -1.04607976e-01
   3.84012380e-01 -2.95516098e-03 -2.48611163e-01 -1.66711989e-01
  -2.78495019e-01 -1.69593791e-01 -9.34354453e-02  7.25575780e-02
   4.55105074e-02 -2.33633416e-02]
 [-1.82948236e-01  7.90598527e-02  1.60846539e-01  4.53678116e-02
   5.15344075e-02 -3.71228547e-01 -1.22233579e-01 -2.94666488e-01
  -3.18173872e-02 -9.38309563e-02 -1.14857365e-01 -5.59789455e-02
  -1.04334542e-01 -4.46788261e-01 -1.04927871e-01  3.78310242e-01
   2.06297065e-01 -3.03813604e-01 -1.87363691e-01  2.54956991e-02
   1.31391402e-01  5.76587610e-02 -2.98080151e-01 -6.31916082e-02
  -6.02026533e-02 -8.70954992e-02  1.05022076e-02  1.74112921e-02
   2.71885337e-02  5.40852558e-02]
 [-1.46095212e-01 -2.35633332e-01  1.65128083e-01 -1.47672075e-01
  -2.96086546e-01 -1.17764861e-01 -2.39824618e-01 -4.50722355e-02
  -1.13123531e-01 -1.42162838e-01 -1.58752504e-03  6.22758786e-01
  -1.40825189e-01  7.91144705e-02  1.41412578e-01 -6.25547928e-02
   1.59518087e-01 -1.53484635e-02 -1.03933021e-02 -3.64541372e-01
```

```
 -1.42538612e-01  1.79738170e-01  1.19240021e-01  3.32362196e-02
 -5.93901148e-02  1.02707765e-01 -2.67903688e-02  1.73414576e-02
 -1.83033571e-02 -8.97201369e-03]
[-1.97452996e-01  1.39336716e-01  1.02094487e-01  8.39858215e-02
  1.63310672e-01 -2.89572724e-01 -1.82834318e-01 -4.48003607e-02
  8.29994395e-02  1.14682515e-02  3.94847977e-02 -1.73680236e-01
 -6.74991763e-02  8.21343947e-02  3.57425254e-01  1.45604051e-01
 -8.48203851e-02 -1.00580040e-01  3.98379684e-01  2.06601161e-01
 -4.78521576e-01  7.16362158e-02  2.38821969e-01  1.98366370e-01
  5.89801546e-03  6.78082202e-02  1.33036816e-01  1.60539213e-02
 -6.30946102e-02 -5.82357076e-02]
[-1.19919988e-01 -2.23112101e-01  1.51155593e-01  2.67306005e-02
 -1.02868807e-01  8.67922243e-02  4.36726637e-01 -2.10429803e-01
  4.50885993e-01 -5.44030744e-01  1.36041819e-01 -7.28376842e-02
 -1.48348088e-03  7.23969937e-02  4.56751259e-02 -1.09466243e-01
 -7.83006967e-02 -2.57997482e-01 -1.11407213e-01  1.25997897e-02
 -1.00742750e-01 -1.02272069e-01  5.06827716e-02 -2.60713565e-02
  1.34170471e-02 -3.66311161e-02  5.44367138e-02 -1.27483646e-02
 -5.45112282e-02  4.31390397e-03]
[-1.20569288e-01 -2.52566420e-01 -2.97002887e-01 -2.97642869e-01
 -1.10666305e-01  3.72947859e-02 -1.24037860e-01 -3.91994006e-01
  2.16181322e-01  2.05028338e-01 -7.19501321e-02 -1.99973212e-01
  2.97810454e-01 -4.10533537e-02  2.19813274e-01  4.76593342e-02
  1.51882676e-01  2.88037713e-01 -3.24219537e-01  5.28592813e-02
 -1.18448340e-01  9.48219907e-02 -1.66364863e-02  1.31361588e-01
  1.12248679e-01  1.36860014e-01  2.73878175e-02 -1.93496332e-05
 -1.02526698e-02 -2.44914606e-02]
[-1.53630166e-01 -2.77113092e-01  1.44000644e-01  1.08457386e-01
  5.70585180e-02  2.27538240e-01 -1.92708264e-01  4.40014367e-02
  3.61170851e-01  2.07948429e-01 -9.77163585e-02  3.90212730e-02
  2.21778431e-01  5.68993133e-02 -1.99613513e-01  1.04086498e-01
  2.55741315e-01 -2.39556478e-01  3.72480527e-01  9.62807658e-02
  3.48506712e-01  1.73574459e-01  2.11805695e-01 -6.36161531e-02
  8.92110493e-02  5.20774206e-02 -3.35535486e-02  3.63665801e-02
  6.88344465e-03  2.61034414e-02]
[-1.74526588e-01 -2.76080441e-02 -2.05150431e-01 -4.86542956e-02
 -2.02779378e-01  2.43523516e-01 -2.16006912e-01 -2.08375152e-01
 -2.90412198e-01  1.88890050e-02  3.49754318e-01 -2.47072721e-01
 -2.94622788e-01  3.07311715e-01 -6.15022493e-02  2.39214476e-01
  8.99998353e-02 -2.38194159e-01 -4.18705453e-02 -1.39922759e-02
  2.34487213e-02 -2.69539101e-01  1.85706990e-01 -1.77688805e-01
  1.87432142e-02 -6.03516585e-02 -9.82132727e-02  5.11374801e-02
 -1.69967747e-02  3.02225960e-02]
[-2.07522958e-01  8.89456905e-02 -3.22340803e-01 -9.96683081e-02
 -1.77444091e-03  1.93867769e-01  8.20236422e-02  2.91737690e-03
  8.70687509e-02  8.97011999e-02  1.62329863e-01  1.55447706e-01
