1a) Planet Mass vs. Semi-major Axis (Confirmed exoplanets from NASA exoplanet archive)

In [7]:

```
# Import relevant modules/packages
import numpy as np
import matplotlib.pyplot as plt
from astropy import units as u
from astropy import constants as const
```

In [8]:

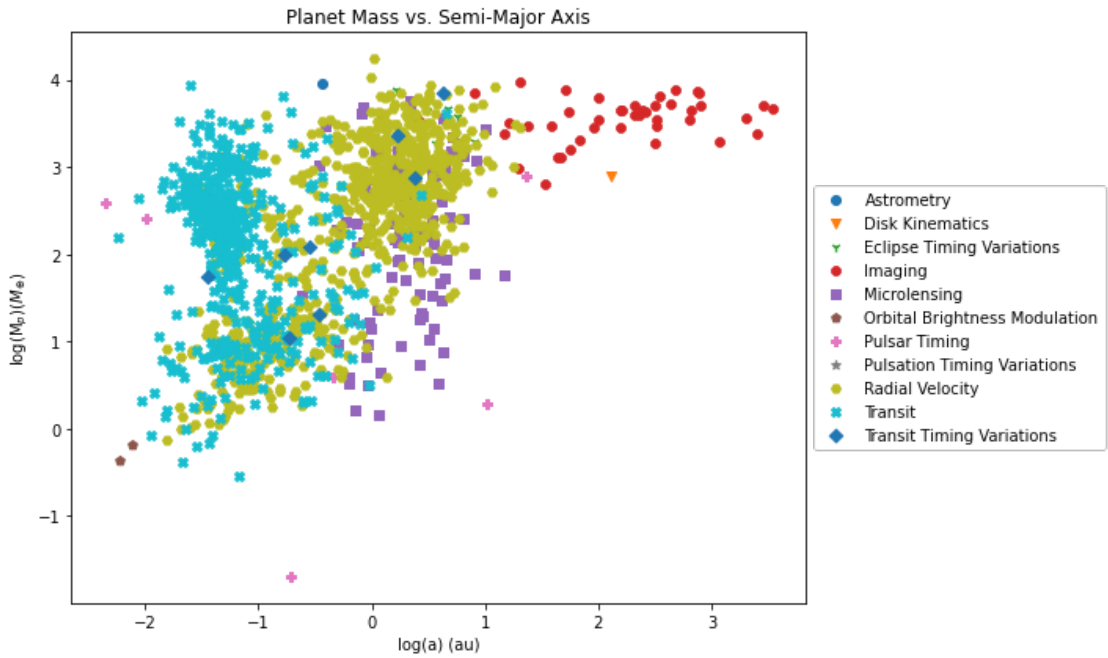
```
# Function to read text files
def Read(filename):

    # Read and return data of confirmed exoplanets from NASA Exoplanet Archive
    data = np.genfromtxt(filename,dtype=None,delimiter=',',skip_header=97,name
    return data
```

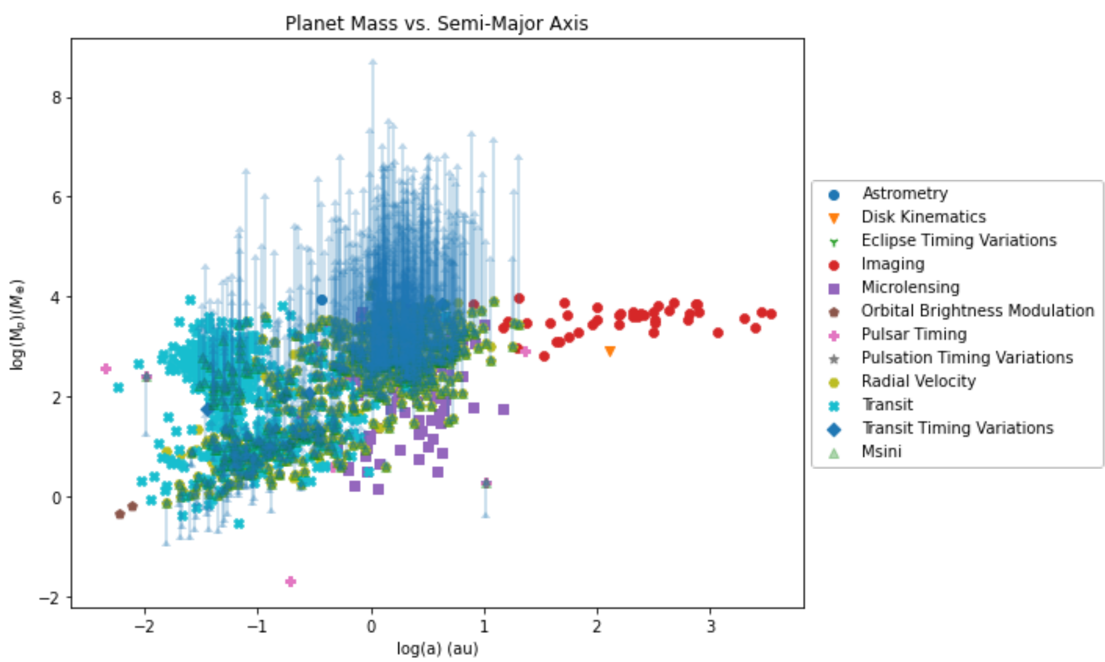
In [153]:

```python
# Function to plot mass vs. semi-major axis (save is parameter to save figure
def MassSemiMajor(save=False):

    # Save data using 'Read' function
    data = Read('table.csv')

    # Define x and y datasets to plot
    xdata = []
    ydata = []

    x_sini = data['pl_orbsmax'][np.where(data['pl_bmassprov']=='Msini')]
    y_sini = data['pl_bmasse'][np.where(data['pl_bmassprov']=='Msini')]
    mass_error = data['pl_bmasseerr1'][np.where(data['pl_bmassprov']=='Msini')
    #print(y_sini[0:5])

    logx_sini = [np.log10(x) for x in x_sini]
    logy_sini = [np.log10(y) for y in y_sini]
    logmass_error = [np.log10(mass) for mass in mass_error]

    # Define list of symbols to use for different discovery methods
    markers = ['o','v','1','8','s','p','P','*','H','X','D']

    # Establish figure and axis object for plotting
    plt.figure(figsize=(10,6))
    ax = plt.subplot(111)

    # Loop through discovery methods
    for i in range(0,len(np.unique(data['discoverymethod']))):

        # Identifty locations of data points for each discovery method
        detections = np.where(data['discoverymethod'] == np.unique(data['disco

        # Define x (mass) and y (semi-major axis) datasets to plot
        x = data['pl_orbsmax'][detections]
        y = data['pl_bmasse'][detections]

        # Add x and y list for each method to a master list
        xdata.append(x)
        ydata.append(y)

        # Make a scatter plot for each datset with a new symbol for each disco
        ax.scatter(np.log10(xdata[i]),np.log10(ydata[i]),label=np.unique(data[

    # Plot lower limits for planets with Msini mass measurements
    #ax.scatter(logx_sini,logy_sini,label='Msini',marker='^',color='g',alpha=(
    #ax.errorbar(logx_sini,logy_sini,yerr=logmass_error,fmt='*',capthick=0.1,b

    # Set axis labels, title, and legend (legend outside of figure)
    ax.set_title('Planet Mass vs. Semi-Major Axis')
    ax.set_xlabel('log(a) (au)')
    ax.set_ylabel(r'log(M$_{p}) (M_{\oplus}$)')
    ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
    plt.tight_layout()

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1a_plot.jpg')

    # Show plot
    plt.show()
```

In [154]:
```
1 MassSemiMajor()
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:18: Runti
meWarning: divide by zero encountered in log10
```





Comments about 1a:

The different clusters in this plot appear to be centered around a central semi-major axis, but spread about wide range of masses.

The teal blue cluster on the left reveals that most exoplanets detected by the transit method have high masses and lie very close to their host star . This makes sense as this combination of properties causes more significant dips in the host star's light curve, thus making them easiest to detect this way.

The dark yellow cluster(s) in the top middle and bottom left of the plot reveal that radial velocity detections span a wide range of masses and semi-major axes, although more of these detections appear to be at higher masses. One would expect more high-mass detections with this method as these planets would have a more pronounced effect on Doppler shifting spectra of their host star.

The red cluster at the top right of the plot reveals that most imaging detections have high masses and large semi-major axes. This agrees with the bias of this method towards finding larger planets that can reflect more light from their host star and further planets that are less likely to to be lost in the glare of the host star.

The purple cluster in the center of the plot indicates that microlensing selects less massive planets that are further from their host star. This method can detect small variations in a star's light curve, but very few systems are aligned precisely enough to observe this, hence the scarcity of microlensing detections.

There is only 1 detection made by astrometric observations. There is also only 1 detection made by disk kinematic observations.

There is an obvious gap in detections on the bottom and right of this plot. There are almost no detections of low mass, low semi-major axis planets which makes sense since they are heavily obscured by their host star and are difficult to detect indirectly since they have a miniscule impact on the star itself. The lack of detections in the right of this plot suggests that for a planet far from its host star to be detected, it needs to be very massive.

