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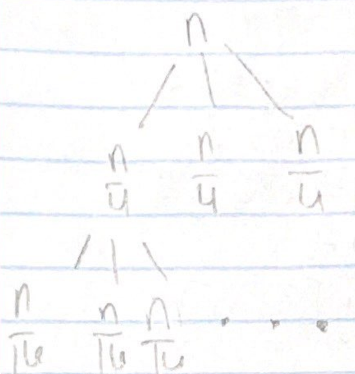
Recitation-03

1a) $W(n)=3W(\frac{n}{4})+n^2$



n^2

$3\left(\frac{n}{4}\right)^2$

$9\left(\frac{n}{4^2}\right)^2$

$3^i\left(\frac{n}{4^i}\right)^2$

$$3^i\left(\frac{n}{4^i}\right)^2 = 3^i\left(\frac{n^2}{4^{2i}}\right) = n^2\left(\frac{3^i}{4^{2i}}\right) = n^2\left(\frac{3^i}{16^i}\right)$$

$$\sum_{i=0}^{\log_4 n} n^2\left(\frac{3^i}{16^i}\right) = n^2\sum_{i=0}^{\log_4 n}\left(\frac{3}{16}\right)^i = n^2\left(\frac{1}{1-3/16}\right)$$

$$n^2\left(\frac{1}{1-3/16}\right) < c\cdot n^2 \boxed{\in O(n^2)}$$

b) $W(n) = W\left(\frac{n}{3}\right) + W\left(\frac{2n}{3}\right) + n\log n$

$$\begin{array}{ccc} & n & \\ \frac{n}{3} & & \frac{2n}{3} \end{array} \qquad \begin{array}{l} n\log n \\ \frac{n}{3}\log\frac{n}{3} + \frac{2n}{3}\log\frac{2n}{3} < \end{array}$$

$$= \frac{n}{3}\log\frac{2n}{3} + \frac{2n}{3}\log\frac{2n}{3}$$

$$= n\log\frac{2n}{3}$$

$$= n\log\left[\left(\tfrac{2}{3}\right)^i n\right]$$

it also $> \frac{n}{3}\log\frac{n}{3} + \frac{2n}{3}\log\frac{n}{3} = n\log\left[\left(\tfrac{1}{3}\right)^i n\right]$

$\to n\left[\log\left(\tfrac{2}{3}\right)^i + \log n\right] = n\left[i\log\tfrac{2}{3} + \log n\right]$

$$n\sum_{i=0}^{\log_{\frac{3}{2}} n} i\log\frac{2}{3} + \log n = n\left[\sum i\log\frac{2}{3} + \sum \log n\right]$$

$$= n\left[\log\frac{2}{3}\sum_{i=0}^{\log_{\frac{3}{2}} n} i + \sum_{i=0}^{\log_{\frac{3}{2}} n} \log n\right] =$$

$$n\left[\lg\frac{2}{3}\left(\frac{\log_{\frac{3}{2}} n\left(\log_{\frac{3}{2}} n + 1\right)}{2}\right) + \log_{\frac{3}{2}} n \cdot \log n\right]$$

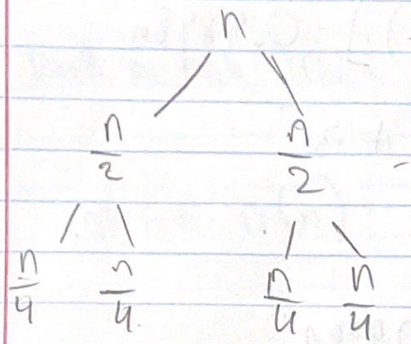
for our analysis it is ok to assume logs all have the same base

$$n\left[c\left(\log n(\log n)\right) + \log n \cdot \log n\right]$$

$$n\left[2c\left[\log^2 n\right]\right] = n\log n \ \boxed{\in O(n\log n)}$$

Since constants don't change this analysis we can simplify and just look where the logs are.

c) $W(n) = 2W(\frac{n}{2}) + n/\log n$



$n/\log n$

$2\left(\frac{n/2}{\log\frac{n}{2}}\right) = \frac{n}{\log\frac{n}{2}}$

$4\left(\frac{\frac{n}{4}}{\log\frac{n}{2^i}}\right) = \frac{n}{\log(\frac{n}{2^2})}$

$\Rightarrow \frac{n}{\log(\frac{n}{2^i})}$

$$\sum_{i=0}^{\log_2 n} \frac{n}{\log(\frac{n}{2^i})} = n\sum_{i=0}^{\lg n} \frac{1}{\lg n - \lg 2^i} = n\sum_{i=0}^{\lg n} \frac{1}{\lg n - i} \approx$$

$$n\sum_{i=0}^{\lg n} \frac{1}{i} = n\left[\ln(1+\lg n)\right] = n\log(\log(n))$$

$$\boxed{\in O(n\lg(\lg(n)))}$$

2) a) $W(n) = 2W(0.49n) + 1.01n$

$C(\text{root}) = 1.01n$

$C(\text{level } 1) = 2[1.01(0.49n)] = 0.9898n$

it is root dominated so it is

$\boxed{\in O(n)}$

b) $W(n) = W(\frac{n}{2}) + W(\frac{n}{4}) + 0.999n$

$C(\text{root}) = 0.999n$

$C(\text{level } 1) = 0.999(0.5n) + 0.999(.25n)$
$= 0.4995n + 0.24975n$
$= 0.74925n$

it is root dominated so,

$\boxed{\in O(n)}$

c) $W(n) = \sqrt{n}\, W(\sqrt{n}) + \sqrt{n}$

$C(\text{root}) = \sqrt{n}$

$C(\text{level } 1) = \sqrt{n}\left[\sqrt{\sqrt{n}}\right]$
$= n^{1/2} \cdot (n^{1/2})^{1/2} = n^{1/2} \cdot n^{1/4} = n^{3/4}$

$\Rightarrow$

it is leaf dominated

num levels: $\log_{\sqrt{n}} n$

leaves: $\sqrt{n}^{\log_{\sqrt{n}} n} = n$

work on each leaf: constant because it will break down to a base case

$n \cdot c \boxed{\in O(n)}$