 -1.14897307e-02 -2.07069273e-01 -1.24963404e-02  2.28825900e-02
 -1.64155720e-01 -4.66722169e-02  1.98006876e-01 -1.52837442e-01
 -1.35186957e-01  3.06646529e-01 -1.71357294e-01 -4.66822105e-02
  1.92302785e-01 -5.76617149e-01  6.09537033e-02  1.82903065e-01
 -1.95763428e-01 -2.29802702e-02]
[-2.26737756e-01  1.51233712e-01  4.60962915e-03 -9.89807105e-03
  1.38188412e-01 -7.98276858e-02  9.45764219e-02  2.62356255e-03
 -2.79536735e-02 -1.81840388e-03  1.78401278e-01  1.89552108e-01
  1.06043990e-01  6.63971534e-02 -8.15004051e-02  9.22263197e-02
  9.72771413e-02  3.53067203e-02 -4.95329784e-02 -1.18635655e-01
  2.40668375e-01 -3.26866789e-01  8.27379875e-02  6.49666264e-01
  3.28993130e-01 -1.15436807e-01  5.45134386e-02 -2.13289177e-01
  2.09723478e-02 -2.13799734e-02]
[-8.55761166e-02 -3.26998980e-01 -2.62989941e-01 -1.29564451e-01
```

```
 -4.06740042e-02 -2.90717478e-01  5.31202128e-02  4.20711177e-01
 -8.62674652e-02 -4.04434181e-01  2.95704740e-02 -1.30902707e-01
  3.43788647e-02  4.76966690e-02 -2.58823529e-01  2.94228343e-01
  2.00945899e-01  3.08940532e-01  1.35693712e-01  1.29115319e-01
 -3.75421579e-02  7.05384603e-02  1.80786993e-02 -3.35379489e-02
 -3.94325066e-03 -7.13038303e-02  1.67911694e-02 -1.07657107e-02
  8.57200973e-03 -5.87906925e-02]
 [-2.06499425e-01  1.03937601e-01 -6.21499483e-03 -3.35911011e-02
  1.64405705e-01 -8.81347243e-02 -1.18645662e-01  1.39525871e-01
  2.75337707e-04  8.12402975e-02 -4.69149149e-02  2.20074535e-01
  4.25546919e-01  2.47606349e-01 -2.70873147e-01 -1.38383073e-01
  1.12410636e-01 -2.35154123e-01 -2.42705812e-01  1.44507730e-01
 -4.21094040e-01 -2.52624829e-01 -1.36902143e-01 -2.62782163e-01
  3.75388379e-02 -7.33006788e-02  2.26691280e-02  2.39577689e-02
  9.70809208e-03 -1.64019849e-02]
 [-1.06207299e-01 -3.11297246e-01 -3.15488315e-01  6.45281363e-02
  1.27237931e-01 -3.45522169e-01 -1.65218302e-01  1.74305338e-01
 -2.07022136e-02  5.61473219e-02  2.58376463e-02 -1.79475160e-01
 -9.22849539e-02 -1.44560278e-01  1.68527908e-01 -5.59736504e-01
 -2.71075024e-02 -3.06923628e-01 -7.09419307e-02 -9.81122465e-02
  2.02943864e-01 -1.01107892e-01  8.73428499e-02  4.92430675e-02
 -3.71023249e-02 -9.54798724e-02 -1.22094093e-02  6.21357349e-02
  3.98208140e-02 -1.16182726e-02]
 [-2.38444554e-01  1.52808551e-01 -5.35460128e-04  1.49544747e-01
  9.23319266e-02  7.51982353e-02 -1.42075705e-02  1.10219855e-01
 -6.23498101e-02 -1.32845982e-01  3.87658641e-02 -4.93891932e-02
 -5.32770855e-02  1.16298186e-01  6.82750336e-02 -1.33757316e-01
  1.45567238e-01  1.08206731e-01 -1.84040367e-01  1.13523607e-01
  4.43361995e-02  2.89188282e-01 -1.57998207e-03  4.87213372e-02
  6.21897196e-02  8.59677405e-02  5.45534268e-04  8.21252641e-02
 -1.36129093e-01  7.76107550e-01]
 [-1.34439435e-01 -2.79029076e-01  1.60872303e-01  2.63735675e-01
 -4.39180279e-02 -3.65111698e-02  4.31805272e-02  1.54177523e-01
  3.18114678e-01  4.40510324e-01  1.45749618e-01 -8.18881413e-04
 -4.38290625e-01  2.02265440e-02 -2.17938687e-01  6.50989336e-02
 -2.52701378e-02  2.23160726e-01 -1.94484507e-01 -1.96024871e-02
 -2.15194824e-01 -1.10289646e-01 -1.41847899e-01  1.01958724e-01
 -1.51625983e-01 -6.31407803e-02  1.09128768e-01  7.96237032e-03
  2.64583636e-02  4.14236008e-02]
 [-2.35145883e-01  9.55006425e-02 -2.43830630e-01 -1.17137925e-01
 -7.48270055e-03  1.01114457e-01  1.23871760e-01  4.43748336e-02
  6.06410225e-02  5.04788725e-02  9.26996318e-02  6.97672182e-02
 -7.97547013e-03 -1.15940758e-01  2.76952538e-02  2.63087999e-02
 -6.76163955e-02 -9.75313909e-02  1.37382178e-01 -5.40758666e-02
 -5.74832166e-02  8.91403747e-02 -3.39720229e-02 -1.01822426e-01
 -2.14700387e-01  1.41219664e-01  9.29703482e-02 -4.88344863e-01
  6.35032356e-01  1.58374707e-01]
 [-2.20468641e-01  1.69647025e-01 -2.24900677e-01 -1.09933286e-01
  3.10110673e-03  1.12224532e-01  1.40790799e-01  1.02063126e-01
