ASTR 5490 Homework #3
Due Oct. 14 (23rd class day)

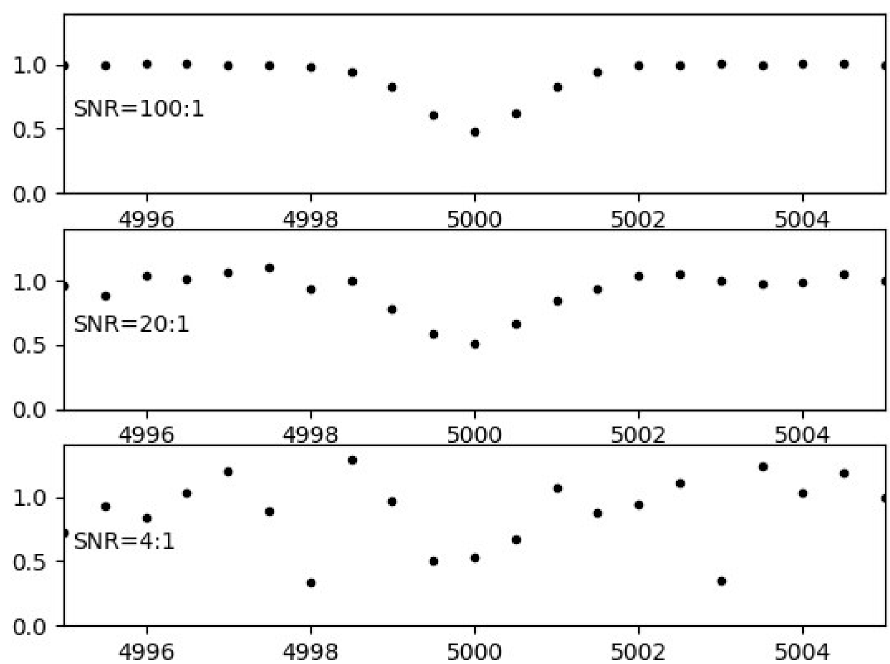
Learning goals: Experiment with Gaussian fitting and errors on parameters, explore the meaning of FALSE Alarm Probabilities using Monte Carlo simulations, generate light curves and velocity profiles for transiting planets.

1. Use the table of measured velocities for 61 Vir b from [Vogt et al. (2009)]() on the [course web page]().
   A) Make a periodigram of the whole dataset. Compare to the upper panel of their Figure 3.
   B) Considering only the 38 d planet in this system, compute the false alarm probability (FAP). Do this by using Monte Carlo methods. As described by Cumming ([2004, MNRAS 354, 1165]()) page 3, first column, "The fraction of trials for which the periodigram power exceeds the observed value gives the FAP." Use the given uncertainties on each velocity and add noise of this magnitude drawn from a Guassian distribution to add noise to each velocity measurement. Then compute the maximum power in the periodigram near 38 d. Do this 10,000 times, or as many times as needed, to determine the FAP for this planet. Compare your FAP to those given by horizontal lines in the Vogt et al. Figure 3. The FAP for this planet is quite small, according to Vogt et al.
   C) Fold you data at this period to see the phased velocity curve and discuss what you can infer from this.
   D) Consider if Vogt et al. had taken data for only any given 100 d interval during their survey (you can pick your favorite 100-day interval). Pick a 100 d interval and compute the FAP for this planet. Try two 100-d intervals that are 1) contiguous and 2) separated by a large time gap and make the periodigram and compute the FAP for each case. Discuss what general guidelines you can infer from these experiments.

2. A) Generate a Gaussian absorption line for a normalized spectrum (continuum level of 1.0) having a peak depth of P=0.5, a width of σ=1 Å, centered at 5000 Å. Sample with pixels every 0.5 Å. The general form to a Gaussian function is

$$f(x) = offset + P\,e^{\frac{-(x-mean)^2}{2\sigma^2}}$$

Then, add Gaussian noise to each data point for a spectrum having a continuum signal-to-noise ratio of 100:1, 10:1, and 3:1 per pixel (this is different than a SNR per resolution element). The plots should looks something like those below. Since the width of your Gaussian is 1 Å then the Full Width at Half Maximum (FWHM) is 2.35 (by definition of a Guassian), and since you sample every half Å your spectrum is approximately Nyquist sampled.



