MATLAB PROJECT 3

Please include this page in your Group file, as a front page. Type in the group number and the names of all members WHO PARTICIPATED in this project.

GROUP # 26

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By including your names above, each of you had confirmed that you did the work and agree with the work submitted.

Exercise 1

```
type rowspace
```

```
function []=rowspace(A,B)
a = rref(A);
b = rref(B);
if(size(A,2) == size(B,2))
    fprintf('Row A and Row B are subspaces of R^%i\n',size(A,2))
    fprintf('The dimension of Row A is %i\n',rank(A));
    fprintf('The dimension of Row B is %i\n',rank(B));
    if(rank(A) >= size(A,2) && (rank(B) >= size(B,2)))
        fprintf('Row A and Row B spans the whole R^%i\n',size(B,2))
        return
    else
        if(rank(A) == rank(B))
                pa = zeros(size(A,2),rank(A));
                ta = transpose(a);
                for i = 1:rank(A)
                    pa(:,i) = ta(:,i);
                end

                pb = zeros(size(A,2),rank(A));
                tb = transpose(b);
                for i = 1:rank(A)
                    pb(:,i) = tb(:,i);
                end

            if(pa == pb)
                disp('A basis for Row A and Row B is')
                P = pa
            else
                disp('Row A and Row B have the same dimensions but are not equal')
            end
        else
            disp('Row A and Row B have diffrent dimensions and cannot be equal')
        end
    end

else
    disp('The Rowspaces of A and B are of diffrent vector spaces')
end
```

```
C=[2 6 2 4 -6;-4 -9 -7 -2 3;-2 -5 -3 -2 3;3 8 9 -1 4]
```

```
C = 4×5
     2     6     2     4    -6
    -4    -9    -7    -2     3
    -2    -5    -3    -2     3
     3     8     9    -1     4
```

```
%(a)
A=C, B=rref(C)
```

```
A = 4×5
     2     6     2     4    -6
    -4    -9    -7    -2     3
    -2    -5    -3    -2     3
     3     8     9    -1     4
B = 4×5
     1     0     0     0    -2
     0     1     0     1    -1
     0     0     1    -1     2
```

```
     0     0     0     0     0
```

```
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^5
The dimension of Row A is 3
The dimension of Row B is 3
A basis for Row A and Row B is
P = 5×3
     1     0     0
     0     1     0
     0     0     1
     0     1    -1
    -2    -1     2
```

```
%(b)
A=A',B=B'
```

```
A = 5×4
     2    -4    -2     3
     6    -9    -5     8
     2    -7    -3     9
     4    -2    -2    -1
    -6     3     3     4
B = 5×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     1    -1     0
    -2    -1     2     0
```

```
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^4
The dimension of Row A is 3
The dimension of Row B is 3
Row A and Row B have the same dimensions but are not equal
```

```
% Since the transpose of the matrices have diffrent
% row spaces, this indicates that the elementry row
% operations done by rref(C) changed the column space
% of C

%(c)
A=[C;zeros(2,5)], B=rref(C)
```

```
A = 6×5
     2     6     2     4    -6
    -4    -9    -7    -2     3
    -2    -5    -3    -2     3
     3     8     9    -1     4
     0     0     0     0     0
     0     0     0     0     0
B = 4×5
     1     0     0     0    -2
     0     1     0     1    -1
     0     0     1    -1     2
     0     0     0     0     0
```

```
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^5
```

```
The dimension of Row A is 3
The dimension of Row B is 3
A basis for Row A and Row B is
P = 5×3
     1     0     0
     0     1     0
     0     0     1
     0     1    -1
    -2    -1     2
```

```
%(d)
A=magic(3);B=magic(4);
rowspace(A,B)
```

```
The Rowspaces of A and B are of diffrent vector spaces
```

```
%(e)
A=magic(4);B=hilb(4);
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^4
The dimension of Row A is 3
The dimension of Row B is 4
Row A and Row B have diffrent dimensions and cannot be equal
```

```
%(f)
A=magic(5);B=hilb(5);
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^5
The dimension of Row A is 5
The dimension of Row B is 5
Row A and Row B spans the whole R^5
```

```
%(g)
A=magic(5);B=[A;eye(3,5)];
rowspace(A,B)
```

```
Row A and Row B are subspaces of R^5
The dimension of Row A is 5
The dimension of Row B is 5
Row A and Row B spans the whole R^5
```

Exercise 2

Part 1:

```
type shrink
```

```
function B=shrink(A)
[~,pivot]=rref(A);
B=A(:,pivot);
end
```

```
A=magic(4)
```

```
A = 4×4
    16     2     3    13
     5    11    10     8
     9     7     6    12
```

```
     4    14    15     1
```

```
rref(A)
```

```
ans = 4×4
     1     0     0     1
     0     1     0     3
     0     0     1    -3
     0     0     0     0
```

```
[r,pivot]=rref(A)
```

```
r = 4×4
     1     0     0     1
     0     1     0     3
     0     0     1    -3
     0     0     0     0
pivot = 1×3
     1     2     3
```

```
P=A(:,pivot)
```

```
P = 4×3
    16     2     3
     5    11    10
     9     7     6
     4    14    15
```

The first is A, the second is the reduced row echelon form of A, then it splits up the result and the pivot columns, and makes a matrix P out of only the pivot columns of A.

```
B=shrink(A)
```

```
B = 4×3
    16     2     3
     5    11    10
     9     7     6
     4    14    15
```

This forms a basis for the column space of A because it only includes the pivot columns of A.

```
R=rref(transpose(A)), BC=colspace(sym(A))
```

```
R = 4×4
     1     0     0     1
     0     1     0     3
     0     0     1    -3
     0     0     0     0
BC =
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & -3 \end{pmatrix}$$

colspace(sym(A)) generates a symbolic matrix by first transforming A into a symbolic matrix, and then finding the columns of A that are pivot columns.

