Portfolio Returns and standard deviations

In the context of portfolios, it is possible for weights to be negative. This is equivalent to selling short an asset. However, we require that the sum of weights is equal to 1. If we assume that $w_A = -0.2$, then $w_B = 1.2$, it's like we shorted asset A and used the proceeds from that short to buy more of asset B. In this case, we would have

$$r_P = -0.2 \times 5\% + 1.2 \times 11\% = 12.20\%.$$

We define this situation as **leveraging the portfolio**. Essentially, we are borrowing money by selling short asset A (the asset with the lower return) and using the proceeds to buy more of asset B (the asset with the higher return). This can be done, but in many situations, a portfolio manager could be prevented from having short positions as clients may consider shorting stocks too risky for their personal risk tolerance. Besides, in our example above, things went well, but we don't usually know what returns we can achieve, and we could actually lose money on asset B, in which case the use of leverage would magnify our negative return.

It's also possible for the weights of a portfolio to sum to zero. In this case, it would be like assuming that we built a portfolio where we are short an asset and we are long the other asset, and the two dollar amounts are the same. We call this portfolio a **dollar-neutral** portfolio. This would be akin to investing without using one's money. Some hedge funds may come close to having positions like this, but for our course now, this is not something to consider.

In a portfolio, weights represent the proportion of market value and not merely the number of shares that we invested in the assets. So let's write the formula in terms of number of shares and prices:

$$r_P = \frac{q_A \times p_A \times r_A + q_B \times p_B \times r_B}{q_A \times p_A + q_B \times p_B}$$

- $q_A =$ number of shares of stock A
- $p_A =$ share price of stock A
- $r_A =$ expected return of stock A
- $q_B =$ number of shares of stock B
- $p_B =$ share price of stock B
- $r_B =$ expected return of stock B

In other words, the weights represent a proportion of market values. A further way to write the formula for the portfolio return is

$$r_P = \frac{MV_A \times r_A + MV_B \times r_B}{MV_A + MV_B}.$$

portfolio variance / portfolio standard deviation

2. Portfolio Standard Deviation

In this section, we are going to look at portfolio variance. The formula for portfolio variance is not as simple as the one for portfolio return. In fact, if we indicate the portfolio variance with the symbol σ_P^2, then

$$\sigma_P^2 = w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2\rho_{A,B} w_A \times w_B \times \sigma_A \times \sigma_B.$$

You will recognize the terms representing weights. There are two new terms:

- σ_A represents the standard deviation (volatility) of stock A;
- σ_A^2 represents the variance of stock A;
- The corresponding terms apply to stock B;
- $\rho_{A,B}$ is a new term that represents the **correlation** between the returns of stock A and stock B.

Know that there are three terms on the right-hand side. Let's examine each term individually:

- The first term on the right-hand side represents the variance of stock A, but note that its coefficient is not merely the weight but rather the weight squared. This square exists because we are in the realm of variance, not standard deviation.
- The second term represents the square-weighted variance coming from stock B. The key point about these terms is that they are always positive. Any number squared here is positive (there are no imaginary weights!). The sum of positives will be positive.
- The third term on the right-hand side represents the interaction of the two stocks. This term itself consists of five inputs: the correlation coefficients, two weights, and two standard deviations.

NOTE. Let's reiterate that we are using long-only portfolios, so all the weights, along with all the standard deviations, are positive.

The correlation value is the only number on the entire right-hand side that does not have to be positive (assuming weights are positive rather than 0 to indicate no position in that stock). It turns out that the correlation plays a key role in determining the portfolio's volatility. We'll have much more to say about this, but for now, let's get some numbers. Since it's helpful to think in terms of standard deviation, let's take the square root of both sides and derive the formula for a portfolio's standard deviation:

$$\sigma_P = \sqrt{w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2\rho_{A,B} w_A \times w_B \times \sigma_A \times \sigma_B}.$$

Let's reiterate that we are using long-only portfolios, so all the weights, along with all the standard deviations, are positive.

3. Portfolio Return and Variance

Recall from Module 2 that we took a stock's volatility and divided it by the stock's expected return. This is the coefficient of variation. We can do the same for a portfolio. Using CV_P to denote the coefficient of variation for the portfolio, we have

$$\mathrm{CV}_O = \frac{\sigma_P}{r_P}.$$

In our previous example, we have a coefficient of variation of

$$\mathrm{CV}_P = \frac{5.162\%}{8\%} = 0.645.$$

We can use this number to compare it to other portfolios and determine the number of volatility points we pay per unit of return. As always, the smaller this number, the greater the "*bang for the buck*." Generally, this number is used in relative comparisons rather than absolute comparisons. Can we compare it to Security B itself? For Security B,

$$\mathrm{CV}_B = \frac{8\%}{11\%} = 0.727$$

Interestingly, the portfolio with A and B has a lower "vol per unit of return cost" than stock B alone. By including stock A in the portfolio, the portfolio somehow has a better risk-return relationship than B alone.

