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Modeling Parking

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This paper presents a simple model of parking congestion focusing on drivers' search for a vacant parking space in a spatially homogeneous metropolis. The mean density of vacant parking spaces is endogenous. A parking externality arises because individuals neglect the effect of their parking on this mean density. We examine stochastic stationary-state equilibria and optima in the model. Due to the model's nonlinearity, multiple equilibria may exist and the effects of parking fees are complex. Several extensions are discussed, including determining the social value of a particular parking information system.

Downtown parking is a significant problem in all major cities. Remarkably, however, there has been very little formal economic analysis of even the most obvious issues. If traffic congestion is efficiently priced, how should parking fees be set? Alternatively, what are the second-best parking fees when traffic congestion is underpriced? Depending on the pricing of auto congestion and public parking, should private, off-street parking fees be taxed or regulated? For various pricing régimes, how much land should be allocated to parking, both on- and off-street? What is the value of information concerning parking availability? In this paper we develop a simple structural model which provides a conceptual basis to answer such questions.

Various aspects of parking have been considered in the literature. Descriptions of parking patterns, the effects of on-street parking on traffic circulation, and the technology of off-street parking appear (e.g., [12] and [14]), as well as discussions of parking policy (e.g., [17], [15], [18], [22], and [1]). Some empirical work has been done identifying the determinants of

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modal choice and parking location (e.g., [9], [10], [25], and [13]). Numerous city-specific parking studies have been undertaken [20]. And there are high-quality, non-technical economic discussions of parking policy, notably Vickrey [24] and Roth [16].

But with the exception of a note by Douglas [8] and papers by Arnott *et al.* [2], Glazer and Niskanen [11], and Verhoef *et al.* [23] no economic model has been developed that considers the potential efficiency gains from parking fees or that incorporates the effects of parking on travel congestion. The effects may be substantial, for in major urban areas the time to find a parking spot and walk from there to work can be an appreciable fraction of total travel time, and parking fees may be comparable to vehicle operating costs [9]. Arnott *et al.* [2] explored the effects of parking fees in a deterministic model of the morning auto commute to the central business district, with bottleneck congestion. They showed that parking fees which vary over location can significantly reduce total travel costs. Glazer and Niskanen [11] examined simple partial equilibrium models to demonstrate that raising parking fees may *increase* both local traffic (by encouraging shorter visits) and through traffic. And Verhoef *et al.* [23] compared parking fees and parking regulations.

This paper presents a model focusing instead on the *stochasticity* of vacant parking spaces. Stochasticity is important to treat for several reasons. First, it results in drivers cruising to find a parking space. It has been claimed, for cities with a severe parking problem such as Boston and major European cities, that over one-half the cars driving downtown in rush hour are cruising for parking. Cruising for parking is both frustrating and time-consuming and congests traffic significantly by increasing traffic volume and slowing traffic down. Second, many cities are exploring parking information systems; to evaluate such systems, it is necessary to treat the stochasticity of vacant parking spaces. Third, recent studies (e.g., [19]) conclude that *unanticipated* travel time is disproportionately costly; variability in the time to find a parking spot is a major component.

Our aim is modest. We do not treat these issues in their full complexity. Rather, we explore perhaps the simplest structural model incorporating the stochasticity of vacant parking spaces. A later section discusses how the model can be extended in the direction of realism. Our basic model is as follows. The city is located on the outside of a circle and is spatially symmetric. There is a fixed number of parking spaces per unit distance. The demand for parking is derived from the demand for trips. Trip opportunities are generated according to an exogenous, stochastic, time-invariant process. A trip opportunity provides a benefit to a specific individual if she travels to a specific location and visits there for a specified period. She sits at home waiting for a trip opportunity. When she receives an opportunity, she decides whether to accept it, and if she does so, what mode of transport to take. If she drives, she decides how far from her destination to start cruising for parking, then takes the first available parking spot and walks to her destination. The expected walking distance depends on the mean density of vacant parking spaces, which is endogenous. A parking externality arises because individuals collectively neglect the effect of their parking on this mean density.

Our main finding is that the model exhibits complex nonlinearity. One consequence is that there may be two stable equilibria which can be Pareto ranked; which obtains depends presumably on the path of adjustment to equilibrium. Another consequence is that the comparative static properties of the model, including its response to policy variables (notably parking fees) are complex. The important policy insight is that even though parking pricing is a potentially powerful tool for regulating traffic congestion, it is intrinsically difficult to determine the appropriate parking fee.

Section I describes the basic model. Section II examines equilibrium with no parking fee, and Section III the social optimum. Section IV treats equilibrium with a parking fee, and explores decentralization of the social optimum via parking fees. Section V considers directions for future research and illustrates how the model can be employed to determine the social value of parking information systems. Concluding comments appear in Section VI.

I. THE BASIC MODEL

The basic model provides a highly stylized and simplified, but structural and general equilibrium, representation of the downtown parking problem. It admits numerous extensions. The model has four modules: spatial structure, trip generation technology, technology of parking and travel, and stationary-state conditions.

I.1. Spatial Structure and Trip Generation Technology

To abstract from complications arising from spatial heterogeneity, we assume the city is spatially symmetric. More specifically, the city lies on the circumference of a circle (see Fig. 1) of arbitrarily large inner radius r and has the same spatial structure at each location. Population density is Γ per unit length. The number of parking spaces per unit distance, the density of parking spaces, is D.

The demand for parking is derived from the demand for trips. An individual takes trips for the benefit obtained at the destination. To avoid complications associated with scheduling, interaction between individuals is ignored and all trips are single-purpose.

An individual receives trip opportunities according to a Poisson process. She receives opportunities only when at home, not when on a trip (the "no cellular telephone" assumption). A trip opportunity states that if she travels immediately from home to a specific location and visits for a fixed period of time l, she will receive a fixed dollar benefit β: l and β are the

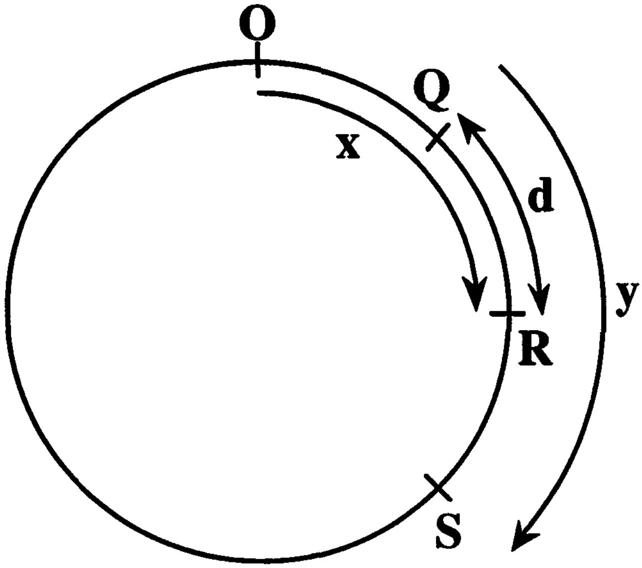


FIG. 1. Geometry of the model: O, home; R, trip destination; Q, location at which individual starts cruising for parking; S, parking location.

same for all trips. If the trip opportunity is accepted, she travels the shorter distance around the circumference of the circle to that location, receives the benefit, and returns home to await her next opportunity. The origin of trip opportunities is uniformly distributed around the circle. The Poisson arrival rate of opportunities is μ per individual, independent of time.

