My First Document

My Name

May 31, 2019

Contents

1 Results

1.1 Output

1.1.1 Typical simluations

A simulation = ... show picture : genes, field, network =¿ show two picture, one with also model SS2 Genes = weighted over 8 bits : values between 0 and $2^8 - 1$

this should be much longer

so as figures don't run into one another...

otherwise learn to skip pages...

Mostly will be working with average for relative value for genes over the pop / after a lot of time One experiment =¿ results for four genes Typically (unless specified overwise), values averaged over 30 experiments

1.1.2 Remembered Heroes

These output values are precise to the percentage point. For **SelfSacrifice** this poses a problem, as final values are expected to be relatively small - with typical parameter values (as in **section 2**), average probability of self-sacrifice is capped at 2-3%.

For this reason, simulations also kept track of **RememberedHeroes**, a measure for the number of heroes in the live memory of a society. At each simulation step (year), the number of "voluntary" martyrs was computed by subtracting the expected number of martyrs which could be solely attributable to mutations in a population that did not engage in self-sacrifice. These were added to the number of heroes which could be assumed to have been *witnessed* by (alive) individuals in the population. In a given year, **RememberedHeroes** corresponds to the number of heroes witnessed by at least *RemThreshold* percent of the population (5 % in practice). In situations where self-sacrificial behavior can be said to be absent, **RememberedHeroes** is close to 0 (e.g. when *Admiration* is null in the first model under).

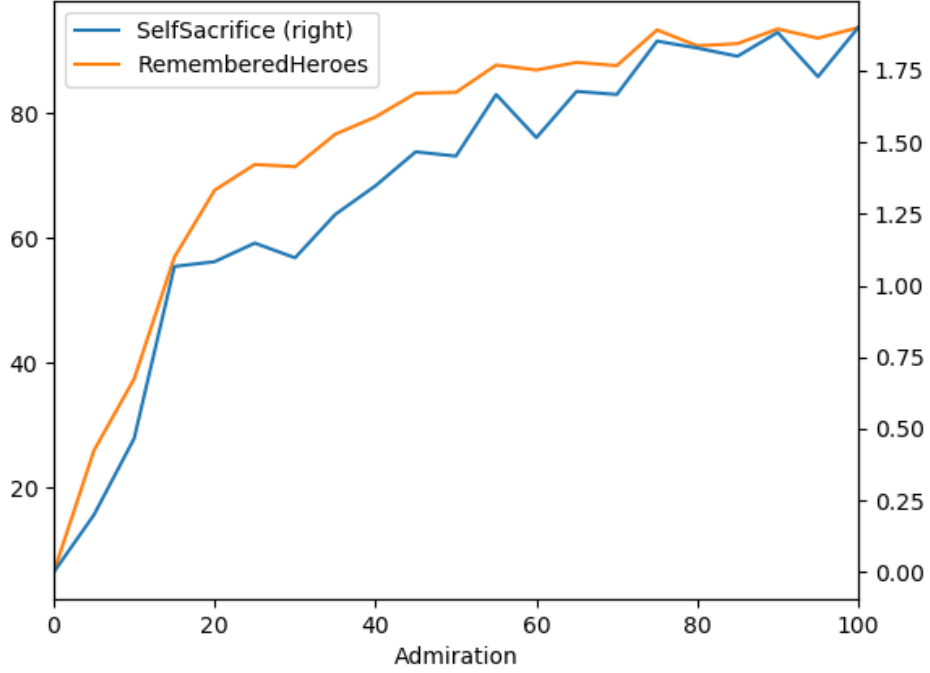## 1.2 Exogenous model: simulation outputs

1.2.1 Main results

Figure 1: **SelfSacrifice** and **RememberedHeroes**, as a function of *Admiration* (typical parameter values).

Figure 1 shows results obtained by averaging over 30 simulations, according to the *Admiration* parameter (all others being kept constant at the previously described values). As could be expected, **RememberedHeroes** rises from 0 with *Admiration*, quickly reaching two thirds of its maximum value of under 100 heroes when *Admiration* exceeds 20.

