15.094J/1.142J Robust Modeling, Optimization and Computation

Instructors:

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Lecture: Wednesdays 4-7pm, E51-372.

Recitation: Monday 1-2pm, E51-085.

Course Content and Objectives: Today the modeling of uncertain phenomena in both theory and practice is done via probability theory, whose foundation is based on the axioms set forth by Kolmogorov in 1933. While it offers insights in understanding uncertainty, probability theory, in contrast to optimization, has not been developed with computational tractability as an objective when the dimension increases. Correspondingly, some of its major areas of application remain unsolved when the underlying systems become multidimensional: Queueing networks, auction design in multi-item, multi-bidder auctions, network information theory, pricing multi-dimensional options, optimization under uncertainty among others. At the current time, the only alternative for modeling uncertain phenomena is probability theory.

The first goal of the class is to propose an alternative via robust optimization (RO) for modeling uncertain phenomena. The key idea is to replace the Kolmogorov axioms and the concept of random variables as primitives of probability theory, with uncertainty sets that are derived from some of the asymptotic implications of probability theory. In this way, the performance analysis questions become highly structured optimization problems (linear, semidefinite, mixed integer) for which there exist efficient, practical algorithms that are capable of solving problems in high dimensions involving hundreds of thousands of variables and constraints.

The second goal of the class is to develop RO as a tractable methodology for solving optimization problems under uncertainty. Specifically we review under what conditions RO problems remain tractable when modeling uncertainty via uncertainty sets. The notion of tractability used, however, is not the same as theoretical efficiency (polynomial time solvability) developed in the 1970s. The Simplex method, for instance, has proven over many decades to be practically efficient, but not theoretically efficient. It is exactly this notion of practical efficiency we use in this class: it is the ability to solve problems of realistic size relative to the application we address. For example, queueing networks with hundreds of nodes, auctions with hundreds of items and bidders with budget constraints, network information theory with hundreds of thousands of codewords, and option pricing problems with hundreds of securities.

The third goal of the class is to expose students to a large number of applications ranging from supply chains, revenue management, energy, portfolio theory, options pricing, risk management, kalman filtering, queueing theory, information theory, statistics and engineering design. Our objective here is to teach students how to model and optimize uncertain phenomena via RO and be able to solve large scale problems involving hundreds of thousands to millions of variables and constraints.

With the availability of very large data sets arising in applications, the key principle we use in this class is that the ability to compute in high dimensions is critical. Correspondingly, **the fourth and final goal of the class is to expose students to the tools necessary for large scale computation for RO.**

Text: Research papers and class notes.

Recitations: The recitations will cover software for RO, computational aspects, and examples and applications that enhance the theory developed in the lectures.

Course Requirements: Problem sets, one midterm examination, and one final team project. Grades will be determined by performance on the above requirements weighted approximately as 30% problem sets, 30% midterm exam, and 40% final team project.

Lecture	Time	Topic	Readings
1	W, 2/05	Probability theory and its limitations	[4]
2	W, 2/05	The new primitives: Uncertainty sets	[4]
3	W, 2/12	Robust linear optimization I	[10]
4	W, 2/12	Robust linear optimization II	[26, 13]
5	W, 2/19	Robust mixed integer optimization	[27, 28]
6	W, 2/19	Robust convex optimization	[9, 29]
7	W, 2/26	Constructing uncertainty sets from data	[20]
8	W, 2/26	Adaptive multistage optimization I	[8, 21, 7]
9	W, 3/05	Adaptive multistage optimization II	[22, 14]
10	W, 3/05	Power of robust policies in adaptive optimization	[19, 18]
11	W, 3/12	Power of affine policies in adaptive optimization	[17, 21]
12	W, 3/12	Robust Kalman Filtering	[11]
13	W, 3/19	RO in supply chains	[30]
14	W, 3/19	RO in energy	[23]
	W, 3/26	Spring break	
	W, 4/02	Midterm	
15	W, 4/09	Robust portfolios	[32, 25]
16	W, 4/09	Robust options pricing	[2]
17	W, 4/16	RO and risk preferences	[12]
18	W, 4/16	Constructing utilities via RO	[24]
19	W, 4/23	Robust steady-state queueing theory	[5]
20	W, 4/23	Robust transient queueing theory	[6]
21	W, 4/30	Robust mechanism design	[1]
22	W, 4/30	RO in statistics	[31, 15, 16]
23	W, 5/07	Robust information theory I	[3]
24	W, 5/07	Robust information theory II	[3]
25	W, 5/14	Project presentations	
26	W, 5/14	Project presentations	

References

[1] C. Bandi and D. Bertsimas. Optimal design for multi-item auctions: A robust optimization approach. To appear in *Mathematics of Operations Research*, 2011.

[2] C. Bandi and D. Bertsimas. Robust option pricing - An ϵ-arbitrage approach. To appear in *European Journal of Operational Research*, 2011.