I.2. Travel and Parking Technologies

There are two travel modes—walking and driving, indexed $i = 1, 2$, respectively. Let x denote the shorter of the clockwise and counter-clockwise distance around the circle of a trip opportunity from home. Each individual owns a car and when at home parks in her garage. The *expected travel time* to x and back home by mode i is $T_i(x)$. Walking speed is a constant w. Thus,

$$T_1(x) = \frac{2x}{w}. \tag{1}$$

Determining auto travel time is more complex. There are two components of car travel time: time spent in the car, and time spent walking from the parking location to the destination and back again. The time spent in the car can in turn be decomposed into time spent cruising for parking and time spent in "regular" car travel. Since congestion caused by cruising for parking is an important aspect of the parking problem, as is walking time when driving, it is important to model car travel with care. Two simplifying assumptions are made, both of which should be relaxed in more realistic models. The first is that cars travel at a constant speed, independent of the density of cars in regular traffic and cruising for parking; that is, there is no travel congestion. The second is that car speed is the same whether in regular traffic or cruising for parking, v.

To provide a primitive treatment of the parking technology and to incorporate cruising for parking, it is important to treat the stochastic nature of finding a parking spot. Since traffic is in a stochastic stationary state, it is reasonable to assume that a driver knows the probability of finding a vacant spot between x and $x + dx$, $P\,dx$, where P is the *average density of vacant parking spaces*. To simplify, we assume that a driver searching for a parking spot can neither stop and wait for a parking spot to become vacant, nor back-track. For short distances, the driver would start cruising for parking as soon as she leaves home. For longer distances, she would start cruising for parking a distance d from her destination, where d, the *cruising distance*, is a decision variable depending on the density of vacant parking spaces. A simple argument establishes that with a non-negative parking fee, which we assume, she walks on shorter trips up to a distance $\tilde{x} \geq d$, where $\tilde{x}$ is the *maximum walking distance*, and drives on longer ones.

Let y be the distance the driver cruises for parking[1]—the distance driven after coming within a distance d of the destination (see Fig. 1). $P\,dy$ is the probability that she finds a vacant parking space in an interval dy. Assuming that the probabilities of adjacent parking spaces being vacant are independent,[2] the probability that she finds her first vacant parking space between y and $y + dy$ is $P\exp(-Py)\,dy$. *Expected driving time* on the round-trip journey is

$$R(x, P, d) = \frac{2(x-d)}{v} + \frac{2}{vP} \qquad \text{for } x \geq d. \tag{2}$$

This is derived as follows. On the journey from home to the destination, she drives a distance $x - d$ before starting to cruise for parking and then cruises for an expected distance $1/P$. Since, by assumption, driving speed is v in regular traffic and while cruising for parking, expected driving time

[1] It is assumed that the radius of the city is sufficiently large that the probability that a driver will drive more than halfway around the circle beyond her destination is negligible.

[2] This assumption is an approximation.

on the outbound journey is $(x - d)/v + 1/(vP)$. On the homeward journey, she travels an expected distance $x - d + 1/P$ in regular traffic at speed v. Summing the times on the outbound and homeward journeys gives $R(\cdot)$.

Expected walking time on a car trip is now computed. If the driver finds a parking spot before reaching her destination, i.e., $y < d$, she must walk a distance $d - y$; if she finds a spot at $y > d$, she must walk a distance $y - d$. Thus,

$$\begin{aligned} W(P, d) &= 2\int_0^d \frac{d - y}{w} Pe^{-Py}\, dy + 2\int_d^\infty \frac{y - d}{w} Pe^{-Py}\, dy \\ &= \frac{2}{w}\left(\frac{2e^{-Pd}}{P} + d - \frac{1}{P}\right). \end{aligned} \tag{3}$$

Finally,[3]

$$\begin{aligned} T_2(x, P, d) &= R(x, P, d) + W(P, d) \\ &= \frac{2x}{v} + \frac{4e^{-Pd}}{wP} + 2\left(d - \frac{1}{P}\right)\left(\frac{1}{w} - \frac{1}{v}\right). \end{aligned} \tag{4}$$

I.3. Decision Variables and Stationary State Conditions

There are three individual decision variables. The first is which offers to accept. Since there is an opportunity cost to her time, she will not accept trip opportunities beyond $\bar{x}$, the *maximum travel distance*. The second is her travel mode; she will walk shorter distances, up to the maximum walking distance $\tilde{x}$, and drive longer distances. And the third is the cruising distance d—the distance from her destination a driver will start cruising for parking.

The *expected trip period* L features prominently in the analysis. This has three components: expected travel time, visit length l, and expected time waiting at home for an accepted trip opportunity. Expected travel time is $\int_0^{\bar{x}} T(x)g(x)\, dx$, where $g(x)$ is the p.d.f. of x on trips taken and $T(x)$ is travel time to x with the chosen mode. Since the location of trip opportunities is uniform on the circle, and since all trips up to $\bar{x}$ are accepted, $g(x) = 1/\bar{x}$. Furthermore, $T(x) = T_1(x)$ for $x \le \tilde{x}$ and $T(x) = T_2(x, P, d)$

[3] The partial derivatives of $W(P, d)$ and $T_2(x, P, d)$ are provided in Arnott and Rowse [5, Appendix 1].

for $x \in (\tilde{x}, \bar{x})$. Hence, expected travel time is

$$\frac{1}{\bar{x}}\left[\int_0^{\tilde{x}} T_1(x)\,dx + \int_{\tilde{x}}^{\bar{x}} T_2(x, P, d)\,dx\right].$$

Since the arrival rate of trip opportunities is μ and the proportion of opportunities accepted is $(2\bar{x})/(2\pi r)$, the arrival rate of accepted opportunities is $\mu(\bar{x}/(\pi r))$. The expected time waiting for an accepted opportunity between trips is therefore $(\pi r)/(\mu\bar{x})$. Thus,

$$L(\tilde{x}, \bar{x}, P, d) = \frac{1}{\bar{x}}\left[\int_0^{\tilde{x}} T_1(x)\,dx + \int_{\tilde{x}}^{\bar{x}} T_2(x, P, d)\,dx\right] + l + \frac{\pi r}{\mu\bar{x}}. \quad (5)$$

Finally, there is a stochastic stationary-state condition, which can be interpreted in various ways. One is that the average rate at which parking spaces become occupied equals the average rate at which they are vacated. In stationary state, the rate at which parking spaces become occupied (per unit distance) equals the rate at which car trips are initiated. The rate at which car trips are initiated (per unit distance) equals the rate at which each individual initiates trips, $1/L$, times population density, Γ, times the proportion of trips by car, $(\bar{x} - \tilde{x})/\bar{x}$. And, also in stationary state, the rate at which parking spaces are vacated (per unit distance), equals the density of occupied spaces, $D - P$, times the rate at which each occupied space is vacated, which equals the reciprocal of the expected time parked on a car trip, $W(P, d) + l$. Thus[4]

$$D - P = \frac{\Gamma(W(P, d) + l)(\bar{x} - \tilde{x})}{L\bar{x}}. \quad (6)$$

II. EQUILIBRIUM WITH NO PARKING FEE

This is the natural base case.

