Financial Challenges

LESSONS 2 & 3

- Collateral related factors
  - Credit
  - Financing risk
- Statistical related factors
  - Volatility
  - Correlations
- Magnifying factors
  - Leverage
  - Non-linearities
- Frictional factors
  - Liquidity
  - Regulation

regulations.

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In the previous modules, we were introduced to several key themes. In this lesson, we will review the valuation of bonds, stocks, options, and ETFs in normal markets. Then, we will understand how these valuations change in light of stresses: changing rates, extreme events, and underestimated volatilities, as well as sector hits. In the case of bonds we devote a good amount of attention to the concepts of duration and convexity which are very important when working with fixed-income securities.

1. Bond Valuation in Normal Markets

Let's begin with bonds. Recall that bonds followed the discounted cash flows. Pricing a bond essentially involves finding the single yield, known as the yield to maturity, by which all cash flows are discounted. For bonds, those coupon sizes are fixed, hence the term fixed-income market, so this becomes a repeated exercise in discounting.

The industry throws an extra complexity in how it calculates time. There are day count conventions that measure how many days have elapsed since the last coupon was paid and how many days in total there are in a period. Not always intuitive, some of these day-count conventions do not use the actual number of days but rather a simplified counting system (e.g., 30 days in each and every month). For now, we will neglect day count conventions and instead focus on simply periods.

Suppose we have a 3% bond that has a 2-year maturity. In total, there are 5 cash flows. The yield-to-maturity is the number that makes the discounted cash flows match the market price. Maturity and coupon are known. Price and yield determine each other.

What if we don't know the size of the cash flow? Well, in the world of fixed income, that is not an issue. Indeed, this is where fixed income gets its name. These periodic payments are fixed in timing and size. (Let's not consider floating rates bonds here). The reliability of a fixed coupon makes bonds a relatively steady and safe investment. But what happens if we relax that assumption? In the next section, we'll cover stocks.

The yield-to-maturity is the number that makes the discounted cash flows match the market price.

2. Stock Valuation in Normal Markets

We are now going to look at how to value a stock. There are several models to value a stock: discounted cash flow model, relative valuation, dividend discount model, etc. Here, we will consider the dividend discount model, which is predicated upon a company paying dividends.

Earlier you learned about growth stocks--that is not the subject here. Instead, we focus on value stocks, which are stocks that provide income. How do these compare to bonds? Well, bonds are required to pay their coupons or they go into default. Stocks are not required to pay a dividend, even value stocks, but historically have paid dividends consistently. If a value stock failed to pay a dividend, or even pays a lower-than-expected dividend, indeed, the stock price takes a hit. This is often why companies report their earnings and dividends after the market closes; otherwise, the sudden and surprising news could cause undesirable volatility. Clearly, then the dividend yield is crucial in valuing the stock.

What are the two key differences between a bond and a stock?

1. The bond has a specified, agreed-upon maturity date. At that date, the final coupon is paid and the notional amount reimbursed. After that date, no further cash flows occur. Stocks, on the other hand, do not have a "maturity" or an "expiration" date. Companies are theoretically meant to last forever. Therefore, the dividend stream of a stock is never supposed to end.
2. Second, bonds have a specified coupon size. Stocks do not. The size of the dividend is random. If a company does not earn profits in a given period of time and does not have retained earnings, or simply decides not to pay a dividend, there is no dividend. Therefore, dividend sizes are random variables.

To summarize, bonds have 2 known parameters: time to maturity and cash flow size. Stocks have neither of these. The problem of not having a time to maturity is solved by realizing the stock does not have one. The question of cash flow size is answered by modeling the size as a constant or variable. The variable itself can be modeled as one that grows at a constant rate or dynamic rate. We simplify the first by assuming there is no end period. How do we simplify the second? We may assume a constant dividend, a dividend that grows at a constant rate, or a dividend that grows at an increasing rate.

The constant dividend model is flawed if for no other reason than inflation. A company failing to pay a higher dividend is seen as not keeping up with inflation. If these were the model we used, then it would be very similar to fixed income valuation—-except that we somehow have to handle the perpetual nature of the cash flows. When we value equities, we'll see that it pays to consider a dividend that grows at a constant rate or even one that grows at a variable rate.

3. Option Valuation in Normal Markets

Stocks and bonds are fundamental assets in the market. Derivatives are derived from these fundamental securities. When we value stocks and bonds, we are using cash flows. When we value derivatives, we tend to use replication arguments, which use stocks and bonds. Think of stocks and bonds as atoms. Yes, they have simpler parts (like protons and electrons). Think of derivatives as compounds containing atoms in different amounts. Indeed, it will be shown later in the program that an option can be thought of as a combination of a stock and a bond. You'll have to wait for Derivative Pricing to see the details, but for now, let's review the basics of option valuation in normal markets.

Suppose we want to value a call option whose underlying is a stock. We saw in a previous module that there were 6 key inputs. The first 5 of these are very easy to obtain from the market: the stock price, the strike level, the time to expiration, the risk-free rate, and the dividend yield. The final input, the volatility, is not as easy to observe. We need a model for it. One model we may use is the standard deviation of return over the last 30 days, or we may use a time-series model that recognizes volatility is mean-reverting and use that long-term (ergodic) volatility as the estimate for the volatility. Alternatively, we may simply use a proprietary model. When we put volatility in, we get a price.

Suppose we get volatility wrong. In this course, we saw that one problem with volatility is that it may not be constant. We examined distributions that we tested for normality, and then we showed the assumptions required for the normal distribution. Specifically, the kurtosis showed that the volatility itself seemed to have uncertainty with it. As a result, we may input a volatility whose value is less than the true volatility. In this sense, we underestimate volatility in normal markets. Nevertheless, a standard deviation may work well. That value becomes our input in pricing the option. In normal (usual) markets, hopefully, this volatility works well. But if we get this wrong, we underprice options: a costly mistake if we sold them and are hedging them to the tune of high volatility.

4. ETF Valuation in Normal Markets

A portfolio has combinations of securities. One way to estimate the return is to find the weights of each of the securities and the expected returns of each of the securities. Fortunately, return is linear, so we can simply take the weighted average of the returns. Using those returns, we can calculate a covariance matrix. The covariance matrix and historical returns can provide an expected measure of future volatility. If the covariance matrix refers to normal markets, then the covariance matrix should indicate future volatility. Of course, the markets may not remain normal but may become turbulent. To what extent would the covariance matrix degrade?

There are two ways the covariance matrix may change significantly. One is that variances get substantially higher. This would result in higher volatilities along the diagonal of the covariance matrix. Here, we have the same problem as in the previous section--the market may exhibit higher volatilities than we anticipated. Second, the correlations may increase, which means we have less diversification than expected.

We will have to wait until Econometrics to discuss issues like stationarity, which relate to variances becoming more or less constant over time.

5. Market Stress

5.1 Bond Markets

There are several things that can frazzle interest rates. In many countries, the central bank can set policy to raise or lower interest rates, in an effort to combat high unemployment and/or high inflation. Indeed, interest rate hikes and cuts can send other asset classes into a frenzy. For example, in the first half of 2022, not only interest rate hikes but also the acceleration at which those hikes were expected sent equities reeling, causing a stock market correction into bear territory.

