Light Beams at Nonlinear Interfaces

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Abstract

We study the behavior of self-confined beams propagating near nonlinear interfaces with saturable electro-optic nonlinearity by applying an equivalent particle theory to simplify the problem to the study of the dynamics of an equivalent particle moving under the influence of a potential which captures the behavior of the beam. The goal is to find agreement between the predictions of the equivalent particle theory and the numerical and experimental findings of the authors of [1].

Background

Photorefractive materials are media - typically crystals - for which the refractive index is a nonlinear function of the intensity of the optical field. Light beams inside photorefractive materials with sufficiently high intensity can exhibit self-trapping, in which the effect of the nonlinear refractive index balances the diffraction of the beam such that the beam creates its own waveguide as it travels through the medium. These waveguides persist and can serve as waveguides for beams of lesser intensities, and can easily be erased. The dynamic control of light beam propagation inside photorefractive materials allows for the creation of addressable waveguides in these media, and several methods to achieve this have been tested. A recently proposed technique uses external electrodes to dynamically create a nonlinear refractive index interface within the material capable of bending the trajectories of solitonic beams in a desired direction. Light beam propagation at such an interface is governed by a nonlinear Helmholtz equation for the optical field, and beam propagation can be predicted by the numerical solution of this equation as in [1]. However, an alternative approach involves the use of analogy to an equivalent particle to simplify the description of beam dynamics at the interface.

Approximations

The nonlinear Helmholtz equation for the propagation of an optical beam inside a medium with saturable electro-optic nonlinearity is given by

$$\nabla^2 A = -\frac{\epsilon_{NL} E_{bias}}{1+\frac{|A|^2}{|A_{sat}|^2}} A \tag{1}$$

where ϵ_{NL} is the nonlinear delectric constant, E_{bias} is an electrical bias induced by external electrodes, and $|A_{sat}|^2$ is the saturation intensity of the nonlinearity.

We restrict our attention to beams traveling in the z direction and confined to the $x-z$ plane, so the Laplacian reduces to $\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2}$. We will also use the slowly-varying envelope approximation, assuming that the field has only a narrow band of z wavenumbers, to write $A(x,z) = F(x,z)e^{i\beta z}$ where F is the envelope of the field and $e^{i\beta z}$ captures the fast variations in A. Inserting into (1), using the unidirectionality of the beam to say $|\frac{\partial^2 F}{\partial z^2}| << |2\beta \frac{\partial F}{\partial z}|$, and making the change of variables $\tau = \frac{z}{2\beta}$ we find that F satisfies

$$i\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial x^2} - \left(\beta^2 - \frac{\varepsilon_{NL} E_{bias}}{1+\frac{|F|^2}{|A_{sat}|^2}}\right) F = 0. \tag{2}$$

Using the approximation $\frac{1}{1+x} \approx 1 - x$ for small x, (2) is approximately

$$i\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial x^2} + \frac{\varepsilon_{NL} E_{bias}}{|A_{sat}|^2} F|F|^2 = (\beta^2 + \varepsilon_{NL} E_{bias})F \tag{3}$$

which is the Nonlinear Schrödinger Equation but with the additional term $(\beta^2 + \epsilon_{NL} E_{bias})F$.

Equivalent Particle Theory

Using an equivalent particle representation of the beam, we can describe the dynamics of the light beam at the interface a dynamical system. By equivalent particle, we mean that we associate to a light beam an analogous particle traveling towards the interface while under the influence of a potential.