II.1. Derivation of Equilibrium

The individual chooses $\tilde{x}$, $\bar{x}$, and d to maximize benefits per unit time, taking P as fixed. Because P depends on everyone's choice of $\tilde{x}$, $\bar{x}$, and d, there is an uninternalized parking externality. Since the benefit per trip is fixed, maximizing benefits per unit time is equivalent to minimizing the

[4] Later we shall determine how P responds to changes in $\tilde{x}$, $\bar{x}$, and d. For this purpose we regard (5) and (6) as two equations in the unknowns L and P. The comparative statics of this pair of equations is given in Arnott and Rowse [5, Appendix 2].

average trip period. Thus, using (5), the individual's optimization problem is[5]

$$\min_{\tilde{x}, \bar{x}, d} \frac{1}{\bar{x}}\left[\int_0^{\tilde{x}} T_1(x)\, dx + \int_{\tilde{x}}^{\bar{x}} T_2(x, P, d)\, dx\right] + l + \frac{\pi r}{\mu \bar{x}}. \tag{7}$$

The first-order conditions are

$$\tilde{x}: \quad \frac{1}{\bar{x}}\left[T_1(\tilde{x}) - T_2(\tilde{x}, P, d)\right] = 0, \tag{8a}$$

$$\bar{x}: \quad -\frac{1}{\bar{x}^2}\left[\int_0^{\tilde{x}} T_1(x)\, dx + \int_{\tilde{x}}^{\bar{x}} T_2(x, P, d)\, dx + \frac{\pi r}{\mu}\right] + \frac{1}{\bar{x}} T_2(\bar{x}, P, d) = 0, \tag{8b}$$

or, using (5),

$$\frac{1}{\bar{x}}\left[-L(\tilde{x}, \bar{x}, P, d) + l + T_2(\bar{x}, P, d)\right] = 0, \tag{8b$'$}$$

$$d: \quad \frac{1}{\bar{x}}\left[\int_{\tilde{x}}^{\bar{x}} \frac{\partial T_2(x, P, d)}{\partial d}\, dx\right] = 0. \tag{8c}$$

Equation (8a) indicates that, absent a parking fee, she chooses the mode with the lower travel time. Equation (8b′) has the following interpretation: She accepts a trip opportunity if the benefit covers the opportunity cost of the expected trip time. The trip benefit is β. Since the opportunity cost of time is β/L, the opportunity cost of expected trip time to $\bar{x}$ is $\beta(T(\bar{x}, P, d) + l)/L$. Hence, she is indifferent between accepting and declining a trip opportunity to $\bar{x}$. Equation (8c) indicates that she chooses d to minimize expected travel time by car. An increase in d decreases expected driving time. Thus, she chooses d so that the decrease in expected driving time from a small increase in d is just offset by an increase in expected walking time. Using (4), (8c) gives

$$d = \frac{\theta}{P}, \qquad \text{where } \theta = -\ln\left[\frac{1}{2}\left(1 - \frac{w}{v}\right)\right]. \tag{8c$'$}$$

[5] The individual's optimization problem is well behaved. The second-order conditions are satisfied so that the optimum is unique. Furthermore, as long as $(\pi r)/\mu > \theta^2/(D^2 w)$, which we assume, $\bar{x} > d \geq \tilde{x}$.

The cruising distance is inversely proportional to the density of vacant parking spaces. The solution to (8a) is

$$\tilde{x} = d = \frac{\theta}{P}. \tag{8a$'$}$$

The intuition is as follows. At a distance d from the destination, she is indifferent between taking a vacant parking spot and continuing driving. Travel time when taking the vacant parking spot is just twice the time spent driving to that location plus twice the time spent walking from that location to the destination. At $x = d$, the travel time from taking the vacant parking spot equals travel time when walking. Thus, at $x = d$ she is indifferent between walking and driving.

The equilibrium with no parking fee is characterized by (8a), (8b′), (8c′), (5), (6), and (3), where the six unknowns are $\tilde{x}$, $\bar{x}$, d, L, P, and W. Unlike the equations characterizing the social optimum and positive parking fee equilibria, which are examined below, these equations can be reduced to two equations in $\tilde{x}$ and $\bar{x}$:

$$H(\tilde{x}, \bar{x}) \equiv \frac{\bar{x}^2}{v} + \tilde{x}^2\left(\frac{1}{w} - \frac{1}{v}\right) - \frac{\pi r}{\mu} = 0, \tag{9}$$

$$\begin{aligned} G(\tilde{x}, \bar{x}) \equiv \left(D - \frac{\theta}{\tilde{x}}\right)\bar{x}\left(2\left(\frac{\bar{x}}{v} + \tilde{x}\left(\frac{1}{w} - \frac{1}{v}\right)\right) + l\right) \\ -\Gamma\left(\frac{2\tilde{x}}{\theta}\left(\frac{\theta}{w} - \frac{1}{v}\right) + l\right)(\bar{x} - \tilde{x}) = 0. \end{aligned} \tag{10}$$

Equation (10) is obtained by substituting (8a′), (8c′), (5), and (3) into (6). Thus, it has the interpretation as the locus of $(\tilde{x}, \bar{x})$ such that parking is in equilibrium. Equation (9) is obtained from (8a) and (8b). Since (10) too incorporates (8a), (9) is appropriately interpreted as the $\bar{x}$ chosen by the individual as a function of the $\tilde{x}$ she chooses. Equation (9) describes an ellipse with the origin as center. Equation (10) has a far more complex form. The substitution of (9) into (10) gives a sixth-order polynomial equation (in $\tilde{x}$ or $\bar{x}$).

The rest of this section explores the characteristics of equilibrium.

II.2. Two Numerical Examples

We have been unable to obtain a complete analytical characterization of the solutions to (9) and (10), though we have proved that there is at least one real solution with $\tilde{x}$ and $\bar{x} > 0$. To gain some insight into the properties of the no-parking-fee equilibrium, we examine two numerical examples.

EXAMPLE 1. We employ the following parameter values:[6]

$$w = 3.0 \text{ miles per hour}, \qquad D = 200 \text{ spaces per mile},$$
$$v = 12.0 \text{ miles per hour}, \qquad \Gamma = 2533.3 \text{ persons per mile},$$
$$\frac{\pi r}{\mu} = 0.79052 \text{ mile-hours}, \qquad l = 0 \text{ hours}.$$

These parameter values imply that $\theta \approx 0.98083$.

Three solutions[7] of economic interest to (9) and (10) were found:

	$\tilde{x}$	$\bar{x}$	P	L	d
①	0.0052382	3.0800	187.25	0.51595	0.0052382
②	0.085619	3.0764	11.456	0.55554	0.085619
③	1.4924	1.6747	0.65722	1.0253	1.4924

Equations (9) and (10) are plotted for this example in Fig. 2. As already noted, (9) ($H(\tilde{x}, \bar{x}) = 0$) is an ellipse. The properties of (10) can be explained by noting that with $\tilde{x}$ fixed, the equation is a quadratic function in $\bar{x}$. The upper curve for (10) corresponds to the positive root of the function and the lower curve to the negative root. Note that $P \leq D$ since the number of vacant parking spaces cannot exceed the number of parking spaces. From (8a′) this implies that $\tilde{x} \geq \theta/D$ so that lower values of $\tilde{x}$ are not of economic interest; relatedly, the singularity in the positive root function of (10) at $\tilde{x} = \theta/D$ is not of economic interest.

Arnott and Rowse [5] argue that equilibria ① and ③ are stable and equilibrium ② is unstable. Their argument is supported by the fact that comparative static results in the neighborhood of equilibria ① and ③ are intuitive, while those in the neighborhood of equilibrium ② are perverse. Following the economic terminology applied to road congestion, we term equilibrium ① the *congested* equilibrium and ③ the *hypercongested* equilibrium.

[6] D was chosen such that there is continuous parking on one side of the street, with each parking space 26.4 feet long. Γ was chosen on the basis of six-story apartment buildings on each side of the street, each apartment having a frontage of 25.01 feet. Thus, there is one household per 25.01/12 feet, corresponding to 2533.3 households per mile. $\pi r/\mu$ was chosen so that, with no congestion in parking, the longest trip taken would be approximately 3.08 miles.

[7] All numerical results are presented with five non-zero digits. The computed accuracy was the maximum allowed by EUREKA, namely, to 13 digits.