SelfSacrifice follows similar dynamics, with values ranging between 0 and 2% (values to a higher precision than the percentage point being obtained artificially by averaging over experimental results). These values are far from negligible: for a typical population of 200, we expect an average of (almost) 4 martyrs each year. Such collective behavior is captured here by individuals all bearing similar genetic probability P of self-sacrifice at equilibrium (and obtaining similar **Reproductive_points** in the long-term),

as seen on **Figure 2**.
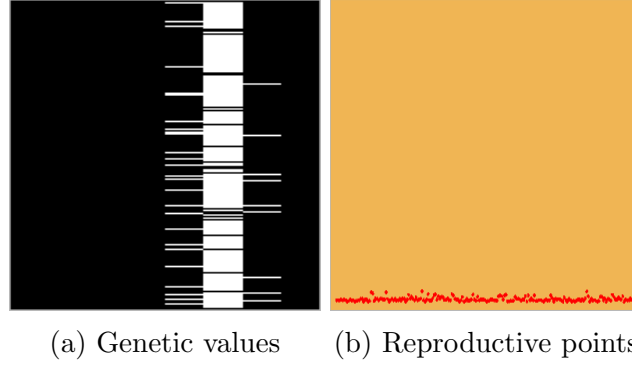

(a) Genetic values   (b) Reproductive points

Figure 2: Snapshot of individual values, $A{=}50$. An individual's genome is represented by a horizontal line on the left: here most bear a **SelfSacrifice** gene of relative value $\frac{2^2}{2^8-1}$ (around 1.6%). Individuals (horizontal axis) and their reproductive points (vertical) are represented on the right.

However, one can also imagine the collectively equivalent situation where only a small fraction f of individuals engage in self-sacrificial behavior with higher probability p (with $P = f * p$), to the much more significant benefit of their families (see **Section 1.3**).

1.2.2   Influence of *ReproGainsThreshold* RGT
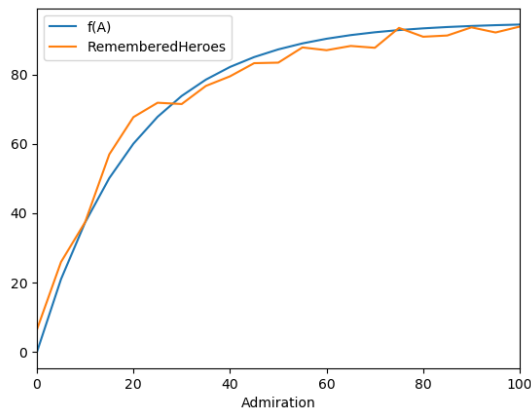

Figure 3: **RememberedHeroes** and corresponding optimal f as functions of *Admiration*, for $RGT = 10$ ($C = 95, \tau = 20$).

As a function of *Admiration*, **RememberedHeroes** resembles a function of the form: $f_{C,\tau}(t) = C * (1 - exp(-t/\tau))$. Very approximately[1], it can best be approached by a function of this form with parameters $C = 95$ and $\tau = 20$, as visible on **Figure 3**.
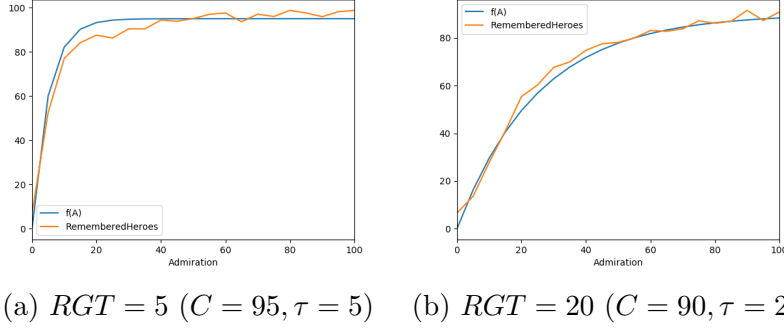

