Uncertainty Quantification of Ice Fog Events

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1. Introduction

Ice fog is a cold-weather phenomenon in which clouds of small water particles freeze in the in the air at around -38°C, usually due to water pollution (from power plants, heaters, cargo planes, etc.). Ice fog poses many challenges for both land and air transportation, as it can severely inhibit visibility. Weather stations currently lack an accurate method of forecasting ice fog since these events are geographically localized and cannot be accurately predicted using a spatially-coarse Global Forecasting System (GFS) predictive grid. The goal of this research is to use deterministic GFS prediction data and corresponding local observation data to build a corrective model for localized weather which captures the uncertainty in this correction.

Figure 1: An ice fog event.

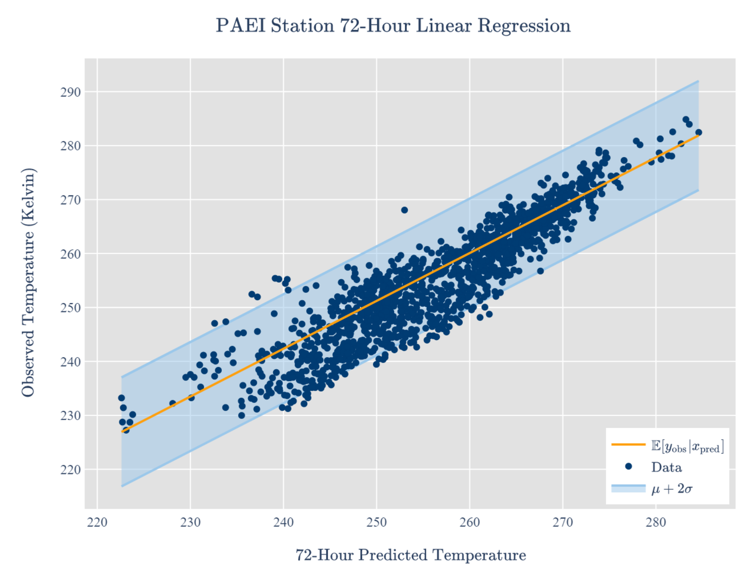
We confined our study to ice fog events near Fairbanks, AK occurring during the winters of 2015 to 2019. There are no weather databases which specifically track ice fog events, so we defined ice fog events to be days where observed temperature is less than -38°C. For the 11 weather stations in Fairbanks, AK, we collected time series of predicted temperatures (interpolated from the GFS grid) and the reported observed temperatures at these stations. We considered predictions made with varying time, ranging from 12 to 72 hours in advance.

Given a vector of independent variables $\mathbf{x} \in R^n$ for a fixed weather station, we seek to construct a model of the R^{n+1}-dimensional joint density $p(\mathbf{x}, y)$ where y denotes an observed temperature value. Here, $\mathbf{x}$ may consist of several meteorological variables, in our case, we consider x to be comprised of the predicted temperatures at multiple horizons in advance. Using the modified joint density, we can then compute the density

$$p(y \,|\, \mathbf{x}) = \frac{p(\mathbf{x}, y)}{\int_R p(\mathbf{x}, y)\, dy} \tag{1}$$

to recover a distribution over the observed temperature value we expect to observe. This formula can be used to post-process each predicted temperature value in a time series of predictions, allowing us to compute a "corrected" predictive time series and its credible intervals in time. To model the joint density $p(\mathbf{x}, y)$, we compare the results of three approaches: Bayesian Regression, Normalizing Flows, and Polynomial Transport Maps.

