MAFS 5250 Computer Assignment1

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Summary

We analyzed the pricing behavior of warrant with call and call value.

In terms of warrant with call, its price decreases monotonically as volatility of either stocks or both stocks increase. Meanwhile, its price decreases as the risk-free interest rate increases. Interestingly, as the correlation coefficient between two stocks increases from -1, smile is found on the curve of the price, due to value of the call increases faster than that of warrant without call when the volatility of both stocks stay at a low level.

The value of call exhibits same trend as that of warrant with call, in cases where the risk-free interest rate increases, or where the volatility of either stock or both stocks increases.

Part1. Implementation

We divided the warrant life into 8 observation periods, P=0,1,2,3,4,5,6,7. Within each observation period , we divided the trading days into 22 time steps, N=0,1,2, ……,21.(The 21^{st} time step of P^{th} observation period is the 0^{th} time step of $(P+1)^{th}$ observation period, therefore we can define the jump in warrant value due to coupon payment)

Let *W(P,N,i,j,k)* denote the warrant value at P observation period from initiation, N time step from the last coupon payment date , *i* up moves for Stock 1, *j* up moves for Stock 2 and *k* days that have been counted from the last coupon payment date up to the current day such that the accrual coupons will be collected on the next coupon payment date.

% Here are pseudo-codes, more readable.

% Apply MATLAB to calculate, source codes are in the /code folder.

Lattice Tree

```
% Constants, including lambda, sigma_1, sigma_2, r, are passed into as arguments for the pricing function.
% Probability formulas come from Page 92, Chapter 1, on the multi-state extension part.
% Stock 1 represents the Wal-Mart Stores Inc.
% Stock 2 represents the Intel Corp.
p_uu = 0.25 * (1/lambda^2 + sqrt(dt)/lambda…
    *((r-(sigma_1^2)/2)/sigma_1 + (r-(sigma_2^2)/2)/sigma_2)…
    + rho/lambda^2);
p_ud = 0.25 * (1/lambda^2 + sqrt(dt)/lambda…
    *((r-(sigma_1^2)/2)/sigma_1 - (r-(sigma_2^2)/2)/sigma_2)…
    - rho/lambda^2);
p_dd = 0.25 * (1/lambda^2 + sqrt(dt)/lambda…
    *(-(r-(sigma_1^2)/2)/sigma_1 - (r-(sigma_2^2)/2)/sigma_2)…
    + rho/lambda^2);
p_du = 0.25 * (1/lambda^2 + sqrt(dt)/lambda…
    *(-(r-(sigma_1^2)/2)/sigma_1 + (r-(sigma_2^2)/2)/sigma_2)…
    - rho/lambda^2);
p_00 = 1 - (p_uu + p_ud + p_dd + p_du);
u_1 = exp(lambda_1 * sigma_1 * sqrt(dt));
d_1 = exp(-lambda_1 * sigma_1 * sqrt(dt));
u_2 = exp(lambda_2 * sigma_2 * sqrt(dt));
d_2 = exp(-lambda_2 * sigma_2 * sqrt(dt));
l1=log(0.87)/log(u_1); % Number of jumps required from the reference price to the exercise price. Do not need to be an integer. How to derive it? S1*u_1^m1 = S1_exer => u1^m1 = 0.87 => m1 = log(0.87/u1) = log0.87 – log u1. Same rule applies to S2.
l2=log(0.87)/log(u_2);
```

Forward Shooting Grid

```
function [ d ] = gcou(i,j,k,l1,l2)
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```
if i>l1&&j>l2
    d=k+1;
else
    d=k;
end
%gcoupon (n,i,j,k) = k + 1{min(S1 (n,i) / S1,exer , S2 (n,j) / S2,exer) > 1}.
```