  5.15542599e-02  2.59647104e-02  7.72932779e-02  2.51538006e-02
  7.13041430e-02 -1.23909668e-01 -1.10612128e-01  2.37245774e-02
  5.17452888e-02 -1.11021791e-01  9.90209502e-02 -1.05993162e-01
  2.27992846e-02 -8.90108890e-02 -1.06409638e-01  2.19172707e-01
 -5.31339914e-01  4.32193584e-01 -6.09539001e-02  2.42231774e-01
 -3.60917266e-01 -9.12049698e-02]
 [-2.31688108e-01  1.94640155e-01  3.50763389e-02  1.41730503e-01
  8.32058207e-02  1.20165310e-01 -1.77530558e-02 -1.39497262e-02
 -2.20236668e-02 -1.48373706e-01  6.59539792e-02 -1.40869035e-02
 -1.05660938e-01  3.24390576e-02 -7.17376996e-02 -1.43892371e-01
  9.93205465e-02  1.19333787e-01 -1.67531478e-01  1.11176783e-01
```

```
  4.31571180e-02  2.44513447e-01  4.68664987e-02  8.79140152e-02
  7.93408067e-02  8.90019600e-02 -2.76691010e-02  5.05591620e-01
  4.88991421e-01 -3.92816719e-01]
 [-5.46238612e-02 -2.13249546e-01 -2.65602257e-01  6.10310585e-01
  1.32805786e-01 -3.44580368e-02  2.66875877e-01 -2.99467122e-01
 -2.23745264e-01  3.07878255e-02  3.51531652e-02  2.66153080e-01
  1.24412476e-01 -2.89436375e-02  1.83219339e-01  5.57828465e-02
  1.40296903e-01  1.66706199e-01  1.82869651e-01  4.09896955e-02
 -8.31487423e-03 -1.52214302e-01 -7.48422225e-02 -1.73889508e-01
  1.26862453e-02  8.97955621e-02 -2.24603921e-02  4.49079459e-02
 -1.30351279e-02 -1.09907854e-02]
 [-2.24909939e-01 -2.78121032e-02  1.57850870e-01  9.83863051e-02
 -2.21334108e-01  1.92804107e-01 -5.99686436e-02  1.62740917e-01
 -2.04286818e-01 -3.69961528e-02 -1.24077236e-01 -1.38437751e-01
  1.55132403e-01 -1.23303333e-01  1.30910945e-01  2.25213672e-03
 -3.99277904e-02  4.88451060e-02 -3.88498348e-02 -9.95897377e-02
  1.05228162e-01 -1.32573902e-01  1.02983679e-03 -9.81877404e-02
 -1.65504293e-03  7.58052170e-03  7.75070693e-01  3.79121023e-02
 -5.83272445e-02 -6.62970695e-02]
 [-3.04039490e-02 -2.50145081e-01  1.08818811e-01 -2.20498670e-01
  6.51439831e-01  3.87422700e-01 -2.03424411e-01  8.96390280e-02
 -1.74319669e-02 -1.87599959e-01 -1.92908792e-02  1.02513387e-01
 -1.47023243e-01 -1.68912982e-01  2.39480190e-01  1.21075603e-01
 -1.90013171e-02  9.88907150e-02 -4.79620831e-02 -8.63205262e-03
 -3.48689110e-02 -2.29649175e-01 -8.61565516e-02 -4.87277443e-02
 -5.84102836e-02 -1.86889333e-02 -4.35354563e-02  3.29408355e-02
  1.82985803e-02  1.65895641e-02]
 [-1.46671487e-01 -3.02760982e-01  1.81674198e-01  2.85532982e-02
  1.32496317e-01 -2.52107177e-02 -2.69902690e-02 -1.48388077e-01
 -3.29075186e-01  2.33477016e-02  2.47066913e-01 -5.11667802e-02
  2.24769582e-01 -1.31469954e-01 -3.41734445e-01  3.55737801e-03
 -5.66516953e-01 -1.01402960e-02 -7.42707823e-02  7.77293606e-03
 -4.19804014e-02  2.48125766e-01  1.20662880e-01  6.80814323e-02
  3.48259168e-02  1.85925557e-01 -6.08951718e-02 -5.71976814e-02
 -5.39592363e-02  1.22820648e-02]
 [-2.13525318e-01  1.36988096e-03  2.01279981e-01  1.02242973e-01
 -2.78668451e-01  1.48456693e-01  2.67592899e-02  1.14711993e-01
 -1.17949299e-01  1.53087913e-02 -9.61237919e-02 -4.40443027e-02
  2.13414522e-01 -3.14484924e-01  1.45802126e-01 -3.00778107e-02
  4.23028604e-02  1.24698614e-01 -6.78022360e-02  1.90760492e-01
 -7.71101219e-02 -1.56380903e-01  3.02980149e-01  8.94507359e-02
 -3.08654860e-01 -3.82145789e-01 -4.04363480e-01 -3.79580104e-02
  5.02766420e-02  2.49983901e-02]
 [-2.19461030e-01 -8.29979717e-02  1.17647880e-01  1.08320502e-01
 -1.75747485e-01  1.13923476e-01  2.12298087e-02  2.22397928e-01
 -7.17992988e-02 -5.64000048e-03 -2.05937620e-01 -2.81735111e-01
  3.41640919e-02  1.71117905e-02  1.28258547e-01  2.19805882e-02
 -5.02700959e-02 -7.58738808e-02  1.44634541e-01 -3.17619000e-01
 -1.48994771e-01 -6.71897647e-02 -4.37794632e-01  4.35616656e-02
  3.77015017e-01  2.14109745e-01 -3.71542078e-01 -1.88910780e-02
  5.24827310e-02 -2.70652735e-02]
 [-1.81648856e-01 -8.16924850e-02 -2.49489426e-01  1.89670265e-02