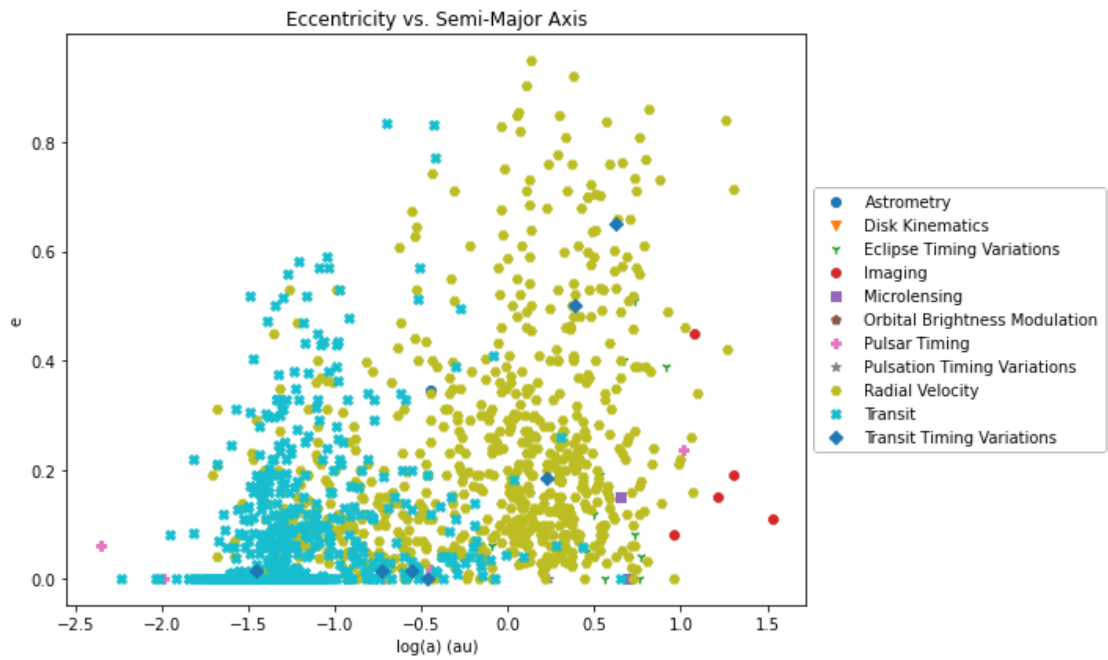
1b) Orbit Eccentricity vs. Semi-Major Axis

In [9]:

```
# Function to plot eccentricity vs. semi-major axis (save is parameter to save
def EccenSemiMajor(save=False):

    # Save data using 'Read' function
    data = Read('table.csv')

    # Define x and y datasets to plot
    xdata = []
    ydata = []

    # Define list of symbols to use for different discovery methods
    markers = ['o','v','1','8','s','p','P','*','H','X','D']

    # Establish figure and axis object for plotting
    plt.figure(figsize=(10,6))
    ax = plt.subplot(111)

    # Loop through discovery methods
    for i in range(0,len(np.unique(data['discoverymethod']))):

        # Identifty locations of data points for each discovery method
        detections = np.where(data['discoverymethod'] == np.unique(data['disco

        # Define x (mass) and y (semi-major axis) datasets to plot
        x = data['pl_orbsmax'][detections]
        y = data['pl_orbeccen'][detections]

        # Add x and y list for each method to a master list
        xdata.append(x)
        ydata.append(y)

        # Make a scatter plot for each datset with a new symbol for each disco
        ax.scatter(np.log10(xdata[i]),ydata[i],label=np.unique(data['discovery

    # Set axis labels, title, and legend (legend outside of figure)
    ax.set_title('Eccentricity vs. Semi-Major Axis')
    ax.set_xlabel('log(a) (au)')
    ax.set_ylabel('e')
    ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
    plt.tight_layout()

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1b_plot.jpg')

    # Show plot
    plt.show()
```

In [10]:
```
1 EccenSemiMajor(True)
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about 1b

<u>Transit</u> detections are correlated with <u>close, minimally-eccentric orbits</u> (a<1 au, most e<0.20). This makes sense because closer planets cause more signficant dips in their host star's light curve, making them easiest to detect when they cross in front of the star. This also makes sense because the tidal forces acting on planets close to their host star circularize (bring e closer 0) their orbits.

<u>Radial velocity</u> detections are correlated with <u>comparitively further orbits than those for the transit method</u>, but occupy a <u>wide range of eccentricities</u> from nearly circular (e≈0) to nearly unbound, parabolic orbits (e≈1).

<u>Imaging</u> detections are correlated with <u>large semi-major axes</u> (a>10au) and <u>low eccentricies</u> (e<0.50) as expected since planets are less obscured further from their host star, making them easier to see

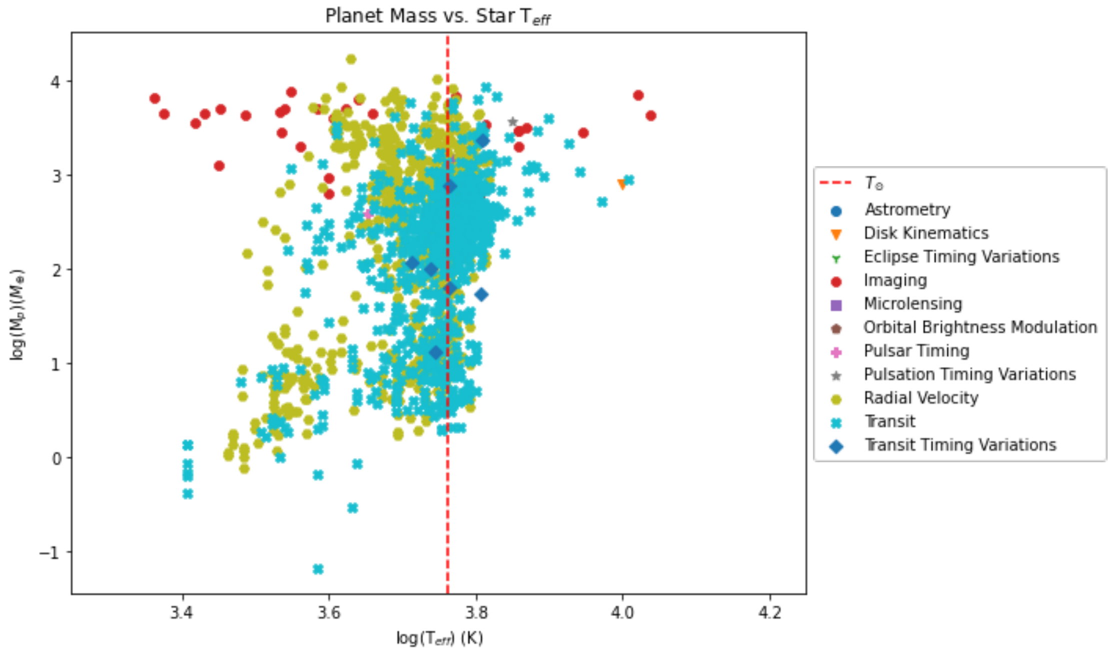
1c) Mass vs. Effective Temperature

In [161]:

```
# Function to plot mass vs. effective temperature (save is parameter to save f
def MassTeff(save=False):

    # Save data using 'Read' function
    data = Read('table.csv')

    # Define x and y datasets to plot
    xdata = []
    ydata = []

    # Define list of symbols to use for different discovery methods
    markers = ['o','v','1','8','s','p','P','*','H','X','D']

    # Establish figure and axis object for plotting
    plt.figure(figsize=(10,6))
    ax = plt.subplot(111)

    # Loop through discovery methods
    for i in range(0,len(np.unique(data['discoverymethod']))):

        # Identifty locations of data points for each discovery method
        detections = np.where(data['discoverymethod'] == np.unique(data['disco

        # Define x (mass) and y (semi-major axis) datasets to plot
        x = data['st_teff'][detections]
        y = data['pl_bmasse'][detections]

        # Add x and y list for each method to a master list
        xdata.append(x)
        ydata.append(y)

        # Make a scatter plot for each datset with a new symbol for each disco
        ax.scatter(np.log10(xdata[i]),np.log10(ydata[i]),label=np.unique(data[

    # Set axis labels, title, and legend (legend outside of figure)
    ax.set_title(r'Planet Mass vs. Star T$_{eff}$')
    ax.set_xlabel(r'log(T$_{eff})$ (K)')
    ax.set_ylabel(r'log(M$_{p}) (M_{\oplus}$)')

    # Exclude outliers to see trends better
    ax.set_xlim(3.25,4.25)

    # Plot vertical line of Sun's effective temperature
    ax.axvline(np.log10(5780),linestyle='--',color='red',label=r'$T_{\odot}$')
    ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
    plt.tight_layout()

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1c_plot.jpg')

    # Show plot
    plt.show()
```

In [162]:
```
1 MassTeff()
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about 1c

With the red vertical dashed-line indicating the surface temperature of the Sun (5780 K), it appears that a <u>large majority of detections</u> across all methods <u>lie near Sun-like stars</u> (4000K<T<8000K). This result suggests that planets are <u>less likely to form around very hot stars</u>.

Many of the large exoplanet surveys sought out planets around habitable stars which we, by default, associate with stars like our Sun. So it is not surpising that a majority of the detected planets orbit around stars with surface temperatures near the Sun's.

