Optimization of the WARP Shoe Company Business Plan

MIE262
Joaquin Lim
Limjoaqu
1008463440
April 13th, 2023

1 Introduction

The WARP Shoe Company is one of the oldest shoe companies in Canada. Over the past three years, the WARP management found it helpful and sometimes profitable to consult with students from the department of Industrial Engineering of the University of Toronto. Please make yourselves familiar with the information on the company's website:

S:\ecfpc\warp\WarpShoes.Inc

All information needed on the WARP Shoe Company, including the database, can be found At

S:\ecfpc\warp\Warp2011W.mdb

The management of WARP Shoe Company would like to thank you in advance for your effort and help.

2 Problem Statement

At the beginning of year 2006, one of the major market competitors of WARP went bankrupt. Thus, WARP market analysts predict that for the month of February the demand for all types of shoes will double. Assume that closing inventory of January 2006 was zero for all types of shoes. Assume also that all sales happen at the end of the month.

The management of WARP wants to know what is the most profitable production plan. You are advised to take the following considerations into account:

- The budget for raw materials is $10,000,000.
- Sales price on shoes remains the same as in the Product Master Table.
- Not meeting the demand for any type of shoe means loss of new potential customers, and hence has been evaluated at a cost of $10/pair. Demand for each month from

- 1997-2003 can be found in the Product_Demand table.
- • It has been decided that during February 2006 the WARP shoe production plant, including all machines, will work up to 12 hours a day, 28 days a month. The cost of operation of each machine is given in the Machine_Master table. Assume that setup times and costs are negligible. The amount of time (s) each shoe requires from each machine is given in the Machine_Assign table.
- Workers are paid on an hourly basis at the rate of $25/hour. Each machine has to be operated by one worker.
- The total warehouse capacity can be obtained from the Warehouse_Master table.
- Transportation costs can be ignored.
- The manufacturing sequence can be ignored

3 Methodology

<u>Assumptions</u>

- The demand for February 2006 can be predicted by finding the average of all demands in the month of February from 1997-2003.
- The average demand will not be considered an upper bound for the number of shoes to be produced since it is an estimate and not representative of the actual number of shoes that may be demanded for in February 2006.
- Since transportation costs are ignored, shoes will be transported to any available warehouse space, regardless of distance, until all warehouse space has been filled.
- The amount of shoe pairs produced can only take on discrete quantities.
- The average demand is continuous since it is only an estimate, used to calculate opportunity cost from potentially lost customers.
- When number of shoes produced is equal to the expected demand for that shoe type in February 2006, opportunity cost stays at $0 and any further increase will not result in the gaining of money (difference between demand and shoes produced cannot be negative)

The goal of the proposed model will be to find a business plan, which maximizes the amount of profit (Revenue - Expenses), earned by the WARP Shoe Company for the month of February 2006, specifically.

Sets

I = {SH001, … , SH557}

Each i-value in the set, I, represents a Product Number which uniquely identifies a shoe type offered by WARP Shoes

J = {1 , ... ,72}

Each j-value in the set, J, represents a Machine Number which uniquely identifies a machin in the WARP Shoes Plant

K = {1, … , 165}

Each k-value in the set, K, represents a RM Number which uniquely identifies a raw material WARP Shoes uses to make their shoes

L = {1, … , 8}

Each l-value in the set, L, represents a Warehouse Number, which uniquely identifies a Warehouse owned by WARP Shoes

Decision Variables

There are 2 types of decision variables created for this business plan:

x_i : the total number of pairs of type-i shoes made for the month

y_j : the total amount of time that machine j operates within a month (in minutes)

Parameters

$priceSales_i$: sale price for selling a single pair of type-i shoes ($\frac{\$}{shoe\ i}$)

$opCost_j$: cost required for machine j to operate for a minute ($\frac{\$}{min}$)

$demand_i$: expected number of pairs of type-i shoes to be sold

$qtyReq_{i,k}$: quantity of raw material k required to produce a single pair of shoe type-i

$qtyAvail_k$: amount of raw material k that is purchaseable within a month

$cost_k$: cost required to purchase a single unit of raw material k ($\frac{\$}{RM\ k}$)

$avgDuration_{i,j}$: average amount of time required to produce a component of shoe type-i with machine j ($\frac{min}{shoe\ i}$)

$capacity_l$: number of pairs of shoes that can be stored within warehouse l

<u>Objective Function</u>

To maximize the profits for WARP Shoes, the monthly inflows and outflows of wealth were first analyzed. 1 source of revenue and 5 expenses were identified from the problem statement.