 -8.49472985e-02  7.01672444e-02  2.22944557e-03 -2.53004768e-02
 -3.28814895e-02 -4.00867531e-02 -6.14775522e-01  2.10677711e-01
 -2.30119098e-01  2.25506554e-01 -1.76967838e-02  1.24937722e-01
 -3.88934754e-01 -1.32327790e-01 -9.78422289e-02  3.44987708e-01
  1.13500954e-01 -7.19148139e-03 -2.64585391e-02  1.62860959e-01
 -1.63520447e-02 -2.35033126e-02 -2.11517699e-03 -1.46909549e-02
 -3.15991520e-02 -2.49179754e-02]
 [-1.89779346e-01  1.25524632e-01  3.53945796e-03  4.84393340e-02
```

```
  1.14606438e-01  7.86704101e-03 -2.13699293e-01 -3.75883383e-01
  5.24600056e-02 -1.91567976e-01 -2.95528490e-01 -1.81891283e-01
 -9.37233926e-02  6.63634414e-02 -3.79583738e-01 -3.11689213e-01
  2.97737796e-02  3.31838944e-01  2.52406917e-01 -2.77951580e-01
 -5.33624065e-02 -1.02124548e-01  2.03904404e-02 -3.13270170e-02
 -1.44770468e-01 -1.31350815e-01  3.05915220e-03 -1.53320045e-01
 -5.49910830e-02  5.12381103e-03]
[-2.10566859e-01  1.70967137e-01 -2.66724533e-02 -1.78064402e-01
 -4.83675892e-02 -2.07611975e-01  9.26113975e-02  4.47447516e-02
  1.99074895e-01  8.06062756e-02 -4.39486320e-02  1.27403083e-01
 -1.08314908e-01 -2.67065750e-01 -2.84081856e-02  5.11822259e-02
 -2.20914406e-01  2.05430434e-01  3.66695814e-02 -3.59908309e-02
  9.89446564e-02 -3.14319733e-01  3.36505845e-01 -3.86046484e-01
  3.18846654e-01  2.30718355e-01 -8.11298298e-02  2.11086459e-01
 -1.45739020e-02  1.28344600e-01]
[-2.35277851e-01  1.66248779e-01 -1.29846415e-02  1.41130231e-01
  9.26506476e-02  5.21925406e-02 -2.44873932e-02  1.32965041e-01
  5.02373444e-02 -2.10227648e-02  8.10486175e-02  3.26748098e-03
 -1.48705214e-01 -2.01062793e-02  1.02357762e-01 -6.60236943e-02
  9.96160172e-02  6.62251968e-02 -2.45133079e-01  9.18602262e-02
  1.04259518e-01  2.48008104e-01  8.04875428e-02 -2.39113445e-01
  9.76425750e-02  1.03116281e-01 -9.90852367e-02 -5.14267193e-01
 -3.71504376e-01 -4.11208429e-01]]
```

In [18]:
```
eigenvalues, eigenvectors = LA.eig(standardized_returns_cov)
idx = np.argsort(eigenvalues)[::-1]
eigenvalues = eigenvalues[idx]
eigenvectors = eigenvectors[:, idx]
eigenvalues
```

Out[18]:
```
array([13.83809818,  2.91172917,  1.5144785 ,  1.14582481,  1.03750683,
        0.96578088,  0.91632498,  0.76395054,  0.7483066 ,  0.62887191,
        0.59908471,  0.53622869,  0.49784749,  0.48426379,  0.43204344,
        0.40979354,  0.37594111,  0.31478778,  0.30061415,  0.28382871,
        0.26930219,  0.22250477,  0.18595305,  0.15665849,  0.13658945,
        0.11823818,  0.07816574,  0.05705594,  0.04586529,  0.02436109])
```

In [19]:
```
eigenvectors
```

```
Out[19]: array([[-1.48310310e-01, -7.49072060e-02, -1.45795230e-01,
                 -2.28376847e-01, -2.72802626e-01,  9.50397224e-02,
                 -5.44412145e-01, -2.45363524e-02, -2.37922403e-01,
                  2.70845810e-01, -2.69316177e-01, -1.52037533e-01,
                 -5.65399470e-02, -2.17234924e-01, -4.50495780e-02,
                  7.48937144e-02, -1.77465507e-01, -8.22425862e-02,
                 -6.69547395e-02,  2.74913936e-01, -1.18427885e-02,
                  1.61206409e-01,  2.43954996e-01, -8.50909001e-02,
                  7.48481499e-02, -1.16443136e-01,  9.57334702e-03,
                  5.89181637e-02, -4.99576244e-04, -3.12670405e-02],
                [-1.78257710e-01, -9.74435360e-02, -2.34714659e-01,
                 -3.67420905e-01,  1.03606197e-01,  3.59333381e-02,
                 -1.64621336e-01, -1.39334693e-01, -2.01596213e-01,
                  1.18293484e-01,  1.38647185e-01,  6.08141184e-02,
                 -1.54166779e-01,  5.01732346e-03, -1.06740701e-02,
                 -3.39273229e-01, -1.40667380e-01,  8.40684170e-02,
                  2.96787901e-01, -4.95399529e-01,  1.04474473e-01,
                 -6.97464766e-02, -3.26800293e-01, -2.66879945e-02,