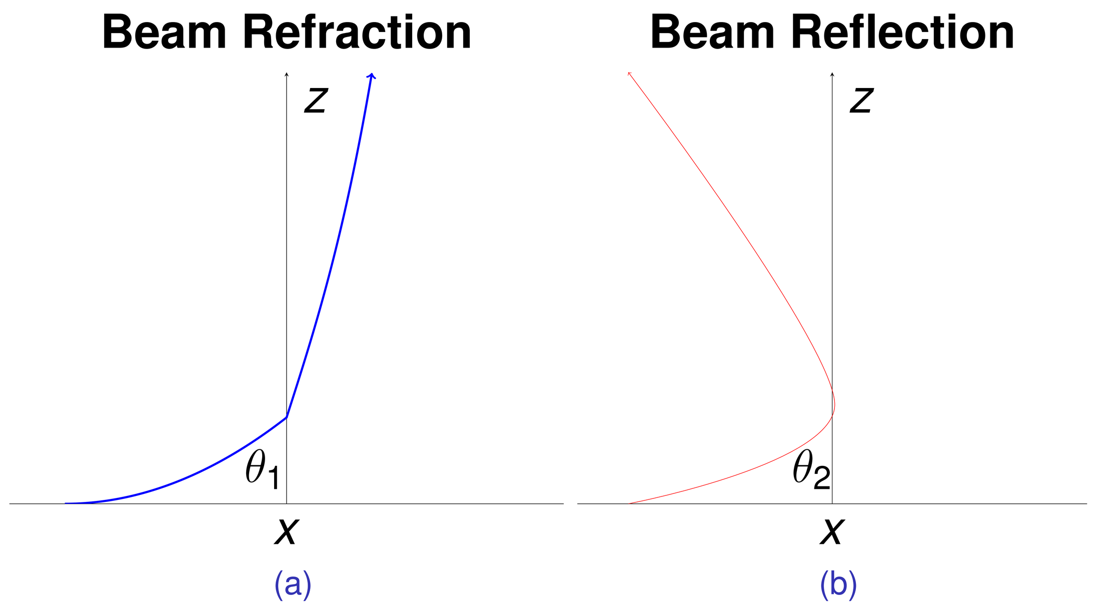

Figure: [1] Light beam refraction and reflection.

Given an equation of the form

$$i\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial x^2} + 2A|A|^2 = VA \tag{4}$$

we make the following definitions:

$$\begin{cases} p = \int_{-\infty}^{\infty} AA^* \, dx, \\ \overline{x} = \frac{1}{p}\int_{-\infty}^{\infty} xAA^* \, dx, \\ v = \frac{i}{p}\int_{-\infty}^{\infty} \left(A\frac{\partial A^*}{\partial x} - A^*\frac{\partial A}{\partial x}\right) dx. \end{cases} \tag{5}$$

Differentiating with respect to τ, we find that

$$\begin{cases} \frac{dp}{d\tau} = 0, \\ \frac{d\overline{x}}{d\tau} = v, \\ \frac{dv}{d\tau} = -\frac{2}{p}\int_{-\infty}^{\infty} \frac{\partial V}{\partial x} AA^* \, dx \end{cases} \tag{6}$$

We interpret the dynamical system as describing the motion of a particle with constant momentum traveling under the influence of the potential $V(x)$ with position $\overline{x}$ and velocity v. Here the velocity v is related to the angle of incidence of the beam at the interface. The goal of this project is to find the perturbation potential $V(x)$ that allows us to put (3) into the form (4) so that (6) can be applied to predict trajectories of light beams at the interface.