[3] C. Bandi and D. Bertsimas. Network information theory via robust optimization. Submitted to *IEEE Transactions on Information Theory*, 2012.

[4] C. Bandi and D. Bertsimas. Tractable stochastic analysis in high dimensions via robust optimization. *Mathematical Programming, Series B*, 134(1):23–70, 2012.

[5] C. Bandi, D. Bertsimas, and N. Youssef. Robust queueing theory. Submitted for publication, 2012.

[6] C. Bandi, D. Bertsimas, and N. Youssef. Robust transient queueing theory. Submitted to *Operations Research*, 2013.

[7] A. Ben-Tal, B. Golany, E. Guslitzer, and A. Nemirovski. Adjustable robust solutions of uncertain linear programs. *Mathematical Programming, Series B*, 99(2):351–376, 2004.

[8] A. Ben-Tal, A. Goryashko, A. Nemirovski, and J.P. Vial. Retailer-supplier flexible commitments contracts: A robust optimization approach. *Manufacturing and Service Operations Management*, 7(3):248–271, 2005.

[9] A. Ben-Tal and A. Nemirovski. Robust convex optimization. *Mathematics of Operations Research*, 23(4):769–805, 1998.

[10] A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs. *Operations Research Letters*, 25(1):1–13, 1999.

[11] D. Bertsimas and D. Brown. Constrained stochastic LQC: A tractable approach. *IEEE Transactions on Automatic Control*, 52:1826–1841, 2007.

[12] D. Bertsimas and D. Brown. Constructing uncertainty sets for robust linear optimization. *Operations Research*, 57(6):1483–1495, 2009.

[13] D. Bertsimas, D. Brown, and C. Caramanis. Theory and applications of robust optimization. *SIAM Review*, 53(3):464–501, 2011.

[14] D. Bertsimas and C. Caramanis. Finite adaptability in linear optimization. *IEEE Transations on Automatic Control*, 55(12):2751–2766, 2010.

[15] D. Bertsimas and A. Fertis. Robust logistic regression. Working paper, 2008.

[16] D. Bertsimas and A. Fertis. On the equivalence of robust optimization and regularization in statistics. Working paper, 2009.

[17] D. Bertsimas and V. Goyal. On the power and limitations of affine policies in two-stage adaptive optimization. *Mathematical Programming, Series B*, 134(2):491–531, 2011.

[18] D. Bertsimas and V. Goyal. On the approximability of adjustable robust convex optimization under uncertainty. *Mathematical Methods of Operations Research*, 77(3):323–343, 2013.

[19] D. Bertsimas, V. Goyal, and X.A. Sun. A geometric characterization of the power of finite adaptability in multi-stage stochastic and adaptive optimization. *Mathematics of Operations Research*, 36(1):24–54, 2011.

[20] D. Bertsimas, V. Gupta, and N. Kallus. Data-driven robust optimization. Submitted to *Operations Research*, 2013.

[21] D. Bertsimas, D. Iancu, and P. Parrilo. Optimality of affine policies in multi-stage robust optimization. *Mathematics of Operations Research*, 35(2):363–394, 2010.

[22] D. Bertsimas, D. Iancu, and P. Parrilo. A hierarchy of near-optimal policies for multi-stage adaptive optimization. *IEEE Transactions on Automatic Control*, 56(12):2809–2824, 2011.

[23] D. Bertsimas, E. Litvinov, X.A. Sun, J. Zhao, and T. Zheng. Adaptive robust optimization for the security constrained unit commitment problem. *IEEE Transactions on Power Systems*, 28(1):52–63, 2013.

[24] D. Bertsimas and A. O'Hair. Learning preferences under noise and loss aversion: An optimization approach. *Operations Research*, 61(5):1190–1199, 2013. Submitted for publication.

[25] D. Bertsimas and D. Pachamanova. Robust multiperiod portfolio management in the presence of transaction costs. *Computers and Operations Research*, 35(1):3–17, 2008.

[26] D. Bertsimas, D. Pachamanova, and M. Sim. Robust linear optimization under general norms. *Operations Research Letters*, 32(6):510–516, 2004.

[27] D. Bertsimas and M. Sim. Robust discrete optimization and network flows. *Mathematical Programming, Series B*, 98(1-3):49–71, 2003.

[28] D. Bertsimas and M. Sim. The price of robustness. *Operations Research*, 52(1):35–53, 2004.

[29] D. Bertsimas and M. Sim. Tractable approximations to robust conic optimization problems. *Mathematical Programming, Series B*, 107(1-2):5–36, 2006.

[30] D. Bertsimas and A. Thiele. A robust optimization approach to inventory theory. *Operations Research*, 54:150–168, 2006.

[31] L. El-Ghaoui and H. Lebret. Robust solutions to least-square problems to uncertain data matrices. *SIAM Journal on Matrix Analysis and Applications*, 18(4):1035–1064, 1997.

[32] D. Goldfarb and G. Iyengar. Robust portfolio selection problems. *Mathematics of Operations Research*, 28(1):1–38, 2003.