                 -6.59788867e-02,  2.26731092e-02,  2.62812864e-02,
                 -2.93006765e-02,  5.21705506e-02, -2.72844140e-02],
                [-2.06758318e-01,  5.91166283e-02, -1.36396592e-01,
                 -2.13106618e-02, -2.93086120e-02,  2.43613662e-01,
                  1.01775820e-01, -2.91165476e-03,  1.53802936e-01,
                  5.28943644e-03,  1.75685720e-01, -5.38443297e-03,
                  2.13744371e-01, -4.14689240e-01, -2.84543974e-01,
                  1.32974432e-01,  2.89714068e-01,  1.95765749e-01,
                  6.02149238e-03,  1.04607976e-01,  3.84012380e-01,
                 -2.95516098e-03, -2.48611163e-01, -1.66711989e-01,
                  2.78495019e-01, -1.69593791e-01, -9.34354453e-02,
                  7.25575780e-02, -4.55105074e-02, -2.33633416e-02],
                [-1.82948236e-01,  7.90598527e-02, -1.60846539e-01,
                  4.53678116e-02, -5.15344075e-02,  3.71228547e-01,
                  1.22233579e-01, -2.94666488e-01, -3.18173872e-02,
                 -9.38309563e-02, -1.14857365e-01, -5.59789455e-02,
                 -1.04334542e-01,  4.46788261e-01,  1.04927871e-01,
                  3.78310242e-01, -2.06297065e-01, -3.03813604e-01,
                 -1.87363691e-01, -2.54956991e-02,  1.31391402e-01,
                  5.76587610e-02, -2.98080151e-01, -6.31916082e-02,
                  6.02026533e-02, -8.70954992e-02,  1.05022076e-02,
                  1.74112921e-02, -2.71885337e-02,  5.40852558e-02],
                [-1.46095212e-01, -2.35633332e-01, -1.65128083e-01,
                 -1.47672075e-01,  2.96086546e-01,  1.17764861e-01,
                  2.39824618e-01, -4.50722355e-02, -1.13123531e-01,
                 -1.42162838e-01, -1.58752504e-03,  6.22758786e-01,
                 -1.40825189e-01, -7.91144705e-02, -1.41412578e-01,
                 -6.25547928e-02, -1.59518087e-01, -1.53484635e-02,
                 -1.03933021e-02,  3.64541372e-01, -1.42538612e-01,
                  1.79738170e-01,  1.19240021e-01,  3.32362196e-02,
                  5.93901148e-02,  1.02707765e-01, -2.67903688e-02,
                  1.73414576e-02,  1.83033571e-02, -8.97201369e-03],
                [-1.97452996e-01,  1.39336716e-01, -1.02094487e-01,
                  8.39858215e-02, -1.63310672e-01,  2.89572724e-01,
                  1.82834318e-01, -4.48003607e-02,  8.29994395e-02,
                  1.14682515e-02,  3.94847977e-02, -1.73680236e-01,
                 -6.74991763e-02, -8.21343947e-02, -3.57425254e-01,
                  1.45604051e-01,  8.48203851e-02, -1.00580040e-01,
                  3.98379684e-01, -2.06601161e-01, -4.78521576e-01,
                  7.16362158e-02,  2.38821969e-01,  1.98366370e-01,
                 -5.89801546e-03,  6.78082202e-02,  1.33036816e-01,
                  1.60539213e-02,  6.30946102e-02, -5.82357076e-02],
```

```
[-1.19919988e-01, -2.23112101e-01, -1.51155593e-01,
  2.67306005e-02,  1.02868807e-01, -8.67922243e-02,
 -4.36726637e-01, -2.10429803e-01,  4.50885993e-01,
 -5.44030744e-01,  1.36041819e-01, -7.28376842e-02,
 -1.48348088e-03, -7.23969937e-02, -4.56751259e-02,
 -1.09466243e-01,  7.83006967e-02, -2.57997482e-01,
 -1.11407213e-01, -1.25997897e-02, -1.00742750e-01,
 -1.02272069e-01,  5.06827716e-02, -2.60713565e-02,
 -1.34170471e-02, -3.66311161e-02,  5.44367138e-02,
 -1.27483646e-02,  5.45112282e-02,  4.31390397e-03],
[-1.20569288e-01, -2.52566420e-01,  2.97002887e-01,
 -2.97642869e-01,  1.10666305e-01, -3.72947859e-02,
  1.24037860e-01, -3.91994006e-01,  2.16181322e-01,
  2.05028338e-01, -7.19501321e-02, -1.99973212e-01,
  2.97810454e-01,  4.10533537e-02, -2.19813274e-01,
  4.76593342e-02, -1.51882676e-01,  2.88037713e-01,
 -3.24219537e-01, -5.28592813e-02, -1.18448340e-01,
  9.48219907e-02, -1.66364863e-02,  1.31361588e-01,
 -1.12248679e-01,  1.36860014e-01,  2.73878175e-02,