Modeling interest rates by merely looking at a five- or 10-year history may not take into account older time periods that are more relevant to the central bank phases of expansion and tightening. More recent time series do not necessarily equate to more relevant data.

5.2 Equity Markets

In normal markets, the returns of stocks are shown to be fairly normal or Gaussian. In highly volatile markets, stock returns may still be Gaussian, but they may have much higher volatilities. Worse, the distributions may look more like a mixture of normals, with excess kurtosis and even jumps in the stock prices that increase the likelihood of an extreme event occurring.

5.3 Option Markets

In normal markets, options are modeled by careful calculation of the volatility of the underlying, the key input into pricing options in the first place. In turbulent markets, volatilities undoubtedly rise. Will we be able to price and hedge options correctly in a world where volatility reaches higher levels and rises and falls more quickly? Indeed, we can see volatilities easily exceed 100% for many equities and other asset classes in these markets.

5.4 Sector Markets

The issue of diversification that we thought was present may not actually be there. Suppose we are exposed to a sector that crashes. We may earn profits in multiple sectors but lose money in others. However, in highly volatile markets, correlations can rise, which lowers diversification. The promise of diversification to help hedge against a downturn turned out to be a false sense of security, as the heightened correlation treats the portfolio more like a single stock than a basket.

6. Interest Rate Risk with Duration and Convexity

By now, we are accustomed to associate volatility with risk when dealing with equities. However, bonds are different creatures. While volatility exists even for the price of bonds, bond investors are much more concerned about interest rate risk. And the reason is simple: when interest rate changes, so do the prices of bonds.

A key metric to gauge interest rate risk for fixed income investment is **duration**, which is a measure of the sensitivity of the price of a bond or other debt instrument to a change in interest rates. In general, the higher the duration, the more the price of a bond will drop as interest rates rise (thus we bear a greater the interest rate risk). For example, if rates were to rise 1%, a bond or bond fund with a five-year average duration would likely lose approximately 5% of its value. We will make this more rigorous in the following paragraphs.

NOTE. Making everything somewhat rigorous requires a bit of Calculus, but not much more than a knowledge of first and second order derivatives and Taylor's expansions. Something that is expected of students who plan to get a Master's degree in Financial Engineering. However, derivations/calculations will not be tested and only the use of these formulae will.

To understand how the duration and convexity formulae work, we start by recalling the formula for the price of a bond; i.e., the price of the bond at time t is given by:

$$P_t(r) = \sum_{i=1}^{N} \frac{\mathrm{CF}_i}{(1+r)^{t_i - t}},$$

where CF_i is the cash flow (coupon payments and principal repayments) at time t_i for $i = 1, 2, \ldots, N$, and r is the relevant yield to maturity or the market discount rate to use at time t. As you can see, $P_t(r)$ is a function of r and calculated at time t. Then, we can calculate the first and second derivatives of $P_t(r)$ with respect to r, which with some tedious but simple calculations can be shown to be:

$$P_t'(r) = \frac{dP_t(r)}{dr} = -\sum_{i=1}^{N} \frac{\mathrm{CF}_i}{(1+r)^{t_i - t + 1}}(t_i - t)$$

and

$$P_t''(r) = \frac{d^2 P_t(r)}{dr^2} = \sum_{i=1}^{N} \frac{(t_i - t)(t_i - t + 1)CF_i}{(1+r)^{t_i - t + 2}}.$$

The formula involving the first derivative, $P_t'(r)$, is known as the **\$-duration** of the bond P, at time t, when the yield to maturity is r. Instead, the second derivative, $P_t''(r)$, is known as the **\$-convexity** of the bond P, at time t when the yield to maturity is r.

How do \$-duration and \$-convexity, play a role in helping us to assess interest rate risk?

Consider initially the case of a first order Taylor's expansion of $P_t(r)$:

$$dP_t(r) = P_t(r+dr) - P_t(r) = P_t'(r)dr + o(dr) \approx \frac{dP_t(r)}{dr}dr \equiv \text{\$-duration} \times dr$$

where $o(dr)$ is a notation to indicate that the additional terms not included go to zero when dr goes to zero, faster than dr does and thus are negligible. Thus, the change in value (dollars or another currency) is approximately equal to $P_t'(r)dr$ plus an infinitesimal term $o(r)$, which can be ignored for small movements in the yield to maturity, r. In other words, the \$-duration measures the change in the price of the bond due to the slope of the relationship between $P_t(r)$ and r, that is it informs an investor about how the price of the bond changes following a small change of the yield to maturity using a first order approximation.

Now, if we divide both sides of the previous equation by $P_t(r)$, we get

$$\frac{dP_t(r)}{P_t(r)} = \frac{P_t'(r)}{P_t(r)}dr + o(dr)$$

and, if we let

$$\text{MD}_{t,r} \equiv -\frac{P_t'(r)}{P_t(r)}$$

which called the **modified duration** of bond P, then $\text{MD}_{t,r}$ provides the relative change in value in the price of a bond at time t for a small change in r; i.e.,

$$\frac{dP_t(r)}{P_t(r)} \approx -\text{MD}_{t,r} \times dr.$$

Summarizing, it is useful to think in terms of profits and losses (P&Ls); i.e.,

$$\text{absolute gain/loss} \approx \text{\$-duration} \times dr$$

and

$$\text{relative/percent gain/loss} \approx -\text{MD} \times dr.$$

NOTE. The last two formulae must be known to answer quizzes! Let's see the formulae at work with a numerical example.

Example. *Consider a bond with a maturity of 10 years, a coupon rate of 6%, and a yield to maturity of 5%. For simplicity, assume that the coupon frequency and compounding frequency are both annual. An analyst has calculated that the bond's \$-duration is -810 and the MD is 7.55. Assume that the yield to maturity decreases by 0.1%. What is the change in price for the bond in dollars and as a percentage?*

Answer. The absolute loss/gain can be computed using the \$-duration. In this case, it is a gain and more specifically $\text{absolute gain} = -810 \times (-0.1\%) = 0.81$ i.e the price of the bonds goes up by \$0.81.
In percentage terms, the gain is $\text{relative gain} = -7.55 \times (-0.1\%) = 0.755\%$.

Remark. Note that if the frequency of coupon payments or the compounding frequencies are not annual, the formula for dollar duration is different. It can (easily) be shown that in the semi-annual payments case we have:

$$P_t'(r) = -\sum_{i=1}^{2N} \frac{\text{CF}_i(t_i - t)}{(1 + r/2)^{2(t_i - t)+1}}.$$

However, we are not going there and leave it for the students to derive a general formula in the case of k-payments a year. Aspiring financial engineers need to be able to do that on their own.