(a) $RGT = 5$ $(C = 95, \tau = 5)$ (b) $RGT = 20$ $(C = 90, \tau = 25)$

Figure 4: **RememberedHeroes** and f for $RGT = 5, 20$

The key parameter governing relative growth of **RememberedHeroes**[2] thus appears to be one equal to two times *ReproGainsThreshold*. The importance of RGT is not surprising since it plays the crucial role of defining the unit in which **Reproductive_points** are counted. The relationship between RGT and final results is however non-trivial, since variation according to A for $RGT = 5$ (resp. $RGT = 20$) are best captured by $\tau = 5$ (resp. $\tau = 25$), as visible on **Figure 4**; suggesting perhaps a quadratic relationship between RGT and τ. **Section 1.3.2** returns to this issue.

1.2.3 Variation with *SacrificeHeredity* h

Contrary to what could be expected, h -¿ 1 is a problem: probs because pop = too small and all end up with same h... - as shown by simu // because same family ? (or just expected in LT?) h = 0 discontinuity = expected

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[1]By choosing optimal C and τ at a precision of 5 units; the idea here being simply to get a "feel" for overall variation with A. Optimal parameters are the ones for which Euclidean distance is minimal.

[2]This output's absolute value is arbitrary, chosen according to *RemThreshold* in order to provide for visible variations (see **Section 1.1.2**).

1.2.4 Variation with *ReproductionRate* r

1.3 Exogenous model: mathematical proof of concept

1.3.1 Simplifying assumptions and characterization of equilibrium

Individuals live to a maximum of *AgeMax* M years, fixed at 40 here. Expected life span is lower however, as individuals face random accidents (in a wide sense) and is equal to $\alpha * M$, where α captures the effects of natural selection. In this model, all individuals face the same α, but may obtain differing reproductive opportunities (**Selectivity** mode).

An individual who engages in self-sacrificial behavior with probability p shortens his/her lifespan by a multiplicative factor of (see **Appedix B.1.2**):

$$\beta(p) = \frac{(1-p) - (1-p)^{(M+1)}}{p * M} \tag{1}$$

As such, said individual's reproductive window is shorter, which should lead to the disappearance of self-sacrificial behavior - unless this is compensated by increased reproductive opportunities. In contrast to how the actual simulation is played, let us assume that:

- Only a *negligible proportion f* of individuals engage in self-sacrificial behavior, with equal probability p;

- These future "heroes" will be the first this society sees (everyone starts off with no **Reproductive_points** RP);

- Generations do not overlap;

- Individuals are granted their lifetime reproductive potential R at birth (in contrast with a year-by-year attribution).

In such a situation, individuals who engage in self-sacrificial behavior obtain on average $\alpha * \beta(p) * M * r$ offspring, with others obtaining $\alpha * M * r$, where r is the population-level *ReproductionRate*. However, the former's children are granted a larger reproduction potential $R_+(A, f)$, which notably depends on *Admiration A*. Since f is assumed to be very small, $R_+(A, f)$ is the same for all children of individuals who self-sacrifice (we neglect the possibility of individuals being born from two would-be martyrs), and the children of non-would-be martyrs obtain a reproductive potential which can be approximated to r.

If, in addition, we assume that *p and M are sufficiently large*[3] for would-be heroes to largely end up actually laying their life for the group (and not dying in another way), such individuals obtain on average $R_+(A, f) * \alpha * \beta(p) * M * r$ grand-children, while others obtain $\alpha * M * r^2$. Thus, a first-order (neglecting the effects on subsequent generations) characterization of equivalence between both strategies can be written as:

$$R_+(A, f) * \beta(p) = r \tag{2}$$

1.3.2 Existence of a self-sacrifice ESS

Let N be equal to *PopulationSize*, implicitly assumed to be large here (since f is negligible). Following the simplifying assumptions made above, total social admiration is borne from each non-hero in the first generation and is thus equal to $A * N * (1 - P)$. For children of martyrs, two cases are possible:

- either $\frac{AN(1-P)}{fN*\alpha\beta(p)Mr} < RGT$, and each receives reproductive potential r;

- or $\frac{AN(1-P)}{fN*\alpha\beta(p)Mr} \geq RGT$, and each receives $R_+(A, f) > r$.