 -1.93496332e-05,  1.02526698e-02, -2.44914606e-02],
[-1.53630166e-01, -2.77113092e-01, -1.44000644e-01,
  1.08457386e-01, -5.70585180e-02, -2.27538240e-01,
  1.92708264e-01,  4.40014367e-02,  3.61170851e-01,
  2.07948429e-01, -9.77163585e-02,  3.90212730e-02,
  2.21778431e-01, -5.68993133e-02,  1.99613513e-01,
  1.04086498e-01, -2.55741315e-01, -2.39556478e-01,
  3.72480527e-01, -9.62807658e-02,  3.48506712e-01,
  1.73574459e-01,  2.11805695e-01, -6.36161531e-02,
 -8.92110493e-02,  5.20774206e-02, -3.35535486e-02,
  3.63665801e-02, -6.88344465e-03,  2.61034414e-02],
[-1.74526588e-01, -2.76080441e-02,  2.05150431e-01,
 -4.86542956e-02,  2.02779378e-01, -2.43523516e-01,
  2.16006912e-01, -2.08375152e-01, -2.90412198e-01,
  1.88890050e-02,  3.49754318e-01, -2.47072721e-01,
 -2.94622788e-01, -3.07311715e-01,  6.15022493e-02,
  2.39214476e-01, -8.99998353e-02, -2.38194159e-01,
 -4.18705453e-02,  1.39922759e-02,  2.34487213e-02,
 -2.69539101e-01,  1.85706990e-01, -1.77688805e-01,
 -1.87432142e-02, -6.03516585e-02, -9.82132727e-02,
  5.11374801e-02,  1.69967747e-02,  3.02225960e-02],
[-2.07522958e-01,  8.89456905e-02,  3.22340803e-01,
 -9.96683081e-02,  1.77444091e-03, -1.93867769e-01,
 -8.20236422e-02,  2.91737690e-03,  8.70687509e-02,
  8.97011999e-02,  1.62329863e-01,  1.55447706e-01,
 -1.14897307e-02,  2.07069273e-01,  1.24963404e-02,
  2.28825900e-02,  1.64155720e-01, -4.66722169e-02,
  1.98006876e-01,  1.52837442e-01, -1.35186957e-01,
  3.06646529e-01, -1.71357294e-01, -4.66822105e-02,
 -1.92302785e-01, -5.76617149e-01,  6.09537033e-02,
  1.82903065e-01,  1.95763428e-01, -2.29802702e-02],
[-2.26737756e-01,  1.51233712e-01, -4.60962915e-03,
 -9.89807105e-03, -1.38188412e-01,  7.98276858e-02,
 -9.45764219e-02,  2.62356255e-03, -2.79536735e-02,
 -1.81840388e-03,  1.78401278e-01,  1.89552108e-01,
  1.06043990e-01, -6.63971534e-02,  8.15004051e-02,
  9.22263197e-02, -9.72771413e-02,  3.53067203e-02,
 -4.95329784e-02,  1.18635655e-01,  2.40668375e-01,
 -3.26866789e-01,  8.27379875e-02,  6.49666264e-01,
 -3.28993130e-01, -1.15436807e-01,  5.45134386e-02,
 -2.13289177e-01, -2.09723478e-02, -2.13799734e-02],
```

```
[-8.55761166e-02, -3.26998980e-01,  2.62989941e-01,
 -1.29564451e-01,  4.06740042e-02,  2.90717478e-01,
 -5.31202128e-02,  4.20711177e-01, -8.62674652e-02,
 -4.04434181e-01,  2.95704740e-02, -1.30902707e-01,
  3.43788647e-02, -4.76966690e-02,  2.58823529e-01,
  2.94228343e-01, -2.00945899e-01,  3.08940532e-01,
  1.35693712e-01, -1.29115319e-01, -3.75421579e-02,
  7.05384603e-02,  1.80786993e-02, -3.35379489e-02,
  3.94325066e-03, -7.13038303e-02,  1.67911694e-02,
 -1.07657107e-02, -8.57200973e-03, -5.87906925e-02],
[-2.06499425e-01,  1.03937601e-01,  6.21499483e-03,
 -3.35911011e-02, -1.64405705e-01,  8.81347243e-02,
  1.18645662e-01,  1.39525871e-01,  2.75337707e-04,
  8.12402975e-02, -4.69149149e-02,  2.20074535e-01,
  4.25546919e-01, -2.47606349e-01,  2.70873147e-01,
 -1.38383073e-01, -1.12410636e-01, -2.35154123e-01,
 -2.42705812e-01, -1.44507730e-01, -4.21094040e-01,
 -2.52624829e-01, -1.36902143e-01, -2.62782163e-01,
 -3.75388379e-02, -7.33006788e-02,  2.26691280e-02,
  2.39577689e-02, -9.70809208e-03, -1.64019849e-02],
[-1.06207299e-01, -3.11297246e-01,  3.15488315e-01,
  6.45281363e-02, -1.27237931e-01,  3.45522169e-01,
  1.65218302e-01,  1.74305338e-01, -2.07022136e-02,
  5.61473219e-02,  2.58376463e-02, -1.79475160e-01,
 -9.22849539e-02,  1.44560278e-01, -1.68527908e-01,
 -5.59736504e-01,  2.71075024e-02, -3.06923628e-01,
 -7.09419307e-02,  9.81122465e-02,  2.02943864e-01,
 -1.01107892e-01,  8.73428499e-02,  4.92430675e-02,
  3.71023249e-02, -9.54798724e-02, -1.22094093e-02,
  6.21357349e-02, -3.98208140e-02, -1.16182726e-02],
[-2.38444554e-01,  1.52808551e-01,  5.35460128e-04,
  1.49544747e-01, -9.23319266e-02, -7.51982353e-02,
  1.42075705e-02,  1.10219855e-01, -6.23498101e-02,