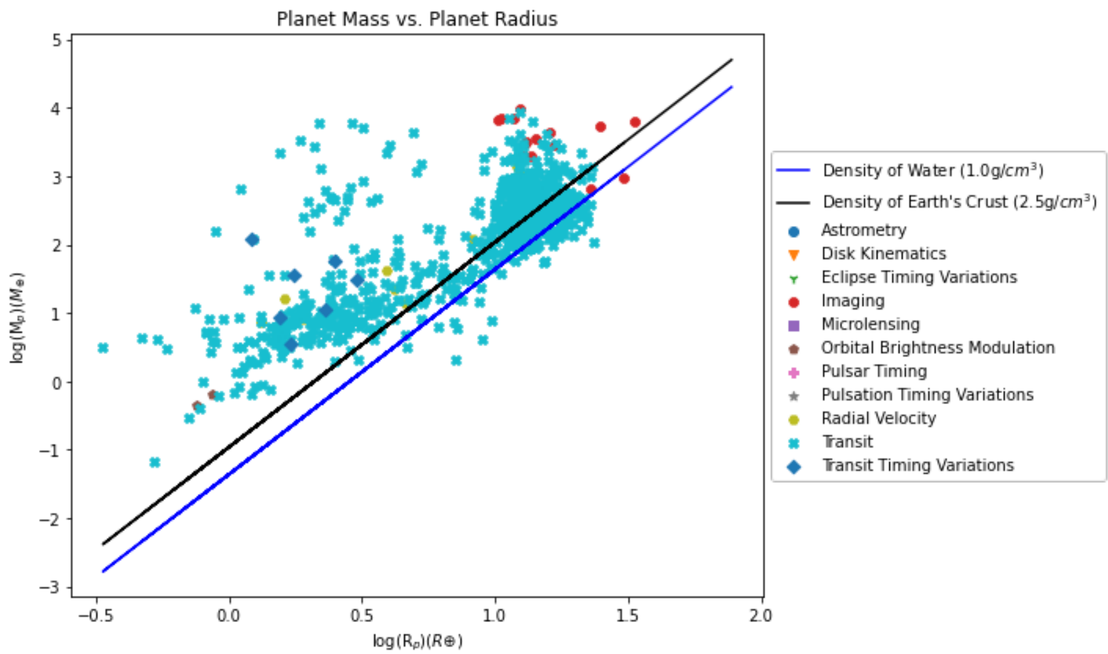
1d) Mass vs. Radius

In [163]:

```
# Function to plot mass vs. radius (save is parameter to save figure or not)
def MassRadius(save=False):

    # Save data using 'Read' function
    data = Read('table.csv')

    # Define x and y datasets to plot
    xdata = []
    ydata = []

    # Define constant densities in CGS units and Earth-based units
    DensWaterCGS = 1.0*(u.g/(u.cm**3))
    DensWaterEarth = DensWaterCGS.to(const.M_earth/(const.R_earth**3))
    DensCrustCGS = 2.5*(u.g/(u.cm**3)) # source: Hyperphysics
    DensCrustEarth = DensCrustCGS.to(const.M_earth/(const.R_earth**3))

    # Define list of symbols to use for different discovery methods
    markers = ['o','v','1','8','s','p','P','*','H','X','D']

    # Establish figure and axis object for plotting
    plt.figure(figsize=(10,6))
    ax = plt.subplot(111)

    # Loop through discovery methods
    for i in range(0,len(np.unique(data['discoverymethod']))):

        # Identifty locations of data points for each discovery method
        detections = np.where(data['discoverymethod'] == np.unique(data['disc

        # Define x (mass) and y (semi-major axis) datasets to plot
        x = data['pl_rade'][detections]
        y = data['pl_bmasse'][detections]

        # Add x and y list for each method to a master list
        xdata.append(x)
        ydata.append(y)

        # Make a scatter plot for each datset with a new symbol for each disc
        ax.scatter(np.log10(xdata[i]),np.log10(ydata[i]),label=np.unique(data

    # Convert xdata from list-of-lists to flattened list
    xflat = [y for x in xdata for y in x]

    # Find corresponding mass of water for different radii and plot line of co
    massWater = [DensWaterEarth.value*(x**3) for x in xflat]
    massCrust = [DensCrustEarth.value*(x**3) for x in xflat]
    ax.plot(np.log10(xflat),np.log10(massWater),color='blue',label=r'Density o
    ax.plot(np.log10(xflat),np.log10(massCrust),color='black',label='Density o

    # Set axis labels, title, and legend (legend outside of figure)
    ax.set_title(r'Planet Mass vs. Planet Radius')
    ax.set_xlabel(r'log(R$_{p}) (R{\oplus}$)')
    ax.set_ylabel(r'log(M$_{p}) (M_{\oplus}$)')
    ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
    plt.tight_layout()

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1d_plot.jpg')

    # Show plot
    plt.show()
```

In [164]:
```
1 MassRadius()
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about 1d

Most of the confirmed exoplanets have densities comparable to that of the Earth's crust, but much greater than that of water. This suggests that many of these detections are terrestrial/rocky planets like the Earth, though most of them are not likely to host liquid water.

1e) Histogram of Metallicity

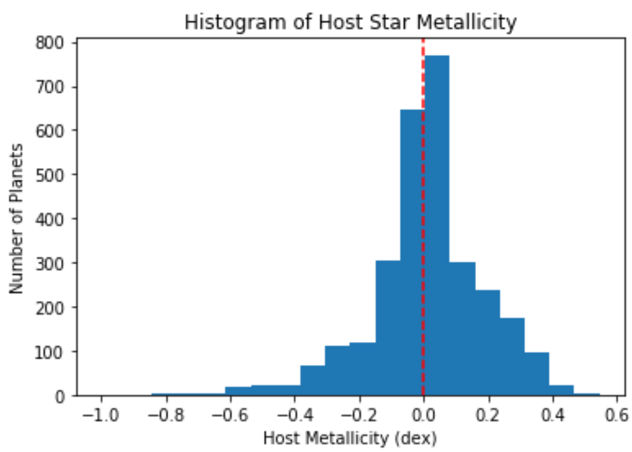
In [116]:

```
# Function to plot mass vs. semi-major axis (save is parameter to save figure
def Metallicity(save=False):
    # Save data using 'Read' function
    data = Read('table.csv')

    # Extract list of metallicities
    metallicity = data['st_met']

    # Plot metallicity of host stars
    plt.title('Histogram of Host Star Metallicity')
    plt.xlabel('Host Metallicity (dex)')
    plt.ylabel('Number of Planets')
    plt.hist(metallicity,bins=20)
    plt.axvline(0,linestyle='--',color='red')

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1e_plot.jpg')

    # Show plot
    plt.show()
```

In [117]:

```
Metallicity(True)
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about 1e

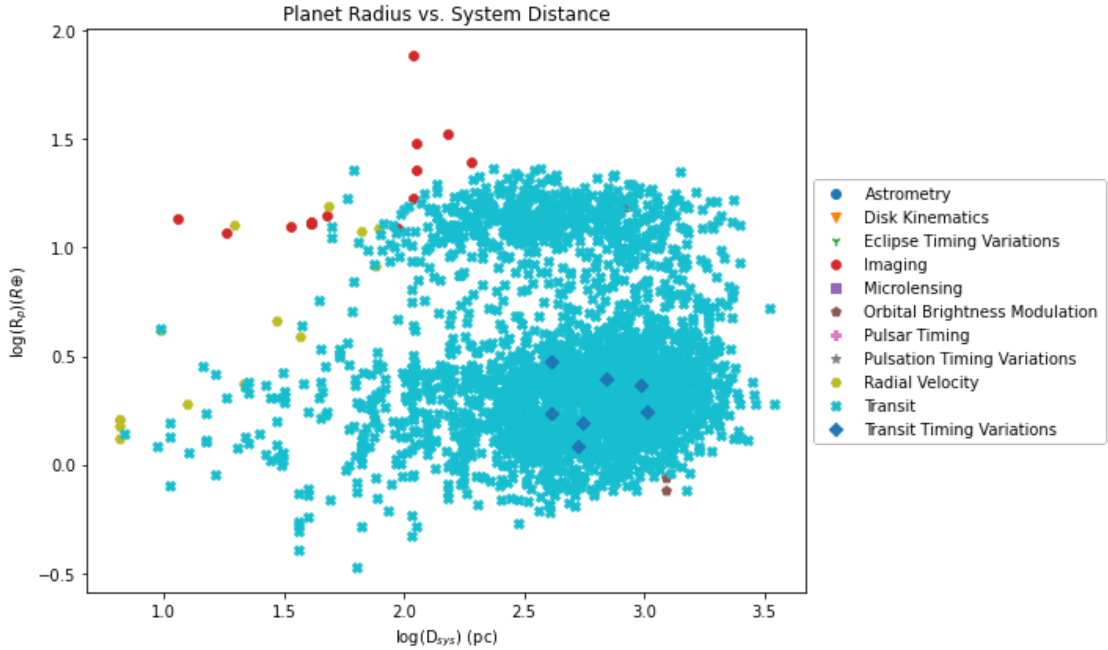
This histogram suggests that planets are more likely to be found around metal-rich stars (dex > 0) and most exoplanet detections were found around stars with metallicites similar to that of the Sun. This data is biased towards planets near stars like the Sun because most detections came from observations of stars in the local galaxy, where the distribution of metals is approximately the same as in our Solar System.