WARP Shoes is only able to earn money through shoe sales, which is represented by the expression below:

Shoe Sale Revenue = $\sum_{i=1}^{557} (x_i * priceSales_i)$

WARP Shoe must pay the operating expenses for using their machines to manufacture shoes; this expense is modeled by the expression below:

Operating Expense = $\sum_{j=1}^{72} (opCost_j * y_j)$

There is also an opportunity cost to be considered, which arises when WARP Shoes fails to meet the demand for any of the shoes they offer and loses potential customers. The opportunity cost is represented by the expression below:

Opportunity Cost = $10 * \sum_{i=1}^{557} (2 * demand - x_i)$

WARP Shoes must pay their workers an hourly wage of $\frac{\$25}{hr}$ as well. The total amount of wages paid can be represented by:

Total Wages Paid = $\frac{\$25}{1\ hr} * \frac{1\ hr}{60\ min} * \sum_{j=1}^{72} y_j$

Lastly, WARP Shoes must purchase the required raw materials in order to produce their shoes. This is modeled by the expression below:

RM Expenses = $\sum_{i=1}^{557} \sum_{k=1}^{165} (qtyReq_{i,k} * x_i * cost_k)$

The above expression can be combined into order to create the objective function:

Profit: $z\ = \sum_{i=1}^{557} (x_i * priceSales_i) - \sum_{j=1}^{72} (opCost_j * y_j) - 10 * \sum_{i=1}^{557} (2 * demand - x_i)$ - $\frac{25}{60} * \sum_{j=1}^{72} y_j$ - $\sum_{i=1}^{557} \sum_{k=1}^{165} (qtyReq_{i,k} * x_i * cost_k)$

Constraints

There were 5 types of constraints identified for the business plan based on the problem statement.

There is a set budget of \$10 000 000 that is to be allocated towards the purchase of raw materials, which cannot be exceeded.

budgetRM : $\sum_{i=1}^{557} \sum_{k=1}^{165} (x_i * qtyReq_{i,k} * cost_k) \leq \$10\ 000\ 000$

There is a maximum amount of time that each machine in the WARP Shoes Plant is allowed to stay active, $(\frac{60\ min}{hr} * \frac{12\ hr}{day} * \frac{28\ days}{month})$

maxMachTime: $y_j \leq 20\ 160$ minutes $\forall j = 1, \ldots, 72$

There is a maximum amount of shoes that can be stored across all warehouses owned by Warp Shoes.

maxWarehouse: $\sum_{i=1}^{557} x_i \leq \sum_{l=1}^{8} capacity_l$

There is a maximum amount of raw materials which is purchasable within a month.

maxRMQty: $\sum_{i=1}^{557} (qtyReq_{i,k} * x_i) \leq qtyAvail_k \ \forall k = 1, \ldots, 165$

The relationship between number of shoes produced and amount of time that a machine spends producing shoes can be represented by the following equality:

machShoeEquality: $\sum_{i=1}^{557} avgDuration_{i,j} * x_i = y_i$ $\forall$j = 1, … , 72

4 Results

The Gurobi Solver in AMPL was implemented in order to solve the integer program (linear program with relaxation). The sets, parameters, decision variables, objective function, and constraints outlined previously were used to formulate the IP in the WARPProject.mod file. The WARP2011W.mdb file was used to obtain the datasets needed to formulate the problem, and tables were formed on AMPL through the use of SQL queries in the WARPProject.dat file. The code was run on the WARPProject.run file, and relevant information was displayed onto the WARPProject.out file.

The optimal profit (after relaxation) was found to be \$14 038 008.10. The optimal decision variables for x_i and y_j , as well as the reduced costs, binding constraints and relaxation violations, are listed in the WARPProject.out file.

