MAFS 5250 Computer Assignment2

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Summary

We evaluated the effects of changing parameter I and J as required, by measuring the price, the relative error and the required CPU time.

We analyzed the pricing behaviors of the participating policy with and without surrender right.

As time to maturity or target budget ratio γ increases, value of the surrender option increases monotonically, while its value decreases as the distribution ratio α increases. Financial interpretations are also given.

Value of the participating policy with surrender option increases monotonically as time to maturity or distribution ratio α increases. Meanwhile, its value decreases as the risk-free interest rate or target budget ratio γ increases.

Part1. Implementation

```
% Here are pseudo-codes, more readable.
% Apply MATLAB to calculate, source codes are in the /code folder.
```

Thomas Algorithm

```
function y = tridiag( a, b, c, f )

%  Solve the  n x n  tridiagonal system for y:
%
%  [ a(1)  c(1)                                 ] [  y(1)  ]   [  f(1)  ]
%  [ b(2)  a(2)  c(2)                           ] [  y(2)  ]   [  f(2)  ]
%  [      b(3)  a(3)  c(3)                      ] [        ]   [        ]
%  [           ...   ...   ...                  ] [  ...   ] = [  ...   ]
%  [                    ...    ...    ...       ] [        ]   [        ]
%  [                        b(n-1) a(n-1) c(n-1)] [ y(n-1) ]   [ f(n-1) ]
%  [                                b(n)  a(n)  ] [  y(n)  ]   [  f(n)  ]
%
%  f must be a vector (row or column) of length n
%  a, b, c must be vectors of length n (note that b(1) and c(n) are not used)

n = length(f);
v = zeros(n,1);
y = v;
w = a(1);
y(1) = f(1)/w;
for i=2:n
    v(i-1) = c(i-1)/w;
    w = a(i) - b(i)*v(i-1);
    y(i) = ( f(i) - b(i)*y(i-1) )/w;
end
for j=n-1:-1:1
   y(j) = y(j) - v(j)*y(j+1);
end
```

Terminal Payoff

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%------------------------Part2: Terminal Payoff-------------------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
payoff=zeros(I,J-j0);
for i=I:-1:0
    for j=J:-1:j0
        payoff(i,j) = max(j*dp,(1+rg)^T*j0*dp);
```

```
    %terminal payoff's lower bound is the bond component.
    end
end
for i = I:-1:0
    for j = J:-1:j0
        jwave = j + max(rg*j, alpha*((i*da/dp-j) - gamma*j));
        jfloor = floor(jwave);
        if jfloor + 1 > J
            VExtra = payoff(i,J) + (jwave - J)*(payoff(i,J)-payoff(i,J-1));
            %Extrapolation
            V(T,0,i,j) = VExtra; %V(T,0) is the value at the end of the policy.
        else
            VInter = (1-(jwave - jfloor))*payoff(i,jfloor) + (jwave -
jfloor)*payoff(i,jfloor+1); %Interpolation
            V(T,0,i,j) = VInter;
        end
    end
end
```

Pricing Kernel

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%-------------------------Part3: Pricing Kernel-------------------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for t =T:-1:2 %Here, (t, k) represents t-k*(fraction of year for each step)
    for k = 0:K %The end of the loop is at t=2,k=K, that is the beginning of year 2.
    i.e., the end of year 1.
        if k == K
            for i = I:-1:0
                for j = J:-1:j0
                    V(t,k,i,j)=max(j*dp,V(t,k,i,j)); %the Early Exercise Decision.
                end
            end
            for i = I:-1:0 %Apply the jump condition at the end of year.
                for j = J:-1:j0
                    jwave = j + max(rg*j, alpha*((i*da/dp-j) - gamma*j));
                    jfloor = floor(jwave);
                    if jfloor + 1 > J
                        VExtra = V(t,k,i,J)+(jwave - J)*(V(t,k,i,J)-V(t,k,i,J-1));
                        %Extrapolation
                        V(t-1,0,i,j) = VExtra;
                    else
                        VInter = (1-(jwave - jfloor))*V(t,k,i,jfloor) + (jwave -
jfloor)*V(t,k,i,jfloor+1); %Interpolation
```

```
                    V(t-1,0,i,j) = VInter;
                end
            end
        end
    else %When it is not the end of the year, apply implicit scheme.
        for j = J:-1:j0
            V(t,k+1,0,j) = (1-r*ds)*V(t,k,0,j); %When the Asset Value=0, i=0;
            B(1) = B(1) - E(1)*(1-r*ds)*V(t,k,0,j);
            V(t,k+1,1:I-1,j)  = tridiag(a,[0;b],[c;0],B);
            %Apply the Thomas Algorithm
            V(t,k+1,I,j) = 2*V(t,k+1,I-1,j) - V(t,k+1,I-2,j);
            % Smooth Parsing Condition;
        end
    end
  end
end

t = 1;
for k = 0:K-1 %Apply implicit scheme within year1.
    for j = J:-1:j0
        V(t,k+1,0,j) = (1-r*ds)*V(t,k,0,j); %When the Asset Value=0, i=0;
        B(1) = B(1) - E(1)*(1-r*ds)*V(t,k,0,j);
        V(t,k+1,1:I-1,j) = tridiag(a,[0;b],[c;0],B);%Apply the Thomas Algorithm
        V(t,k+1,I,j) = 2*V(t,k+1,I-1,j) - V(t,k+1,I-2,j);
       % Smooth Parsing Condition;
    end
end

price = V(1,K,A0/da,P0/dp); %Obtain the price, where A0, P0 are given.
```

Part2. Pricing Behavior

2.1 Effects of increasing I,J on

Table1. Policy value, Relative Error and CPU time with various pairs of I,J.

	(I,J)				
	(100,100)	(200,200)	(400,400)	(800,800)	(1600,1600)
Contract value	112.8115	112.8264	113.1579	113.8576	114.2330
Relative Error	1.24%	1.23%	0.94%	0.33%	0.00%
CPU time(sec)	0.178	0.916	3.052	11.208	45.555

2.2 Surrender Right

(i) Implementation

At the beginning of year 1,2, …… , T-1 (Right after the dividend of this year injects into the policy.) Holder of the contract with surrender right decides get $P(t^+)$ or continue to hold the contract, depending on which one maximizes the contract value. So its value can be denoted as $V_{t^+} = \max(P(t^+), V_{continue})$,where $V_{continue}$ denotes the continuation value of the contract calculated from the fully implicit scheme and $P(t^+)$ denotes the contingent claim . (Since Prof. Kwok emailed us that there is no need to use PSOR, we did not implement PSOR procedure.)

(ii)Influence of Distribution ratio α ,target budget ratio γ and time to maturity on Surrender Right

A: Time to maturity.

As Figure1 shows, the surrender option's value increases as time to maturity increases.

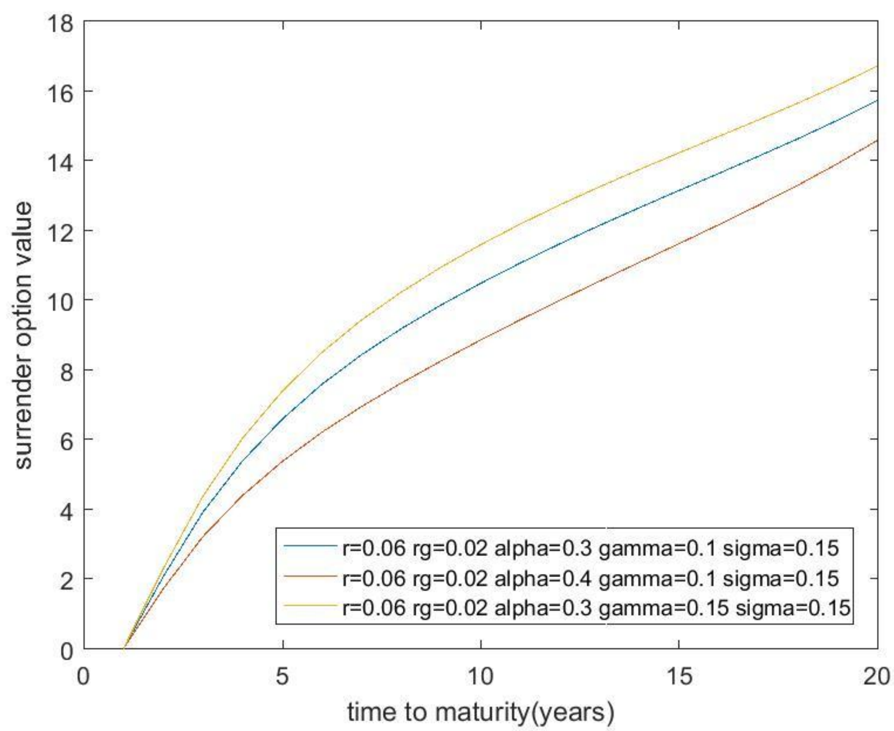

Figure 1:Line graph showing the change of the value of surrender option against time to maturity (up to 20 years) using the set of parameters values on P26.

Financial interpretation:

(1): The result agrees with the financial intuition that insurance value increases as time to maturity of options increases.

(2): The surrender option is a Bermudan option with early exercising rights at the beginning of each year, and the number of rights equals to the year to maturity. As time to maturity increases, the number of early exercising rights increases. Thus, value of the surrender option increases.

B : Distribution ratio α and target budget ratio γ

From figure1, we observe that:

(1) The curve of the surrender option value with $(\alpha, \gamma) = (0.3, 0.1)$ lies above the curve of option with $(\alpha, \gamma) = (0.4, 0.1)$, implying that **the value of the surrender option decreases as distribution ratio α increases**.

(2) The curve of the surrender option value with $(\alpha, \gamma) = (0.3, 0.1)$ lies below the curve of option with $(\alpha, \gamma) = (0.3, 0.15)$, indicating that **the value of the surrender option increases as target budget ratio γ increases.**

To further justify our conjecture on the impact of distribution ratio α and target budget ratio γ on the option value of the surrender right. A surface of option value with respect to α and γ was plotted.

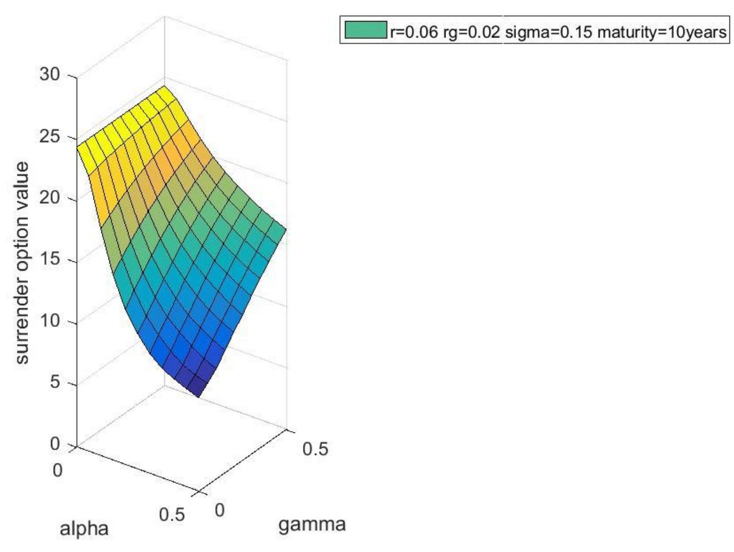

Figure 2: Surface plot of surrender right's value with changes in alpha and gamma.

Figure 2 shows that the value of the surrender option decreases as distribution ratio α increases while the value of the surrender option increases as target budget ratio γ increases. The results match with our conjecture.

Reasoning:

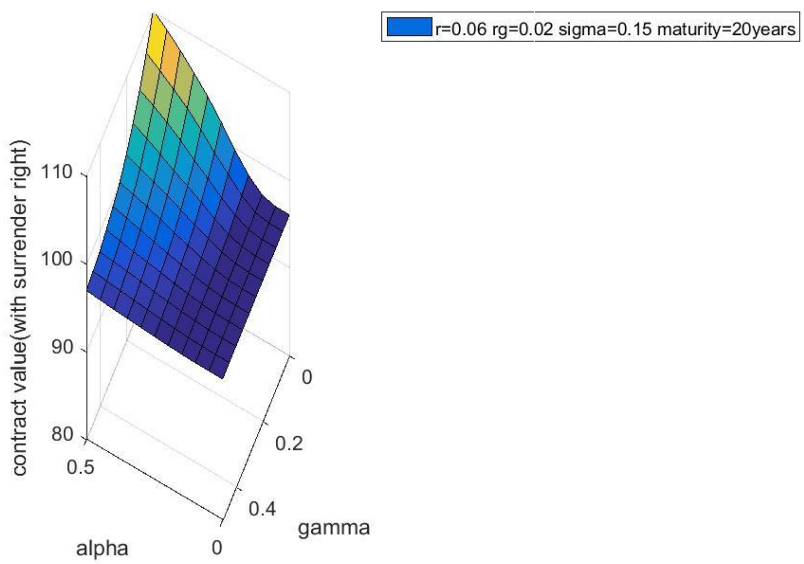

Figure 3: Surface plot of Continuation Value (with surrender right) with changes in alpha and gamma.

Figure 3 shows that the **American style contract increases as α increases while decreases as γ increases. (Higher α has positive impact on $V_{continue}$ and increases the contract value. In contrast, higher γ has negative impact on $V_{continue}$ and will decrease the contract value.)**

Financial Interpretation:

Recall that the value of surrender right at time t^+ equals to $\max(P(t^+), V_{continue}) - V_{continue} = \max(P(t^+) - V_{continue}, 0)$.

Higher continuation value of the contract reduces the value of surrender right.

Therefore, the value of surrender right decreases as α increases. (Since $V_{continue}$ (an American style contract with less time to maturity) increases as α increases, while $\mathrm{P}(t^+)$ is assumed to be fixed during year t. Therefore, $\max(P(t^+) - V_{continue}, 0)$ decreases)

Similarly, the value of the surrender option increases as target budget ratio γ increases. (Since $V_{continue}$ (an American style contract with less time to maturity) decreases as γ

increases, while $\mathrm{P}(t^{+})$ is assumed to be fixed during year t. Therefore, $\max(P(t^{+}) - V_{continue}, 0)$ increases)

2.3 American style contract

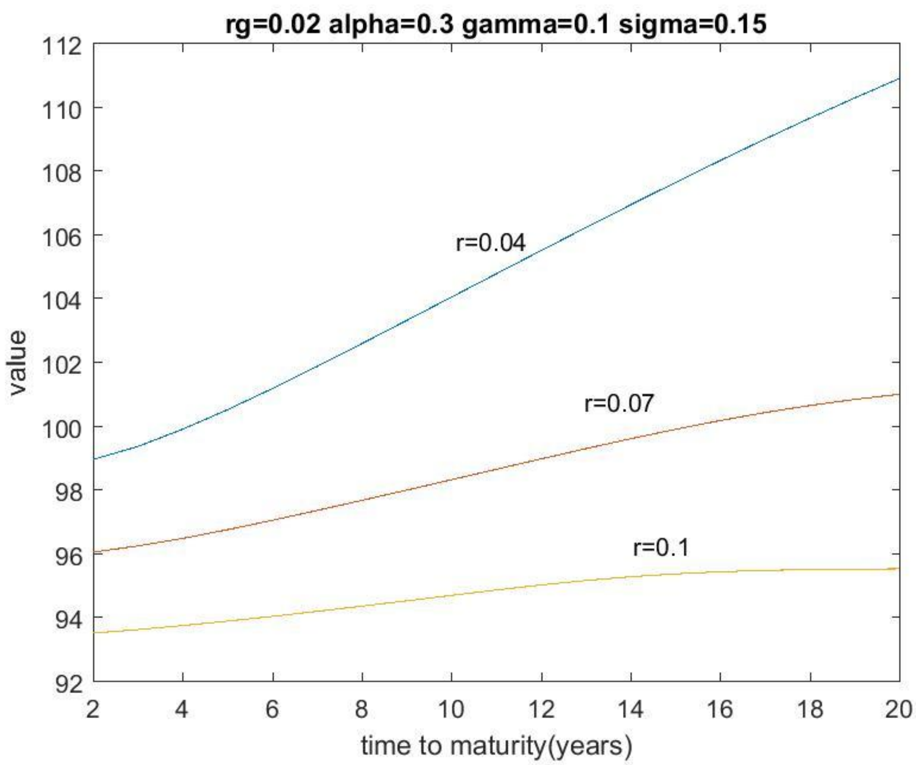

Figure 4: Line graphs of American style contract's value as time to maturity increases

Feature1: The American contract value increases as time to maturity increases.

Feature2: The American contract value decreases as interest rate increases.

Reasoning of Feature 1:

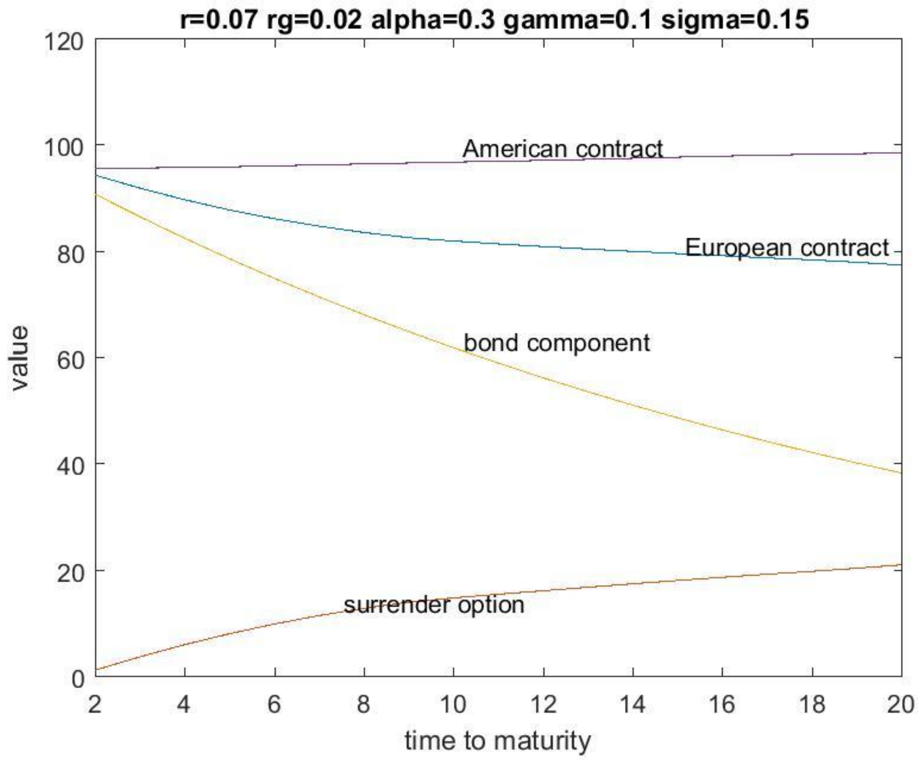

Figure 5: The graph shows the change of the value of American style contract, European contract, bond component and surrender option, as time to maturity increases.

American contract with longer maturity will be protected by the surrender option for longer period. The holder of a American contract with longer maturity will always have more choices than that with shorter maturity. The American contract value increases as time to maturity increases.

Recall that American style contract= bond component+ bonus option(s) + surrender right. **The increment in the surrender option value dominates**. **As a result, the value of American style contract increases as maturity lengthens**.

Reasoning of Feature 2:

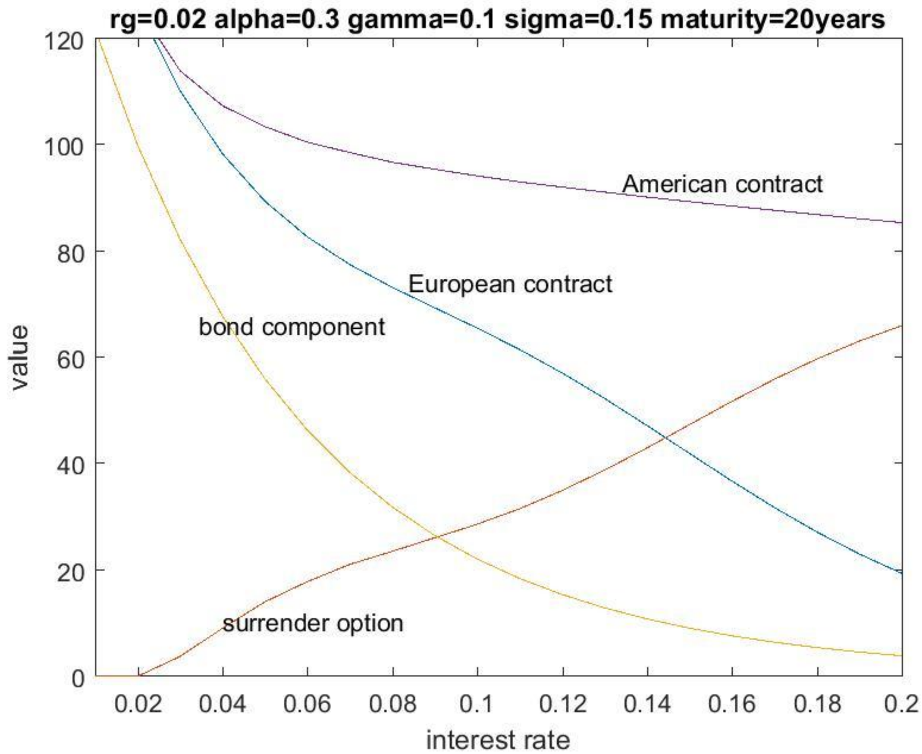

Figure 6: The graph shows the change of the value of European contract, bond component and surrender option, as risk-free interest rate increases.

As shown in Figure 6, **Fall of bond component dominates as risk-free interest rate increases.** Consequently, both American style contract and European style contract's value decreases.