Pricing Kernel

```
% Below are the descriptions of W, the array that stores the computation results
% 1st Tuple: Coupon Period; 2nd Tuple: timesteps within each period;
% 3rd Tuple: Possible states of stock 1 (more than 337 to avoid overflow);
% 4th Tuple: Possible states of stock 2; 5th Tuple: Value of the state variable;
W=zeros(8,22,350,350,22);
% Backward Induction
P=7;for N=21:0; for i=-21*P-N: 21*P+N; for j=-21*P-N: 21*P+N;
        for K = 0:max(0,N-max(0,max(ceil(l1)-i,ceil(l2)-j)));
        % Backward induction implies that you should add the couponed days from 0 to 21.
        % When the stock price is at sufficiently low level, we don't need to consider all
        possible states of coupon paying days.
                if N==21
                    if i>l1&&j>l2
                    % When the stock price is at the high level, we don't need to consider
                    exercising the put, so we simply add the accrual interest on the
                    terminal payoff.
                    % This evaluation reduces the computation time by about 40%.
                        W(P,N,i,j,K)=1+K/21*0.04075*1;
                    else
                        % Incorporate Terminal Payoff of the European Put Part.
                        W(P,N,i,j,K)=min(1,min(u_1^(i)/0.87,u_2^(j)/0.87))+K/21*0.04075;
                    end
                else
                    W(P,N,i,j,K)=exp(-r*dt)*...
                        (p_uu*W(P,N+1,i+1,j+1,gcou(i+1,j+1,K,l1,l2))...
                        +p_ud*W(P,N+1,i+1,j-1,gcou(i+1,j-1,K,l1,l2))...
                        +p_du*W(P,N+1,i-1,j+1,gcou(i-1,j+1,K,l1,l2))...
                        +p_dd*W(P,N+1,i-1,j-1,gcou(i-1,j-1,K,l1,l2))...
                        +p_00*W(P,N+1,i,j,gcou(i,j,K,l1,l2)));
%To save space, 'end' required to finish the 'for' loop is omitted.
for P=6:1; for N=21:0; for i=-21*P-N: 21*P+N; for j=-21*P-N: 21*P+N;
            for K = 0:max(0,N-max(0,max(ceil(l1)-i,ceil(l2)-j)));
                    % On the Observation Date
                    if N==21
                        if i>l1&&j>l2
                            %Incorporate the caller's condition & jump condition.
```

```
                    %To calculate the warrant without call, change the expression
                    to W(P,N,i,j,K)=W(P+1,1,i,j,1) +K/21*0.04075*1;
                    W(P,N,i,j,K)=min(W(P+1,1,i,j,1),1)+K/21*0.04075*1;
                else
                    %Not satisfy the condition to recall. Jump condition between
                    two period is incorporated.
                    W(P,N,i,j,K)=W(P+1,1,i,j,1)+K/21*0.04075*1;
                end
            else
                    %Not on the observation Date
                W(P,N,i,j,K)=exp(-r*dt)*...
                    (p_uu*W(P,N+1,i+1,j+1,gcou(i+1,j+1,K,I1,I2))...
                    +p_ud*W(P,N+1,i+1,j-1,gcou(i+1,j-1,K,I1,I2))...
                    +p_du*W(P,N+1,i-1,j+1,gcou(i-1,j+1,K,I1,I2))...
                    +p_dd*W(P,N+1,i-1,j-1,gcou(i-1,j-1,K,I1,I2))...
                    +p_00*W(P,N+1,i,j,gcou(i,j,K,I1,I2)));
%At the first period, 4.075% coupon is secured.
P=0; for N=21:0; for i=-21*P-N: 21*P+N; for j=-21*P-N: 21*P+N;
    for K = 0:max(0,N-max(0,max(ceil(I1)-i,ceil(I2)-j)));
    % Backward induction implies that you should add the couponed days from 0 to 21.
    % When the stock price is at sufficiently low level, we don't need to consider all
    possible states of coupon paying days.
            % On the Observation Date
            if N==21
                if i>I1&&j>I2
                    %Incorporate the caller's condition & jump condition.
                    %To calculate the warrant without call, change the expression
                    to W(P,N,i,j,21)=W(P+1,1,i,j,21) + 0.04075*1;
                    W(P,N,i,j,K)=min(W(P+1,1,i,j,K),1)+0.04075*1;
                else
                    %Not satisfy the condition to recall. Jump condition between
                    two period is incorporated.
                    W(P,N,i,j,K)=W(P+1,1,i,j,K)+0.04075*1;
                end
            else
                    %Not on the observation Date
                W(P,N,i,j,K)=exp(-r*dt)*...
                    (p_uu*W(P,N+1,i+1,j+1,gcou(i+1,j+1,K,I1,I2))...
                    +p_ud*W(P,N+1,i+1,j-1,gcou(i+1,j-1,K,I1,I2))...
                    +p_du*W(P,N+1,i-1,j+1,gcou(i-1,j+1,K,I1,I2))...
                    +p_dd*W(P,N+1,i-1,j-1,gcou(i-1,j-1,K,I1,I2))...
                    +p_00*W(P,N+1,i,j,gcou(i,j,K,I1,I2)));
price=W(0,0,0,0,0);
```

Part2. Pricing Behavior

Pre-set Parameters

Name	Value
Lambda of Stock1	$\sqrt{3}$
Lambda of Stock2	$\sqrt{3}$
Number of Time steps	168

Results

Correlation Coefficient.

