Nyan Tun (204607621)

1.

$a. P \Rightarrow \neg Q$
$= \neg P \vee \neg Q$
$b. Q \Rightarrow \neg P$
$= \neg Q \vee \neg P$

P	Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

$$M(\neg P \vee \neg Q) \subseteq M(\neg Q \vee \neg P), M(\neg Q \vee \neg P) \subseteq M(\neg P \vee \neg Q)$$
$$\therefore \neg P \Rightarrow \neg Q = Q \Rightarrow \neg P$$

$a. P \Leftrightarrow \neg Q$
$= (P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$
$= (\neg P \vee \neg Q) \wedge (Q \vee P)$
$b. ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$\neg P \vee \neg Q$	$Q \vee P$	$(\neg P \vee \neg Q) \wedge (Q \vee P)$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
F	T	T	T	T	F	T	T
F	F	T	F	F	F	F	F

$$M\big((\neg P \vee \neg Q) \wedge (Q \vee P)\big) \subseteq M\big((P \wedge \neg Q) \vee (\neg P \wedge Q)\big)$$
$$M\big((P \wedge \neg Q) \vee (\neg P \wedge Q)\big) \subseteq M\big((\neg P \vee \neg Q) \wedge (Q \vee P)\big)$$
$$\therefore P \Leftrightarrow \neg Q = ((P \wedge \neg Q) \vee (\neg P \wedge Q))$$

2.

$a. \alpha = (Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
$\neg(\neg Smoke \vee Fire) \vee (Smoke \vee \neg Fire)$
$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$

Smoke	Fire	$(Smoke \wedge \neg Fire)$	$(Smoke \vee \neg Fire)$	$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	F	T	T

$$M(\alpha) \neq \emptyset, M(\alpha) \neq \top$$
$$\therefore \alpha\ is\ neither\ valid\ nor\ unsatisfiable$$

$b. \alpha = (Smoke \Rightarrow Fire) \Rightarrow \big((Smoke \vee Heat) \Rightarrow Fire\big)$
$(\neg Smoke \vee Fire) \Rightarrow (\neg(Smoke \vee Heat) \vee Fire)$
$(\neg Smoke \vee Fire) \Rightarrow ((\neg Smoke \wedge \neg Heat) \vee Fire)$
$\neg(\neg Smoke \vee Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$
$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$

Smoke	Fire	Heat	$(Smoke \wedge \neg Fire)$	$(\neg Smoke \wedge \neg Heat)$	$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$M(\alpha) \neq \emptyset, M(\alpha) \neq \top$$
$$\therefore \alpha \text{ is neither valid nor unsatisfiable}$$

$c. ((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
$(\neg(Smoke \wedge Heat) \vee Fire) \Leftrightarrow ((\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire))$
$((\neg Smoke \vee \neg Heat) \vee Fire) \Leftrightarrow (\neg Smoke \vee \neg Heat \vee Fire)$
$(\neg Smoke \vee \neg Heat \vee Fire) \Leftrightarrow (\neg Smoke \vee \neg Heat \vee Fire)$
$((\neg Smoke \vee \neg Heat \vee Fire) \Rightarrow (\neg Smoke \vee \neg Heat \vee Fire)) \wedge ((\neg Smoke \vee \neg Heat \vee Fire)$
$\Rightarrow (\neg Smoke \vee \neg Heat \vee Fire))$
$(\neg(\neg Smoke \vee \neg Heat \vee Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire))$
$\wedge (\neg(\neg Smoke \vee \neg Heat \vee Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire))$
$((Smoke \wedge Heat \wedge \neg Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire)) \wedge ((Smoke \wedge Heat \wedge \neg Fire)$
$\vee (\neg Smoke \vee \neg Heat \vee Fire))$
$((Smoke \wedge Heat \wedge \neg Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire))$