Coefficient of variation of the portfolio: the number of volatility points we pay per unit of return.

4. Portfolio Sharpe Ratio

The **Sharpe ratio** for a portfolio, which we call SR_P, is also easily calculated:

$$\mathrm{SR}_P = \frac{r_P - r_{rf}}{\sigma_P},$$

where

- r_P is the portfolio return;
- r_{rf} is the risk-free rate; and
- σ_P is the portfolio volatility.

If we assume that $r_{rf} = 1\%$, then the Sharpe ratio for the portfolio of our example is found to be

$$\mathrm{SP}_P = \frac{8\% - 1\%}{5.162\%} = 1.356$$

Let's compare it to the Sharpe ratio of stock A and B this time:

$$\mathrm{SR}_A = \frac{5\% - 1\%}{3\%} = 1.333$$

and

$$\mathrm{SR}_B = \frac{11\% - 1\%}{8\%} = 1.25$$

Interestingly, the portfolio with A and B has a higher Sharpe ratio, that is, a higher return per unit of risk/volatility than stock A alone. By including stock B in the portfolio, the portfolio seems to have a better risk-return profile than stock A alone. The same holds for stock B.

5. Portfolio Return and Variance

The calculations we did are in no way specific to stocks. Indeed, this method can be applied to any asset class. Let's discuss some important issues regarding the weights.

The weights represent proportions of market value. The portfolio returns and variance can be used even if the asset classes are different. The weights should reflect the appropriate market value at a given time. Let's examine the formula for weights.

$$w_A = \frac{MV_A}{MV_A + MV_B} = \frac{q_A \times p_A}{q_A \times p_A + q_B \times p_B}$$

When A and B are two stocks, it is likely very easy to ensure that they have observable prices. But what if we combined one asset that is exchange traded and another that rarely trades, such as real estate? Since exchanges have official closes, it is easy to get the stock price correct. Each day we can mark our position using the market's close. This process is known as **mark-to-market**. We mark our security prices to levels indicated by the market. But how would we know the price of an asset without exchange closes or OTC closing prices? We may need a **mark-to-model** price. We would mark our security at levels indicated by a model, provided by our company or perhaps even regulatory requirements. We would need a model to indicate the price, perhaps as often as daily, so we can compute not only expected returns but also volatilities. Without a consistent price series, it is not possible to compute returns, let alone volatilities.

Once we're sure that we have reliable levels, we can then compute returns and volatilities and combine them into their portfolio equivalents. One other consideration reflects that we live in a global market, with different exchanges opening and closing at different times. To be precise, we should find a time at which we agree to compute returns--for example, at the close of business London time. This is known as Greenwich Mean Time (GMT) and is often used as a reference point for time.

When two assets are negatively correlated, they tend to move in opposite directions, i.e., if one asset grows in value, the other tends to decrease in value and vice versa. This should make the value of the portfolio less volatile.

Example. *Consider the same portfolio that we've used several times lately and assume that the portfolio is equally weighted between asset A and asset B. However, this time, we assume that the returns of A and B have a negative correlation. More specifically, let* $\rho_{A,B} = -0.8$.

Using the formula, we find that

$$\sigma_P = \sqrt{0.50^2 \times 0.03^2 + 0.50^2 \times 0.08^2 + 2 \times (-0.80) \times 0.50 \times 0.50 \times 0.03 \times 0.08} = 2.94\%.$$

In this case, the volatility of the portfolio is lower than that of both asset A and asset B.

This last example tells us that if we can find two securities that are strongly negatively correlated, we would say that one *hedges* the other. That is, by using A and B in this case, we are reducing the overall risk (as well as the return because it's typically not possible to decrease risk without accepting a reduction in return as well). Indeed, the negative correlation is the only way that the variance of a long-only portfolio can be less than the variance of each individual security's variance.

From a pedagogical point of view, we have limited ourselves to considering a portfolio consisting of just two assets because this makes it easier to illustrate the main features that matter to us. However, we could easily consider a portfolio that consists of N assets. In this case, the formula for portfolio return remains more or less equally simple, i.e.:

$$r_P = w_1 r_1 + w_2 r_2 + \ldots + w_N r_N = \sum_{i=1}^{N} w_i r_i.$$

The formula for the standard deviation of the portfolio is a little bit more complex but not dramatically so:

$$\sigma_P = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j}}.$$

The general formulae with $N \geq 2$ assets will be needed in the Portfolio Management course, among other places. In fact, in that course, we will see how to express returns and variances/standard deviation in terms of vectors and matrices.