As the relationship between the price of a bond P and interest, r, is non-linear, it is necessary to consider also the second derivative of the relationship. This is even truer if we consider changes in the yield to maturity that are not really quite small This leads us to need to include **convexity**. From a mathematical point of view, we can approximate $P_t(r)$ using a second order Taylor's expansion:

$$dP_t(r) = P_t'(r)dr + \frac{1}{2}P_t''(r)(dr)^2 + o((dr)^2)$$

where this time, we are including small terms like $(dr)^2$, but we are ignoring terms of order $o((dr)^2)$, which we consider negligible for our purposes.

$$(\text{\$-convexity})_{t,r} \times (dr)^2$$

measures the change in the price of the bond due to the curvature of the relationship between $P_t(r)$ and r.

The last formula above, could be easier to remember as follows:

$$\text{absolute gain/loss} \approx \text{\$-duration} \times dr + \frac{1}{2}\text{\$-convexity} \times (dr)^2.$$

That's enough math for us in this course. Let's review what we learned regarding bonds.

- When the relevant interest rate increases, then the price of the bond decreases. If the interest rate decreases, the price of the bond increases.
- The amount by which the price increases or decreases can be determined using $-duration (first order approximation) or $-duration and $-convexity (second order approximation).
- The formulae provide accurate results only if the change in the interest rates are small. If the change in interest rate is not small, say in the order of a few basis points, then the formulae are in general not accurate and, in any case, using convexity and duration together would provide more accurate approximations.
- It should also be clear that the higher the $-duration is, the more sensitive to changes in interest rates the price of the bond is and the more important it becomes to hedge interest rate risk.
- The change in interest rate are assumed to happen in a parallel fashion, i.e. the entire yield curve moves up or down by the same amount (parallel shifts). However, sometimes, the yield curve steepens, flattens, or twists. These non-parallel movements are not handled well by traditional methods of duration. Other methods, including principal components, can be used to provide risk measures of how much variation is caused by tilt, twist, and other types of yield curve movements.

Duration as a concept is more than a century old! It predates computers, spreadsheets, and software, all of which today can easily compute the risks associated with movements of the yield curve that are not parallel. While duration still has its use, when we want to understand the effects that stressed markets can have on bond prices, and fixed income assets in general, we can use duration and other measures to examine the sensitivity of prices to changes in yield.

7. Equity Risk with Heavy Tails

In a healthy market, volatility of a stock would remain constant. Then, its corresponding options can be priced with a constant volatility as if the equity's returns are Gaussian. Of course, markets do not always follow the assumptions we would like them to have. Therefore, the volatility can change in stochastic ways. We may want to treat volatility as a parameter--a constant. But markets tend to show that it behaves like a variable--a time-varying quantity. Variables can produce extreme values, and this gives rise to heavy tails. Perhaps you will hear someone remark, "Wow, this was a big loss for stocks, a 3-sigma move--such an unlikely move." Keep in mind that they are providing a perspective that views the world as Gaussian. Indeed, in the Gaussian world, 3-sigma events are indeed rare: perhaps we get three events every four years! (99.7% of the time, data lies within three standard deviations. That means only .3% of events are outside that interval. With 250 trading days a year, that equates to about three events every four years). Yet, who says that equity returns have to follow Gaussian distributions? At best, they may be Gaussian, but the volatility simply increased; that is, we underestimated volatility. At worst, they are a more complicated structure, such as non-Gaussian, or a mixture of normal random variables, or even a time-changing distribution that requires regime-switching models or other complicated modeling.

As you read this lesson's required reading "Financial Stability Ratio," you'll see that asset valuation depends very much on the compensation investors demand for taking risk—the so-called risk premium. At times, the appetite for risk is so great that there are high asset valuations; that is, stocks are expensive when comparing their price to earnings or similar metrics. In stressed times, that appetite can drop significantly and abruptly, indicating a very different price investors are willing to pay for risk. As the paper indicates, "The risk premium for an asset varies over time and, unlike the price of an asset, cannot be directly observed." This is just one of many reasons why equity valuation is among the most challenging problems in finance, and perhaps one of the areas with the greatest amount of disagreement. Further discussion of asset valuation can be found in the semi-annual report published by the Federal Reserve.

8. Option Risk with Underestimated Volatility

Following up on the previous section, difficulties in modeling volatility lead to difficulty in pricing options. Volatility could be greater than we think for an untold number of reasons. Let's consider several of them.

First, in a Gaussian world, options can be priced with a constant volatility. We just saw that if volatility is a variable rather than a parameter, we should expect volatility to change over time. So what if we based volatility on a relatively low period of volatility, but then volatility is higher? In this new volatile world, returns could still be Gaussian, but we've underestimated the volatility: a statistical error of inferring too small a value for the statistical estimate.

Second, we may have incorrectly assumed that the stock returns were Gaussian in the first place due to heavy tails: kurtosis. Suppose their distribution is closer to a Student's t-distribution. Then, we would tend to underestimate the frequency of large moves by observing the world with our Gaussian glasses.

Third, we may have incorrectly assumed that the stock returns were Gaussian because we assumed symmetry when in fact there are skewed tails indicating asymmetry. Then, we would tend to underestimate the frequency of a negative return because we didn't take this skewness into account.

Fourth, we may have incorrectly assumed that the stock prices could not jump and therefore assumed that extreme moves could only occur if "3-sigma" or "4-sigma" or similar extreme events occur. The Gaussian distribution, however, shows that these types of events are extremely rare.

Fifth, we may have underestimated the costs and complexities of hedging the position, which adds costs to the volatility itself. In other words, real-world costs of hedging make the implied volatility higher than realized volatilities. That makes option pricing risky and hedging more difficult. At worst, we underestimate the size of volatility and the costs of hedging and sell options into the market too cheaply. Then, the options go into the money, and we find ourselves hedging at considerable expense.

Prudent option writing and buying require us to have a solid grasp of both the complexities of volatility and the dynamics of hedging. For example, hedging a put may require the option writer to sell short the underlying stock—-a transaction that requires a financing cost. That financing cost can vary over time and get prohibitively expensive. So not only is there a risk of underestimating volatility, but there is also a risk that we underestimate the cost of hedging.

9. Sector Risk with Factor Exposure

One takeaway from COVID-19 is how fragile the supply chain can be. For example, an automobile can have 99% of its parts readily available, but one of them—-perhaps even the microchips—-are unavailable due to disruptions in the supply chain. These may arise from the inability to source the materials, new regulation that prohibits importing materials from countries that are not considered "free-trade counterparties," the lack of shipping and trucking to transport goods, etc. The consequences are far-reaching (The Board of the International Organization of Securities Commissions). There can be inadequate supply. In addition, the demand can be pent up--during COVID-19, there was greater demand worldwide for automobiles as people reduced their use of public transportation. You can see how a sector like automobiles can be impacted.

On the other side, business travel was greatly reduced during COVID-19, from regular business travelers to conference attendees. Remote work has become increasingly acceptable for many businesses worldwide. Of course, there are still many people who want to take vacations (holidays) and can still choose to travel, but disruptions to worldwide travel significantly impact the airline and hotel industries.

Sectors can be impacted by a variety of factors, including global pandemics. New regulation, geopolitical risk, and inflation affect sectors and can boost or bust a sector. The classical methods of examining which sectors perform well in specific business cycles need updating to provide better analytics. The key is often to find the underlying factors that affect sectors and test their vulnerabilities.