2. Bayesian Regression

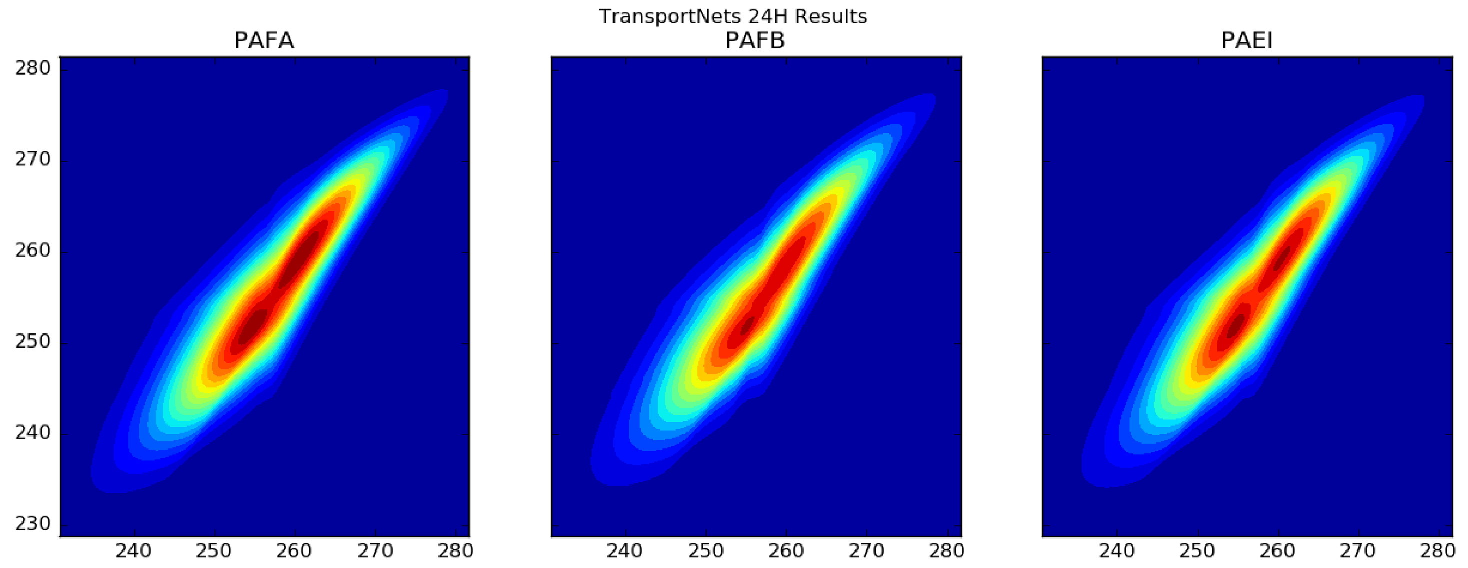


To establish a baseline, we fit Bayesian linear regressions of the form

$$y(\mathbf{x}) = \sum_{j=0}^{N} c_j x_j + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2) \tag{2}$$

where we $\mathbf{x}$ represents the prediction data, y represents the observed data, and ϵ is a multivariate Gaussian noise, and we seek to solve the coefficients c_j. Assuming an unknown fixed variance for the noise, we used MCMC to obtain model for each station.