Smoke	Fire	Heat	$(Smoke \wedge Heat \wedge \neg Fire)$	$(\neg Smoke \vee \neg Heat \vee Fire)$	$((Smoke \wedge Heat \wedge \neg Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire))$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

$$M(\alpha) = \top, \therefore \alpha \text{ is valid}$$

3. *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

Mythical: the unicorn is mythical
Immortal: the unicorn is immortal
Mammal: the unicorn is a mammal
Horned: the unicorn is horned
Magical: the unicorn is magical

(a)
1. $Mythical \Rightarrow Immortal$
2. $\neg Mythical \Rightarrow \neg Immortal \land Mammal$
3. $(Immortal \lor Mammal) \Rightarrow Horned$
4. $Horned \Rightarrow Magical$

(b) CNF:
1. $\neg Mythical \lor Immortal$

$Mythical \lor (\neg Immortal \land Mammal)$

2. $Mythical \lor \neg Immortal$
3. $Mythical \lor Mammal$

$\neg(Immortal \lor Mammal) \lor Horned$
= $(\neg Immortal \land \neg Mammal) \lor Horned$
= $(\neg Immortal \lor Horned) \land (\neg Mammal \lor Horned)$

4. $\neg Immortal \lor Horned$
5. $\neg Mammal \lor Horned$
6. $\neg Horned \lor Magical$

(c)
$KB \vDash^{?} Mythical$
7. $\neg Mythical$

Sentence #	Sentence	Inferred using:
8	$\neg Immortal$	Resolution of 2, 7
9	$Mammal$	Resolution of 3, 7
10	$Horned$	Resolution of 5, 9
11	$Magical$	Resolution of 6, 10

$(KB \vDash^{?} Mythical)$ $cannot\ be\ proved, since\ we\ could\ not\ infer\ any\ rules\ that\ contradicted.$

$KB \vDash^{?} Magical$
7. $\neg Magical$

Sentence #	Sentence	Inferred using:
8	$\neg Horned$	Resolution of 6, 7
9	$\neg Mammal$	Resolution of 5, 8
10	$\neg Immortal$	Resolution of 4, 8
11	$Mythical$	Resolution of 3, 9
12	$\neg Mythical$	Resolution of 1, 10

$(KB \vDash Magical)\ since\ there\ was\ a\ contradiction$

1. $\neg Mythical \lor Immortal$

$Mythical \lor (\neg Immortal \land Mammal)$

2. $Mythical \lor \neg Immortal$
3. $Mythical \lor Mammal$

$\neg(Immortal \lor Mammal) \lor Horned$
$= (\neg Immortal \land \neg Mammal) \lor Horned$
$= (\neg Immortal \lor Horned) \land (\neg Mammal \lor Horned)$

4. $\neg Immortal \lor Horned$
5. $\neg Mammal \lor Horned$
6. $\neg Horned \lor Magical$

$KB \vDash^{?} Horned$
7. $\neg Horned$

Sentence #	Sentence	Inferred using:
8	$\neg Mammal$	Resolution of 5, 7
9	$\neg Immortal$	Resolution of 4, 7
10	$Mythical$	Resolution of 3, 8
11	$\neg Mythical$	Resolution of 1, 9

$(KB \vDash Magical)\ since\ there\ was\ a\ contradiction$

4.

	Figure 1	Figure 2
Decomposable	Yes, all the gates are decomposable	Yes, all the gates are decomposable
Deterministic	No, the top-most OR gate is not deterministic since the initialization of A=1, B=1, C=1, D=1 causes both children to be true	No, there is an OR gate with inputs $(\neg A \land B)$, $(\neg A \land B)$
Smooth	No, the left-most AND gate has inputs $\neg A$ and B	Yes, all the gates are smooth