B) Use the python `curve_fit` function to fit a Guassian to your data to which you've added noise. The co-variance elements which the function returns are used as uncertainties on each parameter. How accurately can you fit the center in each case? Make a plot of the accuracy of the center of the line versus SNR. Allow that all of the four parameters of the Gaussian function are free parameters in your fit. You'll need to do a number of SNR cases, perhaps 8-10 to generalize your result.
C) Repeat part B for the case that the continuum level of 1.0, the width of 1.0, and the depth are fixed parameters and only the center is a free parameter. Compare to part B and make a generalization about the accuracy of fitting line centers in the presence of noise.
D) Convert your accuracy in Å to km/s in each case above, using the non-relativistic form of the Doppler equation.
E) Now what if you have N such absorption lines in your spectrum where N=10,100,1000. The accuracy of a measurement (a centroid in this case) gets better as sqrt(N) (see notes about the error of the mean in lecture notes part 1). If you were trying to detect a planet, use your plot from Homework #1 to see what mass planet you could detect with these velocity precisions around a solar mass star if the period were 0.1 year.

3. Transits can only be observed when the inclination of the orbit is sufficiently large. Make a plot of orbital inclination versus semi-major axis. Using geometrical considerations, plot lines that correspond to a minimally detectable grazing transit for a range of planet sizes from Earth-sized to Jupiter-sized. Make one plot for an M0 dwarf (V) host star and one plot for a K0 III (giant) host star. Shade the approximate habitable zone (assume liquid water surface temps) for each star discuss the probability of detecting transits from habitable zone planets. Comment how these geometrical considerations may bias detection of a planets in plots like those above. Overplot actual planet data, where available, to see if there is evidence of such a bias.

4. Simulate a star as a solid face-on disk by breaking up the stellar surface into square pixels using a grid of 1000x1000 (or more) pixels.
A) Assign each pixel a surface brightness, letting it be 1.0 at the center of the disk and use the quadratic limb darkening tables of Van Hamme (1993) to assign a relative brightess to other pixels, as a function of distance from the center of the star. Render an image of your star to visually check the limb darkening. Assume T_{eff}=5500 for the star (G2V). Assume an observing wavelength of 5000 Å.
B) Let a planet of radius R_p=0.05 R_* pass across your star at impact parameter b=0 (i.e., along the stellar equator). Plot a light curve of you the transit by adding up the visible surface elements of the star at each time during the transit. The np.where or np.argwhere function will be useful to locate pixels that are within a specified distance of the center of your planet at each time step. Renormalize the light curve to unity in each case so that the out-of-transit flux is 1.0.
C) Repeat B for a planet transit impact parameter of 0.5.
D) Repeat B for a planet transit impact parameter of 0.9 (a grazing transit).
E) Overplot on the plots for B/C/D the light curve for a T=10,000 K host star and a 3600 K host star and summarize the differences.
F) Compare your transit model in case C above to that obtained from the batman python package (Kreidberg 2015) which uses the Mandel & Agol (2002) analytic light curve formulae.

5. Now let your star rotate with an equatorial velocity of V_R=10 km/s. The projected radial velocity of each surface element is a function of stellar latitude ϕ and longitude θ.

$$v_{rad} = V_R \sin\theta \cos\phi$$

A) Using your pixelated numerical star surface code, produce an intensity-weighted velocity profile plot for the star with no transit. This is also known as the rotational velocity profile.
B) Produce the same profile for the star for when the planet is at four different positions across the disk of the star at an impact parameter of b=0.5.
C) It will be easier to see the signature of the RM effect if you plot the difference between the non-transit profile in part A and the four transit profiles in part B. Compare your plots quantitatively to those in the Gaudi & Winn (2007, ApJ) paper.

6. Use the light curve data for Kepler 33 from the class web page to fold the data at the period of ONE of the known planets (pick different ones so you all do different ones). Use the batman light curve tool to fit a light transit curve to the data. Document your attempts and show that you can recover the planet parameters of Lissauer et al. (2012).