In a population composed of $N - 1$ non-heroes and one mutant would-be martyr, the latter will thus be able to invade if, in particular[4]:

$$A > A_{min} = \frac{RGT * M * r}{N} \tag{3}$$

When this condition is met, complete absence of self-sacrifice cannot be an ESS.

thus, since

1.3.3 Mathematical characterization of the ESS

As shown in **Appendix B.1.3**, a first-order approximation of R_+[5] in the latter case is (S is equal to *Selectivity*):

$$R_+(A, f) = \frac{S * r}{log(1 + S)} * (1 - \frac{f * \beta(p)}{2}) \tag{4}$$

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[3]We still expect $P = f * p$ to be relatively small, to avoid "martyr over-crowding", but for this to be primarily due to f, in the current mathematical characterization.

[4]α and $\beta(p)$ are smaller than 1.

[5]Note that all equalities in this section are approximations.

When *Admiration* is sufficient large, self-sacrifice with overall probability $P = f * p$ is thus an ESS when p and f verify:

1.4 "Endogenous" model

2 Perspectives

2.1 Limitations

A Python stuff

B Mathematical demonstrations

B.1 Exogenous model

B.1.1 RemTH

B.1.2 $\beta(p)$

Disregarding the effects of natural selection (which are the same for all individuals here), an individual who bears a **SelfSacrifice** gene of relative value p, has a probability p of dying in his first year (before being able to foster any descendants), a probability $(1-p) * p$ of dying at age 1 ... a probability $(1-p)^n * p$ of dying at age $n < M$ ... and is certain to die at age M, should he or she reach it. His/her expected life span is thus:

$$ELS = p * 0 + (1-p) * p * 1 + ... + (1-p)^{M-1} * (M-1) + (1-p)^M * M$$

Let $f \colon \mathbf{R} \to \mathbf{R}$ be the polynomial function defined by the expression:

$$f(x) = \sum_{n=0}^{M-1} p * (1-p)^n * x^n + (1-p)^M * x^M$$

By deriving f, one can note that:

$$ELS = f'(1) \tag{5}$$

$f(x)$ involves a geometric sum and can be simplified to (when $(1 - p) * x$ is different than 1):

$$f(x) = p * \frac{1 - ((1 - p) * x)^M}{1 - (1 - p) * x} + ((1 - p) * x)^M$$

For $x \neq \frac{1}{(1-p)}$ ($p = 1$ trivially yields $ELS = 0$):

$$f'(x) = p * \frac{-M(1 - p)^M x^{M-1} * (1 - (1 - p)x) + (1 - p) * (1 - ((1 - p)x)^M)}{(1 - (1 - p)x)^2} + M(1-p)^M x^{M-1}$$

And thus:

$$f'(1) = p * \frac{(-M(1 - p)^M * p + (1 - p) * (1 - (1 - p)^M)}{p^2} + M(1 - p)^M$$

$$f'(1) = \frac{(1 - p) * (1 - (1 - p)^M)}{p} \tag{6}$$

Combining these two expressions for $f'(1)$ proves equation (**1**). **Figure 5** shows $\beta(p)$ for p between 0 and 1. Factoring in natural selection, an individual's expected life span is therefore equal to: $\alpha * \beta(p) * M$.