```
DC=double(BC)
```

```
DC = 4×3
     1     0     0
     0     1     0
     0     0     1
     1     3    -3
```

Part 2:

```
type noll.m
```

```
function [C,p] = noll(A)
[m,n]=size(A);
C=[];
p=0;
if rank(A) == n
    fprintf('the null space is the zero subspace of R^%i\n',n)
    C = zeros(n,1);
else
    [~,pivot_c] = rref(A);
    S=1:n;
    nonpivot_c = setdiff(S,pivot_c);
    q=numel(nonpivot_c);
    j=1:q;
    C=zeros(n,q);
    Aref = rref(A);
    colOfZero = 0;
    for k = 1:n
        if Aref(:,k) == zeros(m,1)
            colOfZero = 1 + colOfZero;
        else
            break;
        end
    end
    C(1+colOfZero:m+colOfZero,j)= -Aref(1:m,nonpivot_c(j));
    for i = 1:q
     C(nonpivot_c(i),i) = 1;
    end
    p = n - rank(A)
    C = C(1:n,:);
    fprintf('the null space is a %i-dimentional subspace of R^%i\n',p,n)
    fprintf('A basis for Nul A is formed by the columns of the matrix')
    end
end
```

```
A=magic(5)
```

```
A = 5×5
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

```
[C,p]=noll(A)
```

```
the null space is the zero subspace of R^5
C = 5×1
     0
     0
     0
     0
     0
```

```
p = 0
```

```
%(a)
A=[2 -4 -2 3; 6 -9 -5 8; 2 -7 -3 9; 4 -2 -2 -1;  -6 3 3 4]
```

```
A = 5×4
     2    -4    -2     3
     6    -9    -5     8
     2    -7    -3     9
     4    -2    -2    -1
    -6     3     3     4
```

```
[C,p]=noll(A)
```

```
p = 1
the null space is a 1-dimentional subspace of R^4
A basis for Nul A is formed by the columns of the matrix
C = 4×1
    0.3333
   -0.3333
    1.0000
         0
p = 1
```

```
N=null(A,'r' )
```

```
N = 4×1
    0.3333
   -0.3333
    1.0000
         0
```

```
isequal(C,N)
```

```
ans = logical
   1
```

```
%(b)
A=ones(5)
```

```
A = 5×5
     1     1     1     1     1
     1     1     1     1     1
     1     1     1     1     1
     1     1     1     1     1
     1     1     1     1     1
```

```
[C,p]=noll(A)
```

```
p = 4
the null space is a 4-dimentional subspace of R^5
A basis for Nul A is formed by the columns of the matrix
C = 5×4
    -1    -1    -1    -1
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
p = 4
```

```
N=null(A,'r' )
```

```
N = 5×4
    -1    -1    -1    -1
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
isequal(C,N)
```

```
ans = logical
   1
```

```
%(c)
A=[magic(4),ones(4,1)]
```

```
A = 4×5
    16     2     3    13     1
     5    11    10     8     1
     9     7     6    12     1
     4    14    15     1     1
```

```
[C,p]=noll(A)
```

```
p = 2
the null space is a 2-dimentional subspace of R^5
A basis for Nul A is formed by the columns of the matrix
C = 5×2
   -1.0000   -0.0588
   -3.0000   -0.1176
    3.0000    0.0588
    1.0000         0
         0    1.0000
p = 2
```

```
N=null(A,'r' )
```

```
N = 5×2
   -1.0000   -0.0588
   -3.0000   -0.1176
    3.0000    0.0588
    1.0000         0
         0    1.0000
```

```
isequal(C,N)
```

```
ans = logical
   1
```

The null space function works by finding rational basis vectors for the matrix from the RREF.

```
A=[2 -4 -2 3; 6 -9 -5 8; 2 -7 -3 9; 4 -2 -2 -1; -6 3 3 4]
```

```
A = 5×4
     2    -4    -2     3
     6    -9    -5     8
     2    -7    -3     9
     4    -2    -2    -1
    -6     3     3     4
```

```
BR=double(colspace(sym(A')))
```

```
BR = 4×3
    1.0000         0         0
         0    1.0000         0
   -0.3333    0.3333         0
         0         0    1.0000
```

```
q = rank(BR)
```

```
q = 3
```

```
n = rank(A)
```

```
n = 3
```

Exercise 3

```
type polyspace.m
```

```
function P=polyspace(B,Q,rB)
format
n = numel(B);
P=zeros(n);


% Fill the array with the polynomial.
for i = 1:n
    P(:,i) = sym2poly(B(i));
end

P=closetozeroroundoff(P,7);
fprintf('matrix of E-coordinate vectors of the polynomials in B is\n')
display(P)


%Check to see if P forms a basis for R^n.

if (rank(P) == n )

   fprintf('the polynomials in B form a basis for P%d\n',n-1)
   q = sym2poly(Q);
   q = closetozeroroundoff(q, 7);
   B = rref([P transpose(q)]);
   qB = B(:,end);
   fprintf('the B-coordinate vector of Q is\n')
   display(qB)


   R = P*rB;
   R = closetozeroroundoff(R, 7);
   R = poly2sym(R);
   fprintf('the polynomial whose B-coordinates form the vector rB is\n')
   display(R)


else
    sprintf('the polynomials in B do not form a basis for P%d',n-1)
    A=rref(P);
    fprintf('the reduced echelon form of P is\n')
    disp(A)

end
end
```

```
type closetozeroroundoff.m
```

```
function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end
```

```
syms x
%(a)
B=[x^3+3*x^2,10^(-8)*x^3+x,10^(-8)*x^3+4*x^2+x,x^3+x]
```

B =

$$\left(x^3+3\,x^2 \quad \frac{x^3}{100000000}+x \quad \frac{x^3}{100000000}+4\,x^2+x \quad x^3+x\right)$$

```
Q=10^(-8)*x^3+x^2+6*x
```

Q =

$$\frac{x^3}{100000000}+x^2+6\,x$$

```
rB=[2;-3;1;0]
```

```
rB = 4×1
     2
    -3
     1
     0
```

```
P=polyspace(B,Q,rB);
```

```
matrix of E-coordinate vectors of the polynomials in B is
P = 4×4
     1     0     0     1
     3     0     4     0
     0     1     1     1
     0     0     0     0
ans =
'the polynomials in B do not form a basis for P3'
the reduced echelon form of P is
    1.0000         0         0    1.0000
         0    1.0000         0    1.7500
         0         0    1.0000   -0.7500
         0         0         0         0
```

```
%(b)
B=[x^3-1,10^(-8)*x^3+2*x^2,10^(-8)*x^3+x,x^3+x]
```

B =

$$\left(x^3-1 \quad \frac{x^3}{100000000}+2\,x^2 \quad \frac{x^3}{100000000}+x \quad x^3+x\right)$$

```
P=polyspace(B,Q,rB);
```

```
matrix of E-coordinate vectors of the polynomials in B is
P = 4×4
     1     0     0     1
     0     2     0     0
```

```
     0     0     1     1
    -1     0     0     0
the polynomials in B form a basis for P3
the B-coordinate vector of Q is
qB = 4×1
         0
    0.5000
    6.0000
         0
the polynomial whose B-coordinates form the vector rB is
```