Scenario (a): High volatility(volatility>0.25) ->monotone increasing

When the correlation coefficient increases from -1 to 1, the risk-neutral probabilities of both stock prices move on same direction increase, while the risk-neutral probabilities of two stocks' prices move on opposite direction decrease.

As the correlation coefficient increases from -1 to 1, the bond holder should expect more days entitled to receive additional coupons since the probability that min(S1 (n,i) / S1,exer , S2 (n,j) / S2,exer) > 1 increases , resulting a higher bond price. Premium obtained by shorting the European is less likely to loss, resulting a higher expected payoff. The expected continuation value of the warrant increases, thus the value of the warrant increases.

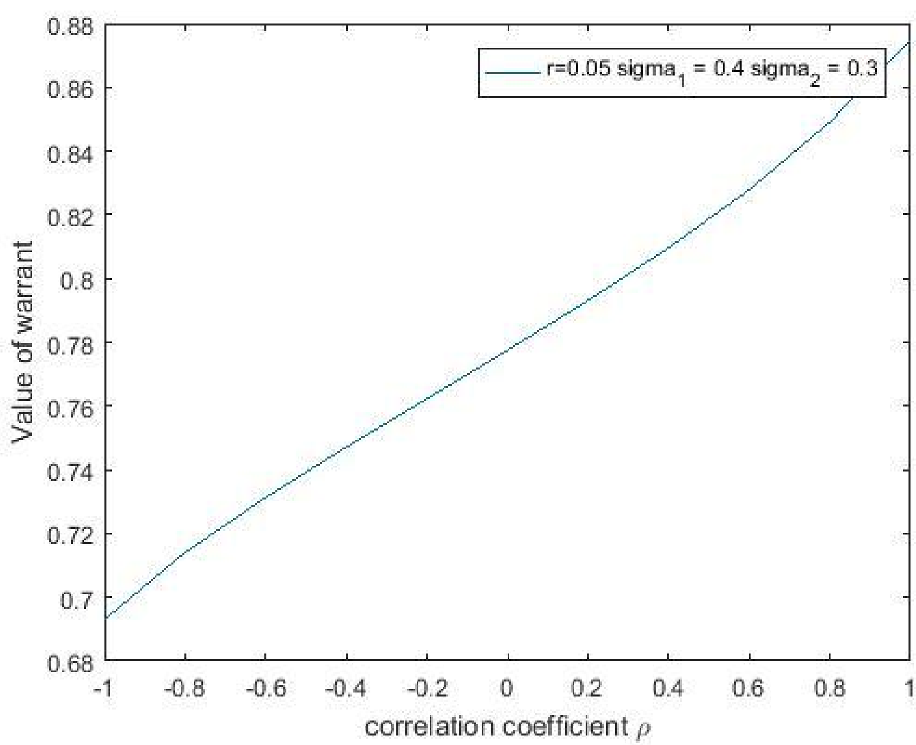

Figure 1:Line graph showing the change of the value of warrant as correlation coefficient increases

Scenario (b): Low volatility(volatility<0.25) ->Smile

- When the volatility of both stock price is relatively low, say, below 0.25. The path of the price exhibits smile pattern. We want to know the reason accounts for such behavior.

