ASTR 5490 Homework #4
Due Nov. 25th

Learning goals: Work with Blackbody spectral energy distributions and photometric bandpasses, create a model debris disk planetary system and try modeling a read debris disk.

1. Create a blackbody spectral energy distribution for a star like the sun with T_{eff}=5780 K and 1 $R_\odot$. Include wavelengths between the X-ray (1 Å) and far infrared (100 microns).
 A. Plot this SED. Integrate it over wavelength and multiply by the surface area of the sun and another factor of π (the angular integral over azimuthal and polar angle) and verify that you recover the luminosity of the sun: $4 \pi R^2 sT^4$ = 2e33 erg/s. Careful with units here!!
 B. What fraction of the energy is emitted shorter than the peak versus longer than the peak?
 C. Spectral energy distributions in astronomy are often plotted as νL_ν on the y-axis versus ν on the x axis because the total energy scales as ν (i.e., $E=h\nu$). Make such an SED and compare it to L_ν on the y-axis versus ν . What changes?
D. Create an SED that consists of two components: a sun-like blackbody and a hot super-Jupiter with radius $R=3R_{Jup}$ at 0.05 au from the star. Plot each component separately. Plot the contrast ratio, the ratio of L_λ (or L_ν) versus wavelength (or frequency) to illustrate where this contrast ratio peaks.
E. Real observations are made through distinct photometric filter systems, each with its own *response function*. Get from the course web page under HW#4 a table defining the response functions of the *Kepler* broad optical band (denoted Kp, not to be confused with the infrared 2.2 micron K band!) and the 4.5 micron *Spitzer* observatory bandpass. Fold (that is, multiply the F_ν by the efficiency at each wavelength and integrate over the bandpass) the SED of the sun (working in frequency units) through each bandpass to add up the energy that would be observed in each bandpass. The magnitude difference is $\Delta m = -2.5 \log (F(K)/F(4.5))$ Compare this to the approximate V-W2 color from Pecaut & Mamajek (2013, ApJS, 208, 9) for a solar type star. [Note the V band is approximately the Kepler bandpass and the WISE spacecraft W2 band is approximately the *Spitzer* I2 bandpass, so this ought to be close.] Do the same for the super-Jupiter to find how many magnitudes of difference there are between the host star and planet at each wavelength.

2. A. Plot a Maxwell-Boltzmann distribution of speeds for He in the Earth's atmosphere. Integrate this distribution between the escape speed for Earth and v=infinity to find what fraction of He atoms at any given time have speeds greater than escape velocity. Do the same for molecular nitrogen and compare fractions.
B. Do the same thing for a hot T=2000 K Jupiter-sized planet, considering both atomic He and molecular CO.

3. Create a model of a protoplanetary debris disk system consisting of a central solar-type star and a disk of dust extending from the dust sublimation radius (where T~2000 K) to 1000 au. Assume that optical depth effects are negligible so that we don't have to do true radiative transfer calculations to account for absorption and scattering (not generally really reasonable, but we'll get something close). Break the disk into thin rings of radius dr and compute the area of each ring and the temperature of each ring under the assumption of radiative equilibrium.
 A. Take 1 Earth mass of dust and spread it uniformly over the model protoplanetary disk. Take the mean mass of an (assumed) spherical grain to be $M_{gr}=4\pi/3\ a^3\ \rho_{gr}$ where the mean grain radius is a=0.1 mm and the mean grain density is ρ_{gr} =2 g cm^{-3} (this is appropriate to interstellar dust but might not be for larger grains that grow within protoplanetary disks). The grain then radiates as a blackbody with that surface area. [In reality the dust grains are described by a power law distribution of sizes between 0.01 μm (interstellar dust sizes) and perhaps 1 cm (pebbles, from agglomeraged material in protoplanetary disks) and they do not emit as perfect blackbodies, especially the smallest ones, but this will serve to get us what we need. Save the details for ISM class...] Given these grain parameters you can compute a grain surface density in number of grains per area in each ring and add up the flux from each grain within each ring. Plot the SED of the star, the disk, and the sum of the two. What is the ratio of luminosity in the disk to the luminosity in the star?

B. Now suppose planets have cleared out all the dust inside 1 au or inside 10 au. Generate and plot the resulting spectral energy distributions and ratios L_{disk}/L_*.   Comment on how these spectral energy distributions might be used to infer the presence of gaps in the disk.
C. Suppose the disk inner edge were now 0.1 au and the outer edge of the disk was truncated at 10 au due to the gravitational effects of a distant planet or perhaps a companion star. How does the resulting SED change compared to the full-disk case?
D. Use actual data from the Beta Pictoris system from the near and far infrared to create a model for an A6V star (use parameters from Pecaut & Mamajek (2013, ApJS, 208, 9)) surrounded by a dusty disk and show a model that you create that roughly reproduces the fluxes observed for that system at an adopted distance of 19 pc. Your model won't be unique. Just show one or two that work and discuss degeneracies between the parameters you can control: the mass of dust, the inner radius of the disk, and the outer radius of the disk.   For data, you can adopt the broadband magnitudes from the optical through near-IR from simbad.   You can convert magnitudes into F_ν units that emerge from your model using the methods of my Observational Methods class, or use the HST/NICMOS online tool. For wavelengths longer than K Band use the IRAS 12/25/60/100 micron fluxes from Table 2 of Pantin et al. (1997). Suppose you adopt an average dust grain size of 0.01 mm instead of 0.1 mm, what changes, and can you see why assumptions about the grain size distribution matter?
See how your total dust masses compare to the Backman & Paresce (1993) rough mass estimate of 0.001-0.01 $M_\oplus$.

Infrared Images of the Beta Pictoris debris disk from Telesco et al. (Nature, 2005).