R = $2x^3 - 6x^2 + x - 2$

```
%(c)
B=[x^3+1,10^(-8)*x^3+x^2+1,10^(-8)*x^3+x+1,10^(-8)*x^3+1]
```

B =

$$\left(x^3 + 1 \quad \frac{x^3}{100000000} + x^2 + 1 \quad \frac{x^3}{100000000} + x + 1 \quad \frac{x^3}{100000000} + 1\right)$$

```
Q=10^(-8)*x^3+2*x^2+x+6
```

Q =

$$\frac{x^3}{100000000} + 2x^2 + x + 6$$

```
rB=[0;-3;1;0]
```

```
rB = 4×1
     0
    -3
     1
     0
```

```
P=polyspace(B,Q,rB);
```

```
matrix of E-coordinate vectors of the polynomials in B is
P = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     1     1     1     1
the polynomials in B form a basis for P3
the B-coordinate vector of Q is
qB = 4×1
     0
     2
     1
     3
the polynomial whose B-coordinates form the vector rB is
```

R = $-3x^2 + x - 2$

```
%(d)
B=[x^4+x^3+x^2+1,10^(-8)*x^4+x^3+x^2+x+1,10^(-8)*x^4+x^2+x+1,10^(-8)*x^4+x+1,10^(-8)*x^4+1]
```

B =

$$\left(x^4 + x^3 + x^2 + 1 \quad \frac{x^4}{100000000} + x^3 + x^2 + x + 1 \quad \frac{x^4}{100000000} + x^2 + x + 1 \quad \frac{x^4}{100000000} + x + 1 \quad \frac{x^4}{100000(}\right.$$

```
Q=10^(-8)*x^4+3*x^3-1
```

Q =

$$\frac{x^4}{100000000} + 3\,x^3 - 1$$

```
rB=diag(magic(5))
```

```
rB = 5×1
    17
     5
    13
    21
     9
```

```
P=polyspace(B,Q,rB);
```

```
matrix of E-coordinate vectors of the polynomials in B is
P = 5×5
     1     0     0     0     0
     1     1     0     0     0
     1     1     1     0     0
     0     1     1     1     0
     1     1     1     1     1
the polynomials in B form a basis for P4
the B-coordinate vector of Q is
qB = 5×1
     0
     3
    -3
     0
    -1
the polynomial whose B-coordinates form the vector rB is
```

R = $17\,x^4 + 22\,x^3 + 35\,x^2 + 39\,x + 65$

Exercise 4

```
type closetozeroroundoff.m
```

```
function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end
```

```
type quer
```

```
function [Q,R] = quer(A)
format
[m,n]=size(A);

[Q,R]=qr(A)
if (closetozeroroundoff(A-Q*R,7)==0)
    disp('the product of Q and R forms a decomposition of A');
else
    disp('no, it cannot be true!');
end

q=0;
r=0;
if(inv(Q) == transpose(Q))
```

```
    q=1;
    disp('Q is a unitary matrix');
    %check that Q is unitary
end
if (istriu(R))
    r=1;
    disp('R is an upper-triangular matrix');
    %check R is an upper-triangular matrix
end
if(q==1 && r==1)
    disp('Q*R forms an orthogonal-triangular decomposition of A');
else
    disp('Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!');
end

if(m == n)
    k=1;
    P=closetozeroroundoff(A-triu(A),7);
while(P~=zeros(m,n))
    A=R*Q;
    [Q,R]=qr(A);
    P=closetozeroroundoff(A-triu(A),7);
    k=k+1;
end
disp('the matrix B');
disp(A);
disp('the number of iterations:');
disp(k);

disp('the main diagonal of the matrix B');
for i=1:m
    disp(A(i,i));
end

E = eig(A);
disp('the eigenvalues of the input matrix A')
disp(E)
end
end
```

```
%(a)
A=randi(10,3,4)
```

```
A = 3×4
     4     7     9    10
     2     5     6     3
     3     4     6     8
```

```
[Q,R] = quer(A);
```

```
Q = 3×3
   -0.7428   -0.0531   -0.6674
   -0.3714   -0.7967    0.4767
   -0.5571    0.6020    0.5721
R = 3×4
   -5.3852   -9.2848  -12.2559  -12.9987
         0   -1.9476   -1.6466    1.8945
         0         0    0.2860   -0.6674
the product of Q and R forms a decomposition of A
R is an upper-triangular matrix
Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!
```

```
%(b)
A=ones(5,4);
```

```
[Q,R] = quer(A);
```

```
Q = 5×5
   -0.4472    0.8944   -0.0000         0    0.0000
   -0.4472   -0.2236    0.8660         0   -0.0000
   -0.4472   -0.2236   -0.2887   -0.5774   -0.5774
   -0.4472   -0.2236   -0.2887    0.7887   -0.2113
   -0.4472   -0.2236   -0.2887   -0.2113    0.7887
R = 5×4
   -2.2361   -2.2361   -2.2361   -2.2361
         0   -0.0000   -0.0000   -0.0000
         0         0   -0.0000   -0.0000
         0         0         0         0
         0         0         0         0
the product of Q and R forms a decomposition of A
R is an upper-triangular matrix
Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!
```

```
%(c)
A=diag([2,2,4,4])
```

```
A = 4×4
     2     0     0     0
     0     2     0     0
     0     0     4     0
     0     0     0     4
```

```
[Q,R] = quer(A);
```

```
Q = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
R = 4×4
     2     0     0     0
     0     2     0     0
     0     0     4     0
     0     0     0     4
the product of Q and R forms a decomposition of A
Q is a unitary matrix
R is an upper-triangular matrix
Q*R forms an orthogonal-triangular decomposition of A
the matrix B
     2     0     0     0
     0     2     0     0
     0     0     4     0
     0     0     0     4

the number of iterations:
     1

the main diagonal of the matrix B
     2

     2

     4

     4

the eigenvalues of the input matrix A
     2
     2
```

```
     4
     4
```

```
%(d)
A=triu(magic(4))
```

```
A = 4×4
    16     2     3    13
     0    11    10     8
     0     0     6    12
     0     0     0     1
```

```
[Q,R] = quer(A);
```

```
Q = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
R = 4×4
    16     2     3    13
     0    11    10     8
     0     0     6    12
     0     0     0     1
the product of Q and R forms a decomposition of A
Q is a unitary matrix
R is an upper-triangular matrix
Q*R forms an orthogonal-triangular decomposition of A
the matrix B
    16     2     3    13
     0    11    10     8
     0     0     6    12
     0     0     0     1

the number of iterations:
     1

the main diagonal of the matrix B
    16

    11

     6

     1

the eigenvalues of the input matrix A
    16
    11
     6
     1
```

```
%(e)
A=triu(magic(4),-1)
```

```
A = 4×4
    16     2     3    13
     5    11    10     8
     0     7     6    12
     0     0    15     1
```

```
[Q,R] = quer(A);
```

```
Q = 4×4
   -0.9545    0.2436   -0.0011    0.1722
   -0.2983   -0.7794    0.0034   -0.5509
         0   -0.5772   -0.0051    0.8166
         0         0    1.0000    0.0062
R = 4×4
  -16.7631   -5.1900   -5.8462  -14.7944
         0  -12.1270  -10.5268   -9.9956
         0         0   15.0003    0.9524
         0         0         0    7.6357
the product of Q and R forms a decomposition of A
R is an upper-triangular matrix
Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!
the matrix B
    16     2     3    13
     5    11    10     8
     0     7     6    12
     0     0    15     1

the number of iterations:
     1

the main diagonal of the matrix B
    16

    11

     6

     1

the eigenvalues of the input matrix A
   25.0011
   12.0417
    7.4036
  -10.4464
```

```
%(f)
A=tril(magic(4),1)
```

```
A = 4×4
    16     2     0     0
     5    11    10     0
     9     7     6    12
     4    14    15     1
```

```
[Q,R] = quer(A);
```

```
Q = 4×4
   -0.8230    0.4186    0.1367   -0.3590
   -0.2572   -0.5155   -0.7600   -0.3009
   -0.4629   -0.1305   -0.0997    0.8711
   -0.2057   -0.7363    0.6275   -0.1478
R = 4×4
  -19.4422  -10.5955   -8.4352   -5.7607
         0  -16.0541  -16.9816   -2.3024
         0         0    1.2138   -0.5692
         0         0         0   10.3048
the product of Q and R forms a decomposition of A
R is an upper-triangular matrix
Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!
the matrix B
    16     2     0     0
```

```
     5    11    10     0
     9     7     6    12
     4    14    15     1

the number of iterations:
     1

the main diagonal of the matrix B
    16

    11

     6

     1

the eigenvalues of the input matrix A
   25.4509
   14.9803
   -7.7521
    1.3209
```

```
%(g)
A=[1 1 4;0 -4 0;-5 -1 -8]
```

```
A = 3×3
     1     1     4
     0    -4     0
    -5    -1    -8
```

```
[Q,R] = quer(A);
```

```
Q = 3×3
   -0.1961    0.1887    0.9623
         0   -0.9813    0.1925
    0.9806    0.0377    0.1925
R = 3×3
   -5.0990   -1.1767   -8.6291
         0    4.0762    0.4529
         0         0    2.3094
the product of Q and R forms a decomposition of A
R is an upper-triangular matrix
Q*R does NOT forms an orthogonal-triangular decomposition of A. What is wrong?!
the matrix B
     1     1     4
     0    -4     0
    -5    -1    -8

the number of iterations:
     1

the main diagonal of the matrix B
     1

    -4

    -8

the eigenvalues of the input matrix A
   -3.0000
   -4.0000
   -4.0000
```