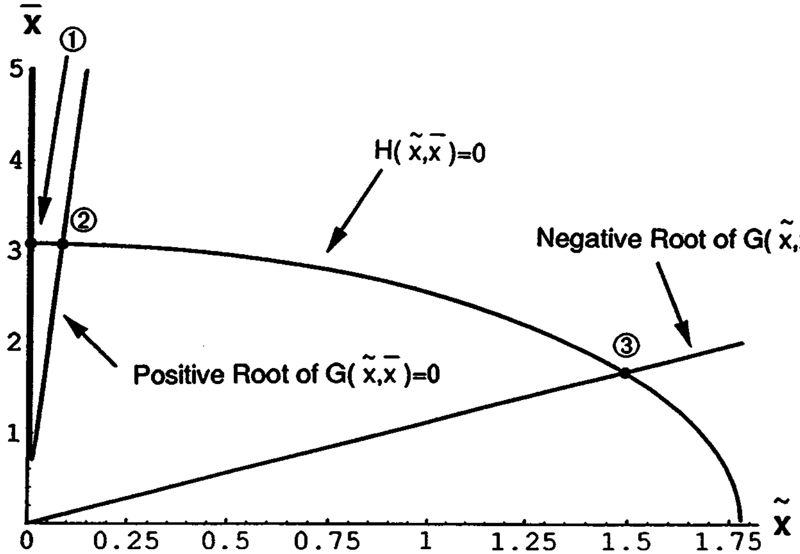


FIG. 2. The zero-parking-fee equilibrium with $l = 0.0$.

In the congested equilibrium ①, over 90% of parking spaces are vacant. On car trips, individuals start cruising for parking only 28 feet before reaching their destination, and walk an average distance of 21 feet to their destination, which takes 4.7 seconds. Since $l = 0$, the average time parked on a car trip therefore equals 9.4 seconds. With a zero parking fee, $\tilde{x} = d$; hence individuals drive on trips exceeding 28 feet. Average trip duration L is about $33\frac{1}{2}$ minutes, of which about $15\frac{1}{2}$ are spent at home with the rest spent traveling. Evidently, this equilibrium entails very little parking congestion.

The hypercongested equilibrium ③ is very different. Only about one in three hundred parking spaces is vacant. On car trips individuals start cruising for parking almost $1\frac{1}{2}$ miles before reaching their destination, and walk an average distance of 1.1 miles to their destination, which takes somewhat over 22 minutes. The average time parked on a car trip therefore equals about 45 minutes. Average trip duration is about $61\frac{1}{2}$ minutes, of which about $28\frac{1}{2}$ minutes are spent at home with the remainder spent in travel. This equilibrium exhibits extreme parking congestion.

Why are there multiple equilibria? To provide an answer, consider a simple model of a supermarket parking lot with D parking spaces. The average time parked T is directly related to the occupancy rate ϕ: $T = Z(\phi)$. The flow demand for shopping at the supermarket, f, is inversely related to T: $f = f(T)$. And in stationary state, the rate of entry to the parking lot f must equal the rate of exit which equals the number of occupied parking spaces, ϕD, times the rate at which each is vacated, $1/T$: $f = \phi D/T$. As the average time parked increases, the flow rate into the parking lot falls while the flow rate out may fall by more or less, which is consistent with multiple equilibria. The explanation for multiple equilibria for the parking model is similar, but more complicated because of its greater complexity. The supermarket parking model shows that the multiplicity of equilibria does not require stochasticity.

EXAMPLE 2. This example has the same parameters, except that $l = 0.25$. Thus, the minimum time parked on a trip is 15 minutes. If $\tilde{x}$ and $\bar{x}$ remained the same as in the congested equilibrium of the above example, parking congestion would increase substantially. Thus, one might expect changing l to cause the congested equilibrium to more closely resemble the hypercongested equilibrium. In fact, raising l eliminates the congested and unstable equilibria. Only the stable, hypercongested equilibrium remains, with $\tilde{x} = 1.4962$, $\bar{x} = 1.6644$, $P = 0.65554$, and $L = 1.2755$. If the analog to Fig. 2 were plotted, $H(\tilde{x}, \bar{x}) = 0$ would remain unchanged, the upper (positive root) portion of $G(\tilde{x}, \bar{x}) = 0$ would lie above $H(\tilde{x}, \bar{x}) = 0$, and the lower (negative root) portion of $G(\tilde{x}, \bar{x}) = 0$ would intersect $H(\tilde{x}, \bar{x}) = 0$ at the hypercongested equilibrium. The equilibrium is very similar to the hypercongested equilibrium ③ of Example 1, except that L is higher by about 0.25.

II.3. Comments

The comparative static properties of the equilibria can be obtained directly from (9) and (10), but the analysis is messy. The only simple comparative static exercise is with respect to μ. In terms of Fig. 1, an increase in μ, the Poisson arrival rate of trip opportunities, shifts (9) inward, toward the origin. Consider, for instance, the congested equilibrium ① in Fig. 1. The inward shift in (9) causes $\bar{x}$ to fall and $\tilde{x}$ to rise. The mechanism is as follows: The immediate effect is that time waiting at home falls, causing the expected trip period L to fall. This in turn has two first-round effects. First, since the opportunity cost of time β/L rises, the individual refuses some longer trips that she previously accepted ($\bar{x}$ falls (Eq. (8b′)). Second, trip frequency rises, increasing the parking occupancy rate (Eq. (6)). The increased parking congestion in turn causes her to walk on some trips on which she would previously have driven ($\tilde{x}$ rises (Eq. (8a)) and increases cruising distance (d rises (Eq. (8c))). The qualitative effects of the full adjustment are the same as for these first-round effects.

The possibility of multiple stable equilibria raises the issue of equilibrium selection. Which equilibrium obtains presumably depends on the path to the stationary state. If the economy were previously highly congested, it should settle at the hypercongested equilibrium, while if congestion built up toward the stationary state it should settle at the congested equilibrium. Unfortunately, this intuition is difficult to make precise because the model's transient behavior is highly complex. For example, in deciding between walking and driving, with perfect foresight an individual would have to take into account that the density of vacant parking spaces would change as she was cruising for parking.

The complexity of the model's solution is discouraging. It is, however, intrinsic to the problem. We chose our assumptions to obtain the simplest structural model capturing the essential elements of the problem. Much of the complexity derives from the stochastic nature of finding a parking space. But, without this stochasticity, there would be no cruising for parking, which we judge to be an essential feature of the problem. Fortunately, the first-best welfare economics is relatively straightforward. However, the second-best welfare economics is comparably complex. This suggests that practical parking policy should be investigated employing realistic simulation models.

III. SOCIAL OPTIMUM

III.1. First-Order Conditions and Interpretation

The planner seeks to maximize trip frequency. Unlike individuals, however, the planner takes into account the dependence of the density of vacant parking spaces on $\tilde{x}$, $\bar{x}$, and d. His optimization problem is

$$\min_{\tilde{x},\bar{x},d,P} L \quad \text{s.t.} \quad \text{(i)} \quad L = \frac{1}{\bar{x}}\left[\int_0^{\tilde{x}} T_1(x)\,dx + \int_{\tilde{x}}^{\bar{x}} T_2(x,P,d)\,dx\right] + l + \frac{\pi r}{\mu \bar{x}}, \tag{11}$$

$$\text{(ii)} \quad \frac{\Gamma(W(P,d)+l)((\bar{x}-\tilde{x})/\bar{x})}{D-P} = L.$$

