Group 4: Robustness of Subway Networks

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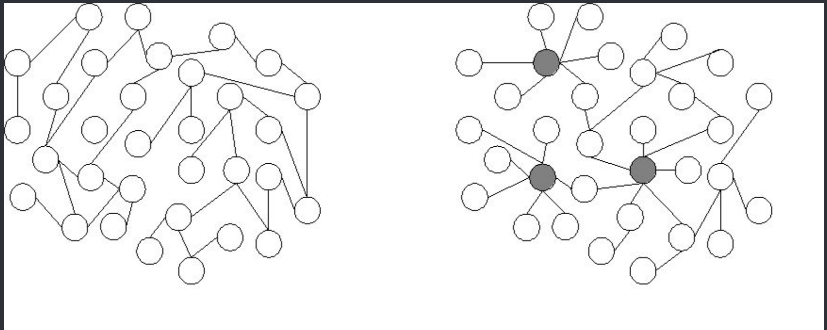
Introduction

- Public transportation makes life in dense urban areas possible—from July to September 2022, public transportation riders took over **1.5 million** distinct trips in the U.S.
- Robustness metrics on a transportation network measure the availability of alternative paths in the event of node or edge failure
- Transportation network robustness is a highly relevant measure—it is resilience of a public transit system to a failure of a line or station
- We examine 14 global subway networks (data from Roth et al.), applying ideas of **core-periphery structure** and "**path score**" in order to provide a more comprehensive picture of robustness.

Attributes of Subway Networks

- Most metro networks are “scale-free” because the degree distribution of their stations follows a Power Law distribution
- Small-world networks due to high clustering and small average geodesic length
- Hub and spoke structure—central “hubs” where subway paths (line graphs) intersect



(a) Random network (b) Scale-free network

New Applications

- Path score previously applied to road networks
  - Metro systems have hub and spoke structure and are much sparser than a grid-like structure, could leading to different/interesting results
- Previous robustness measures are global:
  - Path score analyzes individual nodes, allowing for new insights on alternative paths as measures of transportation network robustness

LITERATURE AND METHODS

Derrible and Kennedy Robustness Metric

- The Derrible and Kennedy robustness metric is essentially **the number of cycles relative to the size of the network**
- Divide nodes into two sets—terminus stations with one connection (V_m) and diatonic with more than one connection (V_d), and do the same for edges (E = {M,D})
- From the diatonic edges and nodes, calculate the number of cycles, **μ**, in a graph by subtracting the number of edges in the transportation network as a tree from the number of edges
- Then, subtract out multi edges and divide by the number of nodes.

