Jonathan Lich Assignment-02

1). • $T(n) = 2T(n/3) + 1$

$C(\text{root}) = 1$
$C(\text{level } 1) = 2(1/3) = 2/3$

it is root dominated so

$T(n) \in O(1)$

• $T(n) = 5T(\frac{n}{4}) + n$

$C(\text{root}) = n$
$C(\text{level } 1) = 5[n/4] = \frac{5n}{4}$

it is leaf dominated

num of leaves is $5^{\log_4 n} = n^{\log_4 5}$

$T(n) \in O(n^{\log_4 5})$

• $T(n) = 7T(n/7) + n$

$C(\text{root}) = n$
$C(\text{level } 1) = 7(n/7) = n$

it is balanced

num of levels $= \log_7 n$
max cost per level $= n$

$T(n) = O(n \log n)$

- $T(n) = 9T(n/3) + n^2$

$C(\text{root}) = n^2$
$C(\text{level } 1) = 9[(n/3)^2]$
$= 9[n^2/9] = n^2$

it is balanced

num of levels $= \log_3 n$
max work is n^2

$T(n) \in O(n^2 \log n)$

- $T(n) = 8T(n/2) + n^3$

$C(\text{root}) = n^3$
$C(\text{level } 1) = 8[(\frac{n}{2})^3]$
$= 8 \cdot n^3/8 = n^3$

it is balanced

num of levels $= \log_2 n$
max work $= n^3$

$T(n) \in O(n^3 \log n)$

- $T(n) = 49\,T(n/25) + n^{3/2}\log n$

$C(\text{root}) = n^{3/2} \log n$

$C(\text{level}1) = 49\left[(n/25)^{3/2} \log(n/25)\right]$

$= 49\,(n^{3/2}/125)\,(\log(n/25))$

$= \dfrac{49n^{3/2}}{125} \cdot \log\left(\frac{n}{25}\right)$

$n^{3/2} < \dfrac{49n^{3/2}}{125}$ $\qquad \log n < \log(n/25)$

both terms are smaller than in the previous expression so

$C(\text{root}) > C(\text{level}1)$ so it is root dominated

$T(n) \in O(n^{3/2} \log n)$

- $T(n) = T(n-1) + 2$

$= T(n-2) + 2 + 2 = T(n-2) + 4$

$= T(n-3) + 2 + 4 = T(n-3) + 6$

$T(n) = T(n-k) + 2k$

when you get to k times $k=n$

$T(n) = T(n-k) + 2k$

$= T(n-n) + 2n$

$1 + 2n$

$= 2n+1$

$T(n) \in O(n)$

$T(n) = T(n-1) + n^c$, with $C \geq 1$

$T(n-1) = T(n-1-1) + n^c$
$= T(n-2) + n^c$

$T(n) = T(n-2) + n^c + n^c$
$= T(n-2) + 2n^c$

$T(n-2) = T(n-2-1) + n^c$
$= T(n-3) + n^c$

$T(n) = T(n-3) + n^c + 2n^c$
$= T(n-3) + 3n^c$

$T(n) = T(n-k) + kn^c$ eventually $k=n$

$T(n) = T(n-n) + n \cdot n^c$
$= 1 + n \cdot n^c$
$= n^1 \cdot n^c + 1$
$= n^{(c+1)} + 1$

$T(n) \in O(n^{c+1})$

- $T(n) = T(\sqrt{n}) + 1$
  $= T(n^{1/2}) + 1$
  $= T(n^{1/2^2}) + 1 + 1$
  $= T(n^{1/2^3}) + 1 + 1 + 1$

$T(n) = T(n^{1/2^k}) + k$

Assume $n = 2^m$, n is in a power of 2

$\to T(2^m) = T(n^{m/2^k}) + k$

To reach base case:

$T(2^{\frac{m}{2^k}}) = T(2^1)$

$\frac{m}{2^k} = 1$

$m = 2^k$ so $k = \log_2 m$

and $m = \log_2 n$

$k = \log(\log(n))$

$T(n) \in O(\log\log n)$

2)

A: $T(n) = 5W(\frac{n}{2}) + n$

$C(\text{root}) = n$

$C(\text{level1}) = 5[\frac{n}{2}] = 5n/2$

it is leaf dominated

there are lgn levels so $5^{\log_2 n}$ leaves.

A is $\in O(n^{\lg 5})$

B: $T(n) = 2T(n-1) + 1$ $\quad k=1$

$T(n-1) = 2T(n-2) + 1$

$T(n) = 2[2T(n-2) + 1] + 1$ $\quad k=2$

$= 4T(n-2) + 3$

$T(n-2) = 2T(n-3) + 1$

$T(n) = 4[2T(n-3) + 1] + 3$ $\quad k=3$

$= 8T(n-3) + 7$

$T(n) = 2^k T(n-k) + 2^k - 1$

when $k=n$

$T(n) = 2^n T(0) + 2^n - 1$

$= 2^1 \cdot 2^n$

$T(n) = 2^{n+1}$ $\quad$ B is $\in O(2^n)$

$C: T(n) = 9T(n/3) + n^2$

$C(\text{root}) = n^2$

$C(\text{level } 1) = 9\left[(n/3)^2\right]$

$= 9\left[n^2/9\right] = n^2$

it is balanced

num of levels is $\log_3 n$

max work per level is n^2

C is $\in O(n^2 \log_3 n)$

B is exponential which is by far the worst. So the comparison is between A and C

A: $O(n^{1.95})$

C: $O(n^2 \log_3 n)$

By graphing these two functions, we see that C is more efficient than A. So we choose C