Assignment 6

Problem 1:

(a) $\mu_{\bar{x}} = \mu = \underline{19.8}$ $\qquad \sigma_{\bar{x}} = \frac{\sqrt{\sigma^2}}{\sqrt{n}} = \frac{\sqrt{7.2}}{\sqrt{30}} = \underline{0.4899}$

(b) $P(19 \leq \bar{X} \leq 20.5) = P(\bar{X} \leq 20.5) - P(\bar{X} \leq 19)$

$\bar{X} \sim N(19.8, 0.4899)\}$ $= P\left(Z \leq \frac{20.5-19.8}{0.4899}\right) - P\left(Z \leq \frac{19-19.8}{0.4899}\right)$

$= P(Z \leq 1.43) - P(Z \leq -1.63)$

$= 0.9236 - 0.0516$

$= \underline{0.872}$

(c) $P(\bar{X} \leq y) = 0.18$

$P(\bar{X} \leq y) = P\left(Z \leq \frac{y-19.8}{0.4899}\right) = 0.18$

By inspection $P(Z \leq -0.92) = 0.1788$
≈ 0.18

$\frac{y-19.8}{0.4899} = -0.92 \Rightarrow y = \underline{19.349}$

Problem 2:

(a) $\mu_x = E(X)$

$= \sum_{i=1} x f(x)$

$= \cancel{(x<4)(0)} + 4(0.2) + 5(0.4) + 6(0.3) + 7(0.1) + \cancel{(x>7)(0)}$

$= \underline{5.3}$

$\sigma_x^2 = E(X^2) - (E(X))^2$

$= (\cancel{0(x<4)})^2 + 4^2(0.2) + 5^2(0.4) + 6^2(0.3) + 7^2(0.1) - (5.3)^2$

$= 28.9 - 28.09$

$= \underline{0.81}$

(b) $\mu_{\bar{x}} = \mu = \underline{5.3}$ $\qquad \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{\sqrt{n}} = \frac{\sqrt{0.81}}{\sqrt{36}} = \underline{0.135}$

(c) $P(\bar{X} < 5.5) = P\left(\frac{\bar{X}-5.3}{\sqrt{0.81}/\sqrt{36}} < \frac{5.5-5.3}{\sqrt{0.81}/\sqrt{36}}\right)$

$= P(Z < 1.33)$

$= \underline{0.9082}$

Problem 3:

(a) $P(\bar{X}_1 - \bar{X}_2 < 2000) = P\left(\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < \frac{2000 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right)$

$$= P\left(Z < \frac{2000 - 0}{\sqrt{\frac{4800^2}{30} + \frac{4800^2}{45}}}\right)$$

$$= P(Z < 1.77)$$

$$= \underline{0.9616}$$

(b) $P(\bar{X}_1 - \bar{X}_2 > 500) = 1 - P(\bar{X}_1 - \bar{X}_2 < 500)$

$$= 1 - P\left(Z < \frac{500 - 0}{\sqrt{\frac{4800^2}{30} + \frac{4800^2}{45}}}\right)$$

$$= 1 - P(Z < 0.44)$$

$$= 1 - 0.6700$$

$$= \underline{0.33}$$

Problem 4:

(a) $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{1000(48 + 53 + 45 + 61 + 59 + 56 + 63 + 49 + 53 + 54)}{10}$

$$= 54100$$

$$s^2 = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \sqrt{\frac{1}{10-1} \sum_{i=1}^{n} (x_i - 54100)^2} = \sqrt{\frac{3.029 \cdot 10^8}{9}} = \sqrt{3.3656 \cdot 10^7}$$

$\underline{s = 5801.341}$

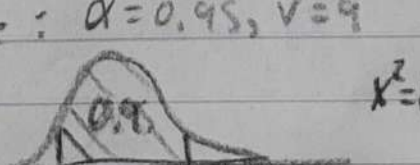
(b) Dist'n $s^2 \sim X^2$

$$X^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(3.3656 \cdot 10^7)}{(6000)^2} = 8.4139$$

$V = n - 1$

$= 9$

From table: $\alpha = 0.95$, $V = 9$



$X^2 = 8.41$ not in $[1.833, 3.325]$

$\therefore$ supplier's claim is false