Remember, though, that good ETFs have strength in numbers: diversification by having pooled idiosyncratic risks. ETFs were shown to be very resilient during the COVID-19 pandemic. See https://www.iosco.org/library/pubdocs/pdf/IOSCOPD682.pdf

Key concepts in financial markets:

1. Collateral-Related Concepts: Credit Risk and Financing

1.1 Long Trades vs. Short Trades

Remember that in financial markets, investors can enter a trade in two distinct ways. They can purchase a security they do not own—-this is known as _going long—-_or they can borrow a security they do not own from someone who does own it and sell it—this is known as *going short*.

- When the investor is long, they anticipate the price of the security increases so that they can sell the security at that higher price. When they do so, they close the position and find themselves with a profit as the net result of the two transactions. The plan is to "*buy low, then sell high*."
- When the investor is short, they anticipate that the price of the security will decrease. The investor hopes that they can buy the security at a lower price later. When they buy it back, known as _covering the short, _they receive the security, return it to the party that loaned it to them, and close their position. The plan is "*sell high, then buy it back low*."

What does an investor need in the situation where they go long? In a word, funds. If this is an individual, they need to have cash in their account. If this is an asset manager, they need to have the funds of their clients. Of course, what if they do not have enough funds? They can finance by borrowing money, paying for a portion of the securities with borrowed funds and the rest with actual funds. Long positions have the flexibility of using financing or not, depending on whether the investor wants to be more conservative (using all cash) or invest more aggressively (using financing/leveraging).

What does an investor need in the situation where they are short? They need to borrow the security. Who loans it to them? Someone who owns that security! Well, why do you think someone would loan a security to another investor with whom they neither met nor have financial relationships? The answer is income: the owner of the security will be paid a financing fee. Thus, financing is not free. Anything borrowed has an associated cost. This cost is assumed to be different things in different models. In Black-Scholes, this cost is assumed to be zero. That is not a great assumption. In a paper by the late quant Marco Avellaneda called "Option Pricing for Stocks that are Hard-to-Borrow," the cost is assumed to be a Brownian motion that can vary with the supply and demand forces for that security. This model for financing costs is more realistic and can lead to better pricing in the options market.

2. Credit Risk and Financing

Credit risk and financing are the first set of challenges we address in financial engineering. Credit risk has been defined several times in different places in our course. Think of credit risk in simple terms: a lender gets less money back than what was originally lent or expected back. Recall from the first few lessons you saw credit risk from the bank's side (in a mortgage) and from the depositor's side (with the lender). In 2022, the failure of FTX to secure funds of investors illustrates the credit risk that exists when funds from brokerage accounts are lent out and not returned. The party that is owed money likes to keep track of their counterparty's credit risk. When collateral is in place, the problem extends to financing and a calculation of market risk (of that collateral). We discussed how collateral, regulation, and insurances (like the FDIC) help to mitigate credit risk.

In subsequent modules, you read that lending rates are a function of several key factors, three of which include the term of the loan, the creditworthiness of the borrower, and the type of securitization, if any, tied to the loan. During markets where the yield curve is upward sloping, the rates for short-term loans tend to be lower than the rates for long-term loans. The rates for securitized loans are lower than the rates for un-securitized loans. For example, home mortgage interest rates are lower than consumer credit card rates. If a homeowner defaults, the bank can take the home as a substitution. If a credit card borrower defaults, the bank may have nothing tangible left—the funds could have been spent on massages, products, and services that were consumed or have lost significant value. Finally, the rates for better creditworthy individuals are more favorable than those borrowers that are considered high risk.

4. Understanding the Motivation of the Curriculum

Before we look at the next set, fast forward two years from now. As you're worked through the program, a few of these factors have become your favorite problems. For example, if you enjoy solving problems in credit risk, you could certainly find work in that area. Within the field of credit risk, there are specializations: counterparty credit risk modeling; credit rating agency factor modeling; credit derivative trading; corporate bond portfolio management; structured credit sales; credit risk management; and so on. When you're getting assessed later in an interview, think about the tools you learned. Rather than thinking, “what did I study in Econometrics?” you can ask yourself “what skills did I build in Econometrics that I applied to solve credit risk problems?”

So you may wonder, why not have one course dedicated to each of these areas, like credit risk and volatility modeling, instead of courses like Econometrics and Machine Learning? This relates to the difference between theory and practice. Econometrics has foundations in linear algebra, probability, statistics, and calculus. These are used to build models. Once you have these statistical, mathematical, and machine learning models, you can apply them in many ways to solve problems in credit risk, financing, leverage, regulation, etc. The educational process in engineering could not possibly teach you every application. That’s its strength and beauty. Instead, we apply the models to solve problems in these areas of estimating credit losses, measuring volatilities, and more. Some courses will emphasize a particular factor. For example, Derivative Pricing will illustrate the power of both leverage and non-linearity. Others can be broadly applied to any areas. Machine Learning can tackle problems across the spectrum.

You are encouraged to think of each metric, methodology, or model as a tool, as if you were a carpenter. Imagine you are training to be a carpenter. You’re going to develop skills in modeling, measuring, cutting, adhering, smoothing, etc. To perform these skills, you would learn to use a variety of tools. Likewise, think of these areas as opportunities for skills development: skills for handling credit risk, addressing financing, etc. As you work through the curriculum, think about how the material illustrates how a tool is used, teaches you to use that skill, and builds a foundation to understand its justification. Don’t miss the forest for the trees! Instead of overemphasizing technicalities, focus on the big picture. Ask yourself, "how did this course improve my mathematical, statistical, or computational background to better understand, model, predict, or interpret credit risk? Volatility? etc."

5. Statistical Factor 1: Volatility

A big part of the Financial Markets course addressed volatility and correlation. Volatility means that we have uncertainty around an expected value. We expect a certain return, a certain payoff, a set of cash flows, etc., but we often get different results. Volatility describes that expected difference.

Considering more than one security at at time, we have a covariance—how two or more securities vary together. This depends not only on their individual variances but also on their correlation. Correlation is the degree to which two securities move together. Of all these financial challenges, most of the attention in your studies will focus on ways to measure and manage volatility and correlation. Let’s define volatility and then discuss a dozen of its properties.

- Volatility can mean risk or uncertainty.
- Volatility is indifferent to direction.
- Volatility is nonlinear.
- There are different types of volatilities.
- There are different ways to measure volatility.
- Securitized portfolios have securities with similar variances.
- Volatilities change over time.
- Volatilities do not necessarily scale with time.
- Volatilities can be implied from derivatives, especially volatility derivatives.
- Volatilities affect other variables.
- Variances are the diagonal of the covariance matrix.