Constraint (i) is the definition of L (Eq. (5)), while constraint (ii) is the parking equilibrium condition (Eq. (6)). The corresponding F-O-C, with L substituted out and λ the Lagrange multiplier on (ii), are

$$\tilde{x}: \quad \frac{1}{\bar{x}}\left[(1-\lambda)(T_1(\tilde{x}) - T_2(\tilde{x},P,d)) - \lambda\frac{\Gamma(W(P,d)+l)}{D-P}\right] = 0, \tag{12a}$$

$$\bar{x}:\quad \frac{1}{\bar{x}}\left[-(1-\lambda)\big(L(\tilde{x},\bar{x},P,d)-l-T_2(\bar{x},P,d)\big)\right.$$

$$\left.+\lambda\frac{\Gamma(W(P,d)+l)}{D-P}\frac{\tilde{x}}{\bar{x}}\right]=0, \tag{12b}$$

$$d:\quad \frac{\bar{x}-\tilde{x}}{\bar{x}}\left[(1-\lambda)\frac{\partial T_2(x,P,d)}{\partial d}+\lambda\frac{\Gamma}{D-P}\frac{\partial W(P,d)}{\partial d}\right]=0, \tag{12c}$$

$$P:\quad \frac{\bar{x}-\tilde{x}}{\bar{x}}\left[(1-\lambda)\frac{\partial T_2(x,P,d)}{\partial P}\right.$$

$$\left.+\frac{\lambda\Gamma}{D-P}\left(\frac{\partial W(P,d)}{\partial P}+\frac{W(P,d)+l}{D-P}\right)\right]=0. \tag{12d}$$

Consider first the interpretation of (12a). When an individual walks to $\tilde{x}$, the social time it takes is $T_1(\tilde{x})$. When instead she drives to $\tilde{x}$, the expected social time of the trip equals her expected travel time plus the expected parking congestion externality she imposes through reducing the density of vacant parking spaces and hence increasing the expected travel time of other drivers. Thus, the condition for the choice of $\tilde{x}$ is

$$T_1(\tilde{x}) = T_2(\tilde{x},P,d) + \text{parking congestion externality}. \tag{12a$'$}$$

The congestion externality is proportional to the length of time parked. Define E to be the *time lost by other drivers per extra minute parked* (E is dimensionless). Hence

$$T_1(\tilde{x}) = T_2(\tilde{x},P,d) + E(W(P,d)+l). \tag{12a$''$}$$

Compare this equation to the analogous equation for the no-parking-fee equilibrium, (8a). The two are the same except that (8a) omits the parking congestion externality term. Comparing (12a) and (12a″) yields

$$E = \frac{\lambda}{1-\lambda}\left(\frac{\Gamma}{D-P}\right). \tag{13a}$$

Equation (12b) is interpreted similarly. A trip opportunity to x is accepted if the expected social time of the trip is not greater than the expected social time until completion of the next accepted trip. Since a trip to $\bar{x}$ is by car, its expected social time is $T_2(\bar{x}, P, d) + l + E(W(P, d) + l)$. If the opportunity to $\bar{x}$ is declined, the expected social time until completion of the next acceptable trip is L plus the expected parking congestion externality. Since a proportion $(\bar{x} - \tilde{x})/\bar{x}$ of trips entail parking, the expected congestion externality is $((\bar{x} - \tilde{x})/\bar{x})E(W(P, d) + l)$. Thus, $\bar{x}$ is characterized by

$$T_2(\bar{x}, P, d) + l + E(W(P, d) + l)$$
$$= L(\tilde{x}, \bar{x}, P, d) + \left(\frac{\bar{x} - \tilde{x}}{\bar{x}}\right)E(W(P, d) + l), \qquad (12b')$$

which, using (13a), is consistent with (12b). Again, the only difference between (12b′) and the analogous equation for the no-parking-fee equilibrium, (8b′), is that (8b′) omits the parking externality term.

Equation (12c) indicates that the planner chooses d to minimize the expected social cost of a car trip, accounting for the parking congestion externality. The equation can be rewritten as $\partial R(x, P, d)/\partial d + (1 + E)\,\partial W(P, d)/\partial d = 0$. The analogous equation for the no-parking-fee equilibrium is $\partial R(x, P, d)/\partial d + \partial W(P, d)/\partial d = 0$; the privately optimal cruising distance minimizes the sum of expected driving time and expected parking time. Since parking generates the externality, for a given P the planner chooses a shorter expected parking time and a longer expected driving time, implying a shorter cruising distance.

Equation (12d) gives the value of λ. Combining (13a) and (12d) yields

$$E = \frac{-\partial T_2(x, P, d)/\partial P}{(W(P, d) + l)/(D - P) + \partial W(P, d)/\partial P}. \qquad (13b)$$

Equation (9) continues to hold at the social optimum and we note in the next section that it holds as well in equilibria with a parking fee. Arnott and Rowse [5] explain why (9) holds in all these situations.

III.2. Examples

We return to our previous examples and report the corresponding social optima.[8]

EXAMPLE 1 ($l = 0, 0$). Recall that the congested equilibrium ① in Example 1 above (shown in Fig. 2) entails the lowest average trip duration of the three equilibria, and very little congestion. Thus, one would expect

[8] To solve for the social optimum, we employed two separate packages (EUREKA and GINO) to check for accuracy. We first optimized with respect to $\tilde{x}$, $\bar{x}$, and d, holding P fixed, and then did a search for the optimal P. In our computational experience, every problem had a unique interior minimum.

the social optimum (s.o.) to closely resemble this equilibrium, and it does:

	$\tilde{x}$	$\bar{x}$	P	L	d
s.o.	0.0056159	3.0800	187.35	0.51595	0.0051148
①	0.0052382	3.0800	187.25	0.51595	0.0052382

Because it takes into account the parking congestion externality, the social optimum entails a larger maximum walking distance and a smaller cruising distance than in equilibrium ①. The level of parking congestion in the no-parking fee equilibrium is, however, so low that the lower average trip duration for the social optimum shows up only in the sixth non-zero digit. Thus, in this example the social loss in the no-parking-fee equilibrium from the uninternalized parking congestion is negligible.

EXAMPLE 2 ($l = 0.25$). Again, Example 2 stands in strong contrast to Example 1.

	$\tilde{x}$	$\bar{x}$	P	L	d
s.o.	1.3874	1.9265	20.966	1.0774	0.036637
e	1.4962	1.6644	0.65554	1.2755	1.4962

The social optimum has several noteworthy features. In contrast to the no-parking-fee equilibrium (e) where $\tilde{x}$ and d are equal, in the social optimum they are very different. There are two reasons: First, the planner takes into account that driving entails the parking externality, whereas walking does not. Recall from the discussion of (12a) that $T_1(\tilde{x}) - T_2(\tilde{x}, P, d)$ quantifies the parking externality in time units. At the social optimum, $T_1(\tilde{x}) = 0.92493$ while $T_2(\tilde{x}) = 0.25520$; thus, the parking externality in time units is 0.66973. Second, the planner favors a shorter cruising distance, since even though this increases travel time, holding P fixed, it reduces how long the driver is parked and hence the parking externality. To ascertain the importance of this, we calculate the cruising distance the driver chooses with $P = 20.966$ and no parking fee; it is $d = 0.046782$, which exceeds the socially optimal value of d, 0.036637. The social optimum also entails a considerably lower walking time when driving; in the no-parking-fee equilibrium it is 0.74323, while in the social optimum it is 0.022128. At the social optimum the average parking duration on a car trip is 0.272128, and hence the parking externality per unit time parked, E, is 2.4611, indicating that for each extra minute a driver parks she inflicts a loss on others of 2.4611 minutes. Finally, the social optimum reduces the average trip duration from 1.2755 to 1.0774 hours, about 15%.