Equivalent Particle

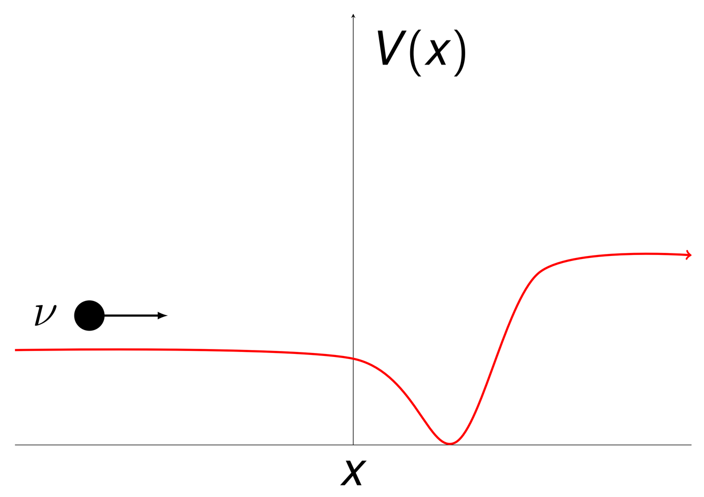

Figure: [2] An equivalent particle in motion under the influence of a potential. The velocity v determines whether the light beam is reflected or refracted at the interface.

Validation

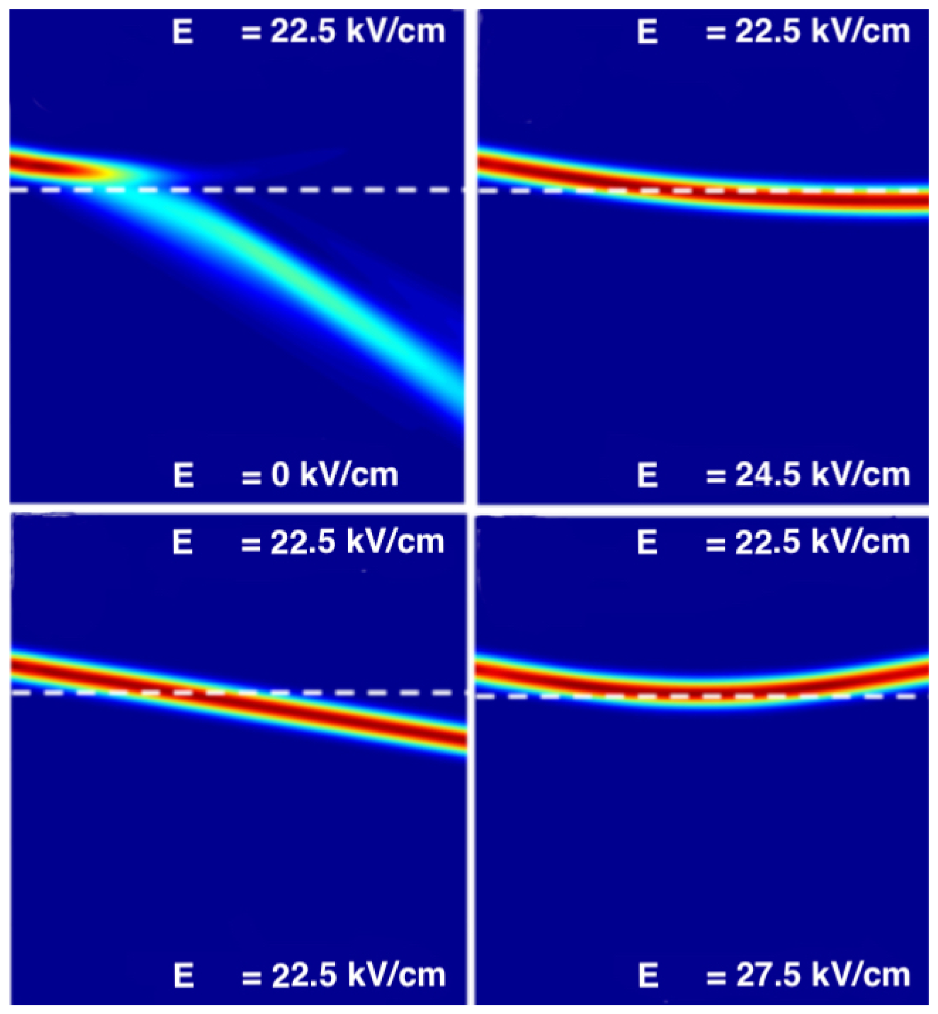

Figure: [3] Numerical solutions of (1), representing a beam with $\lambda = 532$ nm propagating across the nonlinear interface as E_{bias} is varied on either side of the interface (dashed line). Graphic from [1].