5.1 Volatility can mean risk or uncertainty.

Volatility is often used synonymously with risk, a term people use often: risk management, risk aversion, risk premium. Risk typically describes situations where the outcomes—their distributions and probabilities—are known. Let's say we're flipping a coin many times. If all the events are independent and we run lots of them, the long-term results should converge to a normal distribution. Then, we could characterize the risk through metrics like Z scores, which rely on standard deviations. The calculations and methods you know and use from statistics find a home when discussing risk. Although we don't know what a specific outcome is, we have general comfort in knowing what the distribution is. When you studied statistics, you learned of these different distributions. Some were Gaussian. Others were skewed or kurtotic. That is, many have heavy tails compared to the normal distribution. The heavier the tails, the greater the chance of an extreme outcome (assuming we don't have disproportionately more chances of zero outcomes). In simpler terms, volatility is more uncertain in heavy-tailed distributions. For example, in 2022 Q4, the New York Stock Exchange set a record: the market moved down and then up so much that the total distance covered was more than ever before. Although there have been larger moves in one direction on a given day, the record was the comeback within a single day. This reflects very high intra-day volatility. It means that the distribution that we have may not be Gaussian.

One thing that makes volatility difficult is distinguishing risk (known distributions) from uncertainty (unknown distributions and/or outcomes). How do we know that we have the right distribution? If we had some understanding of the shape and the value of the parameters, then we're using classical or Bayesian statistics to draw inferences. For example, parametric Value-at-Risk (VaR) requires a distributional assumption; then, we pick a percentile and report that losses do not exceed a specific value, say, 99% of the time. When we are unsure of the distribution, or even the outcomes and probabilities, we switch the terminology from risk to uncertainty. Perhaps the return of Bitcoin falls into this category. Uncertainty means there's less consensus (if any) on the underlying distribution(s). We may have unstable parameters, changing shapes, or mixtures of distributions. For example, you may have a distribution known as a mixture of normals—a combination of 2 (or more) normals, each with its own mean and variance, plus a proportion of how they mix. Perhaps the second normal has the same mean, but a much larger standard deviation. You put these together, and you have a mixture of normal distributions that introduces some of the heavy-tailed nature that you'll see. The parameters of the mixture (for the bivariate normal there are 5) may be time-varying.

Risk and uncertainty describe what you'll see in this whole master's program as volatility. In the Financial Markets course, Modules 2 and 3 presented to you two different asset classes. You saw both cryptocurrencies and stocks. Those were chosen because some asset classes are a lot more volatile than others. Even within stocks, you have some stocks that are much riskier than others because they are companies that are younger or perhaps just belong to industries that inherently have more uncertainty.

5.2 Volatility is indifferent to direction and semi-deviation.

Indifference to direction can be interpreted by saying that if the order of the elements is changed, their standard deviation does not change. In other words, we could organize the data in ascending or descending order and the value of the volatility would not change. This uncertainty is indifferent to direction. When risk increases, there are greater chances of moves in either direction. Volatility relates to the size of a move rather than its sign. Consider a random variable that is bounded; for example, default probabilities lie between 0 and 1 inclusive. Suppose the current value is 1%. Increasing volatility will mean it can move in either direction, but to be sure the boundary at 0 means there is much less room to move on the downside than on the upside.

However, we could also interpret "*indifference to direction*" as a lack of distinction between "good" and "bad" volatility. For instance, assume that an investor buys shares of XYZ at a price of \50/share. Then, suppose that the price keeps going up almost daily and that 1 year later the shares of XYZ are valued at $120/share. The stock has exhibited a good amount of volatility during the last 12 months, but none of the investors who bought the shares of XYZ would be upset. On the other hand, if the stock kept going down and 1 year later it stood at, say $15 per share, investors in XYZ would be very upset. Volatility does not distinguish the good direction from the bad one and if were were to assess risk from the volatility alone, we might conclude that the investment was riskier when it went from $50/share to $120/share then it was when it went from \50/share to $15/share. Clearly, that does not seem to cut it.

For this reason, portfolio managers, besides regular volatility, may compute what is known as **semi-deviation**. The downside semi-deviation measures the volatility or dispersion of negative returns below a specified threshold. Let B represent the desired target return; the the semple target semi-deviation formula is given by

$$s_B = \sqrt{\sum_{r_i \leq B} \frac{(r_i - B)^2}{N-1}},$$

where N is still the number of observations in our sample, but the sum considers only those returns, r_i's that are below the required threshold B.

Example. *Fund MNO over the last 10 years have achieved the following returns*:

$$4.5\%, 6.0\%, 2.2\%, -2.0\%, 0.0\%, 5.2\%, 4.1\%, 2.5\%, 5.5\%, 4.3\%.$$

- *Compute the standard deviation of returns for fund MNO over the given 10 years*;
- *Compute the semi-deviation of returns over the same period for fund MNO if the target return is* 2%.

Answer. The standard deviation can be computed using the usual formula for the sample standard deviation and it is equal to 2.57%.
To compute the required semi-deviation, we need to use the formula given above with $B = 2\%$, and $N = 10$. Then, since only the returns less than or equal to 2%, matter, we find

$$s_B = \sqrt{\frac{1}{9}[(-2.0\% - 2.0\%)^2 + (0\% - 2\%)^2]} = 1.49\%.$$

The notion of semi-deviation just introduced focuses on the volatility of returns below a specified threshold or target return and allows us to better estimate the likelihood of experiencing negative outcomes.

5.3 Volatility is non-linear.

Volatility is non-linear. If we were to combine the volatilities of 2 securities, it is not simply their weighted average. Correlation affects the total. We'll see this explained in a few points.

5.4 There are different types of volatility.

Sometimes, the word volatility will have another term in front of it. You will see historical volatility, realized volatility, implied volatility, local volatility, stochastic volatility. Some of the distinction is due to the securities used to calculate the volatility. Other distinctions are based on the type of model used.

5.5 There are different ways to measure volatility.

Volatility can be measured with different metrics: variance, semi-variance, inter-quartile range. It may use different data that results in different metrics. For example, using prices we can compute the price range, high price minus low price. Using returns, we can compute variances.

5.6 Securitized portfolios have securities with similar variances.

Recall in discussions of products like MBSs, the securities should have similar risk characteristics. This would facilitate modeling their prepayments. Imagine if the mortgages had very different maturities, interest rates, types (fixed or variable). It would be incredibly difficult to model the overall MBS prepayment. Securitization relies on having securities with similar risk profiles.

5.7 Volatilities change over time.

Sometimes, we'll assume volatilities are constant. (See the Black-Scholes model.) As we develop stronger modeling skills, we'll assume volatilities change. GARCH, by definition, models a changing variance: the H represents heteroskedasticity, from the Greek words for "different variances." In modeling option prices, stochastic differential equations will provide more flexibility to include variance as an additional source of variation aside from the underlying. One of the key choices we'll make in any model is which quantities are parameters versus which ones are variables.

5.8 Volatilities do not necessarily scale with time.

Volatility depends on the timescale on which it was measured. One thing that you really need to understand early and appreciate as you start collecting data and think about models is at what frequency you're measuring data. Most of you will start by looking at daily closes: take the end-of-day price. That's well defined because markets have an official closing price. Some markets trade 24/7 (e.g., Bitcoin), so there is no close. Perhaps you trade high-frequency and need to examine volatility within minutes or seconds. The key point here is that volatility over, say, 1 minute, is a neat fraction of the volatility for a longer period. Sometimes, you may assume volatility scales like a Gaussian, but this is often not advised.