Correlation:

1. Diversification by Lowering Correlation

Diversification is the absence of perfect correlation. You've often heard that people want to have more diversification in their portfolio. In finance, diversification is an investing strategy used to manage risk. Instead of concentrating money in a single company, industry, sector, or asset class, investors diversify their investments across a range of different companies, industries, and asset classes.

Let's be clear about it: diversification is not a strategy aimed at maximizing portfolio returns. In fact, investors who concentrate capital in a limited number of investments might, over a specific time, outperform a diversified investor. However, over time, a diversified portfolio generally outperforms the majority of more focused ones.

One key element of diversification is owning investments that perform differently in similar markets. For example, when stock prices rise, bond yields tend to fall in general. We describe this by saying that stocks and bonds are negatively correlated. While not each and every investment in a well-diversified portfolio will be negatively correlated, the goal of diversification is to buy assets that do not move in lockstep with one another.

Correlation is a statistical measure that can range from -1 to 1, and if correlation equals 1, then there is no diversification. Whenever the correlation is less than 1--by any amount--you have diversification. Although it may not seem like it, there is a diversification benefit even at a correlation of .99. The diversification benefit is not much, but it exists. Now if the correlation decreases, the diversification benefit increases. In fact, the lower correlation goes, the greater this benefit becomes. If correlation goes down to as far as -1, then the diversification benefit is maximized.

In the next section, let's compute the covariance with different values of correlation. Then, we'll quantify the diversification benefit.

A correlation of -1 means that you have perfect negative correlation. In financial terms, this is known as a perfect hedge. In the real world, it would be extremely difficult to find an example where two securities have a correlation that is -1. As you can see from historical correlations, it is hard to get close to either -1 or +1.

In the real world, it's extremely difficult to find an example where two securities have a correlation of -1 because it would mean that any time one security gains, the other is always losing, thereby reducing the volatility of the combined portfolio. A perfect hedge is a trader's dream whereby they can hedge the risk of a particular security, such as a derivative, with a perfect negatively correlated underlying that would effectively guarantee that there's a risk-free component. Indeed, in the theoretical world, such a case does exist. When we derive a formula for option pricing, we will find exactly that: a correlation of -1 that gives us a perfect hedge.

In our last example, we can see that when the correlation is minus 1, we obtain the minimum variance portfolio (for our two assets). Notice that this result is true as long as we require that all assets have nonnegative weights. The result might seem unintuitive, but to emphasize, a portfolio can have a smaller standard deviation (or variance) than either of the individual standard deviations (or variances).

If we were to allow for short positions (negative weights), the result could fail to hold.

If, in the same example, we were to choose $\rho_{A,B} = -0.77$, we would end up with the same standard deviation as the asset with the lower standard deviation:

$$\sigma_P = \sqrt{0.50^2 \times 0.03^2 + 0.50^2 \times 0.08^2 + 2 \times (-0.77) \times 0.50 \times 0.50 \times 0.03 \times 0.08} = 3.00\%.$$

Generally, it's not possible to match the higher standard deviation.

As we found with $\rho_{A,B} = -0.77$, we have the same variance as Stock A. Should we expect a better return with Stock A alone or with a 50% A and 50% B portfolio, given that there is -77% correlation?

Having some amount of Stock B will only increase the return, so clearly, the portfolio with A and B should have a higher expected return. This illustrates the benefit of diversification: a higher expected return for the same amount of risk. Therefore, the portfolio's Sharpe ratio exceeds that of Stock A.

The takeaway is this: portfolios can perform better if we simply add another security to which there's a small amount of correlation but a higher expected return.

Constructing portfolios is not our goal here. That will be the goal of a course devoted to Portfolio Management. But for now, it's important to understand that correlation determines diversification. Let's see what happens when you set correlation to 0.

The takeaway is this: portfolios can perform better if we simply add another security to which there's a small amount of correlation but a higher expected return.

In the case of zero correlation, we should notice that we end up with $\sigma_P = 4.27\%$ This number is less than the straight average of 5% of the standard deviation of assets A and B. This confirms that standard deviation is not a linear entity.