Exercise 5

Part 1

```
type eluinv
```

```
function [L, U] = eluinv(A)
[m,n] = size(A);
[L,U] = lu(A)

if closetozeroroundoff(A-L*U,7) == 0
    disp('Yes, I got a factorization')
end

if m~=n
    return
else

%Code checks to see if U echelon form of A
mat = sum(U~=0,2);
tf = true;
for i=1:size(mat)-1
    if mat(i,:)<mat(i+1,:)
        tf = false;
    end
end

if tf == true
    disp('U is an echelon form of A')
else
    disp('U is not an echelon form of A? What is wrong?!')
    return
end

%Code checks to see if A is invertible
if rank(A)~=n
    sprintf('A is not invertible')
    return
end

%Calculate inverses
L1 = rref([L eye(n)]);
invL = L1(:,n+1:2*n)

U1 = rref([U eye(n)]);
invU = U1(:,n+1:2*n)

invA = invU * invL;
disp('the inverse of A calculated using LU factorization is')
disp(invA)

P = inv(A);
if closetozeroroundoff(invA-P,7) == 0
    disp('Yes, LU factorization works for calculating the inverses')
else
    disp('LU factorization does not work for me!?')
end

end
```

```
type closetozeroroundoff
```

```
function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
```

```
B=A;
end
```

```
% (a)
A = [1 1 4 3;0 -4 0 2;-5 -1 -8 1]
```

```
A = 3×4
     1     1     4     3
     0    -4     0     2
    -5    -1    -8     1
```

```
[L, U] = eluinv(A);
```

```
L = 3×3
   -0.2000   -0.2000    1.0000
         0    1.0000         0
    1.0000         0         0
U = 3×4
   -5.0000   -1.0000   -8.0000    1.0000
         0   -4.0000         0    2.0000
         0         0    2.4000    3.6000
Yes, I got a factorization
```

```
% (b)
A = [1 1 4;0 -4 0;-5 -1 -8;2 3 -1]
```

```
A = 4×3
     1     1     4
     0    -4     0
    -5    -1    -8
     2     3    -1
```

```
[L, U] = eluinv(A);
```

```
L = 4×3
   -0.2000   -0.2000   -0.5714
         0    1.0000         0
    1.0000         0         0
   -0.4000   -0.6500    1.0000
U = 3×3
   -5.0000   -1.0000   -8.0000
         0   -4.0000         0
         0         0   -4.2000
Yes, I got a factorization
```

```
% (c)
A = [1 1 4;0 -4 0;-5 -1 -8]
```

```
A = 3×3
     1     1     4
     0    -4     0
    -5    -1    -8
```

```
[L, U] = eluinv(A);
```

```
L = 3×3
   -0.2000   -0.2000    1.0000
         0    1.0000         0
    1.0000         0         0
U = 3×3
   -5.0000   -1.0000   -8.0000
```

```
         0   -4.0000         0
         0         0    2.4000
Yes, I got a factorization
U is an echelon form of A
invL = 3×3
         0         0    1.0000
         0    1.0000         0
    1.0000    0.2000    0.2000
invU = 3×3
   -0.2000    0.0500   -0.6667
         0   -0.2500         0
         0         0    0.4167
the inverse of A calculated using LU factorization is
   -0.6667   -0.0833   -0.3333
         0   -0.2500         0
    0.4167    0.0833    0.0833

Yes, LU factorization works for calculating the inverses
```

```
% (d)
A = magic(6)
```

```
A = 6×6
    35     1     6    26    19    24
     3    32     7    21    23    25
    31     9     2    22    27    20
     8    28    33    17    10    15
    30     5    34    12    14    16
     4    36    29    13    18    11
```

```
[L, U] = eluinv(A);
```

```
L = 6×6
    1.0000         0         0         0         0         0
    0.0857    0.8893   -0.7306    0.1953    1.0000         0
    0.8857    0.2261   -0.3797   -1.0000   -0.0000    1.0000
    0.2286    0.7739    0.3797    1.0000         0         0
    0.8571    0.1154    1.0000         0         0         0
    0.1143    1.0000         0         0         0         0
U = 6×6
   35.0000    1.0000    6.0000   26.0000   19.0000   24.0000
         0   35.8857   28.3143   10.0286   15.8286    8.2571
         0         0   25.5884  -11.4435   -4.1131   -5.5247
         0         0         0    7.6416   -5.0305    5.2221
         0         0         0         0    5.2718   10.5435
         0         0         0         0         0    0.0000
Yes, I got a factorization
U is an echelon form of A
ans =
'A is not invertible'
```

```
% (e)
A = [2 1 -3 1;0 5 -3 5;-4 3 3 3;-2 5 1 3]
```

```
A = 4×4
     2     1    -3     1
     0     5    -3     5
    -4     3     3     3
    -2     5     1     3
```

```
[L, U] = eluinv(A);
```

```
L = 4×4
```

```
   -0.5000    0.5000         0    1.0000
         0    1.0000         0         0
    1.0000         0         0         0
    0.5000    0.7000    1.0000         0
U = 4×4
   -4.0000    3.0000    3.0000    3.0000
         0    5.0000   -3.0000    5.0000
         0         0    1.6000   -2.0000
         0         0         0         0
Yes, I got a factorization
U is an echelon form of A
ans =
'A is not invertible'
```

```
% (f)
A = magic(5)
```

```
A = 5×5
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