5.9 Volatilities can be implied from derivatives, especially volatility derivatives.

Options imply volatilities. There are different types of derivatives: vanilla options, variance swaps, volatility swaps, etc. Through calibration techniques you'll learn later, you can see how market prices imply volatilities.

5.10 Volatilities affect other variables.

Higher volatility—due to uncertainty in the market—affects liquidity. Similarly, diminishing liquidity can amplify volatility. They affect each other in complex ways. The same can be said for most of the other financial challenges.

5.11 Variances are the diagonal of the covariance matrix.

Within a covariance matrix, the diagonal element contains the individual variances of the securities. So securities that are more volatile will have higher entries. The correlation matrix is scaled, so that can be a more interesting way to see relationships on a more normalized basis.

In short, more volatility means we can expect to be further away from an expected outcome or perhaps even an outcome we never imagined.

6. Statistical Factor 2: Correlation

Let's emphasize some stylized facts about correlation, specifically things that are in the notes for this module and that relate to the previous two modules things. You'll also see these in upcoming modules and even upcoming courses.

Let's consider a dozen properties that are useful to know about correlation from the onset.

- Correlation is bounded and does not have units.
- Correlation does not have a direction.
- Correlation measures linear or monotonic association.
- There are different types of correlation.
- There are different ways to measure correlation.
- Correlation does not imply causality.
- Good regressions have low correlation between predictors.
- Diversified portfolios have low correlation between securities.
- Correlations change over time, especially when stressed.
- Correlation affects over variables (e.g., wrong-way exposure).
- There are alternatives to correlation.

6.1 Correlation is bounded and does not have units.

Correlation is bounded between -1 and 1. Most asset correlations are positive. Extreme correlations (-1 or +1) are rare in practice.

6.2 Correlation does not have a direction.

Correlation does not have a direction. This means that we can swap the order of terms. For example, the correlation of net income and profitability is the same as the correlation of profitability and net income. Note that this is not true for regression. The regression of net income on probability is a different model than the regression of profitability on net income. Regression is directional; correlation is not.

6.3 Correlation measures linear association.

Pearson correlation measures linear association. Recall in our discussion of correlation we also studied Spearman correlation. Spearman measures directional association. Suppose you have two variables, x and y, and they are related through a CAPM-like linear relationship. Graphing y versus x shows a linear relationship. If you switched the relationship to a quadratic, where y contains a squared term in x and a linear term in x, the correlation is not as linear. The higher-order term introduced non-linearity, which would decrease the correlation. You can still have as strong of a relationship, when you include the x-squared term; it is just not linear.

There are examples of nonlinear relationships we have already seen. Recall the graph of a bond's price versus its yield. We get an inverse but convex relationship: the higher the price, the lower the yield. The Pearson correlation of price and yield is not minus one because of convexity. Compare that to the Spearman correlation. Think of Spearman as doing the following: take the correlation of the rank of x and the rank of y. Using ranks instead of values focuses on the direction rather than the size of the move. The Spearman correlation of price and yield is a negative one. In other words, if we increase the price, we lower the yield; we don't do it in a linear way. When the yield is high, small changes in yield cause relatively small changes in price. When the yield is low, small changes in yield cause relatively big changes in price. Those price-yield sensitivities are a result of convexity. When the yield increases, the price increases. Therefore, the ranks of the yield and the ranks of the prices are negatively correlated. Recognize the difference between the Pearson and Spearman correlations: Spearman removes the linear part of correlation and focuses on direction.

6.4 There are different types of correlation.

Depending on the variables we compare, we can calculate different types of correlations. Using two sets of stock returns, we can compute equity correlations. Using two sets of yield, we can compute yield correlations. Using two sets of default probabilities, we can compute default correlations. In credit risk models, we may compute asset correlations.

6.5 There are different ways to measure correlation.

Classical correlation is done using Pearson correlation. There are non-parametric versions like Spearman correlation and Kendall correlation. For time series, we compute autocorrelations and partial correlations. In your studies, be sure to pay attention to any word in front of correlation to understand the specific type of correlation being calculated.

6.6 Correlation does not imply causality.

When we see correlation, we should not assume causality. We don't want to assume when we see correlated events that one causes the other. There may be confounding variables. Let's take a classic example. Suppose you find that a child's reading score and shoe size are correlated. Would you say that having bigger feet improves reading ability? The confounding variable here is age. Age and shoe size are correlated; as kids age, their feet grow. Age and reading ability are correlated; as children learn more, they improve their reading. Sometimes, when examining economic variables this is not so obvious because these variables like interest rates, inflation rates, and unemployment rates affect each other in complex ways. There's not a clear cause-and-effect relationship in observational markets. Finance and economics are physical sciences where we are better suited to conduct controlled experiments. Remember, from property one, correlation is not directional, so we can't get a good sense of what comes first.

6.7 Good regressions have low correlation between predictors.

When you use multiple predictors or factors in regression, you don't want them to be correlated! If you run a linear regression, you don't want to use two factors that are highly correlated with each other. Suppose you're trying to predict the price or return of a company. Say that you include both the market cap and the market share. Imagine for the selection of stocks in your study, these two variables are highly correlated. The predictors are bumping into each other. You don't want to have too high of a correlation of your predictors. They don't have to have a zero correlation, but you don't want one too high; otherwise, you create problems in the regression. The problem is a violation of one of the regression assumptions about residuals being independent of each other. In some examples you'll see, we'll completely eliminate this problem when we use principal components to transform our data into uncorrelated factors. This is a reason to upgrade your math skills so that we're not just measuring correlation.

6.8 Low correlation enhances diversification.

Fifth, just as in our regressors, we prefer to have relatively low correlation in our portfolio. This low correlation facilitates diversification. We'll get to portfolio examples well before the portfolio management course. For now, think that when there is perfect correlation, there is absolutely no diversification. Imagine a vertical line stretching from 1 to -1. Correlation can take any value along that line. As you move down from one to minus one, correlation decreases and diversification increases. If you had a correlation of point nine, you'd still have some diversification, but not much. Recall from Module 3 how diversification benefit is defined. We had an equally weighted portfolio of two stocks, one with a standard deviation of 3 and one with a standard deviation of 8. When the correlation was minus 1, the overall standard deviation was less than the standard deviation of the "safer" one: -2.5. As the correlation increased, the standard deviation increased in a non-linear (i.e., with curvature) way. When the correlation was 1, the standard deviation was just a straightforward average of the two securities. Clearly, as correlation decreases, the diversification benefit increases because the standard deviation decreases. The counterintuitive part is that there is a correlation that makes a combination of the two less risky than either individual security. That's the power of diversification. Incidentally, we call a correlation of -1 a perfect hedge. However, a perfect hedge is not necessarily desirable since it would protect unwanted outcomes but at the cost of wiping out any chance of profits.

Note also you can perform these calculations in your head. The securities have standard deviations of 3 and 8.

With perfect negative correlation, one offsets the other. The difference between eight and three is five: split five in half (with securities equally weighted) and you get a 2.5 standard deviation –remarkably, lower than either security.

With perfect positive correlation, their risks are additive. The sum is 11, making their average 5.5. That is the standard deviation: 5.5.