Finally, in the case where $\rho_{A,B} = 0$, we observe that zero correlation means that if one security has a positive return, the other security can have a positive or negative return with equal probability. Knowing the direction of one return tells us nothing about the direction of the other one. We end up with $\sigma_P = 4.27\%$, a value in between $\sigma_A = 3.00\%$ and $\sigma_B = 8.00\%$, but not equal to the midpoint of σ_A and σ_B.

As our final example, when $\rho_{A,B} = 1$, we end up with $\sigma_P = 5.50\%$, which is exactly the midpoint between σ_A and σ_B. In other words, when there is perfect positive correlation, then the variances combine with each other. 8% + 3% = 11%. The average of these is effectively half that number: 5.5%.

Despite our final example, do not lose sight of the fact that the effects of standard deviation on correlation are not linear. Between correlations of -1 and 1, we do NOT expect to see a straight line but rather a curve.

Students can utilize this Shiny application to observe how optimal portfolios are constructed along a curve called the **Efficient Frontier**.

4. Correlation as Linear Association

There are three ways to measure correlation:

- Pearson correlation
- Kendall correlation
- Spearman correlation

4.1 Pearson Correlation

The first formula, **Pearson correlation**, is the simplest, and it measures the strength of the linear association between two random variables. Its formula is:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$$

Where ρ_{AB} is the correlation coefficient between A and B

σ_{AB} is the covariance between A and B;

σ_A is the standard deviation of A; and

σ_B is the standard deviation of B.

Pearson's coefficient measures the degree of linear association. One way to think of it is to assume you have two variables, X & Y. You graph one versus the other. Then, you fit the best regression line to them. Correlation asks the following question: how tightly wrapped around that line are the data points? Suppose all the points were along the line. Then, you have perfect correlation. If that line is sloping upwards, then you'd have a perfect positive correlation. If the line were downward sloping, then you'd have perfect negative correlation. In both cases, you don't see a cloud; you would just see points on the line. The more the points resemble a cloud that lacks a slope, the closer to 0 that correlation is. In all these cases, correlation is using a line as a reference and measuring the tightness around it.

In finance, however, we may not be merely interested in linear relationships. Sometimes, we may be more interested in co-movements, even if they are not linear. There are other ways to think of correlation, the most important being as a measure of the degree of monotonic association. Note that a linear association is just a simple case of a monotonic association; therefore, the next two measures are more general and more powerful than Pearson's correlation.

Pearson correlation

To be able to use Pearson's correlation, we need the observations, i.e., the pairs (X_i, Y_i), to be independent and normally distributed as well. Furthermore, there must be no outliers in the data.

The two methods that are presented below do not require normality. As such, we sometimes read that Pearson's correlation is a parametric measure while Spearman's and Kendall's correlations are non-parametric.

NOTE (Optional, but worth reading). Why is normality required? This is a question that has been debated in many forums on the internet. It is technical and thus beyond the purpose of this lesson and course. However, it is good to have at least a qualitative idea of why normality is important. We have said that Pearson's correlation coefficient can range from -1 to 1. Well, if the data is bivariate normal, the bounds -1 and 1 are achievable. However, if we assume that the data are bivariate log=normal instead of normal, then the possible range of Pearson's correlation can be shown to be $[-0.37, 1]$. In another instance where one variable is normal and the other is log-normal, it is possible to show that the range for Pearson's correlation would be about $[-0.76, 0.76]$. The author of this lesson learned these insights about the achievable range for Pearson's correlation coefficient from a discussion with Karl Ove Hufthammer.

4.2 Spearman Correlation

Let's consider a functional relation between X and Y such that, for example,
$Y = X^3$. Specifically, let

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8.9, 10\}$$
$$Y = \{0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000\}.$$

In this case, we compute Pearson's correlation, which turns out to be $\rho_{X,Y} = 0.9095$. This is high, but since we have a perfect relationship between X and Y, ideally, the correlation should be 1. The Pearson correlation fails to capture that perfect relationship because it focuses on linear association and penalizes the cubic association for being nonlinear.

Now, to alleviate this issue, we can employ the **Spearman correlation**, also known as Spearman's ρ. This correlation is more work than Pearson's correlation.

Spearman's formula replaces the data points with the ranks of the data points. In other words, imagine if you had 10 data points in each set. You're going to replace the value of X with its rank: 1 refers to the largest data point, 2 to the second largest data point, and so on. Similarly, you do the same for Y : replace the largest value of Y with 1, replace the second largest value of Y with 2, and so on. If there are ties, then all those ties are assigned the same number. In our case, suppose there are two numbers tied for the largest. Ordinarily, these would have ranks 1 and 2. We average the ranks (1.5) and then assign each of these numbers to 1.5. Now, we have a set of ranks for the X values and a set ranks for the Y values. Now we apply the Pearson correlation to the ranks of the data. This method is exactly what the Spearman correlation does.