 -1.32845982e-01,  3.87658641e-02, -4.93891932e-02,
 -5.32770855e-02, -1.16298186e-01, -6.82750336e-02,
 -1.33757316e-01, -1.45567238e-01,  1.08206731e-01,
 -1.84040367e-01, -1.13523607e-01,  4.43361995e-02,
  2.89188282e-01, -1.57998207e-03,  4.87213372e-02,
 -6.21897196e-02,  8.59677405e-02,  5.45534268e-04,
  8.21252641e-02,  1.36129093e-01,  7.76107550e-01],
[-1.34439435e-01, -2.79029076e-01, -1.60872303e-01,
  2.63735675e-01,  4.39180279e-02,  3.65111698e-02,
 -4.31805272e-02,  1.54177523e-01,  3.18114678e-01,
  4.40510324e-01,  1.45749618e-01, -8.18881413e-04,
 -4.38290625e-01, -2.02265440e-02,  2.17938687e-01,
  6.50989336e-02,  2.52701378e-02,  2.23160726e-01,
 -1.94484507e-01,  1.96024871e-02, -2.15194824e-01,
 -1.10289646e-01, -1.41847899e-01,  1.01958724e-01,
  1.51625983e-01, -6.31407803e-02,  1.09128768e-01,
  7.96237032e-03, -2.64583636e-02,  4.14236008e-02],
[-2.35145883e-01,  9.55006425e-02,  2.43830630e-01,
 -1.17137925e-01,  7.48270055e-03, -1.01114457e-01,
 -1.23871760e-01,  4.43748336e-02,  6.06410225e-02,
  5.04788725e-02,  9.26996318e-02,  6.97672182e-02,
 -7.97547013e-03,  1.15940758e-01, -2.76952538e-02,
  2.63087999e-02,  6.76163955e-02, -9.75313909e-02,
  1.37382178e-01,  5.40758666e-02, -5.74832166e-02,
  8.91403747e-02, -3.39720229e-02, -1.01822426e-01,
  2.14700387e-01,  1.41219664e-01,  9.29703482e-02,
 -4.88344863e-01, -6.35032356e-01,  1.58374707e-01],
```

```
[-2.20468641e-01,  1.69647025e-01,  2.24900677e-01,
 -1.09933286e-01, -3.10110673e-03, -1.12224532e-01,
 -1.40790799e-01,  1.02063126e-01,  5.15542599e-02,
  2.59647104e-02,  7.72932779e-02,  2.51538006e-02,
  7.13041430e-02,  1.23909668e-01,  1.10612128e-01,
  2.37245774e-02, -5.17452888e-02, -1.11021791e-01,
  9.90209502e-02,  1.05993162e-01,  2.27992846e-02,
 -8.90108890e-02, -1.06409638e-01,  2.19172707e-01,
  5.31339914e-01,  4.32193584e-01, -6.09539001e-02,
  2.42231774e-01,  3.60917266e-01, -9.12049698e-02],
[-2.31688108e-01,  1.94640155e-01, -3.50763389e-02,
  1.41730503e-01, -8.32058207e-02, -1.20165310e-01,
  1.77530558e-02, -1.39497262e-02, -2.20236668e-02,
 -1.48373706e-01,  6.59539792e-02, -1.40869035e-02,
 -1.05660938e-01, -3.24390576e-02,  7.17376996e-02,
 -1.43892371e-01, -9.93205465e-02,  1.19333787e-01,
 -1.67531478e-01, -1.11176783e-01,  4.31571180e-02,
  2.44513447e-01,  4.68664987e-02,  8.79140152e-02,
 -7.93408067e-02,  8.90019600e-02, -2.76691010e-02,
  5.05591620e-01, -4.88991421e-01, -3.92816719e-01],
[-5.46238612e-02, -2.13249546e-01,  2.65602257e-01,
  6.10310585e-01, -1.32805786e-01,  3.44580368e-02,
 -2.66875877e-01, -2.99467122e-01, -2.23745264e-01,
  3.07878255e-02,  3.51531652e-02,  2.66153080e-01,
  1.24412476e-01,  2.89436375e-02, -1.83219339e-01,
  5.57828465e-02, -1.40296903e-01,  1.66706199e-01,
  1.82869651e-01, -4.09896955e-02, -8.31487423e-03,
 -1.52214302e-01, -7.48422225e-02, -1.73889508e-01,
 -1.26862453e-02,  8.97955621e-02, -2.24603921e-02,
  4.49079459e-02,  1.30351279e-02, -1.09907854e-02],
[-2.24909939e-01, -2.78121032e-02, -1.57850870e-01,
  9.83863051e-02,  2.21334108e-01, -1.92804107e-01,
  5.99686436e-02,  1.62740917e-01, -2.04286818e-01,
 -3.69961528e-02, -1.24077236e-01, -1.38437751e-01,
  1.55132403e-01,  1.23303333e-01, -1.30910945e-01,
  2.25213672e-03,  3.99277904e-02,  4.88451060e-02,
 -3.88498348e-02,  9.95897377e-02,  1.05228162e-01,
 -1.32573902e-01,  1.02983679e-03, -9.81877404e-02,
  1.65504293e-03,  7.58052170e-03,  7.75070693e-01,
  3.79121023e-02,  5.83272445e-02, -6.62970695e-02],
[-3.04039490e-02, -2.50145081e-01, -1.08818811e-01,
 -2.20498670e-01, -6.51439831e-01, -3.87422700e-01,
  2.03424411e-01,  8.96390280e-02, -1.74319669e-02,
 -1.87599959e-01, -1.92908792e-02,  1.02513387e-01,
 -1.47023243e-01,  1.68912982e-01, -2.39480190e-01,
  1.21075603e-01,  1.90013171e-02,  9.88907150e-02,
 -4.79620831e-02,  8.63205262e-03, -3.48689110e-02,