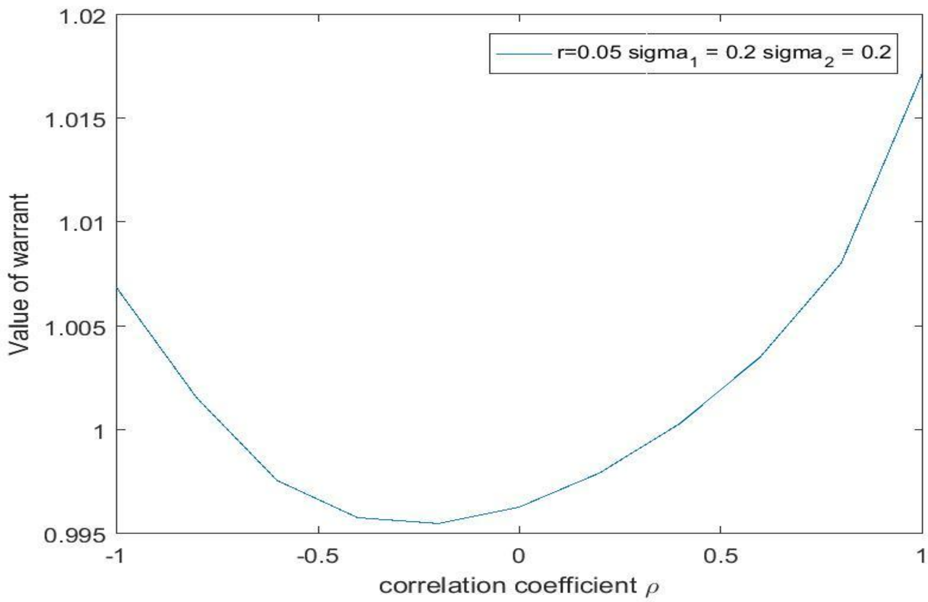

Figure 2:Line graph showing the change of the value of warrant as correlation coefficient increases.

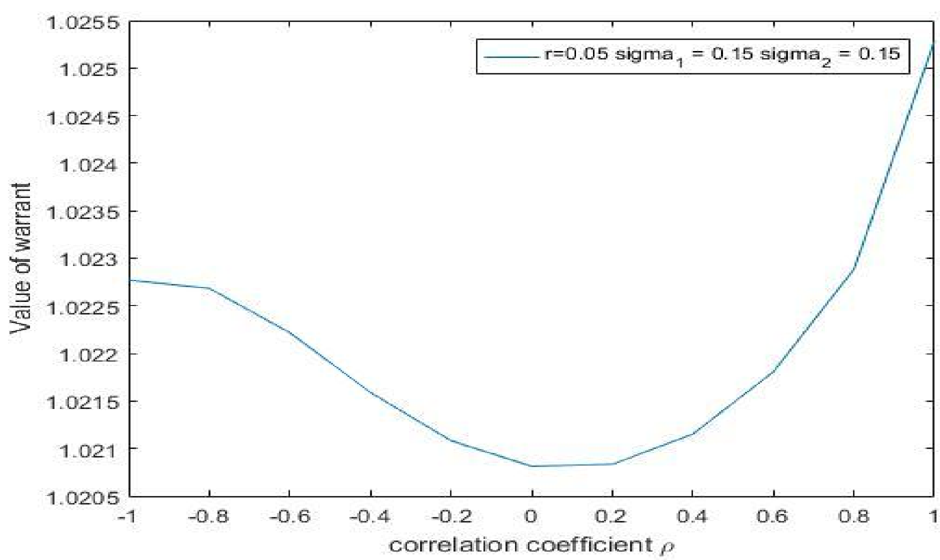

Figure 3:Line graph showing the change of the value of warrant as correlation coefficient increases.

Further analysis into the pricing behavior of warrant without call indicates that, as the correlation coefficient increases, its price increases monotonically. Meanwhile, value of the call increases, as higher correlation coefficient increase the expected value of holding the call option.

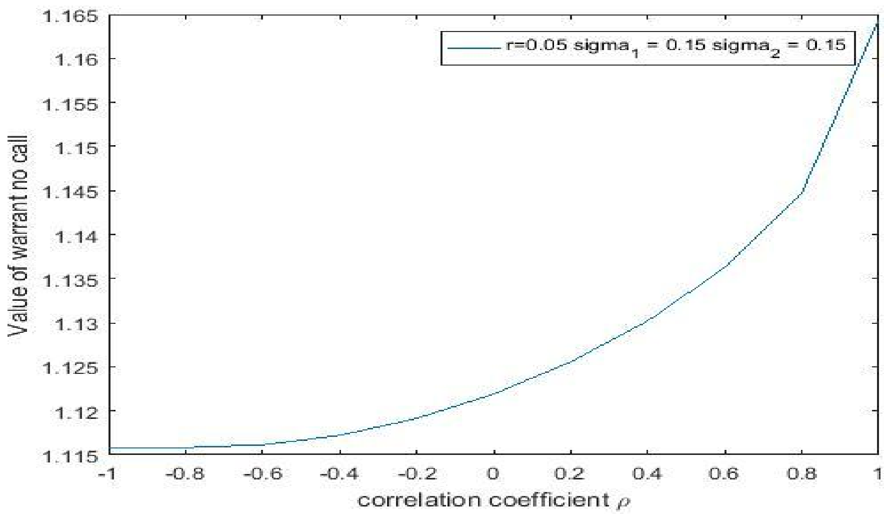

Figure 4:Line graph showing the change of the value of warrant as correlation coefficient increases.