1f) Planet Radius vs. System Distance

In [84]:
```
# Function to plot planet radius vs. system distance (save is parameter to sav
def RadDist(save=False):

    # Save data using 'Read' function
    data = Read('table.csv')

    # Define x and y datasets to plot
    xdata = []
    ydata = []

    # Define list of symbols to use for different discovery methods
    markers = ['o','v','1','8','s','p','P','*','H','X','D']

    # Establish figure and axis object for plotting
    plt.figure(figsize=(10,6))
    ax = plt.subplot(111)

    # Loop through discovery methods
    for i in range(0,len(np.unique(data['discoverymethod']))):

        # Identifty locations of data points for each discovery method
        detections = np.where(data['discoverymethod'] == np.unique(data['disco

        # Define x (mass) and y (semi-major axis) datasets to plot
        x = data['sy_dist'][detections]
        y = data['pl_rade'][detections]

        # Add x and y list for each method to a master list
        xdata.append(x)
        ydata.append(y)

        # Make a scatter plot for each datset with a new symbol for each disco
        ax.scatter(np.log10(xdata[i]),np.log10(ydata[i]),label=np.unique(data[

    # Set axis labels, title, and legend (legend outside of figure)
    ax.set_title(r'Planet Radius vs. System Distance')
    ax.set_xlabel(r'log(D$_{sys}$) (pc)')
    ax.set_ylabel(r'log(R$_{p}) (R{\oplus}$)')
    ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
    plt.tight_layout()

    # Decide whether or not to save file
    if save == True:
        plt.savefig('/d/users/jimmy/Documents/Planets/HW1f_plot.jpg')

    # Show plot
    plt.show()
```

In [85]:
```
1 RadDist()
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conver
sionWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about 1f

I chose these parameters to investigate the sizes of planets that the different surveys and detection methods are sensitive to. I find it remarkable that a significant majority of these detections have radii between 1 and 10 $R_\oplus$ and were observed well beyond 100 pc. To me, this exemplifies the power of the transit method for detecting not only massive planets, but very distant and small planets.

Also, as expected, the imaging detections have a remarkable minimum size of 10 $R_\oplus$ and are detected (relatively) close to Earth. The lack of detections conveys the difficulty of seeing a planet directly amidst the glare of its host.

In [ ]:
```
1
```

2) Planet Period vs. Planet Mass

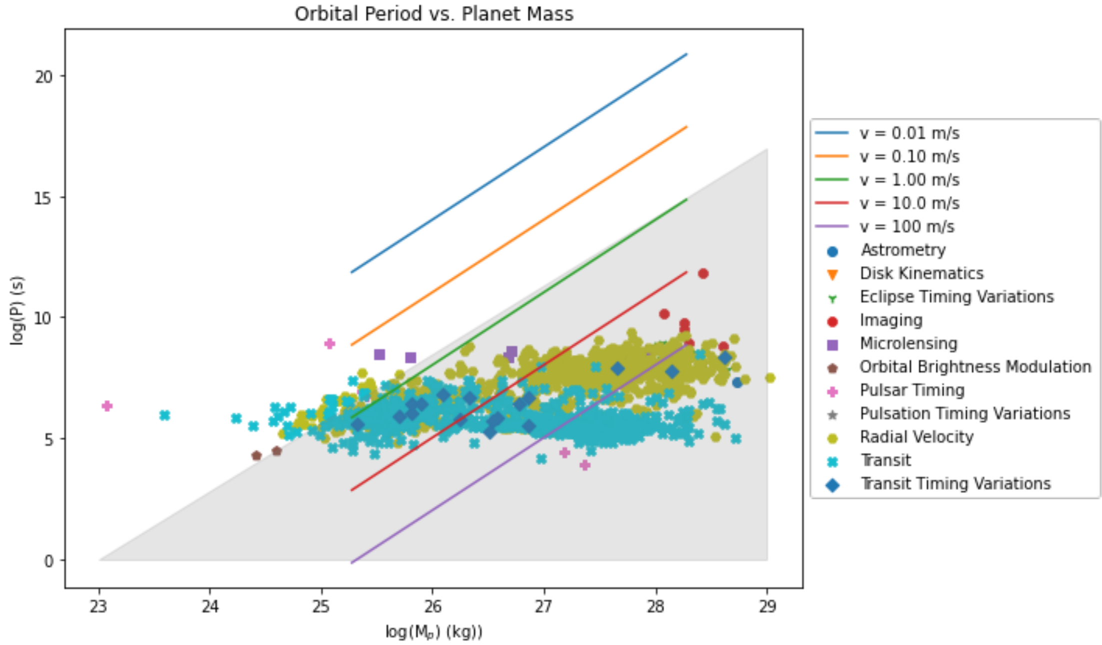
In [652]:
```
# Import relevant modules/packages
import numpy as np
import matplotlib.pyplot as plt
from astropy import units as u
from astropy import constants as const
from matplotlib.patches import Rectangle, Polygon
```

In [322]:
```
# Function to read text files
def Read(filename,skip=97):
    # skip: how many lines before column headers
    # returns: dictionary of data in text file (can access data['column_name'])

    # Read and return data of confirmed exoplanets from NASA Exoplanet Archive
    data = np.genfromtxt(filename,dtype=None,delimiter=',',skip_header=skip,name
s=True,invalid_raise=False,encoding=None)
    return data
```

In [912]:
```
# Define important constants
MJupiter = const.M_jup.value
MSun = const.M_sun.value
G = const.G.value

# Define set of velocities
velocities = [0.01,.1,1,10,100] # in m/s

#Make a list of masses (.1,1,10 Mjup)
mass = [x*MJupiter for x in velocities]
masses = []
```

In [913]:
```
# Define empty list of periods
periods = []

# Function to calculate period from planet mass and orbital velocity
def Period(m,v):
    P = (2*np.pi*G/(v**3))*((m**3)/((MSun+m)**2))
    return P

# Loop to calculate period for each mass
for v in velocities:
    for m in mass:
        masses.append(m)
        periods.append(Period(m,v))

# Create figure and axis object
plt.figure(figsize=(10,6))
ax = plt.subplot(111)

# Plot periods vs. masses of artifical data
ax.plot(np.log10(masses[0:4]),np.log10(periods[0:4]),label='v = 0.01 m/s')
ax.plot(np.log10(masses[5:9]),np.log10(periods[5:9]),label='v = 0.10 m/s')
ax.plot(np.log10(masses[10:14]),np.log10(periods[10:14]),label='v = 1.00 m/s')
ax.plot(np.log10(masses[15:19]),np.log10(periods[15:19]),label='v = 10.0 m/s')
ax.plot(np.log10(masses[20:24]),np.log10(periods[20:24]),label='v = 100 m/s')

# Extract data from NASA exoplanet archive
data = Read('table.csv')

# Define x and y datasets from archive to plot
xdata = []
ydata = []

# Define list of symbols to use for different discovery methods
markers = ['o','v','1','8','s','p','P','*','H','X','D']

# Loop through discovery methods
for i in range(0,len(np.unique(data['discoverymethod']))):

    # Identifty locations of data points for each discovery method
    detections = np.where(data['discoverymethod'] == np.unique(data['discoverymet
hod'])[i])

    # Define x (mass) and y (semi-major axis) datasets to plot
    x = [x*u.M_earth.to(u.kg) for x in data['pl_bmasse'][detections]]
    y = [y*u.d.to(u.s) for y in data['pl_orbper'][detections]]

    # Add x and y list for each method to a master list
    xdata.append(x)
    ydata.append(y)

    # Make a scatter plot for each datset with a new symbol for each discovery me
thod
    ax.scatter(np.log10(xdata[i]),np.log10(ydata[i]),label=np.unique(data['discov
erymethod'])[i],marker=markers[i])

# Flatten x and y lists for shading
xflat = [y for x in xdata for y in x]
yflat = [y for x in ydata for y in x]

# Set axis labels, title, and legend (legend outside of figure)
ax.set_title('Orbital Period vs. Planet Mass')
ax.set_xlabel(r'log(M$_{p}$ (kg))')
ax.set_ylabel('log(P) (s)')
ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
```

```
/usr/local/Anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:5: Conversi
onWarning: Some errors were detected !
    Line #760 (got 11 columns instead of 91)
    Line #4226 (got 11 columns instead of 91)
  """
```



Comments about Problem 2

Modern spectographs can only observe reflex velocities above 1 m/s, so they are currently not sensitive enough to detect any planets with a less signficant impact on their host star than that. With improved sensitivity, less massive planets can be detected with this method since the threshold for detecting their effect on the reflex veloctity of the host star will be lower.

4) Radiative Equilibrium Temperature vs. Semi-Major Axis

$$T_{eq} = \left(\frac{1-A}{\pi}\right)^{1/4}\sqrt{\frac{R_*}{2a}}T_{eff}$$

G0V: T=5920K, R=1.12$R_\odot$

M5V: 3030K, R=.199$R_\odot$

F0V: 7220K, R=1.79$R_\odot$

A0V: 9700K, R=2.09$R_\odot$

G2V: 5770K, R=1.01$R_\odot$

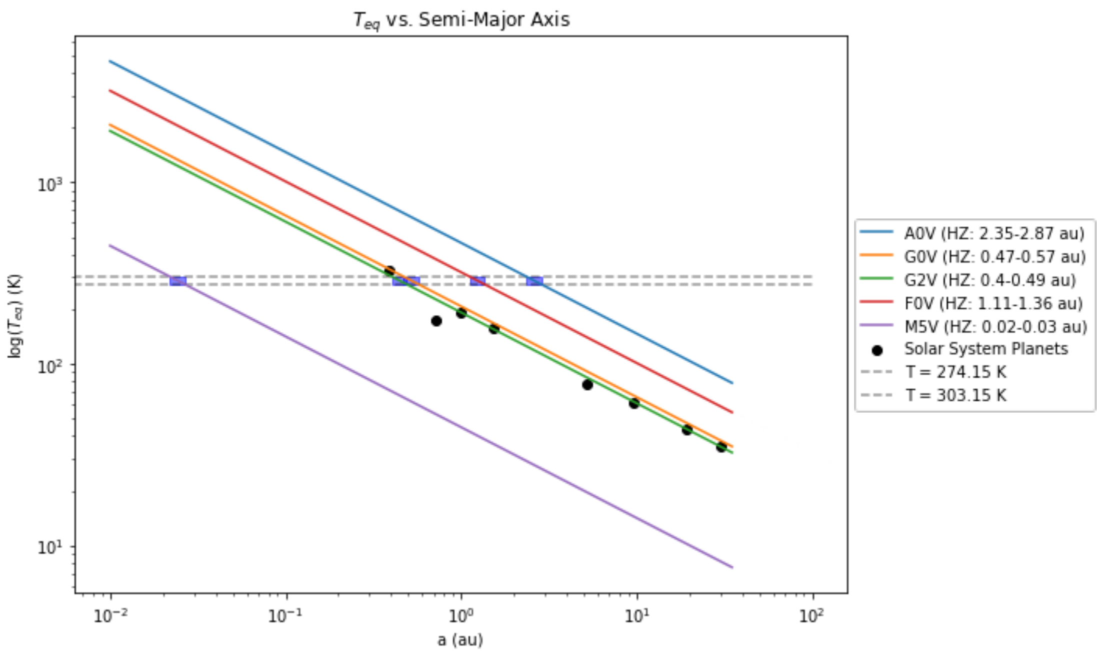
In [76]:

```
# Define value of Solar radius
RSun = const.R_sun.to(u.au).value

# Define min and max liquid water temperatures in K
# Source (slide 3):https://www.astro.umd.edu/~miller/teaching/astr380f09/slides1
4.pdf
TempWaterMin = 274.15
TempWaterMax = 303.15
```

In [87]:
```
# Function to calculate radiative equilibrium temperature for a given host star a
nd semi-major axis
def TeqSMA(startype,SMA):
    # startype: A0V,G0V,G2V,M5V,or F0V
    # albedo: 0-1 (default=0.3)
    # returns: habitable zone range and Teq of planet

    # Define properties of host star
    if startype == 'A0V':
        R = 2.09*RSun # Radius in Rsun
        Teff = 9700 # Effective temperature of host star in K
        albedo = [0.3]*len(SMA)
    elif startype == 'G0V':
        R = 1.12*RSun
        Teff = 5920
        albedo = [0.3]*len(SMA)
    elif startype == 'G2V':
        R = 1.01*RSun
        Teff = 5770
        albedo = [0.3]*len(SMA)
    elif startype == 'M5V':
        R = .199*RSun
        Teff = 3030
        albedo = [0.3]*len(SMA)
    elif startype == 'F0V':
        R = 1.79*RSun
        Teff = 7220
        albedo = [0.3]*len(SMA)
    elif startype == 'Sun':
        R = 1.0*RSun
        Teff = 5780
        # Source wikipedia (references for each value; https://en.wikipedia.org/w
iki/Albedo#Astronomical_albedo)
        albedo = [0.09,0.76,0.31,0.25,0.50,0.34,0.30,0.29]
    else:
        print('Enter valid startype')

    # Calculate minimum and maximum habitable zone distances
    HZMax = R/2*np.sqrt((1-0.3)/np.pi)*((Teff/TempWaterMin)**2)
    HZMin = R/2*np.sqrt((1-0.3)/np.pi)*((Teff/TempWaterMax)**2)

    # Make empty list of Teq
    Teq = []

    # Calculate Teq for each a and add it to list
    for i in range(0,len(SMA)):
        prefactor = ((1-albedo[i])/np.pi)**(1/4)
        Teq.append(prefactor*np.sqrt(R/(2*SMA[i]))*Teff)

    return(HZMax,HZMin,Teq)
```

In [106]:
```
# List names of planets for plotting later
SolSys = ['Mercury','Venus','Earth','Mars','Jupiter','Saturn','Uranus','Neptune']

# Establish semi-major axes of solar system planets (in au)
# Source: https://www.princeton.edu/~willman/planetary_systems/Sol/
aSolSys = [.387,.723,1.00,1.52,5.20,9.54,19.2,30.1]
logaSolSys = [np.log(x) for x in aSolSys]

# Calculate Teq for solar system planets
a,b,TeqSolSys = TeqSMA('Sun',aSolSys)
logTeqSolSys = [np.log10(x) for x in TeqSolSys]
```

In [909]:
```
# Establish figure and axis object for plotting
plt.figure(figsize=(10,6))
ax = plt.subplot(111)

# Make list of semimajor axes|
a = np.linspace(0.01,35,50) # in au

# Make a plot with a line for each startype
startypes=['A0V','G0V','G2V','F0V','M5V']
for i in range(len(startypes)):
    maxD,minD,y = TeqSMA(startypes[i],a)

    # Plot Teq vs. a with habitable zones shaded
    string = ' (HZ: {0}-{1} au)'.format(np.round(minD,2),np.round(maxD,2))
    ax.add_patch(Rectangle((minD,TempWaterMin),maxD-minD,TempWaterMax-TempWaterMi
n,fill=True,color='b',alpha=0.5))
    ax.loglog(a,y,label=startypes[i]+string)

# Plot solar system planets
#for planet in SolSys:
ax.scatter(aSolSys,TeqSolSys,label='Solar System Planets',color='black')

# Illustrate min and max liquid water temps.
plt.hlines(TempWaterMin,-2,100,label='T = {0} K'.format(TempWaterMin),linestyl
e='--',alpha=0.4)
plt.hlines(TempWaterMax,-2,100,label='T = {0} K'.format(TempWaterMax),linestyl
e='--',alpha=0.4)

# Set plot parameters
ax.set_xlabel('a (au)')
ax.set_ylabel(r'log($T_{eq}$) (K)')
ax.set_title(r'$T_{eq}$ vs. Semi-Major Axis')
ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
plt.tight_layout()
```



The planets in our solar system fall very close to the G2V line which verifies that the Sun is a G2V-type star.

5) Angular Shift of Stellar Host vs. Planet Mass (Astrometric Motion)

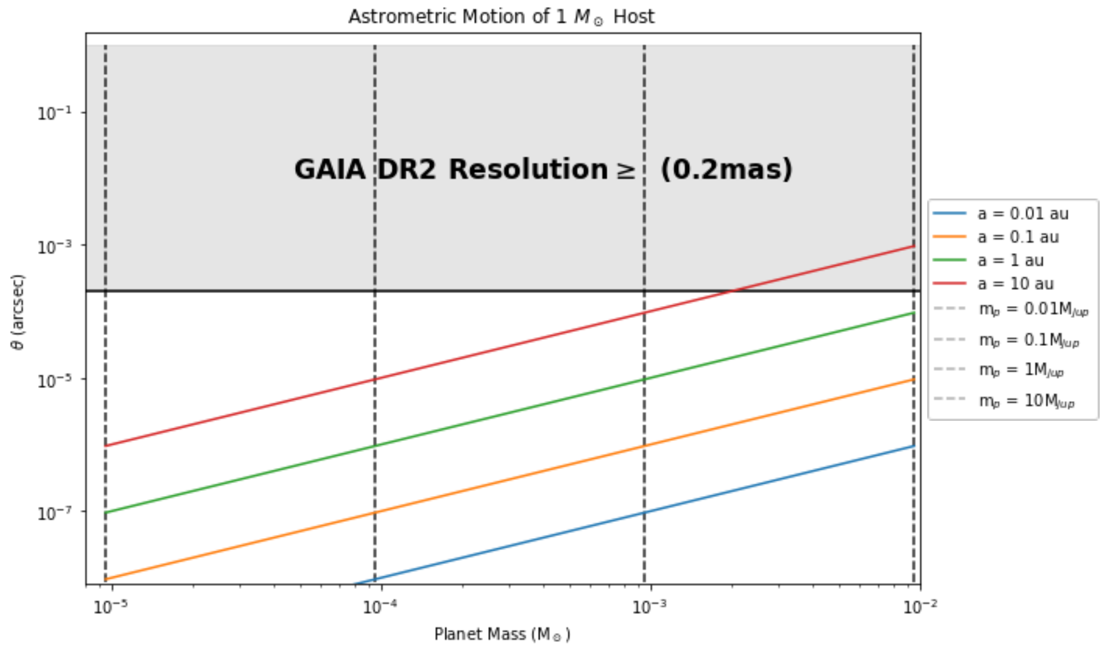
i) For 2-body orbit w/ origin at CoM: $\frac{m_1}{m_2} = \frac{a_2}{a_1}$ --> $\frac{m_*}{m_p} = \frac{a_p}{a_*}$

ii) Angular shift (θ) corresponds to parallax: $\theta = \frac{a_*}{D}$ where D is the distance to the system

iii) We are to choose 4 different a_p values and loop over different masses to calculate values for a_* and thus θ

In [726]:
```
# Define set of planet semi-major axes
a_planet = [0.01,.1,1,10] # in au

# Define distance to system
D = 100*u.pc.to(u.au)
print(D)

#Make a list of masses (.01,.1,1,10 MJup in terms of MSun)
m_planet = [x*const.M_jup.to(u.Msun).value for x in a_planet]

# Empty list-of-lists of masses for plotting purposes
masses = [[],[],[],[]]

# Make empty list-of-lists of angular shifts
theta = [[],[],[],[]]

# Create figure and axis object
plt.figure(figsize=(10,6))
ax = plt.subplot(111)

# Iterate over all planet mass and SMA combinations to calculate theta for each
for i in range(len(a_planet)):
    for m_p in m_planet:

        # Calculate star SMA from 2-body orbit ratio (m1/m2=a2/a1)
        a_star = a_planet[i]*m_p
        shift = (a_star/D)*206265 # convert from radians to arcsec

        # Add mass and angular shift (in mas) to lists
        masses[i].append(m_p)
        theta[i].append(shift)

    # Plot theta vs. mass
    ax.loglog(masses[i],theta[i],label='a = {0} au'.format(a_planet[i]))

    # Plot lines for important masses
    ax.vlines(masses[i],0,1,label=r'm$_{p}$ = '+'{0}'.format(a_planet[i])+r'M$_{j
up}$',linestyle='--',alpha=0.3)

# Include other plot features
title = r'Astrometric Motion of 1 $M_\odot$ Host'
ax.set_title(title)
ax.set_xlabel(r'Planet Mass (M$_\odot)$')
ax.set_ylabel(r'$\theta$ (arcsec)')
ax.legend(loc='center left',bbox_to_anchor=(1, 0.5))
ax.set_xlim(8*10**-6,10**-2)
ax.set_ylim(8*10**-9,1.5)

# Show GAIA resolution (Source: https://www.cosmos.esa.int/web/gaia/dr2)
ax.add_patch(Rectangle((0,0.2/1000),.01,1,color='gray',alpha=0.2))
GAIA_label = r"  GAIA DR2 Resolution$\geq$  (0.2mas)"
ax.text(4*10**-5,10/1000,GAIA_label,fontsize='xx-large',fontweight='bold')
ax.hlines(0.2/1000,8*10**-6,10**-2)
plt.tight_layout()
```

```
20626480.624548033
```



Assuming i=0deg for this problem is not necessary because you could still observe the angular shift of the stellar host at any inclination. The star would still move the same amount, it would just appear to take longer or shorter to shift that amountdepending on the system's inclination

6) Minimum Planet Mass (solving mass function)

i) Mass Function: $\frac{m_2^3}{(m_1+m_2)^2} sin^3 i = \frac{P}{2\pi G} v_{1r}^3$

m_2**: planet mass,** m_1**: host star mass,** i**: inclination,** P**: orbital period,** v_{1r}**: host star reflex velocity**

ii) Known values: P**=3.0 days,** $m_1 = 1M_\odot$**,** $v_{1r} = 50m/s$ **(amplitude is 100 m/s)**

iii) Minimum mass corresponds to i=90; right side is a constant

iv) Loop through m_2**'s to plug into left side to match right**

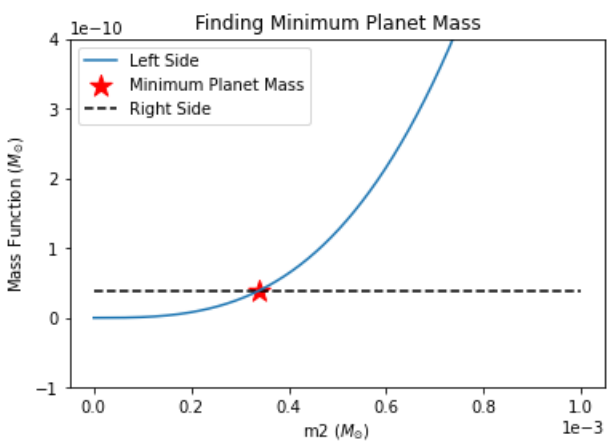
In [458]:
```
# Define constants on right side of mass function
m1 = (1.0*u.M_sun).to(u.kg) # host star mass in kg
v1 = 50*u.m/u.s # host star reflex velocity in m/s
P = (3.0*u.d).to(u.s) # orbital period of planet in s
G = const.G # gravitational constant in mks

# Calculate right side of mass function
right = P/(2*np.pi*G)*(v1**3)
print(right)
print(right.to(u.Msun))
```

```
7.726083868236984e+19 kg
3.8855589997270856e-11 solMass
```

In [300]:
```
# Visualize mass function and value of right side
rightMsun = right.to(u.M_sun).value

# Choose minimum and maximum m2
m2min = 0.0
m2max = 0.001
numSteps = 5000
m2_test = np.linspace(m2min,m2max,numSteps) # in Msun
increment = m2max/numSteps

# Create array to save values of left side for each m2
lefts = []

# Loop over m2's
for m2 in m2_test:
    # Calculate left side for given m2 and add it to list
    left = (m2**3)/((m1.to(u.M_sun).value+m2)**2)
    lefts.append(left)

    # Calculate and print) ratio of left and right side
    ratio = left/right_line

    # Find m2 that makes ratio=1
    if ratio >= .999 and ratio <= 1.0001:
        print('Success! Minimum planet mass is %.2e Msun'%m2)
        plt.scatter(m2,rightMsun,s=200,color='r',marker='*',label='Minimum Planet
Mass')

# Plot value of left side of mass function vs. m2
plt.plot(m2_test,lefts,label='Left Side')

# Plot value of right side
plt.hlines(right_Msun,m2min,m2max,label='Right Side',linestyle='--')

# Set plot features
plt.ylim(-1e-10,4e-10)
plt.ticklabel_format(axis='x',style='sci',scilimits=(0,0))
plt.xlabel(r'm2 ($M_{\odot}$)')
plt.ylabel(r'Mass Function ($M_{\odot}$)')
plt.title('Finding Minimum Planet Mass')
plt.legend()
```

```
Success! Minimum planet mass is 3.39e-04 Msun
```

Out[300]:
```
<matplotlib.legend.Legend at 0x7f00bc2caa90>
```



Solving Mass Function Analytically

(https://math.vanderbilt.edu/schectex/courses/cubic/ (https://math.vanderbilt.edu/schectex/courses/cubic/))

i) General cubic form: $ax^3 + bx^2 + cx + d = 0$

ii) Solutions are:

$$x = (q + (q^2 + (r - p^2)^3)^{1/2})^{1/3} + (q - (q^2 + (r - p^2)^3)^{1/2})^{1/3} + p$$

where $p = \frac{-b}{3a}$; $q = p^3 + \frac{bc-3ad}{6a^2}$; $r = \frac{c}{3a}$

iii) Matching form with mass function: $m_2^3 - \alpha m_2^2 - 2\alpha m_1 m_2 - \alpha m_1^2 = 0$

in this case: $\alpha = \frac{P}{2\pi G} v_{1r}^3 \Big| x = m_2, a = 1, b = -\alpha, c = -2\alpha m_1,$ $d = -\alpha m_1^2$

In [472]:
```
# Define constants from cubic equation
a = 1.0
b = -rightMsun
c = -2.0*rightMsun
d = -rightMsun

# Function to solve cubic equation
def CubicSolver(a,b,c,d):
    # Define substitutions for solving cubic equation
    p = -b/(3.0*a)
    q = (p**3.0)+((b*c)-(3.0*a*d))/(6.0*(a**2))
    r = c/(3.0*a)

    # Calculate solution
    firstTerm = (q+np.sqrt((q**2)+((r-(p**2))**3)))**(1/3)
    secondTerm = (q-np.sqrt((q**2)+((r-(p**2))**3)))**(1/3)
    analytic_solution = firstTerm+secondTerm+p
    print('The analytical solution is also %.2e Msun!'%analytic_solution)
CubicSolver(a,b,c,d)
```

```
The analytical solution is also 3.39e-04 Msun!
```

7) Plotting Binary Orbits

i) Need to solve Kepler equation numerically:
$E - esin(E) = \frac{2\pi}{P}(t - t_0)$

ii) Use Kepler III to find period:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1+m_2)} = \frac{4\pi^2 a^3}{M}$$

iii) Say $t_0 = 0$ and use Newton-Raphson method to converge on E:

$$f(E) = E - esin(E) - \frac{2\pi}{P}t$$

$$f'(E) = 1 - ecos(E)$$

In [896]:
```
# Function to numerically solve differentiable equation
# Resource that helped me: https://www.math.ubc.ca/~pwalls/math-python/roots-optimization/newton/
def NewtonRaphson(f,df,x0,precision,numSteps):
    # Inputs:
    #    f: function to evaluate
    #    df: derivative of function
    #    x0: initial guess at solution
    #    precision: answer won't exactly be 0, so set a tolerance
    #    numSteps: maximum number of times to iterate

    # Establish first guess at solution
    x = x0

    # Iterate over number of steps
    for i in range(0,numSteps):

        # Evaluate function
        func = f(x)

        # If f(x) is within precision, declare that value of x as the solution
        if abs(func) <= precision:
            #print('A solution of {0:.2e} was found in {1} iterations'.format(x, i))
            return x