The reader may wonder why we investigate the social optimum at length when the planner controls none of $\tilde{x}$, $\bar{x}$, d, and P directly. Obviously, doing so provides an insightful benchmark. But more than this, subject to a strong qualification, the social optimum is decentralizable. The only distortion is that the individual fails to account for the externality associated with her parking, and this can be corrected via a parking fee. Some analytical comparative static results can be derived. Given the complexity of the analysis, however, it seems preferable to determine the comparative static properties of the social optimum numerically.

IV. THE EQUILIBRIUM WITH A PARKING FEE

This section examines both the equilibrium with a positive parking fee[9] and the decentralizability of the social optimum.

IV.1. Derivation of Equilibrium

The *parking fee per unit time*, p, is specified in money units. The individual aims to maximize trip benefits net of parking fees per unit time. The average benefit per trip is $\beta - p((\bar{x} - \tilde{x})/\bar{x})(W(P,d) + l)$ since $(\bar{x} - \tilde{x})/\bar{x}$ is the proportion of trips taken by car and the average parking fee per car trip is the parking fee per unit time multiplied by the average time parked. Thus, the maximand is $[\beta - p((\bar{x} - \tilde{x})/\bar{x})(W(P,d) + l)]/L$. If parking revenues are redistributed, the individual regards the payment as a fixed sum per unit time. The analysis is therefore unaffected by the redistribution of parking revenue.

Consequently, the individual's maximization problem is

$$\max_{\tilde{x},\bar{x},d} V(\tilde{x},\bar{x},d;p,P)$$

$$= \frac{\beta - p((\bar{x} - \tilde{x})/\bar{x})(W(P,d) + l)}{(1/\bar{x})\left[\int_0^{\tilde{x}} T_1(x)\,dx + \int_{\tilde{x}}^{\bar{x}} T_2(x,P,d)\,dx\right] + \pi r/(\mu\bar{x}) + l}. \tag{14}$$

$V(\cdot)$ is the *private value of time*. She takes P as given; thus, the FOC are

$$\tilde{x}:\quad \frac{1}{\Delta}\left[p(W(P,d) + l) - V(T_1(\tilde{x}) - T_2(\tilde{x},P,d))\right] = 0, \tag{15a}$$

$$\bar{x}:\quad \frac{1}{\Delta}\left[\beta - p(W(P,d) + l) - V(T_2(\bar{x},P,d) + l)\right] = 0, \tag{15b}$$

[9] One can solve for equilibrium with a negative parking fee. But the analysis needs to be altered somewhat to take into account that a person cannot start cruising for parking before she leaves home. We do not investigate negative parking fees since their economic relevance is dubious. For example, individuals would have an incentive to park their cars on the street when at home so as to collect the parking subsidy.

$$d: \quad \frac{\bar{x} - \tilde{x}}{\Delta}\left[-p\frac{\partial W(P,d)}{\partial d} - V\frac{\partial T_2(\bar{x},P,d)}{\partial d}\right] = 0, \quad (15c)$$

where $\Delta \equiv L\bar{x}$. Each of these conditions has a straightforward interpretation. For example, (15a) indicates that she chooses the mode costing less *in money terms*, where the price of time is its private value. The maximization problem has a unique, interior maximum (see [5, Appendix 4]). And the equilibrium is obtained by combining (15a)–(15c) with (5) and the definition of V, giving five equations in five unknowns, $\tilde{x}$, $\bar{x}$, d, P, and V. Equation (9) continues to hold. Comparative static analysis of this system of equations is messy, so numerical determination of the comparative static properties of the model is justified.

IV.2. Decentralization of the Social Optimum

We argued previously that there is only one distortion—individuals do not pay the full social cost of their parking. By levying an appropriate parking fee, this distortion should be correctable. We proceed by solving for the parking fee which supports the optimum and then turn to a fuller analysis of decentralizability.

Comparing (12a)–(12c) with (15a)–(15c) indicates that the two sets of first-order conditions can be made consistent by setting

$$\frac{p^*}{V^*} = \frac{\lambda^*\Gamma}{(1-\lambda^*)(D-P^*)} \quad \text{or} \quad p^* = V^*E^*, \quad (16)$$

where $*$ denotes evaluation at the social optimum (see [5, Appendix 5]). Equation (16) states that the parking fee should equal the parking externality in time units multiplied by the *private* value of time, both evaluated at the social optimum. Observe that the parking fee causes a divergence between the private and social value of time; the latter is β/L. Since V is a function of p, (16) is an implicit equation. Combining (16) and (14) gives the following explicit equation for p^*:

$$p^* = \frac{E^*\beta}{L^* + E^*((\bar{x}^* - \tilde{x}^*)/\bar{x}^*)(W^* + l)}. \quad (16')$$

Now recall Example 1. With no parking fee, there are two stable equilibria, one congested and one hypercongested. There is very little parking congestion in the congested equilibrium. Also, the social optimum is very similar to the congested equilibrium. These observations suggest that levying the optimal parking fee, computed per (16′), should cause the congested equilibrium to coincide with the social optimum but, since the optimal parking fee is so low, should not eliminate the hypercongested

TABLE 1

		$\tilde{x}$	$\bar{x}$	P	L	d
①	$p = p^*$	*0.0056162*	*3.0800*	*187.35*	*0.51595*	*0.0051149*
	$p = 0$	0.0052382	3.0800	187.25	0.51595	0.0052382
②	$p = p^*$	0.093515	3.0757	11.315	0.55608	0.084541
	$p = 0$	0.085619	3.0764	11.456	0.55554	0.0085619
③	$p = p^*$	1.4878	1.6967	0.75598	1.0132	1.2425
	$p = 0$	1.4924	1.6747	0.65722	1.0253	1.4924

equilibrium. As we shall see, such is indeed the case. Thus, imposing the optimal parking fee given by (16′) does not necessarily attain the social optimum.

We argued earlier that the hypercongested equilibrium should occur if parking is highly congested in the adjustment to the stationary state, and the congested equilibrium should occur otherwise. This argument suggests that the social optimum can always be attained by an appropriate dynamic parking fee. By setting the parking fee sufficiently high during transition to the stationary state, the planner should be able to ensure convergence to the social optimum. Investigating this conjecture requires exploration of the non-stationary dynamics of the model, however, which we do not explore here.[10]

IV.3. *Examples*

We return to our previous examples and explore the effects of parking fees.

EXAMPLE 1 ($l = 0, 0$). We compute the optimal parking fee and then solve for the corresponding equilibria. With $\beta = \$10$, the optimal parking fee is $p^* = \$1.4232$ per hour. We list our results in Table 1. An italicized number indicates that a value is socially optimal. There are few surprises. Since the congested no-parking-fee equilibrium is very little congested, the optimal parking fee is low. Levying the fee does indeed cause the congested equilibrium to replicate the social optimum. But since the optimal fee is low, imposing it does not substantially alter the other equilibria. It is,

[10] Support for this conjecture is provided by Fig. 3a, which plots equilibria as a function of the parking fee for $l = 0$. If the parking fee is set sufficiently high, the hypercongested equilibrium disappears.

however, noteworthy that applying the parking fee *increases* trip duration for the unstable equilibrium.[11]