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For each data point, Spearman computes a value, d, between the 2 ranks.

$$d_i = \text{rank}(X_i) – \text{rank}(Y_i)$$

Then, we square and sum this value

$$d^2 = \sum_{i=1}^{n} d_i^2$$

The Spearman correlation is then

$$\text{Spearmans } \rho = 1 - \frac{6 \times d^2}{n(n^2 - 1)}$$

Spearman correlation (use ranks of the data)

Where n is the number of points. Since it is also typical to use the symbol ρ for Pearson's coefficient, we can use ρ for Pearson's correlation coefficient and ρ^S for Spearman's correlation coefficient.

Think of Spearman as the correlation of ranked data. Ranking doesn't care about linear or non-linear; it simply cares about maintaining the order of the data points. In our previous example, we would find that

$$\text{rank(X)} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}, \text{ and}$$

$$\text{rank(Y)} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}.$$

Then

$$d = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

hence, $d^2 = 0$ and $\rho_{Spearman} = 1 - \frac{6 \cdot 0}{11(121-1)} = 1.$
This is what we were hoping for.

In our example, the ranks of X all line up with the ranks of Y. There are no discrepancies in ranks. When we have the nth largest X value, we have the nth largest Y value as well.

The advantage of Spearman's correlation over Pearson's correlation is that we can simply see if things move together instead of adding linearity to the requirement. Imagine that you are a trader and you want to determine whether one security is likely to go up if another security goes up. Which correlation would be more useful? Well, Pearson's correlation would help; however, Spearman's correlation focuses entirely on ranks, so it would be a more useful way to measure co-movements. This method of replacing data with ranks is a technique from non-parametric statistics and can be helpful if the data contains outliers or skewness, problems that we discussed in the previous module.

What we've learned so far should not make us think that Spearman is the solution to all of our problems. In fact, consider the example immediately below.

Example. *Consider the following simple dataset:*

$$X = \{-3, -2, -1, 0, 1, 2, 3\}$$

and

$$Y = \{9, 4, 1, 0, 1, 4, 9\}.$$

There is clearly a perfect relationship between X and Y, i.e., $Y = X^2$. However, we could show that both Pearson's and Spearman's correlations are zero.

Why is that?

Well, the relationship between X and Y is quadratic and thus not linear, which explains why we get no correlation with Pearson's formula. It is not in Pearson's formula's job description to detect quadratic relationships in general.

Why is Spearman failing then? Well, the relationship between X and Y in the range of the X values is not monotonic either, and that explains why we get zero using Spearman's formula as well. Again, it is not Spearman's formula's job to detect non-monotonic relationships in general.

We just encountered an example where X and Y exhibit a perfect relationship, but according to either correlation formula, no correlation is actually detected.

Students can experiment with several functional forms of the relationships between X and Y and check which formula provides the best answer.

4.3 Kendall Correlation

This is even more complicated to compute. Kendall's correlation coefficient is usually indicated with the Greek letter τ. We can describe it as follows:

- Here, the points X_i and Y_i are considered as a pair (X_i, Y_i) for $i = 1, 2, \ldots, n$;
- Compute the ranks of X and Y so that now we have the pairs $(\text{rank}(X_i), \text{rank}(Y_i))$;
- Organize the points into two columns, the first column for X and the second for Y. The data is organized based on the ranks of X so that the first column will contain the ranks $1, 2, 3, \ldots, n$ but the second column will contain the ranks of the point Y_i associated with X_i. Hence, the second column will not show the ranks as $1, 2, 3, \ldots, n$ in general.
- We now calculate the concordant pairs and discordant pairs as follows: for the columns of the Y ranks, start from the first value and count how many ranks are above the rank of Y_1. Then, do the same for Y_2, and so on. This creates a new column of the number of concordant pairs. Once the concordant pairs have been computed for all ranks of Y, we create another column where we count the number of discordant pairs instead, i.e., we start by looking for the number of ranks that are below the rank of Y_1, then we look for the number of ranks that are below the rank of Y_2, etc.;
- At this point, we have two columns $\{c_1, c_2, c_3, \ldots, c_n)$ and $(d_1, d_2, d_3, \ldots, d_n\}$;
- Now let $C = \sum_{i=1}^{n} c_i$ and $D = \sum_{i=1}^{n} d_i$;
- Finally, Kendall's τ is given by

$$\text{Kendalls } \tau = \frac{C - D}{C + D}.$$

One important advantage of the Spearman and Kendall correlations is that they can be used with ordinal data as they rely on ranks. On the other hand, Pearson's correlation requires that the random variables are quantitative (continuous).