```
[L, U] = eluinv(A);
```

```
L = 5×5
    0.7391    1.0000         0         0         0
    1.0000         0         0         0         0
    0.1739    0.2527    0.5164    1.0000         0
    0.4348    0.4839    0.7231    0.9231    1.0000
    0.4783    0.7687    1.0000         0         0
U = 5×5
   23.0000    5.0000    7.0000   14.0000   16.0000
         0   20.3043   -4.1739   -2.3478    3.1739
         0         0   24.8608   -2.8908   -1.0921
         0         0         0   19.6512   18.9793
         0         0         0         0  -22.2222
Yes, I got a factorization
U is an echelon form of A
invL = 5×5
         0    1.0000         0         0         0
    1.0000   -0.7391         0         0         0
   -0.7687    0.0899         0         0    1.0000
    0.1443   -0.0336    1.0000         0   -0.5164
   -0.0613   -0.1111   -0.9231    1.0000   -0.2464
invU = 5×5
    0.0435   -0.0107   -0.0140   -0.0343    0.0012
         0    0.0493    0.0083    0.0071    0.0127
         0         0    0.0402    0.0059    0.0031
         0         0         0    0.0509    0.0435
         0         0         0         0   -0.0450
the inverse of A calculated using LU factorization is
   -0.0049    0.0512   -0.0354    0.0012    0.0034
    0.0431   -0.0373   -0.0046    0.0127    0.0015
   -0.0303    0.0031    0.0031    0.0031    0.0364
    0.0047   -0.0065    0.0108    0.0435   -0.0370
    0.0028    0.0050    0.0415   -0.0450    0.0111

Yes, LU factorization works for calculating the inverses
```

```
% (g)
A = hilb(5)
```

```
A = 5×5
    1.0000    0.5000    0.3333    0.2500    0.2000
    0.5000    0.3333    0.2500    0.2000    0.1667
    0.3333    0.2500    0.2000    0.1667    0.1429
    0.2500    0.2000    0.1667    0.1429    0.1250
    0.2000    0.1667    0.1429    0.1250    0.1111
```

```
[L, U] = eluinv(A);
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000    1.0000    1.0000         0         0
    0.3333    1.0000         0         0         0
    0.2500    0.9000   -0.6000    0.5000    1.0000
    0.2000    0.8000   -0.9143    1.0000         0
U = 5×5
    1.0000    0.5000    0.3333    0.2500    0.2000
         0    0.0833    0.0889    0.0833    0.0762
         0         0   -0.0056   -0.0083   -0.0095
         0         0         0    0.0007    0.0015
         0         0         0         0   -0.0000
Yes, I got a factorization
U is an echelon form of A
invL = 5×5
    1.0000         0         0         0         0
   -0.3333         0    1.0000         0         0
   -0.1667    1.0000   -1.0000         0         0
   -0.0857    0.9143   -1.7143         0    1.0000
   -0.0071    0.1429   -0.6429    1.0000   -0.5000
invU = 5×5
    0.0000   -0.0001   -0.0004   -0.0007   -0.0140
         0    0.0001    0.0019    0.0084    0.2688
         0         0   -0.0018   -0.0210   -1.1760
         0         0         0    0.0140    1.7920
         0         0         0         0   -0.8820
the inverse of A calculated using LU factorization is
   1.0e+05 *

    0.0002   -0.0030    0.0105   -0.0140    0.0063
   -0.0030    0.0480   -0.1890    0.2688   -0.1260
    0.0105   -0.1890    0.7938   -1.1760    0.5670
   -0.0140    0.2688   -1.1760    1.7920   -0.8820
    0.0063   -0.1260    0.5670   -0.8820    0.4410

LU factorization does not work for me!?
```

Part 2

```
type msystem.m
```

```
function [X1,X2,X] = msystem(A,B)
[~,n] = size(A);
[~,p] = size(B);
[L,U] = lu(A);

%first algorithm
X1 = inv(A) * B;
%second algorithm
X2 = A\B;
%third algorithm
y = rref([L B]);
Y = y(:,n+1:n+p);
x = rref([U Y]);
```

```
X = x(:,n+1:n+p);

if closetozeroroundoff(X1-X2,0) == 0
    if closetozeroroundoff(X1-X,0) == 0
    disp('The solutions calculated by different methods match')
    end
else
    disp('There is a problem with my code?!')
end


end
```

```
% (a)
A = [1 1 4; 0 -4 0; -5 -1 -8], B=magic(3)
```

```
A = 3×3
     1     1     4
     0    -4     0
    -5    -1    -8
B = 3×3
     8     1     6
     3     5     7
     4     9     2
```

