

1.

(a) $P(A, B, C), P(x, y, z)$	$\{ x/A, y/B, z/C \}$
(b) $Q(y, G(B, A), D), Q(G(x, x), y, D)$	Not unifiable because x cannot bind to both B and A
(c) $R(x, z, A), R(A, z, y)$	$\{ x/A, y/A \}$
(d) $Older(Father(y), John), Older(Father(x), x)$	$\{ y/John \}$
(e) $Knows(y, y), Knows(Father(x), x)$	Not unifiable

2.

1. John likes all kinds of food.
2. Apples are food.
3. Chicken is food.
4. Anything someone eats and isn't killed by is food.
5. If you are killed by something, you are not alive.
6. Bill eats peanuts and is still alive. *
7. Sue eats everything Bill eats.

(a) First order logic:

1. $\forall x Food(x) \Rightarrow Likes(John, x)$
2. $Food(Apple)$
3. $Food(Chicken)$
4. $\forall y (\exists x Eats(x, y) \wedge \neg Kills(y, x)) \Rightarrow Food(y)$
 $= \forall x \forall y Eats(x, y) \wedge \neg Kills(y, x) \Rightarrow Food(y)$
5. $\forall x \forall y Kills(x, y) \Rightarrow \neg Alive(y)$
6. $Eats(Bill, Peanuts) \wedge Alive(Bill)$
7. $\forall x Eats(Bill, x) \Rightarrow Eats(Sue, x)$

(b) CNF:

1. $\forall x \text{ Food}(x) \Rightarrow \text{Likes}(\text{John}, x)$
 $= \neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$
2. **$\text{Food}(\text{Apple})$**
3. **$\text{Food}(\text{Chicken})$**
4. $\forall x \forall y \text{ Eats}(x, y) \wedge \neg \text{Kills}(y, x) \Rightarrow \text{Food}(y)$
 $= \neg (\text{Eats}(x, y) \wedge \neg \text{Kills}(y, x)) \vee \text{Food}(y)$
 $= \neg \text{Eats}(d, e) \vee \text{Kills}(e, d) \vee \text{Food}(e)$
5. $\forall x \forall y \text{ Kills}(x, y) \Rightarrow \neg \text{Alive}(y)$
 $= \neg \text{Kills}(f, g) \vee \neg \text{Alive}(g)$
6. **$\text{Eats}(\text{Bill}, \text{Peanuts})$**
7. **$\text{Alive}(\text{Bill})$**
8. $\forall x \text{ Eats}(\text{Bill}, x) \Rightarrow \text{Eats}(\text{Sue}, x)$
 $= \neg \text{Eats}(\text{Bill}, h) \vee \text{Eats}(\text{Sue}, h)$

(c) Prove $\alpha = \text{Likes}(\text{John}, \text{Peanuts})$

9. $\neg \text{Likes}(\text{John}, \text{Peanuts})$	$\neg \alpha$
10. $\neg \text{Food}(\text{Peanuts})$	Resolution of 1, 9 with binding list: { a/Peanuts }
11. $\neg \text{Eats}(d, \text{Peanuts}) \vee$ $\text{Kills}(\text{Peanuts}, d)$	Resolution of 4, 10 with binding list: { e/Peanuts }
12. $\neg \text{Eats}(d, \text{Peanuts}) \vee \neg \text{Alive}(d)$	Resolution of 5, 11 with binding list: { f/Peanuts, g/d }
13. $\neg \text{Eats}(\text{Bill}, \text{Peanuts})$	Resolution of 7, 12 with binding list: { d/Bill }
14. \emptyset	Resolution of 6, 13

Contradiction! $\therefore \text{Likes}(\text{John}, \text{Peanuts})$ is true.

(d) Use resolution to answer the question, "What food does Sue eat?"

9. $\text{Eats}(\text{Sue}, \text{Peanuts})$	Unification of 6, 8 with binding list: { h/Peanuts }
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$\therefore \text{Sue eats peanuts.}$

(e) Use resolution to answer (d) if, instead of the axiom marked with an asterisk above, we had:

1. $\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$
2. $\text{Food}(\text{Apple})$
3. $\text{Food}(\text{Chicken})$
4. $\neg \text{Eats}(d, e) \vee \text{Kills}(e, d) \vee \text{Food}(e)$
5. $\neg \text{Kills}(f, g) \vee \neg \text{Alive}(g)$
6. $\neg \text{Eats}(\text{Bill}, h) \vee \text{Eats}(\text{Sue}, h)$
7. $\forall x (\forall y \neg \text{Eats}(x, y)) \Rightarrow \text{Die}(x)$
 $\quad = \text{Eats}(\mathbf{i}, \mathbf{j}) \vee \text{Die}(\mathbf{i})$
8. $\forall x \text{Die}(x) \Rightarrow \neg \text{Alive}(x)$
 $\quad = \neg \text{Die}(\mathbf{k}) \vee \neg \text{Alive}(\mathbf{k})$
9. $\text{Alive}(\text{Bill})$

“What food does Sue eat?”

10. $\neg \text{Die}(\text{Bill})$	Resolution of 8, 9 with binding list: { k/Bill }
11. $\text{Eats}(\text{Bill}, j)$	Resolution of 7, 10 with binding list: { i/Bill }
12. $\text{Kills}(j, \text{Bill}) \vee \text{Food}(j)$	Resolution of 4, 11 with binding list: { d/Bill, e/j }
13. $\neg \text{Alive}(\text{Bill}) \vee \text{Food}(j)$	Resolution of 5, 12 with binding list: { f/j, g/Bill }
14. $\text{Food}(j)$	Resolution of 9, 13 with binding list: {}
15. $\text{Eats}(\text{Sue}, j)$	Resolution of 6, 11 with binding list: { h/j }

\therefore Sue eats food but we cannot conclude which ones.