ETF:

2. ETFs: An Introduction

Exchange-traded funds (ETFs) are baskets or collections of securities. The first ETF was launched nearly 30 years ago, the SPY, and its goal was to track the Standard & Poor's 500 Index, known more commonly as the **S&P500**. These represent the largest 500 stocks by market capitalization. Its ticker is **SPY** and it still exists today.

Initially, ETFs were created to follow benchmarks like the NASDAQ-100 (QQQQ) and the Dow Jones Industrial Average (DIA). The idea was to make broad market coverage available in a convenient, low-cost way. If someone only had $10,000 to invest in stocks, they likely could not afford to purchase 100 shares of the stocks on any of these indices. It was not possible to buy fractional shares of stocks. So, one would have to choose among many possibilities of stocks. Broadly owning the market avoids the problem of stock selection and instead allows for participation in the general market.

ETFs have a price that is given by their **net asset value**. The net asset value, or NAV, is defined as:

$$\text{NAV} = \frac{\text{Assets - Liabilities}}{\text{ETF Shares Outstanding}}$$

For a traditional equity ETF, the NAV is usually calculated every day, at the end of day, when all the markets being tracked by a particular ETF's index have closed. If the ETF is tracking U.S. equities, then the NAV is calculated after the U.S. markets close at 4:00pm ET. However, there are ETFs that trade in the U.S. but track a European index, say the FTSE 100 in England, and the London Stock Exchange closes at 11:30am ET. As such, the ETF's NAV is calculated shortly after 11:30am, but the ETF keeps trading in the U.S. until 4:30am. In this instance, we could say that the NAV is *stale*, and it is possible that the price at which an ETF trades in the U.S. is different from the NAV between the hours of 11:30am and 4:00pm ET.

Over time, market makers realized the popularity of these indices and began to create additional ETFs that focused on individual sectors. Indeed, there are 11 sectors that comprise the S&P500. Each of these sectors has a corresponding ETF. The first round of these were called select SPDRs.

3. ETF Benefits

The first benefit of ETFs is simply the ability to access broad market diversification. In Lesson 2 of this module, we learned that there are benefits to having a diversified portfolio. Essentially, owning a select SPDR gives a broad diversification across a particular industry. Determining which technology company to invest in is certainly more difficult than investing in the technology sector as a whole. The XLK gives broad market exposure with the purchase of a single security. Here is why it makes sense again to distinguish securities from portfolios: technically, the ETF is a single security, but within it, there is a portfolio of other securities. From a small investor's perspective, however, it provides one of the only mechanisms by which they can own a percentage of every technology company in the S&P500. There are even ETFs that specialize in cryptocurrencies. As such, market exposure could actually mean different securities within a specialized market like cryptocurrencies. Examples of ETFs that specialize in cryptocurrencies are BITW, BLOK, BLCN, LEGR, and KOIN.

A second benefit is professional management. Some ETFs have full-time managers who actively make decisions. Other ETFs have full-time managers who strive to track a benchmark. The decisions they make are more passive, in the sense that they are not trying to beat the market, or benchmark, but rather to track it. Either way, it is prudent to have a trained and dedicated professional managing the complexities of the securities comprising the ETF. If any investor, small or large, were managing the portfolio with stocks, they would likely be spending a lot of time deciding when and how to re-balance the portfolio.

A third benefit is trading flexibility. Some people compare ETFs to open-ended mutual funds. An open-ended mutual fund looks a lot like an ETF, but the open-ended mutual fund has one drawback: you can only buy or sell once a day, at the closing price. If there are market moves that occur during the day that cause you to want to sell, your sell order only gets processed at the closing price. A lot can happen between when you submit your sell order and the closing price. ETFs avoid that problem because they trade like stocks. That is, they trade on exchanges (which is part of their name); they can be bought or sold short throughout the day. This flexibility includes the fact that ETFs can be shorted. If an investor believes a sector to be overvalued, then that investor can sell short the ETF, wait for the price to drop, and cover (buy back) the ETF at the lower price, achieving a capital gain.

The fourth benefit is tied to the third one. ETFs tend to have low fees. The transaction costs, from commission to market impact, tend to be relatively small for ETFs. Surprisingly, not all 500 stocks have similar market impact. The top 50 stocks, say, are much easier to trade in terms of having better execution or less market impact than, say, the bottom 100 stocks of the S&P500.

4. ETF Disadvantages

Keep in mind that ETFs are not without their disadvantages. There are several main disadvantages.