The authors of [1] carry out a joint numerical and experimental analysis of beam propagation inside Lithium Niobate crystals. Specifically, the authors simulate the propagation of a beam with extraordinary polarization that excites the optical nonlinearity, as well as a secondary beam that does not excite the nonlinearity and propagates through the waveguide created by the first beam.

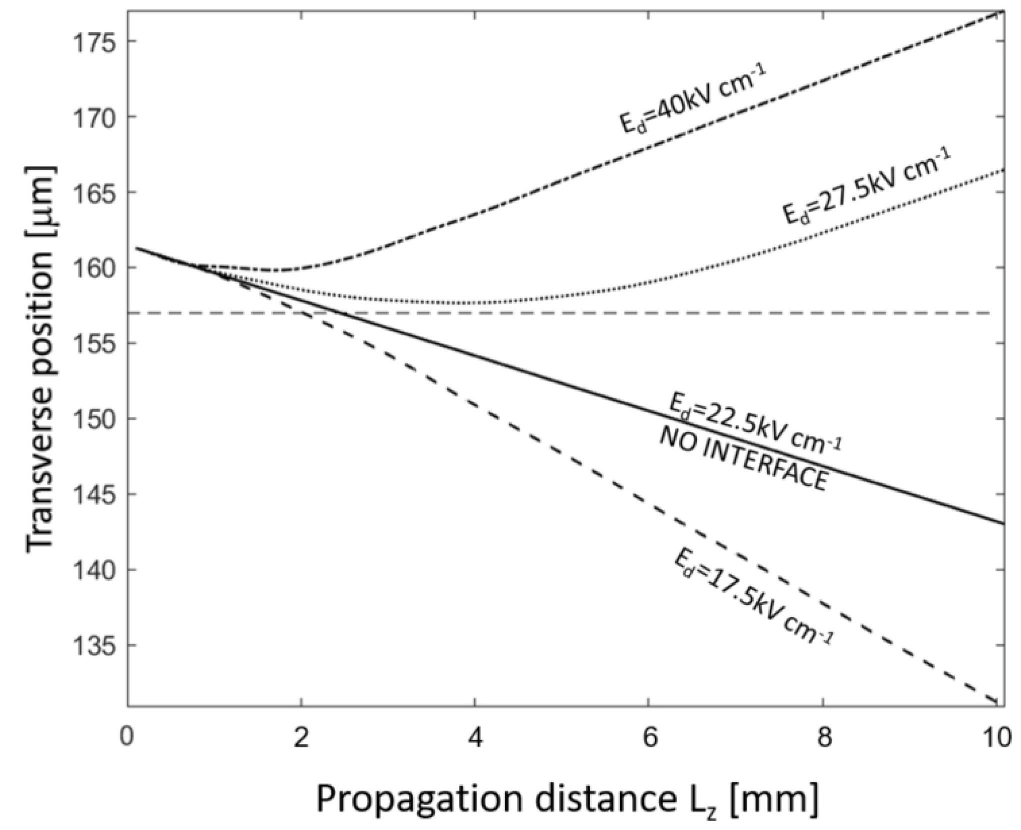

Figure: [4] Experimental behavior of beams at the interface as E_{bias} is varied. Graphic from [1].

Our next steps will be to work towards finding the appropriate perturbation potential $V(x)$ in the case of saturable electro-optic nonlinearity, and proceed to compare the equivalent partcle model to the findings presented [1].

References

[1] ALONZO, M., ET AL. "Solitonic Waveguide Reflection at an Electric Interface." Optics Express, Vol. 27, no. 15, 2019, p. 20273.

[2] ACEVES, ALEJANDRO. Snell's Law at the Interface Between Nonlinear Dielectrics. 1988. The University of Arizona, PhD dissertation.

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