CS 161 Homework 5

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1)

Proof that $P \Rightarrow \neg Q$ and $Q \Rightarrow \neg P$ are equivalent

$P \Rightarrow \neg Q = \neg P \vee \neg Q$; $Q \Rightarrow \neg P = \neg Q \vee \neg P$

P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Proof that $P \Leftrightarrow \neg Q$ and $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent

$P \Leftrightarrow \neg Q = (\neg P \vee \neg Q) \wedge (Q \vee P)$

P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

2)

$\alpha = (Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

(Smoke ⇒ Fire) = ¬Smoke v Fire ; (¬Smoke ⇒ ¬Fire) = Smoke v ¬Fire

Smoke	Fire	(Smoke ⇒ Fire)	(¬Smoke ⇒ ¬Fire)	α
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	1	1	1

Since α = (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) is not true for all assignments of Smoke and Fire, then α is **neither valid nor unsatisfiable**.

α = (Smoke ⇒ Fire) ⇒ ((Smoke v Heat) ⇒ Fire)

(Smoke ⇒ Fire) = ¬Smoke v Fire ; ((Smoke ∨ Heat) ⇒ Fire) = ¬(Smoke ∨ Heat) v Fire = (¬Smoke ∧ ¬Heat) v Fire

Smoke	Fire	Heat	(Smoke ⇒ Fire)	((Smoke ∨ Heat) ⇒ Fire)	α
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	1	1	1

Since α = (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire) is not true for all assignments of Smoke, Fire, and Heat, then α is **neither valid nor unsatisfiable**.

α = ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) v (Heat ⇒ Fire))

$((Smoke \wedge Heat) \Rightarrow Fire) = \neg(Smoke \wedge Heat) \vee Fire = (\neg Smoke \vee \neg Heat) \vee Fire$

$((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)) = (\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire) = \neg Smoke \vee Fire \vee \neg Heat$

Smoke	Fire	Heat	$((Smoke \wedge Heat) \Rightarrow Fire)$	$((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$	α
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	1	1	1

Since $\alpha = (Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$ is true for all assignments of Smoke, Fire, and Heat, then α is **satisfiable / valid**.

3)

a)

Let M = unicorn is mythical, O = unicorn is immortal, L = unicorn is mammal, H = unicorn is horned, G = unicorn is magical.

$KB = (M \Rightarrow O) \wedge (\neg M \Rightarrow (\neg O \wedge L)) \wedge ((O \vee L) \Rightarrow H) \wedge (H \Rightarrow G)$

b)

$KB = (M \Rightarrow O) \wedge (\neg M \Rightarrow (\neg O \wedge L)) \wedge ((O \vee L) \Rightarrow H) \wedge (H \Rightarrow G)$

$KB = (\neg M \vee O) \wedge (M \vee (\neg O \wedge L)) \wedge (\neg(O \vee L) \vee H) \wedge (\neg H \vee G)$

$KB = (\neg M \vee O) \wedge (M \vee (\neg O \wedge L)) \wedge ((\neg O \wedge \neg L) \vee H) \wedge (\neg H \vee G)$

$KB = (\neg M \vee O) \wedge (M \vee \neg O) \wedge (M \vee L) \wedge (\neg O \vee H) \wedge (\neg L \vee H) \wedge (\neg H \vee G)$

KB:

1. (¬M v O)
2. (M v ¬O)
3. (M v L)
4. (¬O v H)
5. (¬L v H)
6. (¬H v G)

c)

For the three proofs below, we will use resolution with refutation. Specifically, we will show that KB ⊨ α by showing that KB ^ ¬α is inconsistent.

Proof that unicorn is mythical

α = M, so ¬α = ¬M

7. ¬O (¬α, 2)
8. L (¬α, 3)
9. True (1, 2)
10. O v L (1, 3)
11. (¬M v H) (1, 4)
12. (¬O v G) (4, 6)
13. (¬L v G) (5, 6)
14. ¬M (1, 7)
15. H (5, 8)
16. G (8, 13)

Since we did not find a contradiction, then we **cannot prove that the unicorn is mythical**.

Proof that unicorn is magical:

α = G, so ¬α = ¬G

7. ¬H (¬α, 6)
8. ¬L (5, 7)
9. ¬O (4, 7)
10. ¬M (1, 9)
11. M (3, 8)

Since we found a contradiction between rules 10 and 11, we **can prove that the unicorn is magical**.

Proof that unicorn is horned:

α = H, so ¬α = ¬H

7. ¬L (¬α, 5)
8. ¬O (¬α, 4)
9. ¬M (1, 8)
10. M (3, 7)

Since we found a contradiction between rules 9 and 10, we **can prove that the unicorn is horned**.

4)

Decomposable: the left and right branches for each AND gate cannot share the same variables

Deterministic: the left and right branches for each OR gate cannot both be true at the same time

Smooth: the left and right branches for each OR gate must have the same variables

Figure 1

Based on the definition of decomposable above, by observation, we know that Figure 1 is **decomposable**.

For the root OR node, the left branch is (((¬A AND B) OR (¬B AND A)) AND (C OR (¬D AND ¬C))). The right branch is ((A OR (¬A AND ¬B)) AND (C AND ¬D) OR (D AND ¬C)). If we set A = 1, B = 0, C = 1, D = 0, then we have both sides of the root OR node as true. Thus, Figure 1 is **not deterministic**.

In the third level of the tree, the second-from-the-left OR gate has variable C in its left branch and variables C and D in its right branch. Thus, Figure 1 is **not smooth**.

Figure 2

Based on the definition of decomposable above, by observation, we know that Figure 2 is **decomposable**.

In the third level of the tree, the leftmost OR gate has the same input, ¬A AND B, for both the left and right branches. Thus, this OR gate can have both the left and right branches be true at the same time, so Figure 1 is **not deterministic**.

Based on the definition of smooth above, by observation, we know that Figure 1 is **smooth**.

5)

a)

A	B	(¬A ∧ B) ∨ (¬B ∧ A)	Weight

0	0	0	N/A
0	1	1	0.27
1	0	1	0.07
1	1	0	N/A

The Weighted Model Count for $(\neg A \land B) \lor (\neg B \land A)$ is 0.27 + 0.07 = **0.34**

b)

The count on the root is the same as the Weighted Model Count for the formula $(\neg A \land B) \lor (\neg B \land A)$.

c)

We will compute the Weighted Model Count using the method described in part b.

$(((0.9)(0.3) + (0.7)(0.1)) * ((0.5)(0.7) + (0.3)(0.5))) + (((0.9)(0.7) + (0.3)(0.1)) * ((0.5)(0.3) + (0.7)(0.5))) = \mathbf{0.5}$