Like other exchange-traded products, ETFs require investors to trade through a broker that charges commissions and other fees. These fees do not exist for the mutual fund buyer. Buyers of mutual funds typically buy directly from the fund itself. Also, if there is not much trading activity for a particular ETF, it may show up as having a wide gap between the price at which you buy the ETF and the price at which you sell the ETF. This is known as the bid-ask spread, which will be discussed in a later module in this course. Some ETFs are much more popular than others, so this liquidity problem will only exist for some names.

ETFs may have high operating expenses. Each ETF has an expense ratio. This expense ratio is fixed regardless of the fund's performance. If the fund were flat, the expense ratio would mean the net return on the investment is negative. If the ETF were negative on the year, the expense ratio would make it more negative. Prudent investors select a good combination of superior performance and reasonable fees.

ETFs often set out to track a particular benchmark. However, given the inexperience or poor performance of management, the ETF may fail to do so. ETFs can and do deviate from the very benchmark or index they attempt to track. Tracking errors can result in not only higher expense fees but also missed opportunities that create significant underperformance.

ETFs can also suffer from a lack of diversification. The select SPDRs we've discussed are each concentrated within a single industry. So although these ETFs provide diversification against idiosyncratic risk of individual companies, they are still very much exposed to industry risk.

ETFs also have an unusual effect on stocks. The more ETFs that include a stock among its constituency, the greater the demand for that stock (among the ETF portfolio managers). However, the converse is true. If stocks are no longer part of an index or perhaps receive a credit downgrade, then they may have much less demand. Such actions can cause ETFs to have volatility around index re-balancing.

To understand the pros and cons of ETFs, it helps to divide them into two categories: passively managed ETFs and actively managed ETFs.

5. Passive Management

ETF mangers seek to track an index. There is a great amount of transparency in these funds, as their constituency mirrors the weights and members of the index it tracks. Due to the stability of indices, the ETFs do not need to re-balance or turn over often. This stability results in both lower transaction costs and a lower management fee. Similarly, because of low turnover, there are not many capital gains.

Who buys passive funds? In today's market, it helps to think of two types of investors. The first group are traditional investors: small investors wanting broad market exposure who would otherwise not own a wide portfolio of stocks. The second group are robo-advisors: algorithmic trading systems that use artificial intelligence to make investment decisions.

Robo-advisors, one of the most well-known FinTech areas, disrupt the market share and profitability of the portfolio management world. These bots determine when to enter a particular sector, how long to hold it, and when to exit. Providing low-cost and strategic decision-making, robo-advisors are among the largest investors of ETFs. Most (not all) robo-advisors exclusively focus on passive ETFs for their low costs and broad exposure. The value the robo-advisor adds is their expertise in timing the market. Typically, this strategy is known as sector rotation. Robo-advisors use AI to gauge the economy, the markets, and even the individual's specific risk, cash flow, and tax situation to create portfolios that maximize returns and/or income and minimize costs and/or taxes.

6. Active Management

Compare this to actively managed funds. An actively managed ETF seeks to outperform the competition, whether it is a benchmark, an index, or the other ETFs in its category. Active managers hire specialized teams, build infrastructure, and perform analytics, all incurring higher costs. For this extra fee, they are expected to earn a higher rate of return. While active managers often have a benchmark to beat, they have liberty to deviate as well under different market conditions. Clearly then, the success of actively managed funds depends heavily on the experience and skill of the manager. While their fees are certainly higher than those for passive funds, their returns may not be. Not all active mangers outperform the market. Indeed, an active manager who has underperformed the market has both higher costs and lower returns. Surprisingly, only 1% of active ETF managers outperformed the S&P 500 (Chen).

Figure 2: Expense Ratios of ETFs

*The expense ratios of passive ETFs are shown in purple while the ones for active ETFs are shown in green.

7. Sector Selection vs. Stock Selection

What are the differences between picking individual sectors vs. individual stocks? Sector selection tends to involve macro-economic modeling and an understanding of business and economic cycles. It involves the study of monetary policies, employment trends, housing markets, and foreign exchange models and considers the big picture. By avoiding a focus on specific companies, sector selection seeks to determine the best combination of market and sector and looks to rotate among sectors and regions. Once sectors are identified, there are several choices of ETFs from which to choose.

Compare that to stock selection. Given the sheer number of companies, stock selection has more choices than sector selection or a specific ETF sector selection. Once you pick a company, you may compare it to a peer (relative analysis) or to the industry at large. The stock will inevitably have idiosyncratic risk because anything bad happening to the company will affect that stock price, without necessarily affecting the overall market.