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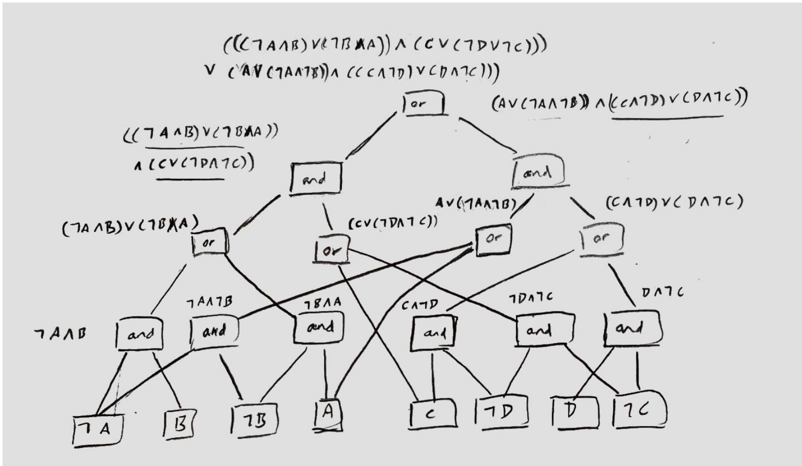

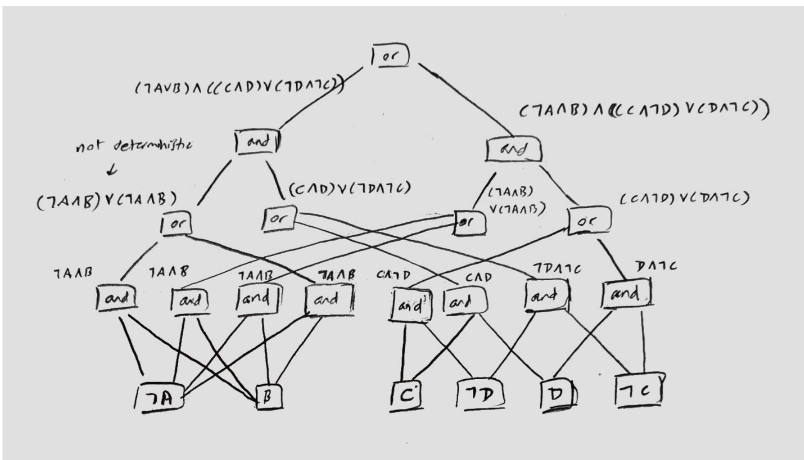

5.

A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	F	F	F

(a) Satisfiable truth assignments: A=T, B=T; A=F, B=T

$W(A, \neg B) = W(A) * W(\neg B) = 0.07$

$W(\neg A, B) \ = \ W(\neg A) * W(B) \ = \ 0.27$

= 0.03 + 0.27 = 0.34

(b) The count on the root is the same as the Weighted Model Count for a decomposable, deterministic and NNF circuit.

(c)
AND gates:
$W(\neg A, B) = 0.27$
$W(\neg A, \neg B) = 0.63$
$W(B, A) = 0.03$
$W(\neg B, A) = 0.07$
$W(C, \neg D) = 0.15$
$W(C, D) = 0.35$
$W(\neg D, \neg C) = 0.15$
$W(D, \neg C) = 0.35$

OR gates:
$W(\neg A, B) + W(\neg B, A) = 0.34$
$W(C, D) + W(\neg D, \neg C) = 0.5$
$W(\neg A, \neg B) + W(B, A) = 0.66$
$W(C, \neg D) + W(D, \neg C) = 0.5$

AND gates:
$\left(W(\neg A, B) + W(\neg B, A)\right) * (W(C, D) + W(\neg D, \neg C)) = 0.17$
$\left(W(\neg A, \neg B) + W(B, A)\right) * \left(W(C, \neg D) + W(D, \neg C)\right) = 0.33$

OR gate:

$$\left(W(\neg A, B) + W(\neg B, A)\right) * \left(W(C, D) + W(\neg D, \neg C)\right) + \left(W(\neg A, \neg B) + W(\neg A, B)\right) * (W(C, \neg D) + W(D, \neg C)) = 0.5$$