 -2.29649175e-01, -8.61565516e-02, -4.87277443e-02,
  5.84102836e-02, -1.86889333e-02, -4.35354563e-02,
  3.29408355e-02, -1.82985803e-02,  1.65895641e-02],
[-1.46671487e-01, -3.02760982e-01, -1.81674198e-01,
  2.85532982e-02, -1.32496317e-01,  2.52107177e-02,
  2.69902690e-02, -1.48388077e-01, -3.29075186e-01,
  2.33477016e-02,  2.47066913e-01, -5.11667802e-02,
  2.24769582e-01,  1.31469954e-01,  3.41734445e-01,
  3.55737801e-03,  5.66516953e-01, -1.01402960e-02,
 -7.42707823e-02, -7.77293606e-03, -4.19804014e-02,
  2.48125766e-01,  1.20662880e-01,  6.80814323e-02,
 -3.48259168e-02,  1.85925557e-01, -6.08951718e-02,
 -5.71976814e-02,  5.39592363e-02,  1.22820648e-02],
```

```
[-2.13525318e-01,  1.36988096e-03, -2.01279981e-01,
  1.02242973e-01,  2.78668451e-01, -1.48456693e-01,
 -2.67592899e-02,  1.14711993e-01, -1.17949299e-01,
  1.53087913e-02, -9.61237919e-02, -4.40443027e-02,
  2.13414522e-01,  3.14484924e-01, -1.45802126e-01,
 -3.00778107e-02, -4.23028604e-02,  1.24698614e-01,
 -6.78022360e-02, -1.90760492e-01, -7.71101219e-02,
 -1.56380903e-01,  3.02980149e-01,  8.94507359e-02,
  3.08654860e-01, -3.82145789e-01, -4.04363480e-01,
 -3.79580104e-02, -5.02766420e-02,  2.49983901e-02],
[-2.19461030e-01, -8.29979717e-02, -1.17647880e-01,
  1.08320502e-01,  1.75747485e-01, -1.13923476e-01,
 -2.12298087e-02,  2.22397928e-01, -7.17992988e-02,
 -5.64000048e-03, -2.05937620e-01, -2.81735111e-01,
  3.41640919e-02, -1.71117905e-02, -1.28258547e-01,
  2.19805882e-02,  5.02700959e-02, -7.58738808e-02,
  1.44634541e-01,  3.17619000e-01, -1.48994771e-01,
 -6.71897647e-02, -4.37794632e-01,  4.35616656e-02,
 -3.77015017e-01,  2.14109745e-01, -3.71542078e-01,
 -1.88910780e-02, -5.24827310e-02, -2.70652735e-02],
[-1.81648856e-01, -8.16924850e-02,  2.49489426e-01,
  1.89670265e-02,  8.49472985e-02, -7.01672444e-02,
 -2.22944557e-03, -2.53004768e-02, -3.28814895e-02,
 -4.00867531e-02, -6.14775522e-01,  2.10677711e-01,
 -2.30119098e-01, -2.25506554e-01,  1.76967838e-02,
  1.24937722e-01,  3.88934754e-01, -1.32327790e-01,
 -9.78422289e-02, -3.44987708e-01,  1.13500954e-01,
 -7.19148139e-03, -2.64585391e-02,  1.62860959e-01,
  1.63520447e-02, -2.35033126e-02, -2.11517699e-03,
 -1.46909549e-02,  3.15991520e-02, -2.49179754e-02],
[-1.89779346e-01,  1.25524632e-01, -3.53945796e-03,
  4.84393340e-02, -1.14606438e-01, -7.86704101e-03,
  2.13699293e-01, -3.75883383e-01,  5.24600056e-02,
 -1.91567976e-01, -2.95528490e-01, -1.81891283e-01,
 -9.37233926e-02, -6.63634414e-02,  3.79583738e-01,
 -3.11689213e-01, -2.97737796e-02,  3.31838944e-01,
  2.52406917e-01,  2.77951580e-01, -5.33624065e-02,
 -1.02124548e-01,  2.03904404e-02, -3.13270170e-02,
  1.44770468e-01, -1.31350815e-01,  3.05915220e-03,
 -1.53320045e-01,  5.49910830e-02,  5.12381103e-03],
[-2.10566859e-01,  1.70967137e-01,  2.66724533e-02,
 -1.78064402e-01,  4.83675892e-02,  2.07611975e-01,
 -9.26113975e-02,  4.47447516e-02,  1.99074895e-01,
  8.06062756e-02, -4.39486320e-02,  1.27403083e-01,
 -1.08314908e-01,  2.67065750e-01,  2.84081856e-02,
  5.11822259e-02,  2.20914406e-01,  2.05430434e-01,
  3.66695814e-02,  3.59908309e-02,  9.89446564e-02,
 -3.14319733e-01,  3.36505845e-01, -3.86046484e-01,
 -3.18846654e-01,  2.30718355e-01, -8.11298298e-02,
  2.11086459e-01,  1.45739020e-02,  1.28344600e-01],
[-2.35277851e-01,  1.66248779e-01,  1.29846415e-02,
  1.41130231e-01, -9.26506476e-02, -5.21925406e-02,
  2.44873932e-02,  1.32965041e-01,  5.02373444e-02,
 -2.10227648e-02,  8.10486175e-02,  3.26748098e-03,
 -1.48705214e-01,  2.01062793e-02, -1.02357762e-01,
 -6.60236943e-02, -9.96160172e-02,  6.62251968e-02,
 -2.45133079e-01, -9.18602262e-02,  1.04259518e-01,
  2.48008104e-01,  8.04875428e-02, -2.39113445e-01,
 -9.76425750e-02,  1.03116281e-01, -9.90852367e-02,
 -5.14267193e-01,  3.71504376e-01, -4.11208429e-01]])
```

```
In [ ]:
```