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1. Proof:

$$\Pr(\alpha_1, \dots, \alpha_n | \beta) = \frac{\Pr(\alpha_1, \dots, \alpha_n, \beta)}{\Pr(\beta)}$$
$$Pr(\alpha_1 | \alpha_2, \dots, \alpha_n, \beta) = \frac{\Pr(\alpha_1) * \Pr(\alpha_2, \dots, \alpha_n, \mathrm{B} | \alpha_1)}{\Pr(\alpha_2, \dots, \alpha_n, \mathrm{\beta})}$$
$$\Pr(\alpha_2, \dots, \alpha_n, \mathrm{\beta}) = Pr(\alpha_3, \dots, \alpha_n, \beta | \alpha_2) * \Pr(\alpha_2)$$
$$\therefore Pr(\alpha_1 | \alpha_2, \dots, \alpha_n, \beta) = \frac{\Pr(\alpha_1) * \Pr(\alpha_2, \dots, \alpha_n, \mathrm{B} | \alpha_1)}{Pr(\alpha_3, \dots, \alpha_n, \beta | \alpha_2) * \Pr(\alpha_2)}$$

$$\Pr(\alpha_1 \mid \alpha_2, \dots, \alpha_n, \beta) \Pr(\alpha_2 \mid \alpha_3, \dots, \alpha_n, \beta) \dots \Pr(\alpha_n \mid \beta)$$
$$= \frac{\Pr(\alpha_1) * \Pr(\alpha_2, \dots, \alpha_n, \mathrm{\beta} | \alpha_1)}{Pr(\alpha_3, \dots, \alpha_n, \beta | \alpha_2) * \Pr(\alpha_2)} * \frac{\Pr(\alpha_2) * \Pr(\alpha_3, \dots, \alpha_n, \mathrm{\beta} | \alpha_2)}{Pr(\alpha_4, \dots, \alpha_n, \beta | \alpha_3) * \Pr(\alpha_3)} * \dots * \frac{\Pr(\alpha_n) * \Pr(\mathrm{\beta} | \alpha_n)}{\Pr(\beta)}$$

$$= \frac{\Pr(\alpha_1) * \Pr(\alpha_2, \dots, \alpha_n, \mathrm{\beta} | \alpha_1)}{\Pr(\beta)}$$
$$= \frac{\Pr(\alpha_1, \dots, \alpha_n, \beta)}{\Pr(\beta)}$$
$$\therefore \Pr(\alpha_1, \dots, \alpha_n | \beta) = \Pr(\alpha_1 \mid \alpha_2, \dots, \alpha_n, \beta) \Pr(\alpha_2 \mid \alpha_3, \dots, \alpha_n, \beta) \dots \Pr(\alpha_n \mid \beta)$$

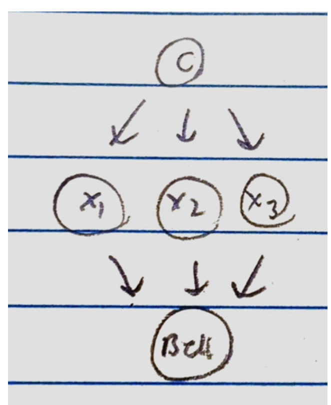
2. Pr(Oil) = 0.5, Pr(Gas) = 0.2, Pr(Neither) = 0.3, Pr(Positive|Oil) = 0.9, Pr(Positive|Gas) = 0.3, Pr(Positive|Neither) = 0.1

$$Pr(Oil|Positive) \ = ?$$
$$Pr(Oil|Positive) \ = \frac{Pr(Oil) * \ Pr(Positive|Oil)}{\Pr(Positive)}$$

$$\Pr(Positive) = \Pr(Positive|Oil) * \Pr(Oil) + \Pr(Positive|Gas) * \Pr(Gas) + \Pr(Positive|Neither) * \Pr(Neither)$$
$$= \ 0.9 * 0.5 + 0.3 * 0.2 + 0.1 * 0.3$$
$$= 0.54$$

$$\therefore \Pr(Oil|Positive) = \frac{0.5 * 0.9}{0.54}$$
$$= 0.83\overline{3}$$

3. The DAG looks like:



We can create a new RV Coin for the coin that is drawn.
CPT Coin:

Coin	Pr(Coin)
A	1/3
B	1/3
C	1/3

$$For\ each\ of\ the\ outcomes\ X_i, the\ CPT\ is:$$

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Coin	X_i	$\Pr(X_i \mid Coin)$
A	T	0.2
A	F	0.8
B	T	0.4
B	F	0.6
C	T	0.8
C	F	0.2

CPT Bell:

X_1	X_2	X_3	Bell	$\Pr(Bell \mid X_1, X_2, X_3)$
T	T	T	T	1
T	T	T	F	0
T	T	F	T	0
T	T	F	F	0
T	F	T	T	0
T	F	T	F	0
T	F	F	T	0
T	F	F	F	0
F	T	T	T	0
F	T	T	F	0
F	T	F	T	0
F	T	F	F	0
F	F	T	T	0
F	F	T	F	0
F	F	F	T	1
F	F	F	F	0

4.
(a)

$$I(A, \emptyset, \{B, E\})$$
$$I(B, \emptyset, \{A, C\})$$
$$I(C, \{A\}, \{B, D, E\})$$
$$I(D, \{A, B\}, \{C, E\})$$
$$I(E, \{B\}, \{A, C, D, F, G\})$$
$$I(F, \{C, D\}, \{A, B, E\})$$
$$I(G, \{F\}, \{A, B, C, D, E, H\})$$
$$I(H, \{F, E\}, \{A, B, C, D, G)\}$$

(i) False, the path A,D,B,E is open because D is convergent and B is divergent.
(ii) True, B is closed because B is divergent and is part of the evidence. H cannot be used in a path either since it is convergent.
(iii) True, CDE is given as evidence, and no path exists.

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(b) $Pr(a,b,c,d,e,f,g,h) =$
$Pr(a)Pr(b)Pr(c|a)Pr(d|a,b)P(e|b)Pr(f|c,d)Pr(g|f)Pr(h|f,e)$

(c) $Pr(A = 1, B = 1)\ and\ \Pr(E = 0|A = 0)$

$Pr(A = 1, B = 1) = \Pr(A = 1) * \Pr(B = 1) = 0.2 * 0.7 = 0.14$
$\Pr(E = 0|A = 0) = \Pr(E = 0)\ (independence\ due\ to\ d - separation)$
$= \Pr(E = 0|B = 1)\Pr(B = 1) + * \Pr(E = 0|B = 0)P(B = 0)$
$= 0.9 * 0.7 + 0.1 * 0.3$
$= 0.66$

5.

	A	B	$\neg A \vee B$	Pr(A, B)
w_0	T	T	T	0.3
w_1	T	F	F	0.2
w_2	F	T	T	0.1
w_3	F	F	T	0.4

(a) w_0, w_2, w_3
(b) $\Pr(\alpha) = 0.3 + 0.1 + 0.4 = 0.8$
(c) $\Pr(A, B|\alpha) = \frac{\Pr(A,B,\alpha)}{P(\alpha)}$ = 0.375

A	B	$P(A,B|\alpha)$
T	T	= 0.3 / 0.8 = 0.375
T	F	0
F	T	= 0.1 / 0.8 = 0.125
F	F	= 0.4 / 0.8 = 0.5

(d) $\Pr(\neg A \vee \neg B|\alpha) = 0.5 + 0.125 = 0.625$