        # If f(x) is not within precision, continue searching for solution
        elif abs(func) > precision:

            # Evaluate derivative
            deriv = df(x)

            # Adjust guess of solution by subtracting quotient of function and derivative from the last x
            x -= func/deriv
```

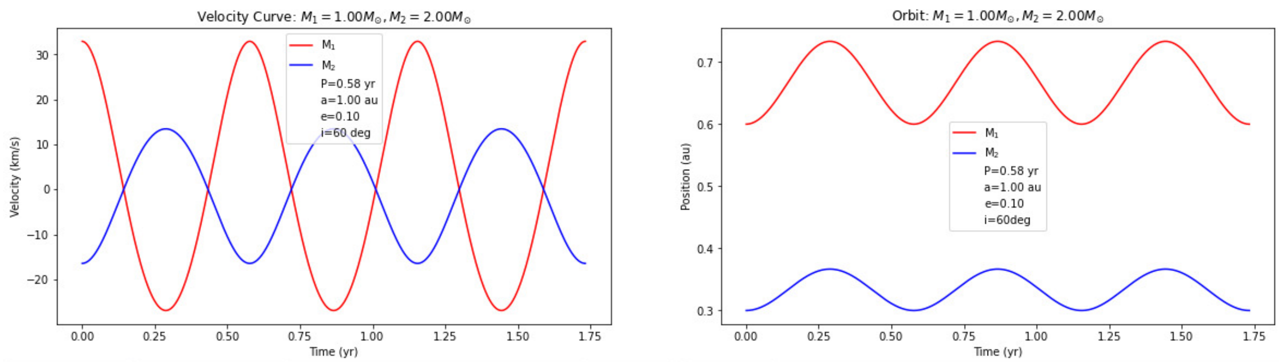
In [895]:
```
# Function and derivative to test NewtonRaphson function
f = lambda x: x**2 - 2
df = lambda x: 2*x
NewtonRaphson(f,df,.1,1e-6,100)
```

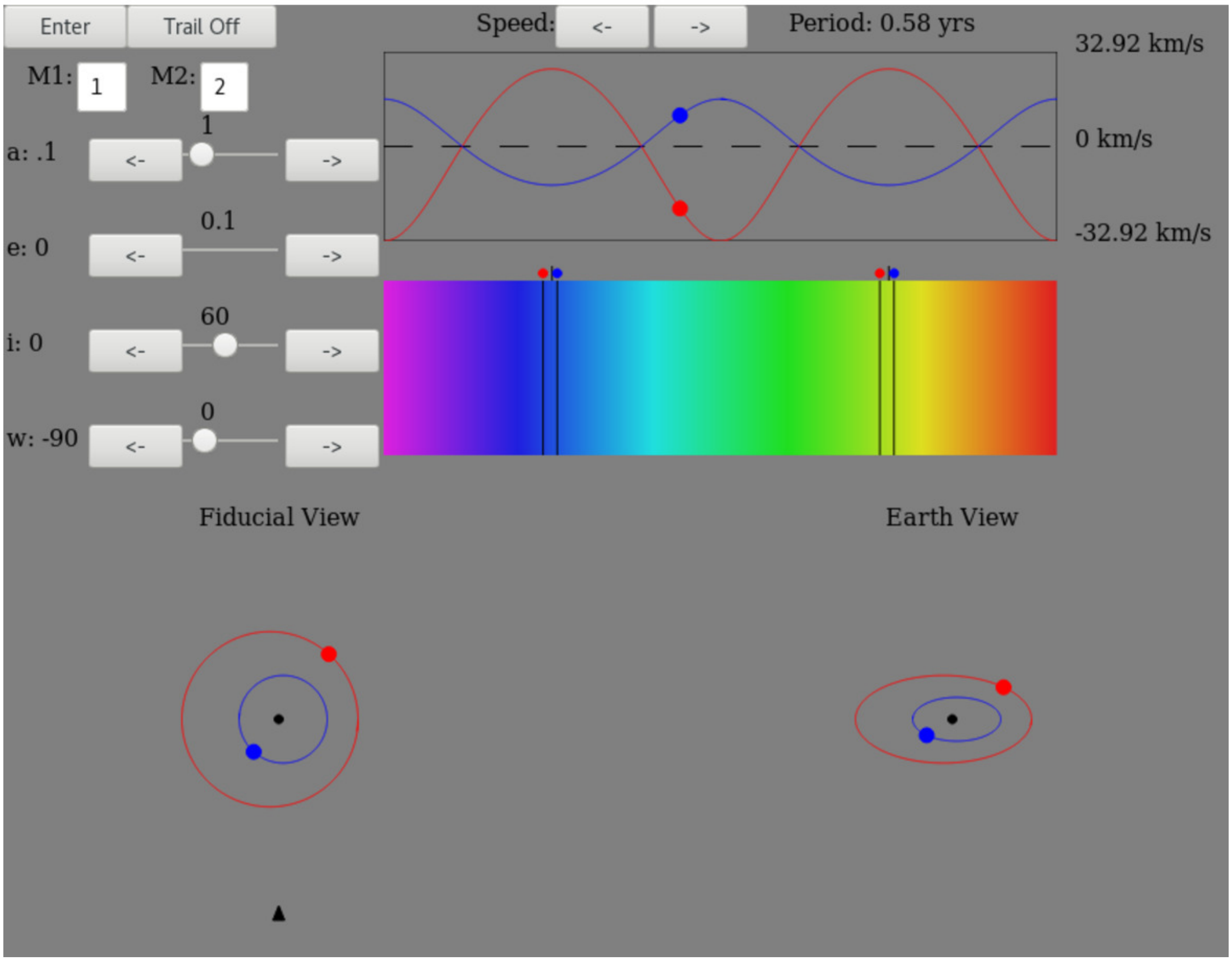
```
A solution of 1.41e+00 was found in 7 iterations
```

Out[895]:
```
1.4142136001158032
```

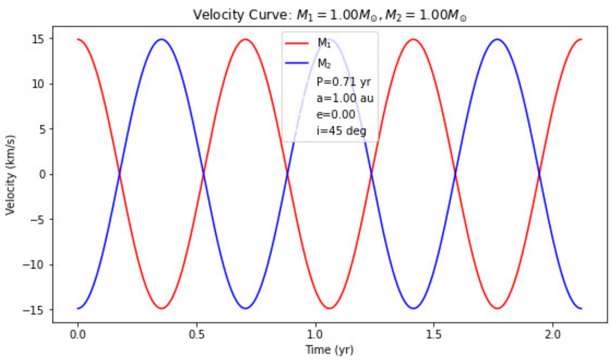
In [884]:
```
# Function to calculate properties of binary orbit
def Orbit(m1,m2,a,eccen,i):
    # Inputs:
    #     m1 = mass of 1st body in system (in Msun)
    #     m2 = mass of 1nd body in system (in Msun)
    #     a = system semi-major axis (in au; a=a1+a2)
    #     e = eccentricity of orbits (0<e<1)
    #     i = inclination of binary system (0 to 90 deg)
    #     precision = precision of solution found by Newton-Raphson method
    # Returns:
    #     velocity curve and astrometric orbit data (times,velocities,positions)

    # Convert G, m, a, and i to appropriate units; then calculate M
    G = const.G.to(u.au**3/(u.Msun*u.yr**2))
    mSolar = (m1+m2)*u.M_sun
    a = a*u.au
    i *= np.pi/180 # convert inclination to radians for numpy
    M = G*mSolar # in au^3/yr^2

    # Find period (in yr)
    P = np.sqrt(4*(np.pi**2)*(a**3)/M).value

    # Find individual semi-major axes (use m1/m2=a2/a1)
    a1 = a.value/(1+m1/m2)
    a2 = a.value-a1

    # Define set of times to evaluate E over (~0 to 3*Period)
    times = np.linspace(0.001,3*P,1000)

    # Define empty list of eccentric and true anomalies to fill
    EA_list = []
    TA_list = []

    # Define empty list of positions and velocities for each body
    r1_list = []
    r2_list = []
    v1_list = []
    v2_list = []

    # Calculate first term of E and theta relationship
    term1 = ((1+eccen**2)/(1-eccen**2))**(1/2)

    # Find E at each time
    for time in times:

        # Define Kepler equation and its derivative
        f = lambda E: E - eccen*np.sin(E) - (2*np.pi/P)*time
        df = lambda E: 1 - eccen*np.cos(E)

        # Use NewtonRaphson function to find E at each time
        EA = NewtonRaphson(f,df,1e-2,1e-6,100)
        EA_list.append(EA)

        # Convert eccentric anomaly (eA) to true anomaly (theta)
        TA = 2*np.arctan(term1*np.tan(EA/2))
        TA_list.append(TA)