As you will see in a later module, stock selection can be done using fundamental analysis, technical analysis, behavioral analysis, sentiment analysis, or statistical analysis. Selecting stocks seems to be a more complex problem than selecting sectors. Nevertheless, the reward for selecting the stock can be greater, as choosing the right stocks or cryptocurrencies can achieve returns in the thousands of percent that are simply implausible for sector-wide securities.

Volatility and correlations:

1. Market Corrections

In the previous two lessons, we discussed the benefits of diversification and saw how ETFs can provide those benefits. One type of ETF we have not yet discussed is asset allocation ETFs. These ETFs own different asset classes. For example, AOR is the iShares Core Growth Allocation ETF with investments in equities and fixed income. Managed by iShares, this fund invests in other iShares ETFs. We started this module thinking about the difference between securities and portfolios. Here is a single security that is essentially a portfolio of *other* portfolios of securities. Such a security is often referred to as a fund-of-funds. This fund tracks a proprietary index and has a mix of equities and fixed incomes.

The benefit is an extra level of diversification. An investor could decide for themselves how to construct a set of ETFs that performs asset allocation. However, the do-it-yourself approach requires insight and experience both to select and to manage these portfolios. The benefit of these asset allocation funds is extra diversification, but at the added expense of an extra layer of management fees. Whether these fees are justified remains a study for the prudent investor.

Moreover, does having multiple asset classes really protect an investor? Let's suppose you held bonds, equities, and Bitcoin at the beginning of 2022. For many asset classes, the financial markets started the year off with quite a bit of turbulence.

2. Correlations

Clearly, we know that there are high correlations within the same asset class. In Financial Econometrics, you will look at the correlations of different fixed-income maturities and find that maturities close together are highly correlated. You'll see that there is even a statistical technique, called principal component analysis, that exploits the high correlations in a correlation matrix to provide insight into common sources of variation.

Even within the same asset class, there are high correlations within sectors, especially when the industry as a whole is impacted by a common factor. For example, the pandemic brought the travel industry to a near halt. Stocks from the travel industry suffered big losses at the outbreak of the pandemic in March 2020.

Less clear, however, is that there is correlation across asset classes. Note in the examples above that the drops happened in three different asset classes, spread out over a few weeks. One important observation is that volatility was also high during that time. Indeed, correlations can change when volatility is high.

2.2 Volatility and Correlation

So what is the relationship between volatility and correlation? When discussing volatility, most investors agree that volatility changes over time. There's even an index called the VIX that measures the volatility of members of the SP100. Indeed, excess kurtosis is a reminder that standard deviations are not good representatives of volatility because they don't express volatility well for non-normal distributions.

2.3 Correlations Change

Is there a simple measure for changing correlations? Not exactly. Variances and standard deviations are properties of individual distributions. Correlations only come into play when discussing two distributions. Correlations and variances combine to tell us covariances. Covariances can be on different scales, depending on the sizes of the individual variances (think Bitcoin volatility compared to fixed income volatility). Correlations are scaled: their values always lie within minus 1 to 1. However, correlations can change over time. One way to understand this is to examine the relationship between correlation and volatility. When reading Loretan and English, notice that the relationship between volatility and correlation depends on the level of volatility: when volatility is low, correlations tend to be lower between two series than when those series have high volatility.

This sobering fact affects what we know about diversifying a portfolio. Diversification, the very heart of what low correlation promises, can be yanked away when it is most needed: in highly volatile markets.

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Other notes:

An **ETF (Exchange-Traded Fund)** trades on secondary markets (such as stock exchanges) throughout the trading day, just like individual stocks. Investors can buy and sell ETFs at market prices that fluctuate throughout the day based on supply and demand.

An **open-ended fund** (like a mutual fund), however, does **not trade on secondary markets**. Instead, it is bought and sold directly from the fund company at the **end-of-day Net Asset Value (NAV)**. Investors place orders during the trading day, but the actual price is determined at the close of the market based on the fund's NAV, not on intraday market prices.

So, only ETFs trade in secondary markets, while open-ended mutual funds do not.

Open-ended mutual funds usually have **higher management fees** than ETFs. This is primarily due to the following reasons:

1. **Active Management**: Many open-ended mutual funds are actively managed, meaning fund managers and analysts make regular investment decisions to try to outperform the market. This active management generally requires more resources, resulting in higher fees.
2. **Operational Costs**: Mutual funds also incur additional administrative and distribution costs, which are often passed on to investors as part of the expense ratio.
3. **ETFs and Passive Management**: In contrast, many ETFs are passively managed and simply track an index, which typically requires less active management and, consequently, lower fees.