```
[X1,X2,X] = msystem(A,B)
```

```
The solutions calculated by different methods match
X1 = 3×3
   -6.9167   -4.0833   -5.2500
   -0.7500   -1.2500   -1.7500
    3.9167    1.5833    3.2500
X2 = 3×3
   -6.9167   -4.0833   -5.2500
   -0.7500   -1.2500   -1.7500
    3.9167    1.5833    3.2500
X = 3×3
   -6.9167   -4.0833   -5.2500
   -0.7500   -1.2500   -1.7500
    3.9167    1.5833    3.2500
```

```
% (b)
A = magic(5), B = randi(10,5,4)
```

```
A = 5×5
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
B = 5×4
     8     6     5     4
     4     8     1     6
     6    10     4     2
     1     2     2     7
     1     6     8     3
```

```
[X1,X2,X] = msystem(A,B)
```

```
The solutions calculated by different methods match
X1 = 5×4
   -0.0426    0.0485   -0.0856    0.2347
    0.1824   -0.0515    0.1971    0.0328
```

```
   -0.1718    0.0985    0.1615    0.0344
    0.0824   -0.0515   -0.1490    0.1943
    0.2574    0.4485    0.1837   -0.1576
X2 = 5×4
   -0.0426    0.0485   -0.0856    0.2347
    0.1824   -0.0515    0.1971    0.0328
   -0.1718    0.0985    0.1615    0.0344
    0.0824   -0.0515   -0.1490    0.1943
    0.2574    0.4485    0.1837   -0.1576
X = 5×4
   -0.0426    0.0485   -0.0856    0.2347
    0.1824   -0.0515    0.1971    0.0328
   -0.1718    0.0985    0.1615    0.0344
    0.0824   -0.0515   -0.1490    0.1943
    0.2574    0.4485    0.1837   -0.1576
```

```
% (c)
A = magic(3), B = [magic(3), eye(3)]
```

```
A = 3×3
     8     1     6
     3     5     7
     4     9     2
B = 3×6
     8     1     6     1     0     0
     3     5     7     0     1     0
     4     9     2     0     0     1
```

```
[X1,X2,X] = msystem(A,B)
```

```
The solutions calculated by different methods match
X1 = 3×6
    1.0000         0   -0.0000    0.1472   -0.1444    0.0639
         0    1.0000         0   -0.0611    0.0222    0.1056
         0    0.0000    1.0000   -0.0194    0.1889   -0.1028
X2 = 3×6
    1.0000         0         0    0.1472   -0.1444    0.0639
         0    1.0000         0   -0.0611    0.0222    0.1056
         0         0    1.0000   -0.0194    0.1889   -0.1028
X = 3×6
    1.0000         0         0    0.1472   -0.1444    0.0639
         0    1.0000         0   -0.0611    0.0222    0.1056
         0         0    1.0000   -0.0194    0.1889   -0.1028
```

Exercise 6

```
type markov
```

```
function q = markov(P,x0)
format
n = size(P,1);
q=[];

%Determine stochastic
if sum(P,1) == ones(1,n)
    Q = null(P-eye(n), 'r');
    q = Q/sum(Q,1);
    counter = 0;
    x = x0;
    while closetozeroroundoff(q-x,7) ~= 0
        x = P * x0;
        x0 = x;
        counter = counter + 1;
```

```
    end
    fprintf("Number of iterations = %d", counter)
else
    disp('P is not a stochastic matrix')
    return
end

end
```

```
%(a)
P=[.6 .3;.5 .7], x0=[.4;.6]
```

```
P = 2×2
    0.6000    0.3000
    0.5000    0.7000
x0 = 2×1
    0.4000
    0.6000
```

```
q=markov(P,x0);
```

```
P is not a stochastic matrix
```

```
%(b)
P=[.5 .3;.5 .7],x0=[.5;.5]
```

```
P = 2×2
    0.5000    0.3000
    0.5000    0.7000
x0 = 2×1
    0.5000
    0.5000
```

```
q=markov(P,x0);
```

```
Number of iterations = 9
```

```
%(c)
P=[.9 .2;.1 .8], x0=[.11;.89]
```

```
P = 2×2
    0.9000    0.2000
    0.1000    0.8000
x0 = 2×1
    0.1100
    0.8900
```

```
q=markov(P,x0);
```

```
Number of iterations = 44
```

```
%(d)
P=[.9 .2;.1 .8], x0=[.90;.10]
```

```
P = 2×2
    0.9000    0.2000
    0.1000    0.8000
x0 = 2×1
    0.9000
    0.1000
```

```
q=markov(P,x0);
```

```
Number of iterations = 42
```

```
%(e)
P=[.90 .01 .09;.01 .90 .01;.09 .09 .90], x0=[.5; .3; .2]
```

```
P = 3×3
    0.9000    0.0100    0.0900
    0.0100    0.9000    0.0100
    0.0900    0.0900    0.9000
x0 = 3×1
    0.5000
    0.3000
    0.2000
```

```
q=markov(P,x0);
```

```
Number of iterations = 71
```

```
%(f)
P=magic(5); P=P.*1./sum(P), x0=randi(10,5,1);x0=x0.*1./sum(x0)
```

```
P = 5×5
    0.2615    0.3692    0.0154    0.1231    0.2308
    0.3538    0.0769    0.1077    0.2154    0.2462
    0.0615    0.0923    0.2000    0.3077    0.3385
    0.1538    0.1846    0.2923    0.3231    0.0462
    0.1692    0.2769    0.3846    0.0308    0.1385
x0 = 5×1
    0.2500
    0.2500
    0.2857
    0.1786
    0.0357
```