$$\mu = |D| - |V_d| + 1$$

$$r^T = \frac{\mu - |D^m|}{|V|}$$

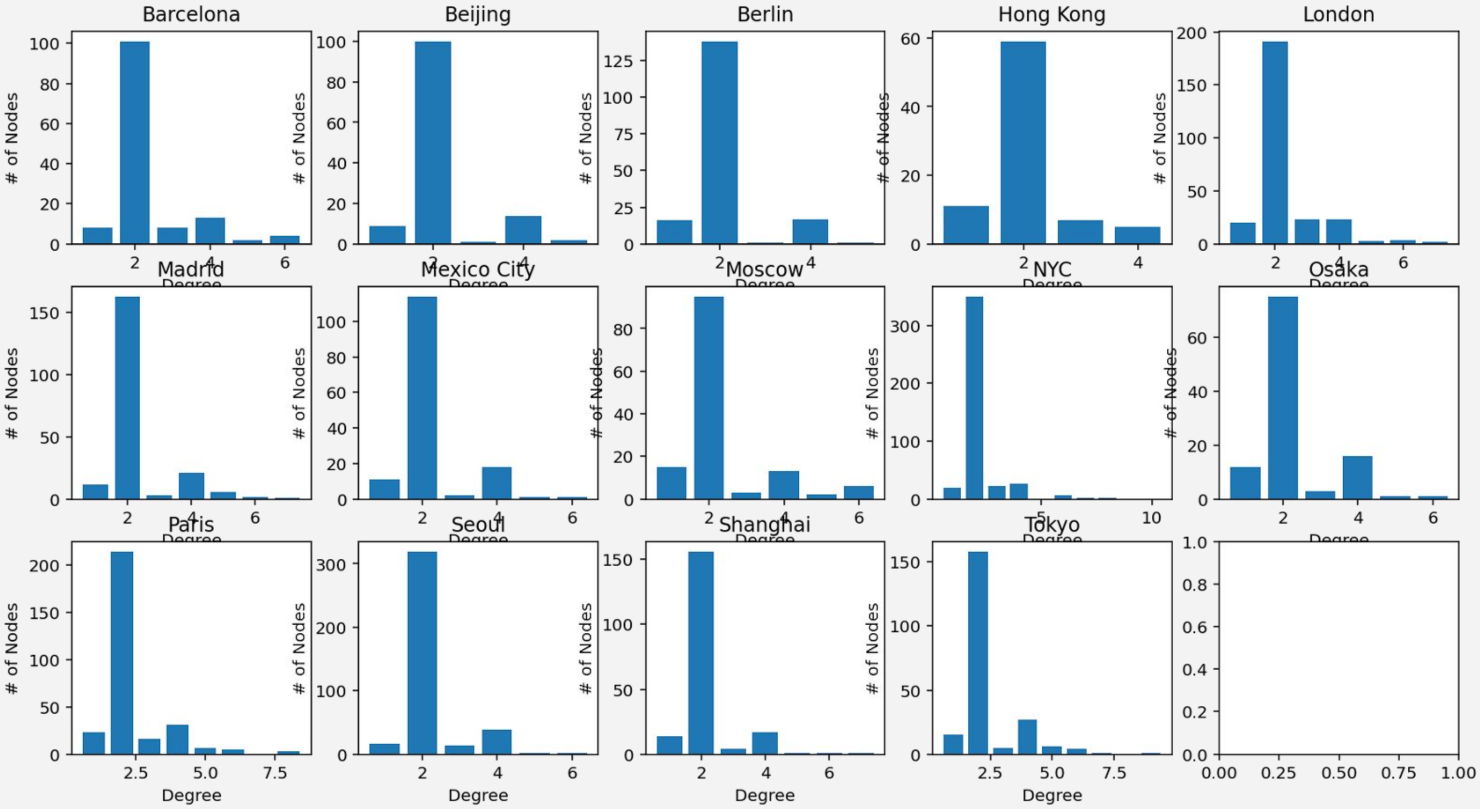
Lee et. al Path Score

- The path score is essentially a sum of **“optimal backup paths” from node j to node k for all j,k in an edgelist when the edge (j,k) is removed** from the edgelist
- Optimal backup path is defined as the shortest path based on network distance
- Each node will have a path score, but looking at the mean path score or proportion of nonzero path scored nodes is telling about the robustness of the network

$$\mathrm{PS}(i) = \frac{1}{|\mathbb{E}|} \sum_{(j,k)\in\mathbb{E}} \sum_{\{p_{jk}\}} \sigma_{jik}[\mathbb{E} \setminus (j,k)]$$

ANALYSIS

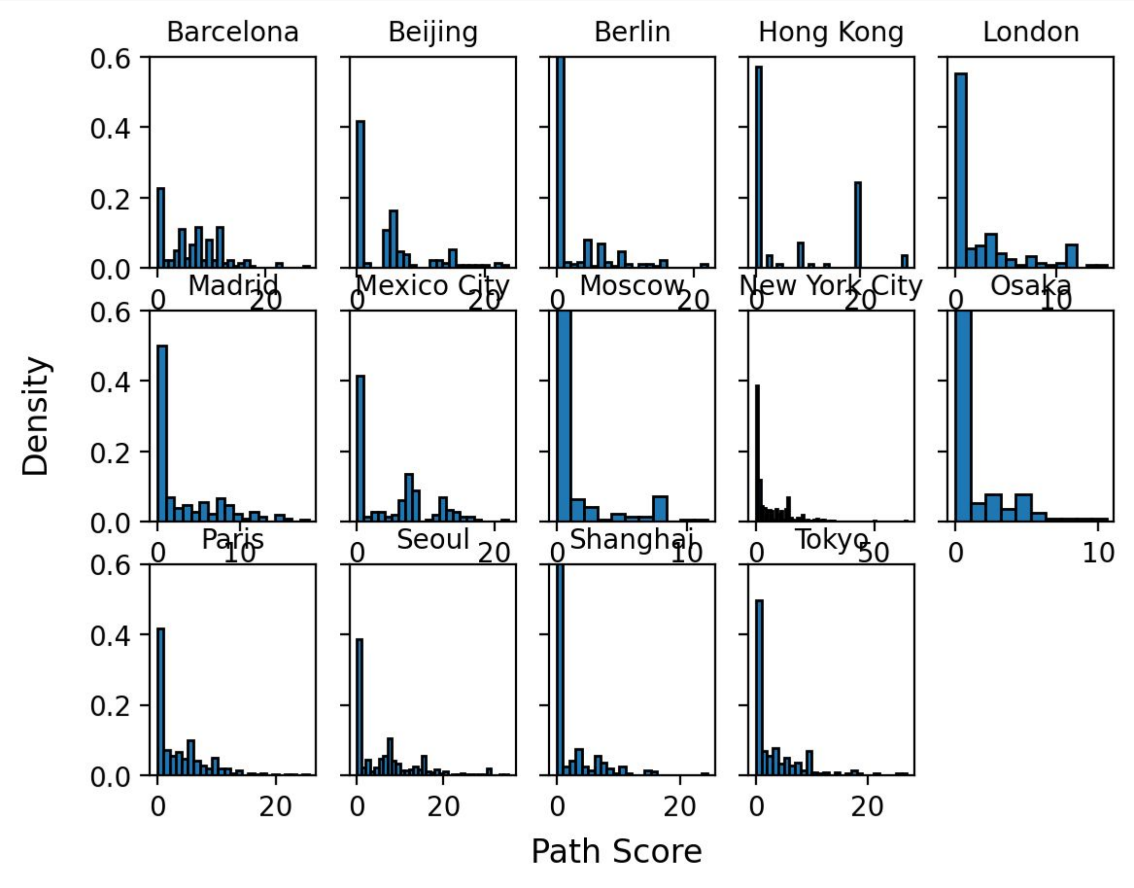
Degree Distribution Plots for all Subways



Degree Distribution

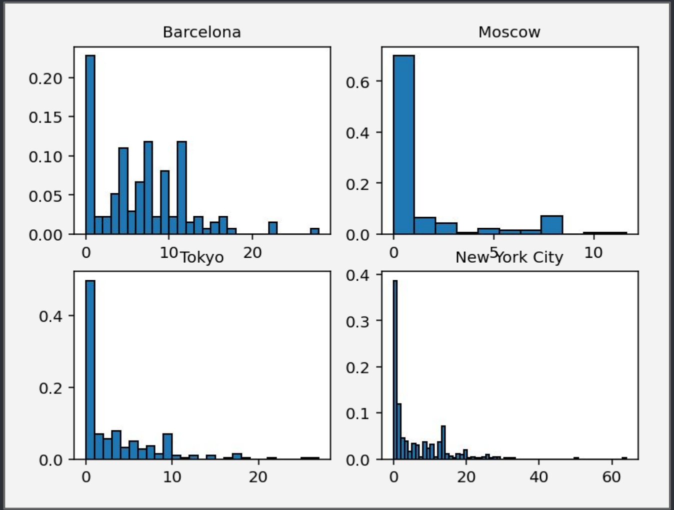
- The degrees seem to be more heavily distributed on even-numbered degrees—most nodes have a degree of two
  - The network structure is mostly line graphs connected at “hubs”. All nodes on a line graph besides the two ends of the line have a degree of 2
  - Unless a node is the end point on a line graph, or the “end of the line” on a transportation route, then it will connect a multiple of 2 nodes per route
- “Hubs” can be defined as nodes that connect more than one transportation route, or serve as an intersection of multiple line graphs—these nodes will have a degree higher than 2

Path Score Distribution Plots for all Subway Networks

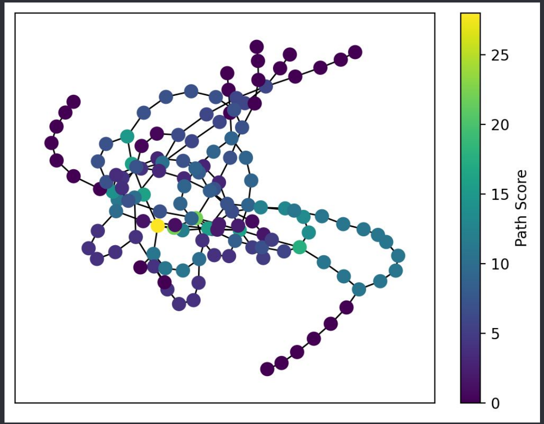


Path Score Distribution

- The most common path score is 0 for all 14 networks
    - Path score of 0 indicates a “spoke,” where no backup paths exist
- Minimum: Barcelona - 20.6% of nodes with PS = 0
- Maximum: Moscow - 70.9% of nodes with PS = 0

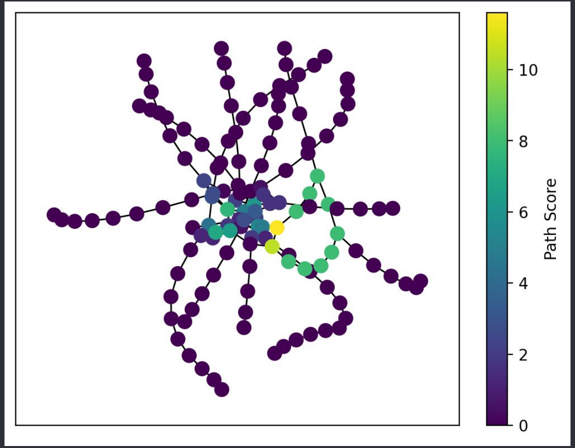


Path Score Heat Maps



Barcelona

- Larger, well-connected core area with nonzero path scores
- Shorter peripheral lines with 0 path score

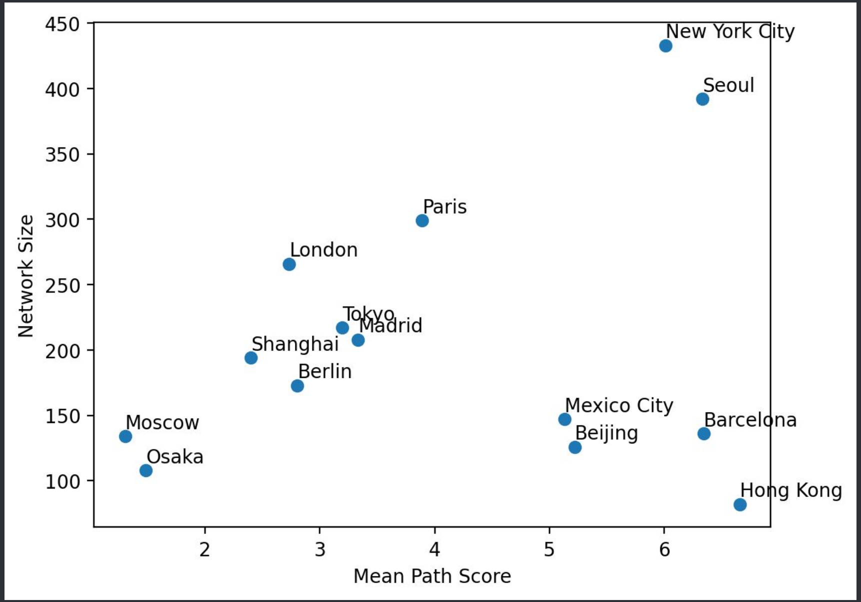


Moscow

- Central hub with long, line graphed spokes protruding
- Only the center has non-zero path scores; the peripheral nodes on the spokes could be inaccessible with one station shut down

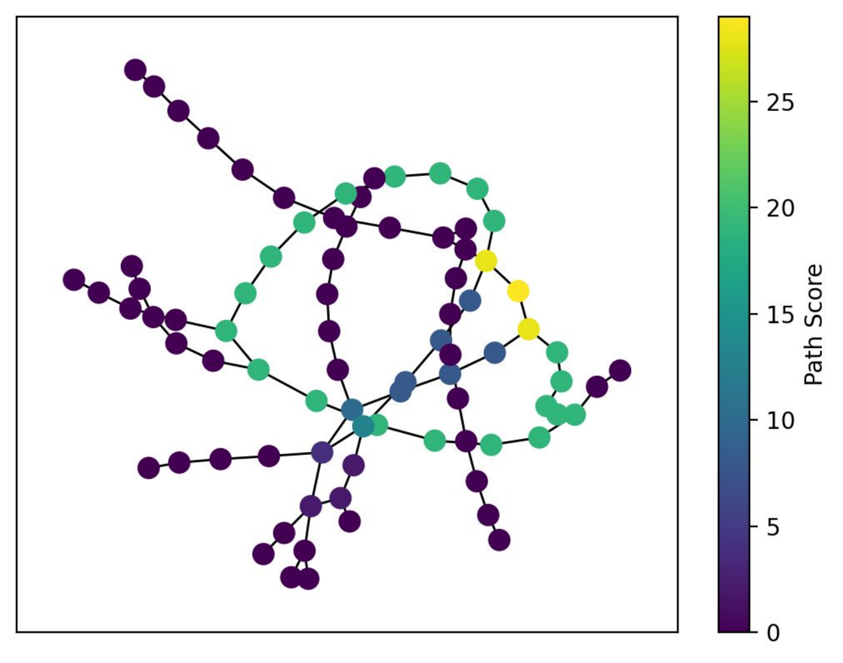
Mean Path Score vs Network Size

- There exists a positive correlation between size and path score
- There appears to be a cluster deviating from the general trend with a smaller network size but a large path score



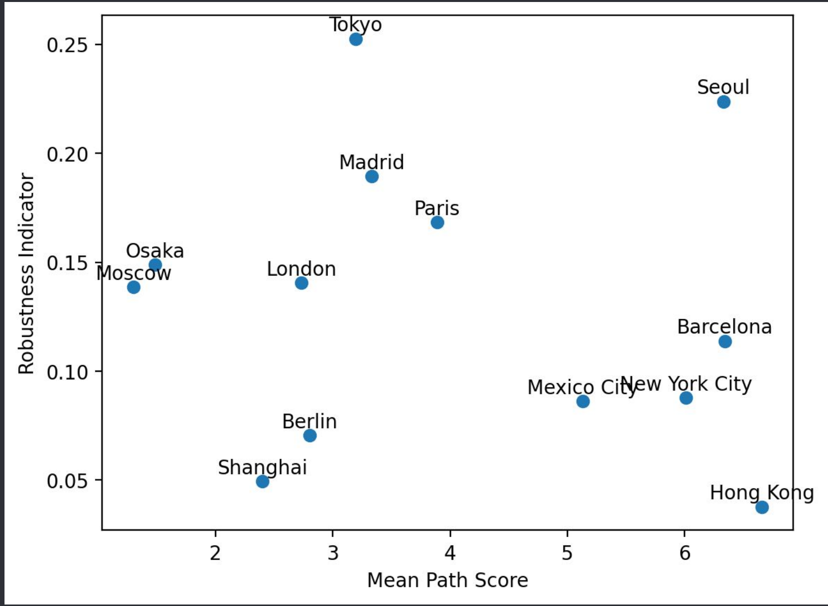
Hong Kong - Small Network, High mean path score

- The structure of the Hong Kong subway network has a centralized loop of stations
- The loop stations have high path scores since they are necessary on backup paths
- Loop creates a more well-connected core, increasing PS and robustness



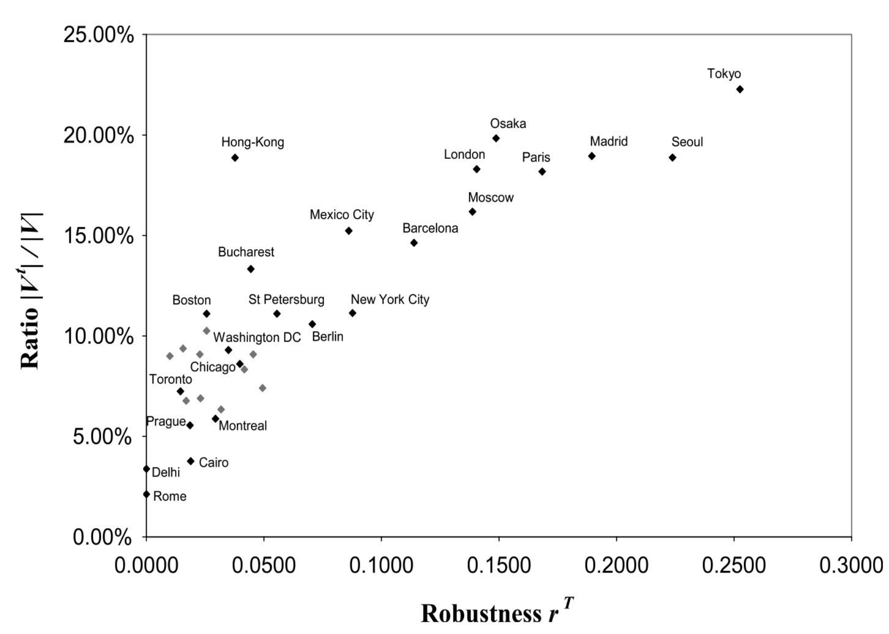
Robustness Indicator (Derrible et al.) vs. Mean Path Score

- Low correlation
- Hong Kong has high PS but low DK Robustness
- There is a general positive correlation between the two metrics, but there are several outlying points



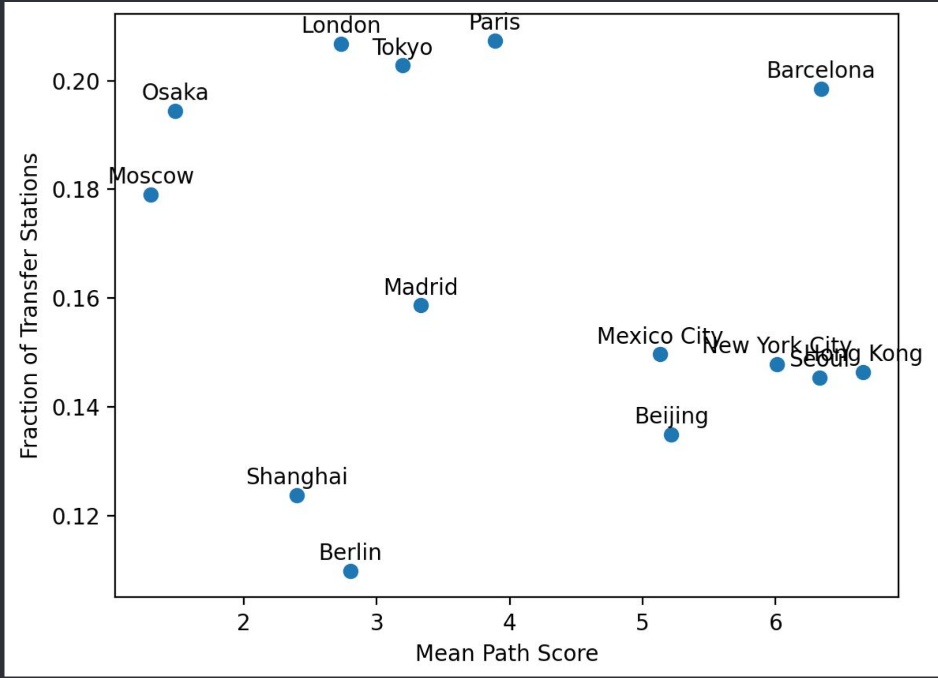
Back to Derrible and Kennedy

- Derrible and Kennedy found a strong positive correlation between robustness and the proportion of “transfer stations” (nodes with degree larger than 2)



FRACTION OF TRANSFER STATIONS VS. MEAN PATH SCORE

- Did not find correlation between fraction of transfer stations and mean path score
- Suggests structure/location of transfer stations is important to robustness
- Revisiting Moscow vs Barcelona:
  - Similar fraction of transfers, but one has a larger core area than the other



CONCLUSIONS

Discussion

- We studied the structure of 14 public transportation networks and explored how their structure impacted robustness
- A high proportion of nodes with path score of 0 indicates a **large periphery** without any alternative paths between nodes, lowering network robustness
- **Larger** networks tend to be **more robust** due to the larger availability of backup paths
    - However, smaller networks with larger, although sparser core structures, still had high mean path scores
- Overall, the core periphery structure of a network seems to be important for robustness

Future Works

- **Larger sample** of real-world metros
- Mathematical properties of mean path score
- Comparison to other metrics of robustness: assortativity, attack metrics
- Directed and weighted networks, bus networks

References

[1] BERTRAND BERCHE et al. "TRANSPORTATION NETWORK STABILITY: A CASE STUDY OF CITY TRANSIT". In: *Advances in Complex Systems* 15.supp01 (June 2012), p. 1250063. DOI: 10.1142/s0219525912500634. URL: https://doi.org/10.1142%5C%2Fs0219525912500634.

[2] Claude Berge. *Theory of graphs*. London, England: Methuen young books, Dec. 1962.

[3] Sybil Derrible and Christopher Kennedy. "The complexity and robustness of metro networks". In: *Physica A: Statistical Mechanics and its Applications* 389.17 (2010), pp. 3678–3691. ISSN: 0378-4371. DOI: https://doi.org/10.1016/j.physa.2010.04.008. URL: https://www.sciencedirect.com/science/article/pii/S0378437110003262.

[4] Sang Hoon Lee, Mihai Cucuringu, and Mason A. Porter. "Density-based and transport-based core-periphery structures in networks". In: *Physical Review E* 89.3 (Mar. 2014). DOI: 10.1103/physreve.89.032810. URL: https://doi.org/10.1103%5C%2Fphysreve.89.032810.

[5] Abbas Masoumzadeh and Tilemachos Pechlivanoglou. *Network-Based Analysis of Public Transportation Systems in North American Cities*. 2020. DOI: 10.48550/ARXIV.2001.01838. URL: https://arxiv.org/abs/2001.01838.

[6] *Public Transportation Ridership Report*. 1300 Eye St NW, Suite 1200 E Washington, DC 20005, Nov. 2022.

[7] Camille Roth et al. "A long-time limit for world subway networks". In: *Journal of The Royal Society Interface* 9.75 (2012), pp. 2540–2550. DOI: 10.1098/rsif.2012.0259. eprint: https://royalsocietypublishing.org/doi/pdf/10.1098/rsif.2012.0259. URL: https://royalsocietypublishing.org/doi/abs/10.1098/rsif.2012.0259.

*path scores were calculated using Matlab code provided by Prof. Porter