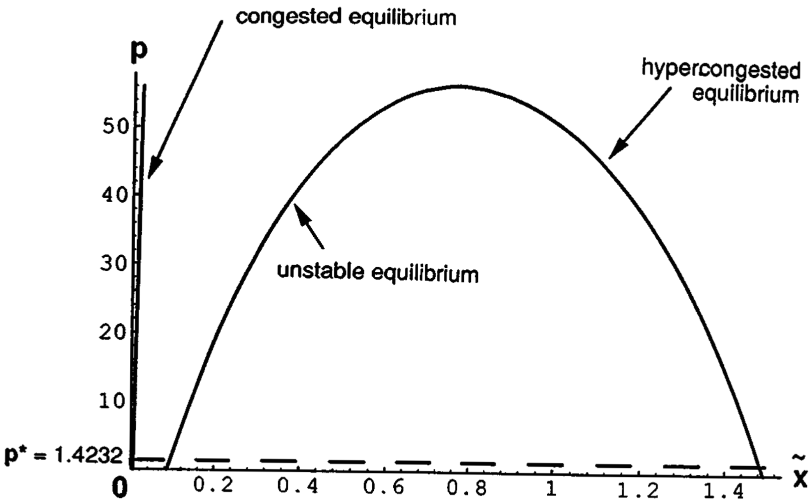
We next investigate the effects of imposing a non-optimal parking fee on the equilibrium.[12] These effects can be explained using Fig. 3a, which plots the equilibrium $\tilde{x}$ as a function of p. Of particular interest is that for $p > \$56.45$ per hour, the unstable and hypercongested equilibria disappear. Thus, a sufficiently high parking fee "unlocks" the hypercongested equilibrium. As the parking fee rises, for each stable equilibrium type the density of vacant parking spaces increases. Whether $\tilde{x}$ is positively or negatively related to p depends on two competing effects: a higher parking fee makes driving less attractive; but it also makes parking less congested, which makes driving more attractive. In the examples the former effect dominates for the congested equilibrium (since there is little congestion) and the latter effect dominates for the hypercongested equilibrium (since an increase in the parking fee reduces congestion substantially).

EXAMPLE 2 ($l = 0.25$). The optimal parking fee is $p^* = \$19.459$ per hour. Imposing the optimal parking fee decentralizes the social optimum. Prior to imposing the parking fee, benefit per hour is $V = \beta/L = \$7.843$. With the parking fee, benefit per hour without redistributing parking fee revenues is $V = [\beta - p((\bar{x} - \tilde{x})/\bar{x})(W + l)]/L = \$7.906$. Thus, even without redistributing parking fee revenues, the parking fee makes individuals better off. In contrast, in Example 1 individuals are worse off in the decentralized social optimum if toll revenues are not redistributed than in the no-parking-fee equilibrium ①. These results accord with the economics of congestion pricing, where unredistributed optimal tolls help drivers if the pre-toll equilibrium is hypercongested but hurt them if it is congested [3].

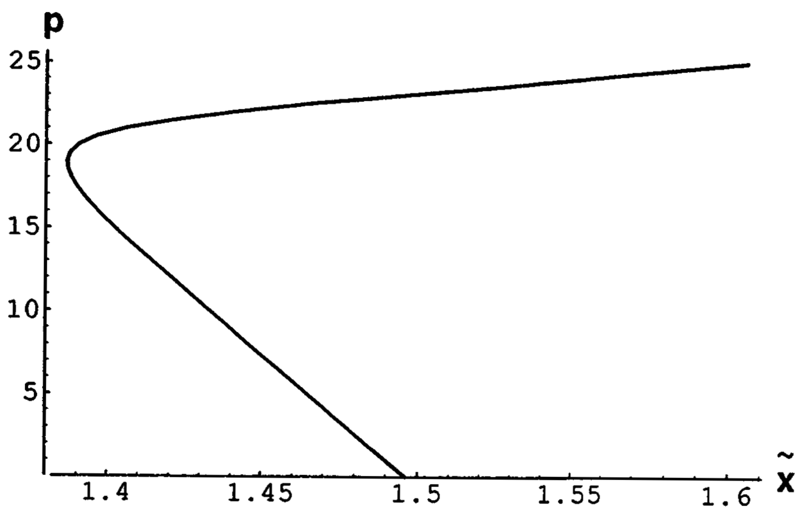
The effects of a non-optimal parking fee can be explained using Fig. 3b. For p between 0 and \$18.90 per hour, $\tilde{x}$ decreases with p—driving becomes more attractive since the reduction in congestion more than compensates for the higher parking fee. For p between \$18.90 per hour and \$23.80 per hour, $\tilde{x}$ increases with p—the reduction in congestion does not compensate for the higher parking fee. At a fee slightly above \$23.80 per hour, no one drives. The fee is so high that even though there is no

[11] Since perverse comparative static results are characteristic of unstable equilibria, this result supports the argument that the intermediate equilibrium is unstable.

[12] We determined these equilibria by employing a combination of MATHEMATICA and numerical comparative statics. Using MATHEMATICA alone, we encountered serious difficulties in the numerical solution. We developed a solution procedure supplementing MATHEMATICA which circumvented these numerical problems. The procedure is described in Arnott and Rowse [4].

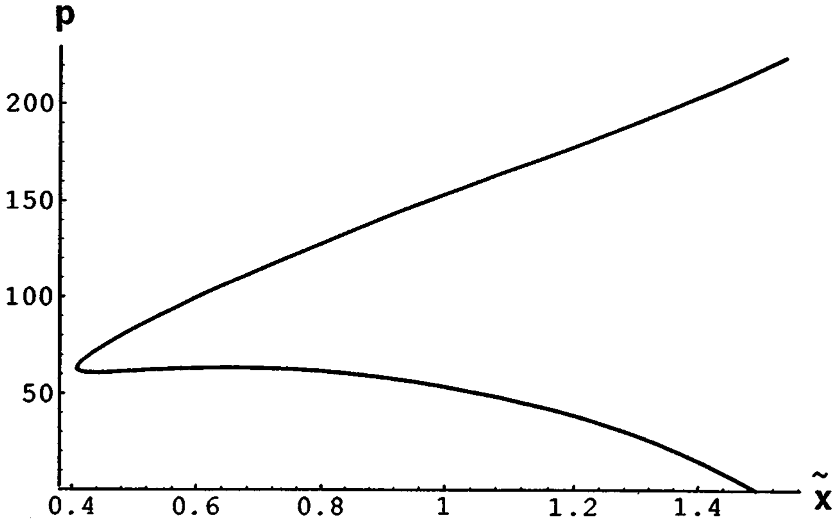


(a)



(b)

FIG. 3. (a) Equilibrium as a function of the parking fee, $l = 0.00$; (b) equilibrium as a function of the parking fee, $l = 0.25$; (c) equilibrium as a function of the parking fee, $l = 0.03$.



(c)

FIG. 3. *Continued.*

congestion, the cost of parking on a car trip ($pl \approx \$5.95$) makes walking cheaper, even on the longest trips.

The effects of the parking fee were so different for $l = 0.0$ and $l = 0.25$ that we explored an intermediate case, $l = 0.03$.

EXAMPLE 3 ($l = 0.03$). Figure 3c plots p against $\tilde{x}$ for this example. With a zero parking fee, there is only a single equilibrium which is hypercongested. As the fee rises, a critical fee $\hat{p}$ is reached where the congested and unstable equilibria appear. As p rises further, another critical fee $\hat{\hat{p}}$ is reached where the unstable and hypercongested equilibria disappear.[13] For fees above $\hat{\hat{p}}$, there is only a single equilibrium, which is congested. This example demonstrates that raising the parking fee may not only cause a hypercongested equilibrium to disappear, as in Example 1, but may also cause a congested equilibrium to appear where one did not exist without a parking fee. The results for this example are broadly analogous to those obtained in analyzing steady-state flow congestion on highways.

[13] Thus, there are three equilibria for $p \in (\hat{p}, \hat{\hat{p}})$. For example, with $p = 61.5$, $\tilde{x} = 0.804$ for the hypercongested equilibrium, 0.489 for the unstable equilibrium, and 0.414 for the congested equilibrium.

V. EXTENSIONS

We have deliberately kept the model as simple as possible to elucidate basic points and to keep the algebra manageable. In this section, we touch on several extensions, and discuss one in some detail since the model is so well suited to treating it—providing information on parking availability.