```
q=markov(P,x0);
```

```
Number of iterations = 11
```

```
%(g)
x0=q
```

```
x0 = 5×1
    0.2000
    0.2000
    0.2000
    0.2000
    0.2000
```

```
q=markov(P,x0);
```

```
Number of iterations = 0
```

Exercise 7

```
type transf_1.m
```

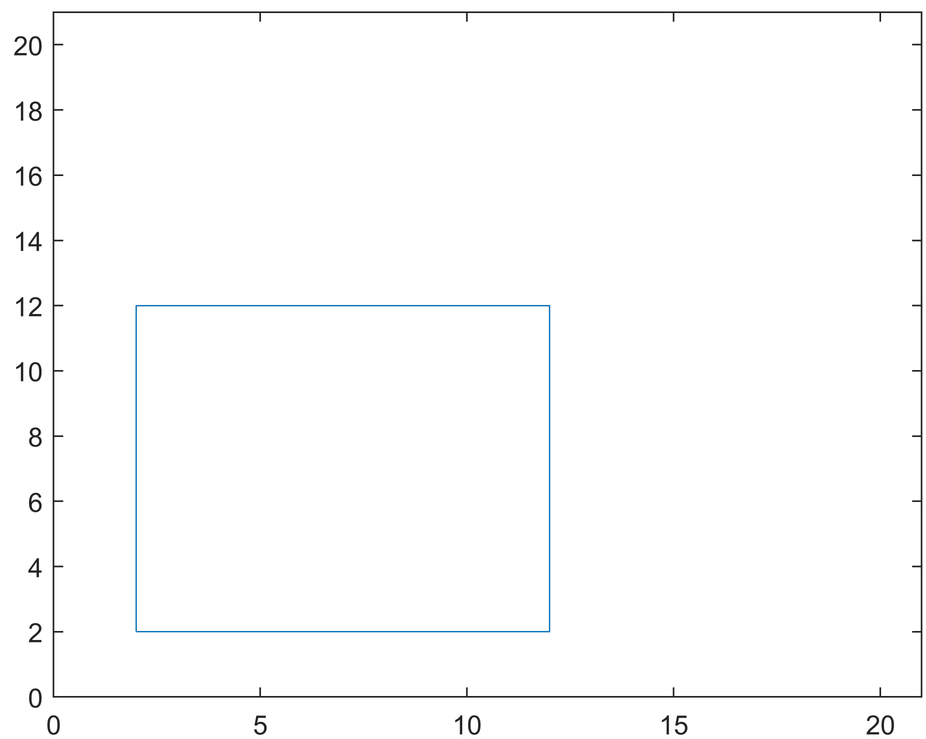
```
function C=transf_1(A,E)
C=A*E;
```

```
x=C(1,:);
y=C(2,:);
plot(x,y)
v=[0 21 0 21];
axis(v)
end
```

```
type polygon.m
```

```
function Area = polygon(E)
A=eye(2);
C=transf_1(A,E);
n=size(E,2)-1;
Area = 0;
for i=1:n
Area = Area + (E(1,i)*E(2,i+1) - E(2,i)*E(1,i+1));
end
Area= Area/2;
end
```

```
%(a)
E=[2 12 12 2 2;2 2 12 12 2];
Area = polygon(E)
```

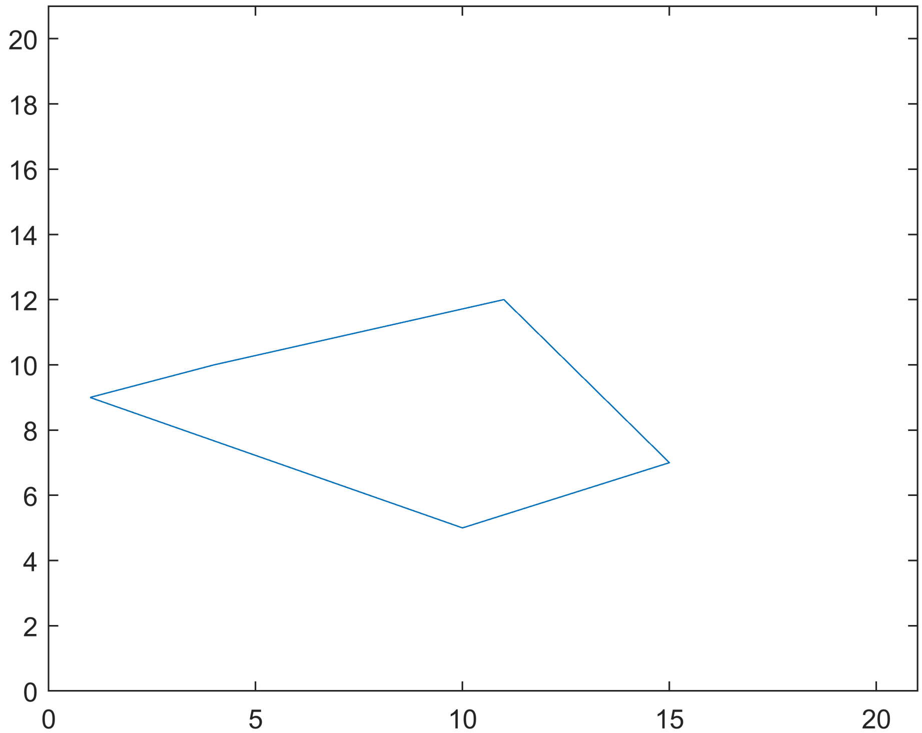



```
Area = 100
```

```
%(b)
E=[1 10 15 11 4 1;9 5 7 12 10 9]
```

```
E = 2×6
     1    10    15    11     4     1
     9     5     7    12    10     9
```

```
Area = polygon(E)
```

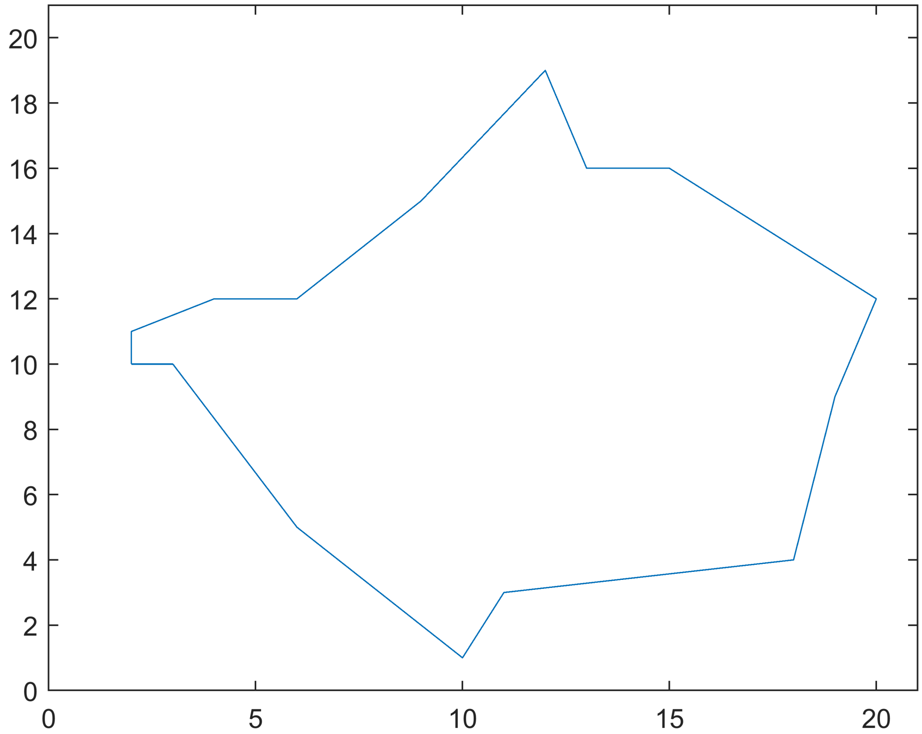



```
Area = 50.5000
```

```
%(c)
A1=randi(10,1,5);
A1=sort(unique(A1), 'ascend');
B1=randi(10,1,size(A1,2));
B1=sort(B1,'descend');
A2=randi([11 20],1,5);
A2=sort(unique(A2), 'ascend');
B2=randi(10,1,size(A2,2));
B2=sort(B2,'ascend');
A3=randi([11 20],1,5);
A3=sort(unique(A3), 'descend');
B3=randi([11 20],1,size(A3,2));
B3=sort(B3,'ascend');
A4=randi(10,1,5);
A4=sort(unique(A4), 'descend');
B4=randi([11 20],1,size(A4,2));
B4=sort(B4,'descend');
E=[A1 A2 A3 A4 A1(1,1);B1 B2 B3 B4 B1(1,1)]
```