1. How should you estimate the demand for the month of February?

From the dataset containing demands, for all shoe products, for all months within the years 1997-2003, the demands specifically for the February months within this time range were concluded to be the most insightful for predicting the demand of February 2006. This conclusion was made based on seasonal trends and how the population's fashion trends shift depending on the weather and time of the year.

The following SQL Query was used within AMPL, in order to find average demand:

```
"SQL=SELECT Product_Num,AVG(Demand) AS avgDemand
FROM Product_Demand
WHERE Month=2
GROUP BY Product_Num"
```

The Product_Demand table was divided into groups, in which all members of a group share the same Product Number. From each of these groups, the shared Product Number as well as the average demand (sum of all demands in the group divided by the number of members in group) was selected. Only rows containing Month = 2 (i.e. Demands for February) were selected for the new table. These average demand values for each type-i product were then used to calculate the opportunity cost in the objective function

2. How many variables and constraints do you have?

There are 2 types of decision variable for this business plan:
There are a total of (x: i = 557)+(y: j = 72) = 629 decision variables.

There are 5 types of constraints for this business plan:
There are a total of (budgetRM: 1) + (maxMachTime: j = 72) + (maxWarehouse: 1) + (maxRMQty: k = 165) + (machShoeEquality: j = 72) = 311 constraints.

3. If you had to relax your integer program to an LP, how many constraints were violated after rounding the LP solution to the closest integer solution?

Since the solver required over 10 minutes to find an optimal solution for the problem, it was relaxed by removing the integer constraints from x_i and y_j then rounding all of the elements within these 2 decision variables after the LP has finished being solved. After rounding these variables, all constraints were checked for violations. It was found that 50 constraints from maxRMQty and all 72 machShoeEquality constraints were violated after relaxation, totalling 122 violated constraints.

4. Which constraints are binding, and what is the real-world interpretation of those binding constraints?

36 maxRMQty constraints are binding. These binding constraints represent the event that the maximum amount of raw material k that is purchasable within a month has been reached, and no further amount of raw material can currently be obtained.

5. Assume that some additional warehouse space is available at the price of $10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

A third decision variable, z_i, was created which represents the additional pairs of shoes of type-i that will be produced this month after additional warehouse space has been purchased for it. It was found that for all type-i shoes, it was optimal not to produce any additional pairs $z_i^* = 0 \ \forall i = 1, \dots, 557$). This is a logical result because maxWarehouse is not a binding constraint, and there still remains additional free warehouse space to be used before the need to purchase additional space.

6. Imagine that machines were available for only 8 hours per day. How would your solution change? Which constraints are binding now? Does the new solution seem realistic to you?

$original\, z^{*} \;=\; \$14038008.10$

$new\, z^{*} \;=\; \$14038525.08$

The difference is -$516.98, with the new objective function value being greater than the original. This result is not realistic because reduction in the upper bound of the hours that a machine can operate should always result in a loss, since less shoes can potentially be produced. After further inspection, this illogical result stems from the relaxation of integer constraints. The values before relaxation, which are visible from the console, are:

$original\, z^{*} \;=\; \$14039059.70$

$new\, z^{*} \;=\; \$14038754.96$

The difference between these 2 objective functions is $304.73. This could potentially be a realistic solution because the amount of time that a machine operates is associated with many of the expenses in the objective function but reduction of hours also leads to reduction in shoes produced, and shoe sales by extension. Therefore, the relatively small difference might be plausible.

The number of maxRMQty constraints that are binding has been reduced to only 23 constraints, with there now being 2 macMachTime constraints that are binding as well.

7. If in addition there were a $7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve again.

There were no differences between the solutions to the original and Q7 formulations. This could be a result of how budgetRM is not a binding constraint, so even at the most optimal business plan, not all of the original $10 000 000 is used.

5 Conclusion

Thus, the most profitable business plan for WARP Shoe in February 2006, with the expected doubling of demand, would result in the company making $14 038 008.10. The optimal number of shoe pairs to be produced for each shoe type, and the optimal amount of time, in minutes, that each machine operates, was identified using the Gurobi Solver on AMPL, which results in the most optimal (maximum) profit.