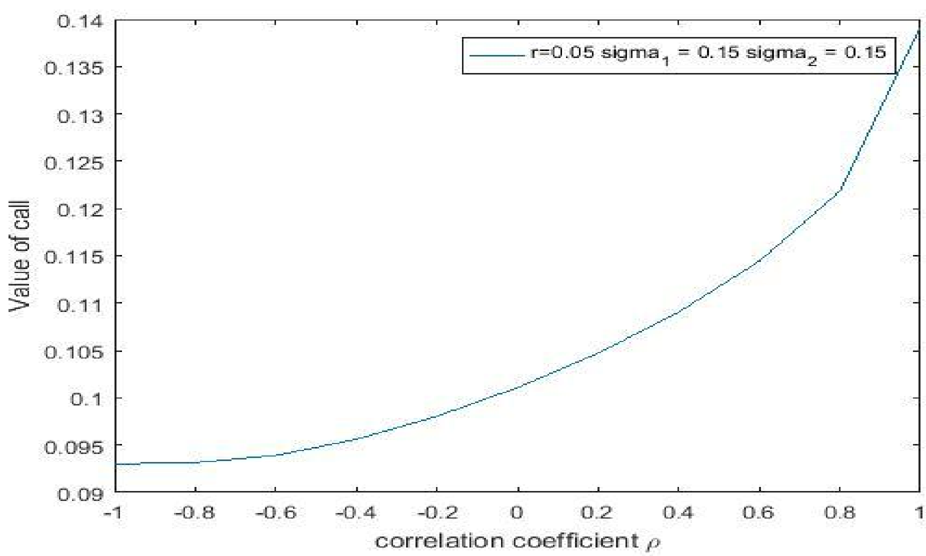

Figure 5:Line graph showing the change of call value as correlation coefficient increases.

If the correlation coefficient is at a relatively low level (depending on the size of the volatility of both stocks), size of increment of the value of the call outweighs that of warrant without call. As a result, value of the warrant with call decreases.

As volatility increases, the increment of the 'slope' of the warrant without call is more than that of the value of the call. Consequently, when volatility is at a relatively high level, the smile disappears. (As shown in Figure 1:Line graph showing the change of the value of warrant as correlation coefficient increases)

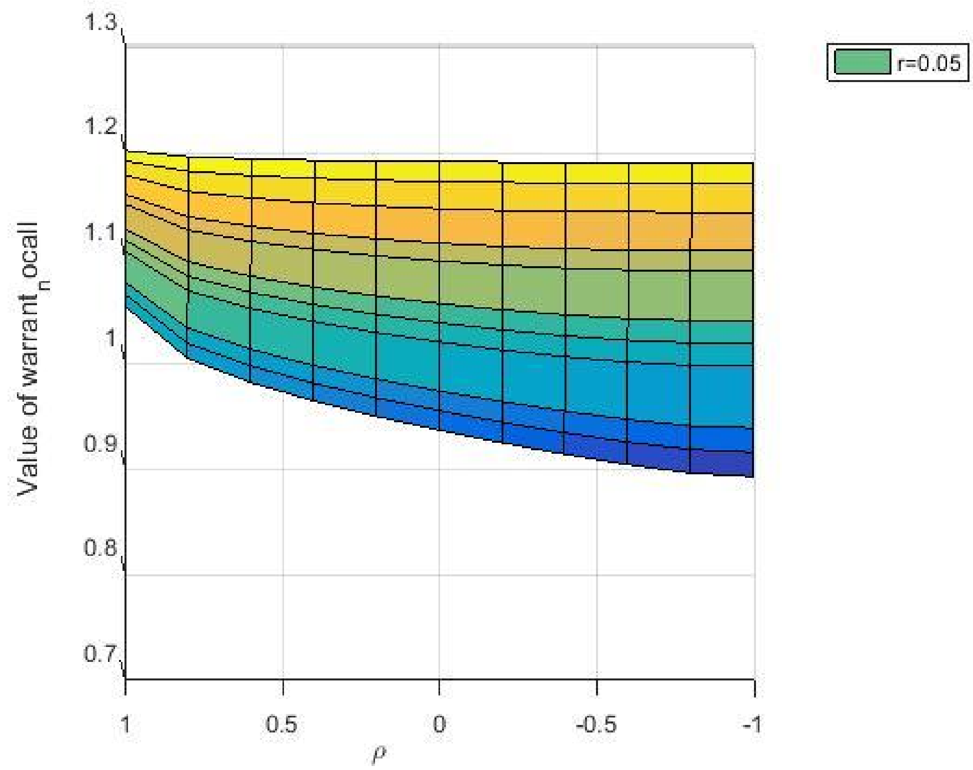

Figure 6:Surface plot showing the change of the value of warrant without call as correlation coefficient increases, the volatility of both stocks change from 0.1(yellow area) to 0.35(blue area).

Volatility

As the volatility of either one of the stock increases, the warrant's price decreases monotonically.

Recall that

warrant = bond (series of binary options due to the accrual feature of coupons)

– European put on minimum of two uncorrelated stocks

– issuer's calling right (Bermudan call option with multiple call dates)

Reasoning:

1.Both stock prices stay far above the exercise value at beginning. If the volatility of either stock increases, the probability that min(S1 (n,i) / S1,exer , S2 (n,j) / S2,exer) > 1 decreases, chance of receiving coupon decreases. The value of the bond decreases.

2.If volatility of both stock increases, the value of the European put shorted by warrant holder increases, which leads to the decrease of the warrant price.

Similar argument holds when the volatility of one stock fixed and the volatility of the other stock increases.

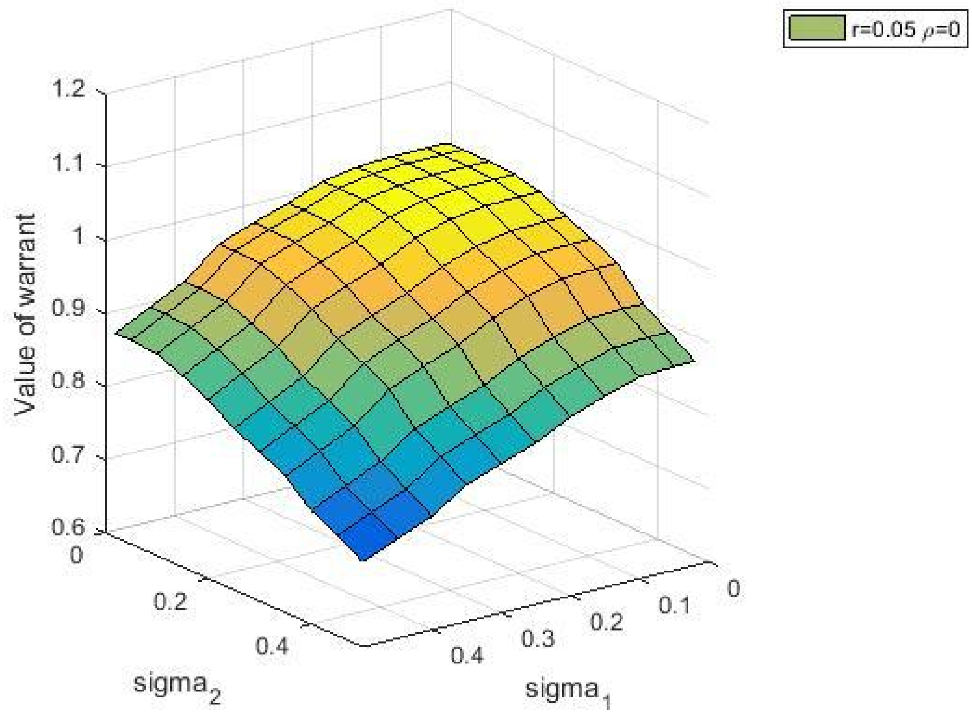

Figure 7: Surface plot showing the change of the value of warrant as volatility increases.

Risk-free Interest Rate

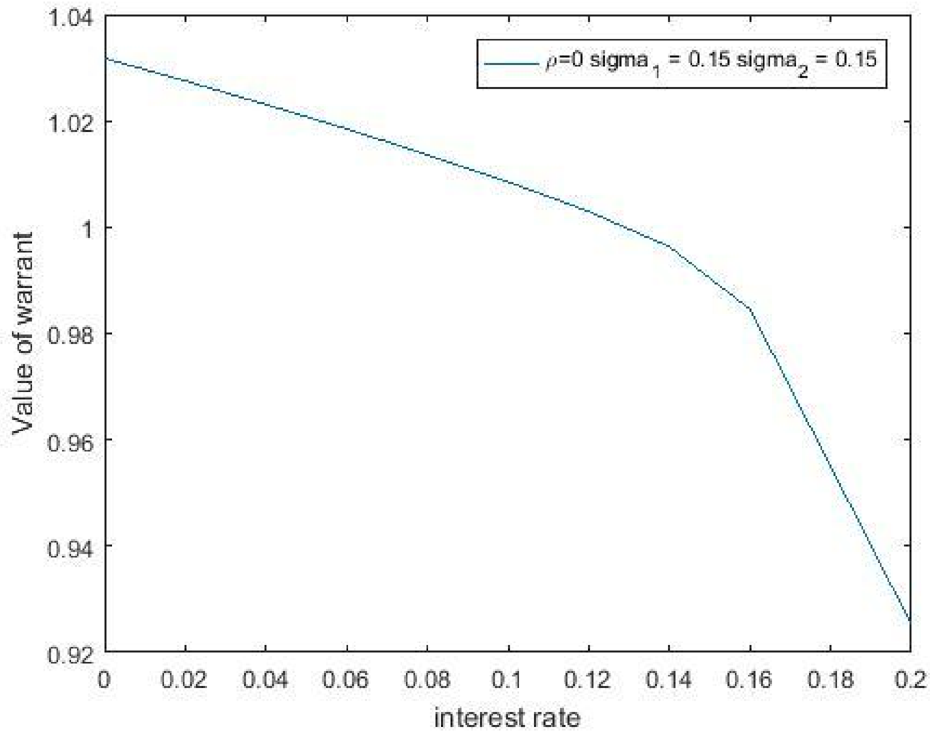

Figure 8: Line graph showing the change of the warrant as the risk-free interest rate increases

As the risk-free interest rate increases, price of warrant declines. The result matches the financial intuition as lower risk-free interest rate appeals investor purchase more warrant.
A further analysis by analyzing how increasing interest rate affects the pricing behavior of each component of the warrant is required.

As the risk-free interest rate increases, the risk-neutral probability that both stock price goes up increases, while probability that either stock drop decreases, and holder of the bond is more likely receive more coupons. Downside loss due to shorting a European put is likely to decrease. Continuation value of the warrant can be higher, and the expected payoff of shorting caller's right can be higher. However, discount factor decreases and dominates the pricing behavior.

Part3. Caller’s Right

Pre-set Parameters

Same as that of Part1.

Results

We define the spread to be the spread between warrant without call and warrant with call. The spread is value of one unit of Bermudan call option, with strike price equal to the call price, and the call price is 100% the notional value. For convenience, the notional value is set to be unit dollar.

The continuation value of the warrant is determined by the expected payoff of shorting one unit of European put option and additional unit dollar. The terminal payoff for shorting the European put is the premium minus the downside loss, which is determined by the difference between unit dollar and the smallest ratio of each stock’s price divided by corresponding exercise price at maturity.

Volatility

As the volatility of either one of the stock increases, the value of issuer’s calling right decreases monotonically.

Reasoning:

1. Similar to what we discussed in Part2, as the volatility of either stock increases, the probability that min($S1$ (n,i) / $S1,exer$, $S2$ (n,j) / $S2,exer$) > 1 decreases, chances to receive coupon decrease, the continuation value of the warrant decreases. Recall that Bermudan call $= W_{cont}\text{-}\min(W_{cont}, K) = \max(W_{cont} - K, 0)$. The value of issuer’s calling right decreases since the continue value of the warrant decreases.

2. As the volatility of either stock increases, the probability that min($S1$ (n,i) / $S1,exer$, $S2$ (n,j) / $S2,exer$) > 1 decreases, the issuer gets less chances to exercise the call, therefore the call value decreases.

Further calculations validate the argument above.