        # Calculate velocities at each time
        v1 = (2*np.pi*a1*np.sin(i))/(P*np.sqrt(1-eccen**2))*(np.cos(TA)+ecce
n)*(u.au/u.yr).to(u.km/u.s)
        v2 = (2*np.pi*a2*np.sin(i))/(P*np.sqrt(1-eccen**2))*(np.cos(TA)+ecce
n)*(u.au/u.yr).to(u.km/u.s)
        v1_list.append(v1)
        v2_list.append(-v2)
```

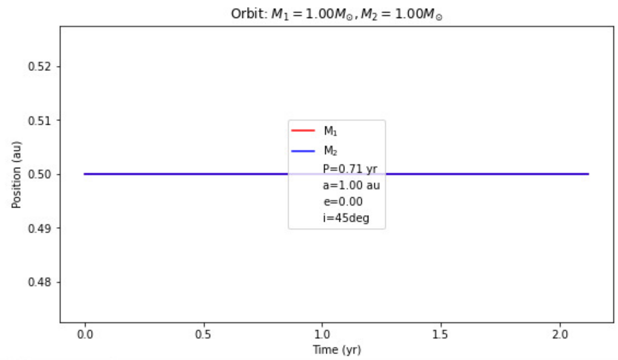
In [ ]:
```
Orbit(1,2,1,0.1,60)
```

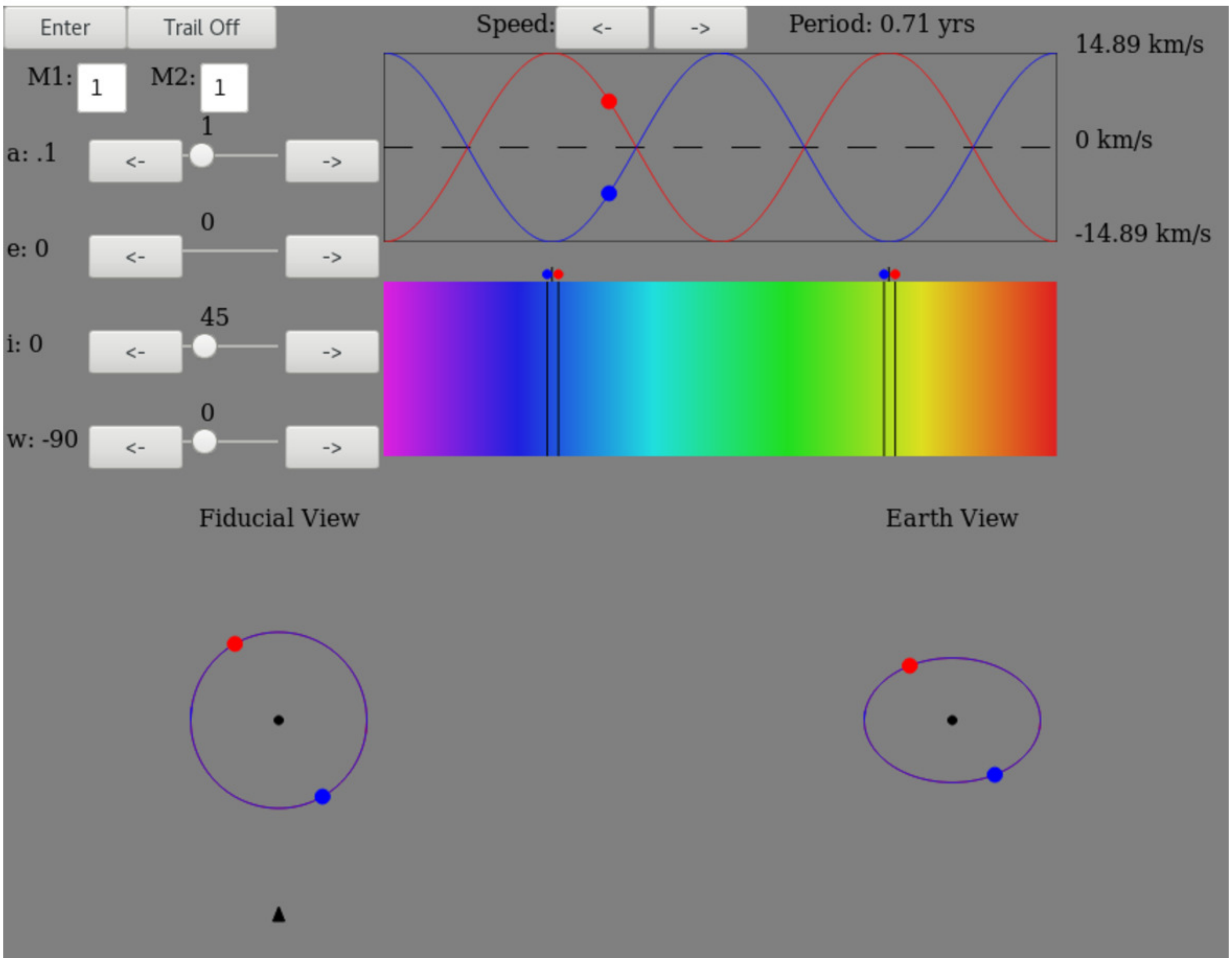




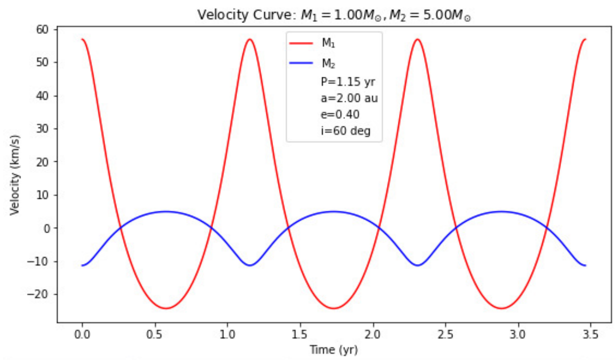
In [ ]:
```
Orbit(1,1,1,0.0,45)
```

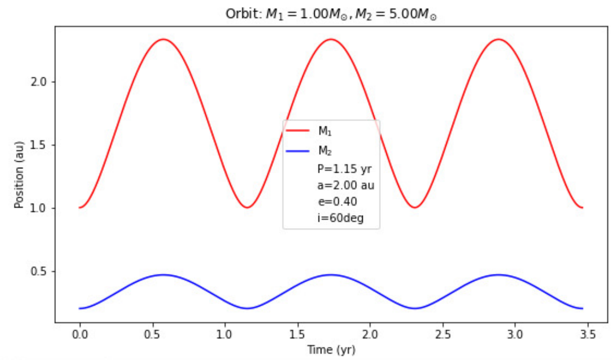


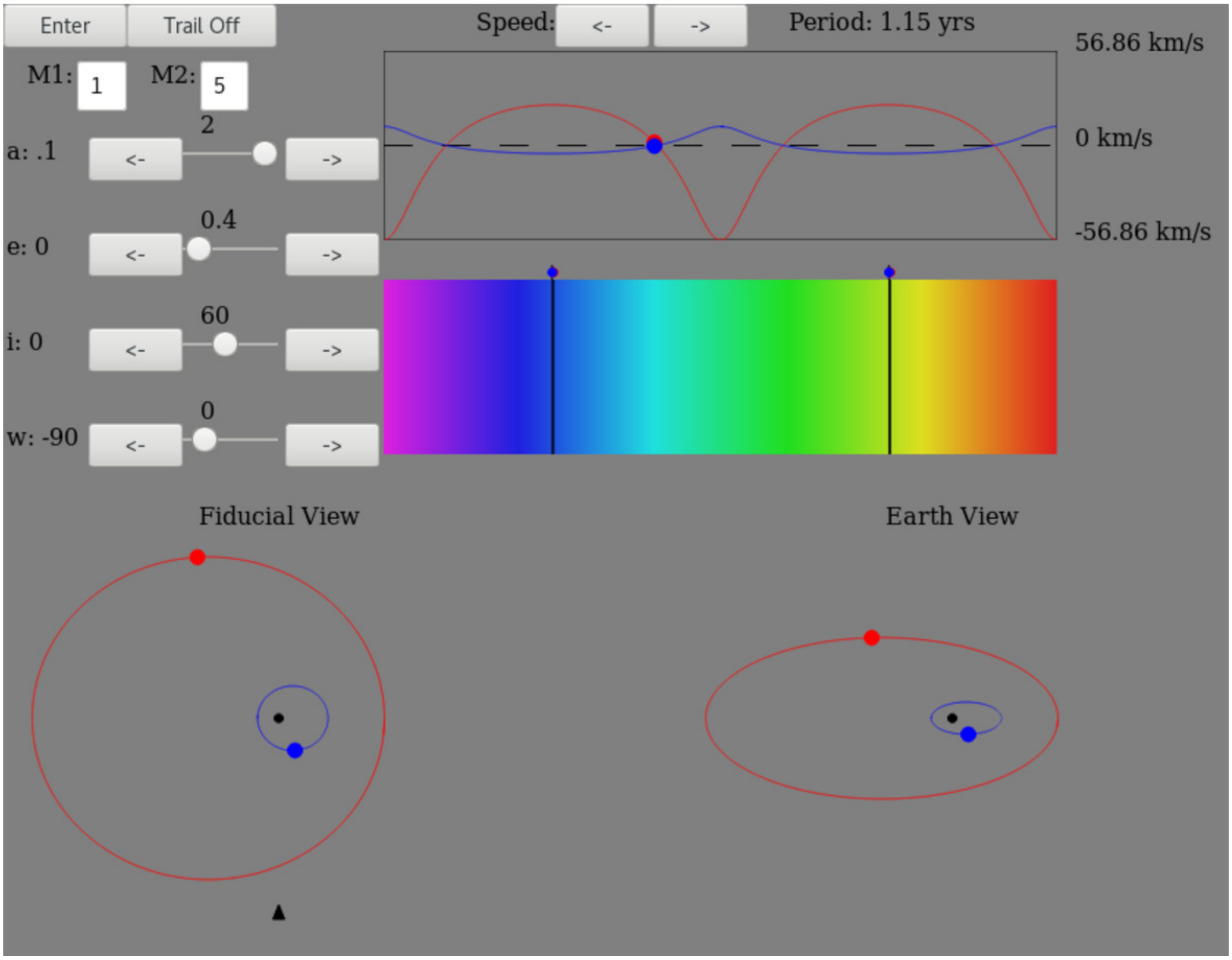





```
In [ ]: Orbit(1,5,2,0.4,60)
```


Velocity Curve: $M_1 = 1.00M_\odot, M_2 = 5.00M_\odot$
M_1
M_2
P=1.15 yr
a=2.00 au
e=0.40
i=60 deg
Velocity (km/s)
Time (yr)


Orbit: $M_1 = 1.00M_\odot, M_2 = 5.00M_\odot$
M_1
M_2
P=1.15 yr
a=2.00 au
e=0.40
i=60deg
Position (au)
Time (yr)


Enter
Trail Off
Speed: <- ->
Period: 1.15 yrs
56.86 km/s
0 km/s
-56.86 km/s
M1: 1
M2: 5
a: .1
2
e: 0
0.4
i: 0
60
w: -90
0
Fiducial View
Earth View

In [ ]: