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CS 161 Hw 5

1) (a) $P \to \neg Q$, $Q \to \neg P$

P	Q	$P \to \neg Q$		$Q \to \neg P$
T	T	F	=	F
T	F	T	=	T
F	F	T	=	T
F	T	T	=	T

(b) $P \Leftrightarrow \neg Q$, $((P \land \neg Q) \lor (\neg P \land Q))$

P	Q	$P \Leftrightarrow \neg Q$		$((P \land \neg Q) \lor (\neg P \land Q))$
T	T	F	= F∨F =	F
T	F	T	= T∨T	T
F	F	F	= F∨F	F
F	T	T	= F∨T	T

2) (a)

smoke	fire	smoke ⇒ fire	$\overline{\text{smoke}} \Rightarrow \overline{\text{fire}}$	smoke ⇒ fire ⇒ $\overline{\text{smoke}} \Rightarrow \overline{\text{fire}}$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(smoke ⇒ fire) ⇒ ($\overline{\text{smoke}}$ ⇒ $\overline{\text{fire}}$) is $\boxed{\text{neither}}$ valid nor unsatisfiable

(b)

smoke	heat	fire	smoke ⇒ fire	(smoke ∨ heat) ⇒ fire	(smoke ⇒ fire) ⇒ ((smoke ∨ heat)
T	T	T	T	T	T
T	F	T	T	T	T
T	T	F	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T
F	T	F	T	F	F
F	F	F	T	T	T

(smoke → fire) → ((smoke ∨ heat) → fire) is $\boxed{\text{neither}}$ valid nor unsatisfiable

(c)

smoke	heat	fire	(smoke ∧ heat) → fire	(smoke → fire) ∨ (heat → fire)	((smoke ∧ heat) → fire) ↔ ((smoke → fire) ∨ (heat → fire))
T	T	T	T	T	T
T	F	T	T	T	T
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

((smoke ∧ heat) → fire) ↔ ((smoke → fire) ∨ (heat → fire)) is $\boxed{\text{valid}}$

3) mythical = M, immortal = I, mammal = A, horned = H, magical = G

(a) $\boxed{M \rightarrow I}$

$\boxed{\neg M \rightarrow (\neg I \wedge A)}$

$\boxed{(I \vee A) \rightarrow H}$

$\boxed{H \rightarrow G}$

(b) $\boxed{\neg M \vee I}$

$\neg M \vee (\neg I \wedge A)$

$= \boxed{(M \vee \neg I) \wedge (M \vee A)}$

$\neg(I \vee A) \vee H$

$= (\neg I \wedge \neg A) \vee H$

$= \boxed{(\neg I \vee H) \wedge (\neg A \vee H)}$

$\boxed{\neg H \vee G}$

(c) * see next page

(c) ① To prove mythical, assume not mythical.

$$\frac{\cancel{\neg M}, \quad (\cancel{M} \vee \neg I) \wedge (\cancel{M} \vee A)}{\neg I \wedge A}$$

$$\frac{\neg I \wedge \cancel{A}, \quad (\neg I \vee H) \wedge (\cancel{\neg A \vee H})}{\text{contradiction}}$$

$\Rightarrow \Leftarrow$ Cannot satisfy both CNF at the same time,
therefore mythical.

② To prove magical, assume not magical

$$\frac{\neg G, \quad \neg H \vee G}{\neg H}$$

$$\frac{\cancel{\neg H}, (\neg I \vee \cancel{H}) \wedge (\neg A \vee \cancel{H})}{\neg I \wedge \neg A}$$

$$\frac{\neg I \wedge \cancel{\neg A}, \quad (M \vee \neg I) \wedge (M \vee \cancel{A})}{(M \vee \neg I) \wedge M = \cancel{(M \wedge M)} \vee (M \wedge \neg I)}$$

$$\frac{M \wedge \neg I, \quad \neg M \vee I}{\text{contradiction}}$$

$\Rightarrow \Leftarrow$ $M \wedge \neg M$ contradicts, $\neg I \wedge I$ contradicts,
given the first sentence the second sentence
$\neg M \vee I$ is unsatisfiable.
Therefore, magical.