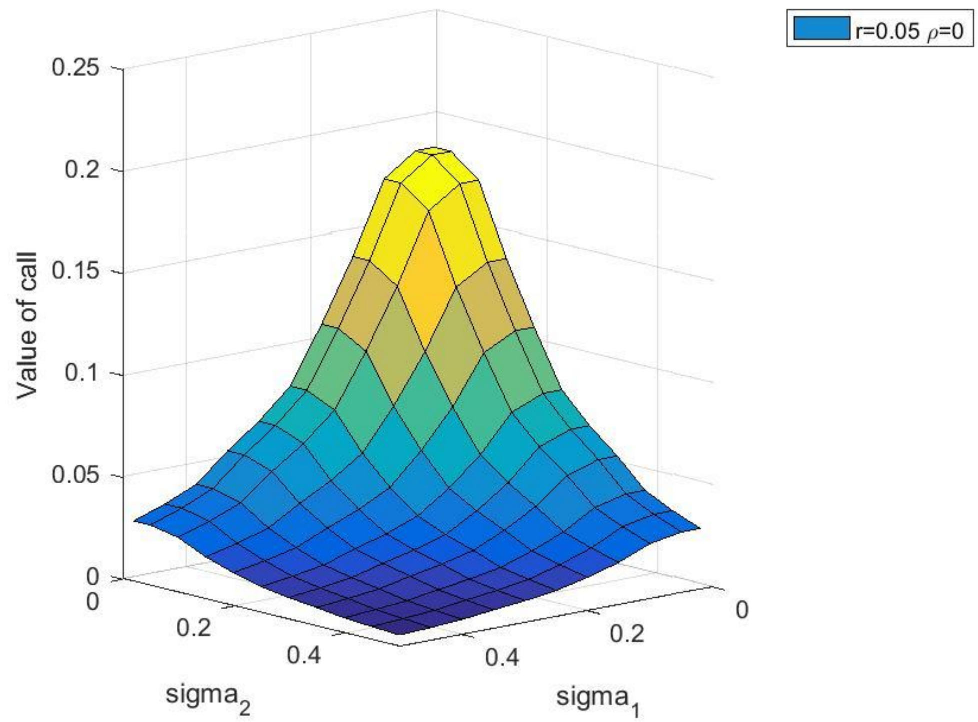

Figure 9: Surface plot of change of call option's value as volatility of both stock increases.

Risk-free Interest Rate

As risk-free interest rate increases, the value of the call decreases.

As risk-free interest rate increases, discount factor become smaller, while the risk-neutral probability of price of both assets goes up increases. However, decrease of discount factor should overwhelm the increase of the price of the call and the value of the Bermudan call is expected decrease to 0.

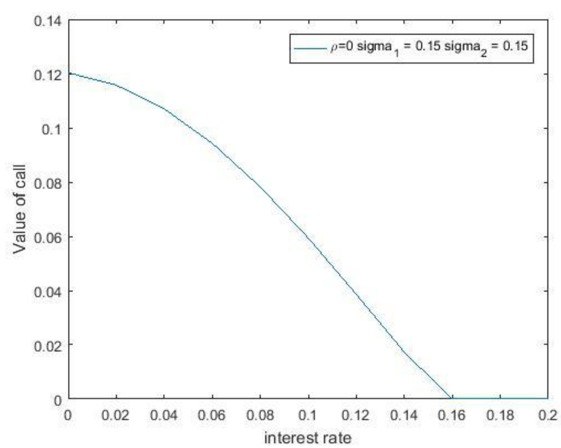

Figure 10: Line graph of change of call option's value as risk-free interest rate increases.

Correlation Coefficient

As correlation coefficient increases, the value of the call increases.

When the correlation coefficient increases from -1 to 1, the risk-neutral probabilities of both stock prices move on same direction increase, while the risk-neutral probabilities of two stocks' prices move on opposite direction decrease.

Recall that the Bermudan call = W_{cont}-$\min(W_{cont}, K) = \max(W_{cont} - K, 0)$. As the correlation coefficient increases from -1 to 1, the continuation value of the warrant increases, value of the Bermudan call increases,

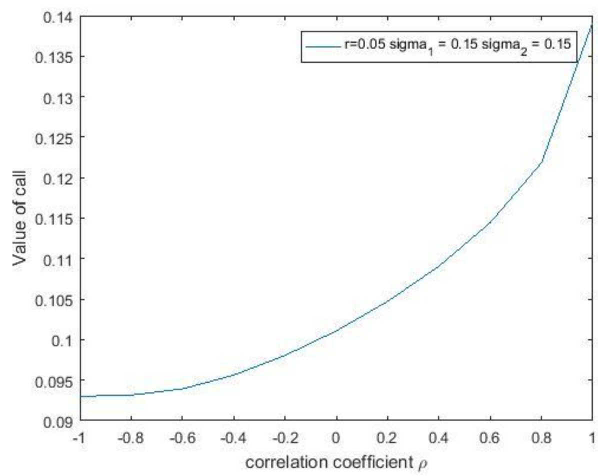

Figure 11: Line graph of change of call option's value as correlation of both stocks increases.

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