```
E = 2×18
     2     3     6     9    10    11    18    19    20    15    13    12     9 ···
    10    10     5     2     1     3     4     9    12    16    16    19    15
```

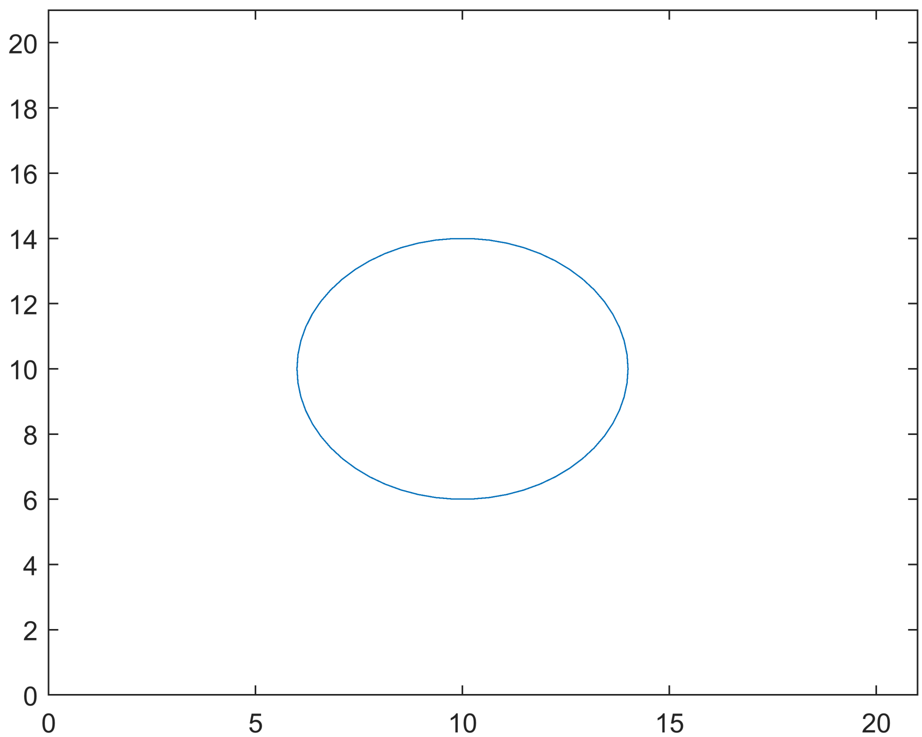
```
Area = polygon(E)
```




```
Area = 170
```

Bonus:

```
clear
R=4;
r0=[10;10];
n=3;
Area = 0;
ACircle = pi*R^2;
while (closetozeroroundoff(Area - ACircle,1)~=0)
   E=zeros(2,n+1);
   for i=1:n+1
       t= 2*pi*(i-1)/n;
       E(1,i)= r0(1,1)+ R*cos(t);
       E(2,i)= r0(2,1) + R*sin(t);
   end
   disp("");
   Area =polygon(E);
   n=n+1;
end
```




```
disp("(1) Approximation of polygon = " +Area);
```

(1) Approximation of polygon = 50.1672

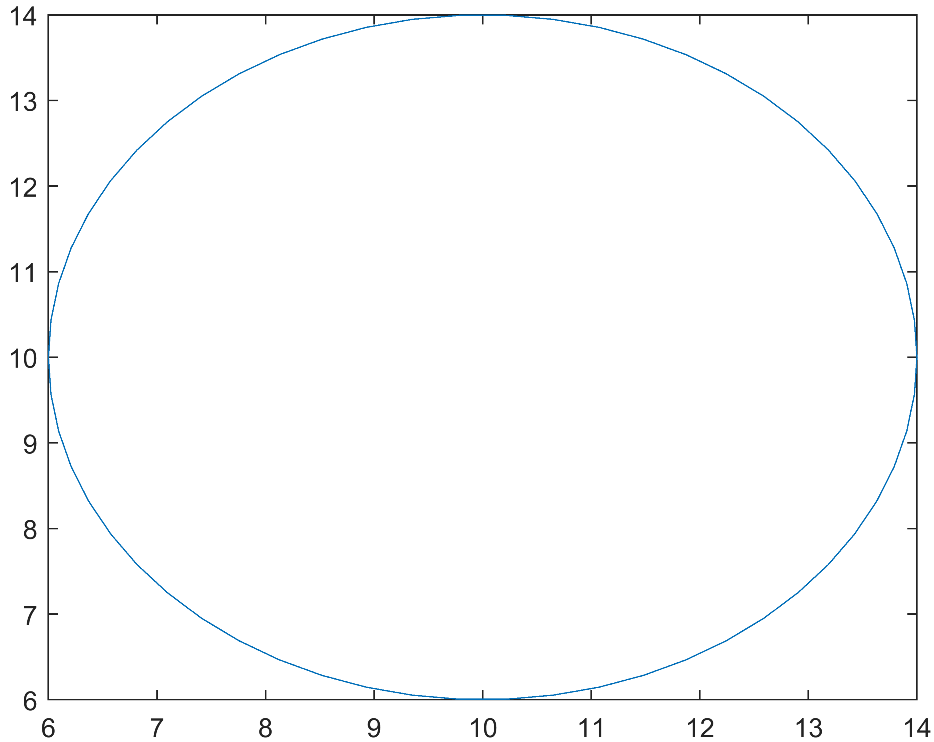
```
disp("(2) Minimum number of vertices needed = " +n);
```

(2) Minimum number of vertices needed = 59

```
x = E(1,:);
y = E(2,:);
disp("(3) Graph of inscribed polygon: ");
```

(3) Graph of inscribed polygon:

```
plot(x,y)
```


14
13
12
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10
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6
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10
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12
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14