Due to the relationship between standard deviation and correlation being nonlinear, that's all the calculations you can do in your head. If the correlation is 0—centered between -1 and +1—the standard deviation is NOT 4.0, the average of 2.5 and 5.5. That is because it is nonlinear.

The fact that a combination of securities can be less risky than either security individually, even if both are held long (and not short), is a common interview question: "Can a 2-stock portfolio be less risky than either security in it?" *The answer is "Yes, depending on the correlation!" At minus one, that's possible.*

6.8 Correlations change over time.

Like volatilities, correlations change over time, especially in stressed markets. Ironically, when correlation is the most important—in stressed periods where you would really want diversification and therefore low correlation—correlations may increase. A common example is equity portfolios that contain domestic and foreign stocks. Suppose the average correlation between a domestic stock and foreign stock is 40%. The foreign stocks are there to diversify the portfolio with other economic regions. Suppose the markets are shocked, and the correlation increases because common global macroeconomic factors have affected both equity markets. Now, the average correlation is 70% instead of 40%. Therein lies the problem: the sense of security from the lower correlation and higher diversification is false because in a stressed market the correlation is higher and diversification is lower. As we've seen, lower diversification means the risks are greater. But even if markets aren't stressed, correlations—just like volatilities—can change over time. There's a model that allows you to look at volatility conditionally changing over time. This is a model from the same inventor of GARCH, Rob Engel, and the model for correlations is called the dynamic conditional correlation model. This is just to say that it could be treated as a variable.

6.9 Correlation affects other variables.

Let's revisit credit risk. Recall the formula for expected loss. The formula contains three variables. The expected loss is the product of the probability of default, the loss given default, and the exposure at default. Correlation seems like it doesn't come into the picture, but probability of default and loss given default are positively correlated. Get one wrong and you are likely to get the other one wrong too. In a relatively safe state of the world, there are low default probabilities and low losses given defaults. In a less safe state of the world, there are high default probabilities and relatively high recoveries. Estimate correctly and the expected loss is accurate. Underestimate and the loss is very understated.

6.10 There are alternatives to correlation.

The last property to mention about correlation is that sometimes you don't even want to use it. For example, in a non-stationary environment, you may prefer to find co-integrating variables that can be used instead of correlation. You may want to model joint distributions using copulas; in this case, a correlation value is still needed, but the bivariate relationship is modeled through the lens of a particular copula distribution.

Correlation is not the only way to describe how two variables behave together, but it is often used to discuss how things are related. No one on the financial news discusses cointegration, and even fewer people mention copulas. That's why we're in financial engineering: to enhance our toolkit. These are not difficult topics compared to other mathematics you will see. Once you've had some econometrics and stochastic models and machine learning, your toolkit equips you to eliminate assumptions and model more insightfully.

7. Statistical Factors Combined: Structured Finance Tranches

In our tour of these financial challenges, we categorized the concepts in groups of two. They could occur in any combination. Let us take an example of how correlation and credit risk combine. To do so, let us examine the world of structured finance tranches. For simplicity, suppose you have a collateralized portfolio whose cash flows feed three different tranches:

- Senior tranche
- Junior tranche
- Mezzanine tranche.

The mezzanine tranche absorbs the first 15% loss of the portfolio.

The junior tranche absorbs the next 10% loss of the portfolio.

The senior tranche absorbs the final 5% of the portfolio.

Now, suppose there is LOW default correlation in the collateralized portfolio. Low default correlation means if one security defaults, the others are not affected in any direction. Zero correlation means uncorrelated. If the default probability is low, then we should have a distribution that is skewed. Consider now the situation of a stressed market. Now there is HIGH default correlation in the collateralized portfolio. Suppose it goes to a default correlation of one. Now, all the securities either survive or default.

Let us consider how the default correlation affects these two tranches. The equity tranche is adversely affected by low default correlation. A few defaults and the equity tranche could be wiped out. However, the equity tranche benefits from a high default correlation. Why? Suppose the default correlation is one. This puts the equity tranche in the same risk category as the senior tranche. Either everyone defaults and no one is paid, or no one defaults, in which case equity receives a higher return. The senior tranche benefits from a lower default correlation because it has the subordinate equity tranche top-subsidize the first set of portfolio losses. Thanks to low default correlation, it is highly unlikely that a massive number of securities default, so the senior tranche's payments are protected. However, the senior tranche suffers from a high default correlation. An extremely high default correlation concentrates the risk, putting all tranches into the same position: either all securities survive or they all default. In that case, the senior has the same risk but paid much more for the tranches because it was considered safer. Recall that senior tranches could be rated AAA, but equity tranches are not even rated.

Leverage and non-linearity:

1. Magnifying-Related Factors

Leverage and non-linearity are magnifying-related factors. They help to amplify returns. Leverage increases the absolute value of returns by simply having a greater size of investment. Non-linearity can amplify returns by having an influence that exceeds linear increments. There are different methods by which leverage and non-linearity operate. Let's begin with leverage and see different contexts in which it is used.

2. Magnifying-Related Factors: Leverage

Leverage can refer to different concepts or have different contexts. Let's visit each in turn.

2.1 Leverage as Force

Sometimes, people use the term leverage to describe a force or an exertion. Consider our example of a seesaw (or using the British term, a teeter-tooter). You could have an adult and child stand on opposite sides of the pivot point of a seesaw. Yet, if the child sits further away, the extra distance provides a leverage that allows the child's force to equal that of the adult. How does this example apply to bonds?

Consider a bond's duration. Suppose you treated each of the five cash flows of a two-year note (four coupon payments, one principal payment). Imagine you took a two-year note that has cash flows and a principal repayment. Now let's balance these along the seesaw. Bear in mind that the cash flows are proportional to their discounted values (so the discounted cash flows are not the same size). How does this example apply to a bond?

Consider a bond's **Macaulay duration**, or simply *duration*, (for a bond that pays coupons semiannually) assuming that the relevant time is $t = 0$:

$$\mathrm{MacD}_{t=0,r} = -(1+r)\frac{P'0(r)}{P_0(r)} = \sum_{i=1}^{N} \frac{CF_i}{(1+r/2)^{t_i}} \times \frac{t_i}{P(r)}$$

where, as before, r is the yield to maturity, $P_0(r) =$ present value of all cash flows (i.e., the current price (current as in $t = 0$) of the bond), CF_i is the cash flow at time t_i, with t_i being the time of i-th cash flow.

This formula for duration was first proposed in 1938 by the Canadian economist Frederick Robertson Macaulay and perhaps the careful student might have noticed that we have (in the case of bonds that pay annual coupons):

$$\mathrm{MacD} = (1+r) \times \mathrm{MD}.$$

The duration is the weighted average time to receive the cash flows. In fact, you should notice that the formula can also be written as

$$\mathrm{MacD}(P)_{t=0} = \sum_{i=1}^{N} t_i w_i$$

with

$$w_i = \frac{\mathrm{CF}_i}{P(r)(1 + r/2)^{t_i}}.$$

Let's make a few important considerations.