③ To prove horned, assume not horned.

$$\frac{\neg H, \quad (\neg I \vee H) \wedge (\neg A \vee H)}{\neg I \wedge \neg A}$$

$\neg I \wedge \neg A$, $(M \vee \neg I) \wedge (M \vee A)$

$(M \vee \neg I) \wedge M = (M \wedge M) \vee (M \wedge \neg I) = (M \wedge \neg I)$

$M \wedge \neg I$, $\neg M \vee I$

contradiction

$\Rightarrow \Leftarrow$ Given $M \wedge \neg I$, $\neg M \vee I$ is unsatisfiable since M must be T and I must be false, thus $\neg T \vee F = F \neq T$

Therefore the unicorn is horned. ✓

4) Figure 1:

Decomposable: Yes because for each AND, the variables in the left subcircuit and the variables in the right subcircuit that feed into the AND do not overlap.

For depth 2 ANDs,

AND: {A,B} {C,D} AND: {A,B} {C,D}

depth 4 ANDs,

AND: {A} {B} AND: {A} {B} AND: {B} {A} AND: {C} {D} AND: {D} {C} AND: {D} {C}

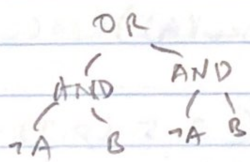
Deterministic: Yes because for each OR, the left subcircuit and right subcircuit are mutually exclusive, meaning they cannot both be high at the same time. This is due to the fact the left subcircuit always has term X as an atomic term or under AND, and the right subcircuit always has term $\neg X$ as atomic or nested under AND.

<u>Smooth</u>: No, it is not smooth because not all OR gates satisfy the property where variables in left subcircuit are exactly equal to variables in right subcircuit. For example, for the second OR to the right in depth 3, the left subcircuit variables are $\{C\}$ and the right subcircuit variables are $\{C, D\}$. $\{C\} \neq \{C, D\}$ $\therefore$ not smooth.

Figure 2:

<u>Decomposable</u>: Yes, because for all AND gates, the left subcircuits have no overlapping variables with the right subcircuits. For example, at depth 2 AND gates left sides are $\{A, B\}$ while right sides are $\{C, D\}$. At depth 4, the disjoint sets are either $\{A\} \wedge \{B\} = \emptyset$ or $\{C\} \wedge \{D\} = \emptyset$.

<u>Determinism</u>: No, because for some OR gates, the left and right subcircuits may be high at the same time. For example,



<u>Smoothness</u>: Yes, because for all OR gates, the left and right subcircuits have the same set of variables. i.e. the OR gates either have on the left and right subcircuits the variable set $\{A, B\}$, or $\{C, D\}$, or $\{A, B, C, D\}$

5) (a) $f(A,B) = (\neg A \wedge B) \vee (\neg B \wedge A)$

<u>Satisfying models:</u>

$\{A, \neg B\}$
$\{\neg A, B\}$

$\Downarrow$

$\omega(A, \neg B) = \omega(A)\omega(\neg B) = 0.1(0.7) = \boxed{0.07}$

$\omega(\neg A, B) = \omega(\neg A)\omega(B) = 0.9(0.3) = \boxed{0.27}$

$WMC(f(A,B)) = 0.07 + 0.27 = \boxed{0.34}$

(b) There is an $\boxed{\text{equality}}$ relation between the count on the root and WMC for the formula.

(c) $WMC(f(A,B,C,D)) =$

$(\omega(\neg A, B) + \omega(A, \neg B))(\omega(C, D) + \omega(\neg D, \neg C)) + (\omega(\neg A, \neg B) + \omega(B, A))(\omega(C, \neg D) + \omega(D, \neg C))$

$= (\omega(\neg A)\omega(B) + \omega(A)\omega(\neg B))(\omega(C)\omega(D) + \omega(\neg D)\omega(\neg C)) + (\omega(\neg A)\omega(\neg B) + \omega(B)\omega(A))(\omega(C)\omega(\neg D) + \omega(D)\omega(\neg C))$

$= (0.9(0.3) + 0.1(0.7))(0.5(0.7) + (0.3)(0.5)) + (0.9(0.7) + 0.3(0.1))(0.5(0.3) + 0.7(0.5))$

$= (0.27 + 0.07)(0.35 + 0.15) + (0.63 + 0.03)(0.15 + 0.35)$

$= (0.34)(0.50) + (0.66)(0.50)$

$= (0.50)(0.34 + 0.66) = 0.50(1) = \boxed{0.5}$