V.1. Parking Supply and Strategies

In our model, all parking is operated by the planner and the number of parking spaces is fixed. A more sophisticated treatment of parking supply could readily be made. At each location the supply of land for roads and for other uses would be specified. The land for roads would be used for either traffic or on-street parking. Allocating more land to on-street parking would increase parking availability, but would exacerbate traffic congestion. Land for other uses would be allocated between housing and off-street parking. Increasing the amount of off-street parking would cause construction of housing at higher density, thereby increasing housing costs. Allocation of land between these uses could be optimized.

V.2. Traffic Congestion

Our model ignores traffic congestion. Flow congestion can be treated by assuming that travel speed in regular traffic depends on the density of cars in regular traffic as well as on effective capacity which is influenced by the amount of on-street parking, the rate at which cars enter and leave on-street parking spaces, the volume of pedestrian traffic, and the density of cars cruising for parking. Cruising-for-parking congestion is also important and can be modeled similarly to flow congestion. Other forms of congestion which may be desirable to incorporate include parking entry-and-exit congestion, both for on- and off-street parking, pedestrian congestion and, in network models, intersection congestion and gridlock.

The incorporation of traffic congestion is not only quantitatively very important, but also, when traffic congestion is underpriced, qualitatively changes the economics since parking policy is then an exercise in the second best. Suppose, for example, that the only forms of congestion are traffic flow congestion and parking congestion, and that the distribution of parking duration is independent of the distance driven on the trip. Then the second-best parking fee would include a fixed component equal to the average flow congestion externality associated with a marginal car trip plus a component linear in time parked to cover the parking congestion externality. More generally, the second-best parking fee would be set to minimize the deadweight loss associated with the myriad forms of unpriced congestion.

V.3. Value of Parking Information

Parking may be modeled at varying degrees of sophistication. At the simplest, one can suppress the stochasticity associated with finding a parking space and posit a function $s = s(\phi)$ which gives time spent parked on a car trip as a function of the occupancy rate of parking spaces, ϕ. With such a specification, one would obtain most of the qualitative results we found—the optimal parking fee equals the parking congestion externality, there may be multiple equilibria, etc. The danger of such an implicit approach is that important insights may be lost. In the current context, for example, if we had started with the function $s(\phi)$ and assumed it to be technological, we would have overlooked the efficiency gains from congestion pricing deriving from its impact on modal choice and cruising-distance decisions. At the other extreme, one can provide an exact mathematical treatment of the stochasticity associated with finding a parking spot by employing stochastic queuing theory [7, 21]. Each parking spot can be regarded as a separate server, with the service time equal to the length of time parked and the distribution of service times endogenously determined. Servers are spatially ordered and the customer travels from server to server at a specified speed until she finds one that is idle. This is not a conventional stochastic-queuing-theoretic problem. While it could presumably be solved from first principles, doing so would be difficult and the solution would be messy. We adopted an intermediate strategy, explicitly treating individuals' behavioral decisions and the stochasticity of parking availability, but making the approximation that the probabilities of adjacent parking spaces being vacant are statistically independent, which permits application of the simple mathematics of Poisson processes. An analogous approximation is made in intersection congestion analysis [14], which assumes that the arrival rate of cars at an intersection is generated by a Poisson process, even though it evidently is not (if cars are delayed by a traffic light at an upstream intersection, for example, they will arrive as a platoon at the intersection under consideration).

In our work, parking fees are independent of the particular realization of vacant parking spaces. Vickrey [24] proposed *responsive* pricing for parking, whereby the parking fee for a parking space would depend on the *realized* availability of nearby vacant parking spaces. The efficiency gains from responsive parking pricing are larger the better-informed are individuals concerning the actual pattern of vacant parking spaces when making their usage decisions.

Because it treats the stochasticity of vacant parking spaces, our model is well suited for examining the value of parking information. Many cities in Europe and Japan put signs on major arterial roads indicating the availability of parking at the major parking lots. And there is talk of providing parking information to drivers via computer either before they start a trip or when they are in transit [6].

To illustrate adapting our model to deal with "informatics," we now consider a particularly simple parking information system (PIS). Each driver is informed, at the time she receives a trip opportunity, of the available parking spot closest to her destination in terms of travel time. If she decides to drive, the parking spot is assigned to her. There is no parking fee. Since the individual is tentatively assigned a parking spot when she receives a trip opportunity, she knows exactly the duration of the trip opportunity—the lesser of the travel times walking and driving. Thus, she chooses the reservation travel time t' to minimize expected trip duration, taking P as fixed. Combining this equation with the appropriate stationary-state condition yields two equations in t' and P. Arnott and Rowse [5] provide details of the solution procedure.

We solve for equilibrium for this PIS using the parameters of Example 1. Recall that, in the absence of PIS, there are three equilibria: a stable, congested equilibrium with $(P, L) = (187.25, 0.51595)$, an unstable, congested equilibrium with $(P, L) = (11.456, 0.55554)$, and a stable, hypercongested equilibrium with $(P, L) = (0.65722, 1.0253)$. With the PIS, $(P, L) = (0.042052, 0.98042)$. Introducing the PIS causes the two congested equilibria to disappear, but reduces expected trip time by about 4.4% in the hypercongested equilibrium. The PIS is very inefficient when without it the parking occupancy rate would be low. To see why, suppose that the economy is in the stable, congested equilibrium without the PIS, and that *one* individual is provided with the PIS. Her trip length would shrink by about three seconds but this would come at the social cost of holding an unoccupied parking spot reserved on each of her outbound car trips, for an average of about nine minutes, which would generate a parking externality of about 40 seconds (calculated as E times nine minutes). In fact, the PIS does even worse than this calculation suggests—it eliminates the stable, congested equilibrium. But when the parking occupancy rate is high without the PIS, the PIS is welfare improving. The gains from directing drivers to the closest parking spaces and from having them reject time-consuming trips they would have taken without parking information more than offset the costs of unoccupied but reserved parking spaces.

Intuition suggests that this parking information system would be improved by reserving a parking spot for an individual only if parking close to her destination is scarce, and would be further improved by updating the reserved parking spot as parking spots become available while she is en route. A possible further refinement would entail the planner assigning the individual the parking spot with the lowest expected social cost rather than the lowest expected private cost.

VI. CONCLUSION

This paper has developed a model of parking. The model was constructed with four principal considerations in mind. First, the model was primitive or structural, rather than reduced-form, viz. the demand for parking and the congestion cost function were derived rather than assumed. Second, the model was designed so that it can be extended to incorporate a host of realistic complications. Third, the model was general equilibrium to ensure a rigorous conceptual basis for welfare analysis. Fourth, the model was designed to focus on stochastic aspects of parking, especially cruising for parking. The model was the simplest we could think of satisfying these four criteria. This simplicity was achieved by assuming spatial and temporal homogeneity and by ignoring the flow congestion of cars.

Despite these simplifying assumptions, the model's behavior is complex. Multiple stable equilibria are possible; and which equilibrium obtains depends presumably on history. This is discouraging since it suggests that non-stationary-state analysis is needed, but this will be difficult. The comparative static properties of the model are complex as well. The welfare economics of the model is, however, relatively straightforward. The parking fee should be set at the value of the parking congestion externality. Setting the parking fee at that level which decentralizes the social optimum does not, however, guarantee attainment of the optimum since the economy may remain stuck at a hypercongested equilibrium. Unfortunately, the complexity of the model appears intrinsic to the parking problem rather than an artifact, which suggests that sound *analytical* work on parking will be discouragingly difficult. Despite computational problems deriving from the "highly" nonlinear nature of the model, numerical solution appears a fruitful avenue for future research, and our model should provide insights into the results of practical parking simulation models.

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