Macaulay duration is always less than or equal to the maturity and, more precisely, it is equal to the maturity of the bond only when the bond is a zero-coupon bond.

Note also that if we increase the coupon size, we're adding heavier members in the front, which then shortens the duration. If we increase the maturity, we're adding distance to the principal payment, and this increases the duration.

Duration represents an equivalence: a zero-coupon bond that behaves like the original bond. Suppose you were to replace all these cash flows with a single zero-coupon bond with a maturity equal to the duration.

Then, we have two bonds that would experience similar sensitivities when the yield curve changes in small, parallel shifts. This risk equivalence was the original intention of the Macaulay duration.

Even though the units of duration are time, duration was intended to measure risk, as we stated in Lesson 1 of this module already, and it says that we can replace a particular bond with a zero-coupon bond whose maturity matches the bond's duration.

Of course, if the yield curve is shifting, if it's tilting as it is right now in many countries, we have an inverted yield curve. If it's going down, then that analogy doesn't hold, and that's why we can't just use duration to measure interest rate risk. Do not ever forget that all duration formulae presented so far require that interest rates along the yield curve move via parallel shifts or the formulae are inappropriate.

2.2 Leverage as Financing

While we're discussing bonds, let's consider another way to think of leverage: borrowing money to increase the size of your investment. Most of the time when people think of leverage, they might think of borrowing cash. Suppose a trader has a strategy on which they make a ten percent return over a year. A $100 investment would be worth $110, earning a $10 capital gain. This is just a straight cash investment. Imagine you can borrow $50, combine that with $50 of capital, and have that same $100 invested. (For simplicity, let's assume 0% interest on borrowed funds). The $100 investment is still worth $110, earning a $10 capital gain.

Now, when we compute the returns of each scenario, we notice that the first scenario returns 10%, while the second returns 20% ($10 capital gain on a $50 initial investment). The 10% return was doubled because the investment was leveraged 2-to-1. The two to one means you double the ten percent return you would have gotten, and that takes the ten percent return to twenty.

Recall the lesson in the module discussing leverage where you buy a home, say it costs $200,000. Let's say that you put twenty percent down, and you borrowed eighty percent. In fractional terms, you're putting one fifth down ($40,000) and financing 80% ($160,000). Suppose the home appreciates ten percent to a value of $220,000. You sell the home. You pay back the mortgage. Now you've made $20,000 on an initial investment of $40,000. That's a fifty percent return. That's because you were leveraged five to one. These results look very promising.

In fact, let's say that you're a hedge fund, and you're investing in some type of futures. Contract futures are among the most leveraged investments you can make. Suppose the fund invests $5 down and borrows $95. Suppose the investments makes 10%, or $10. Calculating the return on the initial $5 investment, we have a 200% return. Thanks to a leverage factor of 20-to-1.

The more leverage you have by borrowing funds, the more a return is amplified. Like volatility, leverage is agnostic to direction and works to improve gains or aggravate losses. Leverage is like time-lapse photography. Have you ever seen a nature show where the film is sped up so you can see the blooming of a flower over a period of seconds instead of weeks? That's what leverage does to returns; it speeds them up. Leverage helps small returns become large returns. The news is great for gains but terrible for losses. A leverage of 10-to-1 means a ten percent return becomes a 100% gain, but a -10% return means a 100% loss! Worse, a 20-to-1 leverage means a 10% drop becomes a 200% loss. Such losses could wipe out a trader, portfolio, or business! Where leverage is involved, risk management needs to provide checks and balances.

2.3 Leverage through Derivatives

Options do not allow financing: their premiums must be paid in full. Yet, options offer a second way to enact leverage: through the multiplier factor. For example, one equity call option grants the holder the right to buy 100 shares of the stock. Suppose you bought one option. Using boundary conditions, we've seen that this costs less than 100 shares of stock. Now suppose the stock price increases. This results in a much larger increase in the call price. There is not a simple number (e.g., 100-fold) due to the non-linearity and time value, but it is much larger than the amount the stock increases. Thus, derivatives provide leverage through a multiplier factor. Depending on the derivative, one contract tends to control many shares of the underlying securities.

This multiplier factor is true for futures. For example, one future contract could also be for 1,000 barrels of oil. Moreover, futures have financing built in as well. Many futures contracts typically require only a 2% to 5% deposit. So a bullish investor would be able to take an even larger position by entering a futures contract, which uses both financing and a factor multiplier to access a greater number of underlying securities by using less capital.

2.4 Leverage and Credit Risk

Everything that we've described previously is affected by leverage. Let's go through them one by one. As an example with credit risk, suppose you apply for a mortgage. One of the factors discussed was the debt to equity. Prior to the Great Financial Crisis (GFC), it was relatively easy to get mortgages where the homeowner puts down less than 20%. Say the home is $100, and the down payment is $20. The problem is if the value of housing drops more than 20%, the homeowner is effectively underwater. Recall the option analogy, where the option is out of the money. We discussed this in Module 4. Prudent credit risk recognizes the leverage (debt-to-equity) of the borrower and ensures it can withstand temporary disruptions in income, housing corrections, or higher interest rates.

So if leverage is used and the result is a huge capital loss, there's a double whammy—the capital loss, and the need for payment of whatever funds were borrowed—the financing costs of using cash or other securities. Suppose the loss is enough to trigger a credit downgrade—after all, an extreme loss wipes out equity and increases debt. This downgrade could make the financing rate more expensive! For example, did you ever read the fine print on your credit card? If you miss a payment on a different card, and the credit card company finds out, they can raise the rate on their card. Why? They have learned the borrower is riskier, and the higher credit risk warrants the lender charging a higher rate.

Since the global financial crisis, many mortgage lending requirements have become stricter to prohibit debt-to-equity ratios that would be excessive. Fifteen years ago, the market was more competitive and relaxed about those; today, that has been made tighter. In part, these stricter controls helped prevent a housing crisis during the COVID-19 pandemic. If you look at the housing crisis during the GFC, there weren't the safety mechanisms in place that are in place today.

2.5 Leverage and Volatility

Equity exchanges have avoided the problem of excessive leverage by setting upper limits on the amount that can be financed. Now let's add in volatility. If volatility is 20%, leverage can turn that into multiples. Imagine in investment that is leveraged 20-to-1. A 1% return looks like a 20% return. But if volatility is sufficiently high to produce a short-term return of 10% then, with simple multiplication, the return is 200%! Extreme values that can make or break the portfolio. Using more volatility or using more leverage will get a portfolio to more extreme values than without. From the outside, it may be difficult to distinguish whether a return was made using more volatility ("risk") or more leverage. One difference is that leverage will always incur some type of financing costs, which still exist even in the case of huge capital losses.

2.6 Leverage and Correlation

Correlation can amplify losses, particularly in credit risk. Consider the securitization examples you studied in previous modules. Higher default correlation causes the shape of the loss distribution to widen. Higher default correlation causes the securities in the pool to either survive together or default together. In a way, this leverages the returns to be extreme. Suppose default correlation increases to 1. Now we would have either complete default or no defaults. Indeed, here, default correlation is behaving like volatility. In your GWP3, think about the option position of the equity tranche.