B.1.3 $R_+(A)$

In **Selectivity** mode, individuals obtain reproductive potential R according to their **Reproductive_points** RP, each individual obtains a rank k according to RP, and receives reproductive potential:

$$R = \frac{r}{2} * (\frac{S}{(S * k + N') * log(1 + S)} + \frac{S}{(S * (k + 1) + N') * log(1 + S)} * N')$$

where $N' < N$ is the number of eligible parents (non-martyrs) and S is equal to *Selectivity* (the average reproductive potential over eligible parents being *ReproductionRate r*). When $N >> 1$, R verifies:

$$R \in [R_{min}; R_{max}], \text{with } R_{max} \approx \frac{S * r}{log(1 + S)} \text{ and } R_{min} \approx \frac{R_{max}}{1 + S} \tag{7}$$
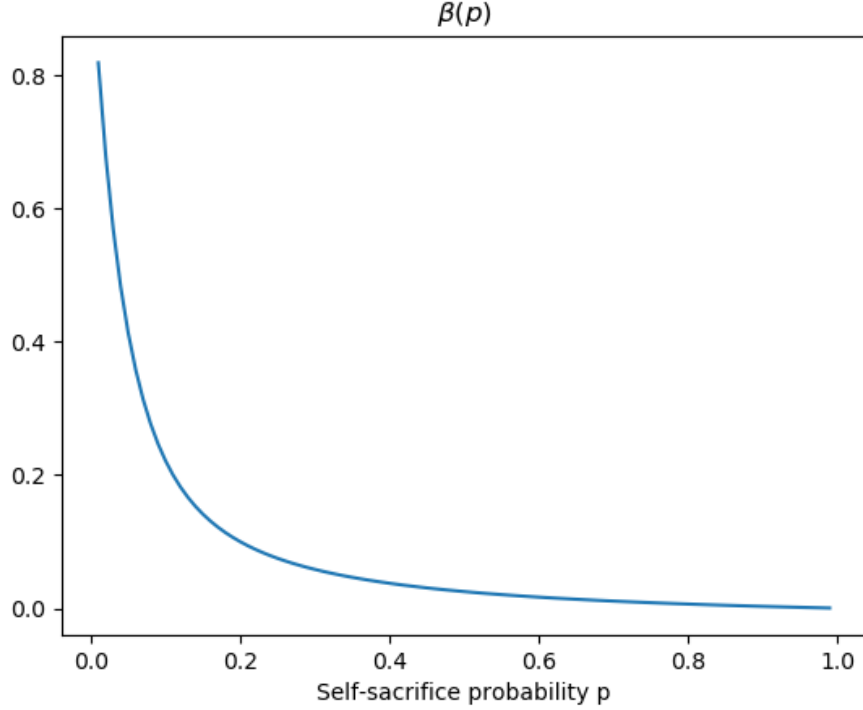
8

Figure 5: Loss of expected life-span due to self-sacrifice.

With typical parameters ($S = 10$ and $r = 15\%$), we obtain: $R_{max} \approx 63\%$ and $R_{min} \approx R_{min} \approx 5,6\%$.

In a case where a negligible proportion f of individuals engage in self-sacrificial behavior (which is assured to end up in their martyrdom), their children each receive, on average:

- reproductive potential r, when $\frac{AN(1-P)}{fN*\alpha\beta(p)Mr} < RGT$;

- $R_+(A, f)$ otherwise, as seen in **Section 1.3.2**.

When $N >> 1$, expected $R_+(A, f)$ is equal to the average between the "luckiest" ($k = 0$) and "unluckiest" child, which is approximately:

$$R_+(A, f) \approx \frac{S*r}{2*log(1+S)} * (1 + \frac{N'}{S*fN*\alpha\beta(p)Mr + N'})$$

$$R_+(A, f) \approx \frac{S*r}{2*log(1+S)} * (1 + \frac{1}{\frac{S*fN*\alpha\beta(p)Mr}{(1-f)N*\alpha Mr+fN*\alpha\beta(p)Mr} + 1})$$

9

$$R_+(A, f) \approx \frac{S * r}{2 * log(1 + S)} * (1 + \frac{(1 - f) + f\beta(p)}{(S + 1) * f * \beta(p) + (1 - f)})$$

Which yields, for $f << 1$ (neglecting terms of order 2 and above):

$$R_+(A, f) \approx \frac{S * r}{log(1 + S)} * (1 - \frac{f * \beta(p)}{2}) = R_{max} * (1 - \frac{f\beta(p)}{2}) \qquad (8)$$