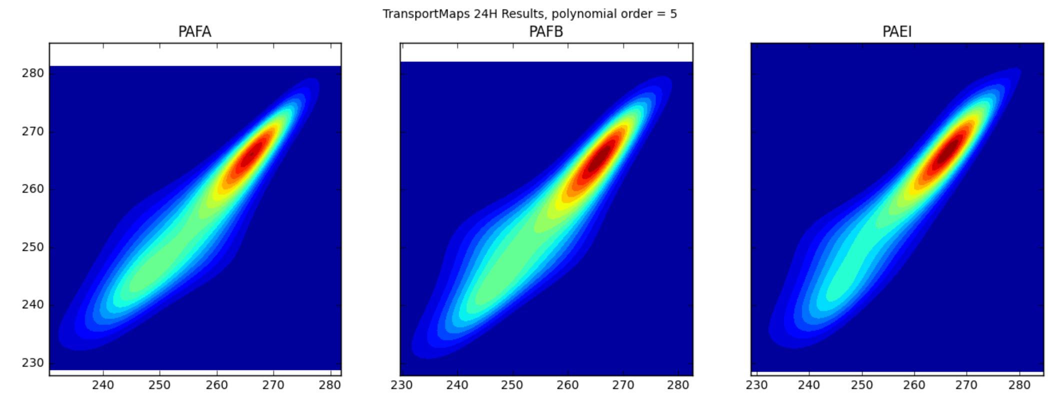
3. Normalizing Flows



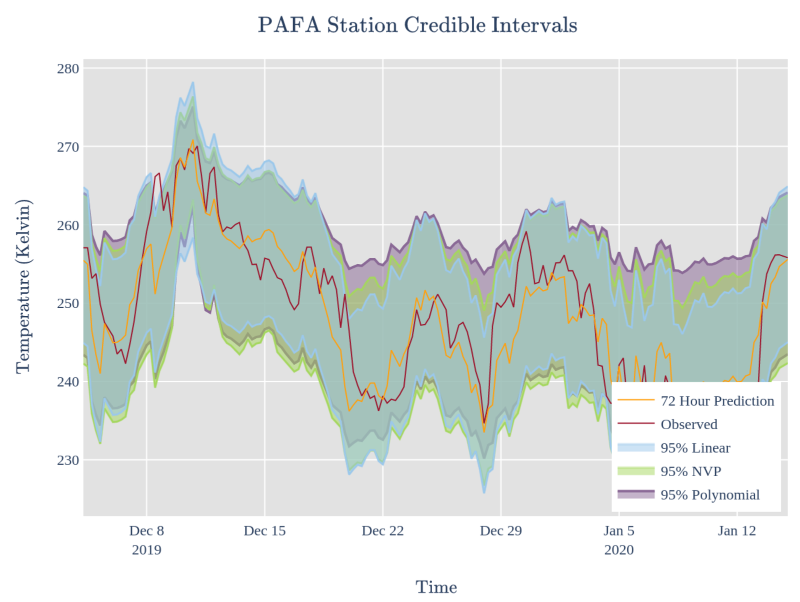
Normalizing flows are a chain of transformations that take us from the reference to the target. The chain of invertible functions $T_{i\,i=1}^{N}$ is implemented in a neural net such that each of the functions T_i has its own set of parameters θ_i. The neural net is trained to optimize the choice of parameters that maximizing the log-likelihood for the observations. For our model, we used a Gaussian distribution as our reference, and the observation versus forecast data as our target. The plots below shows a potential probability distribution for the observed temperature verses the 24H predicted temperature.

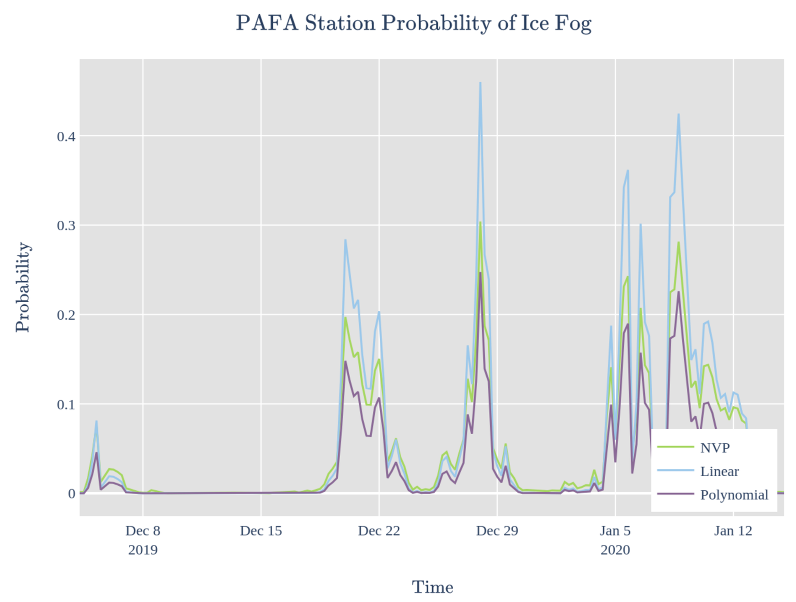
4. Transport Maps

A transport map pushes forward a reference measure, here, a Gaussian distribution, to a target measure. The map is parameterized by expanding it as multivariate polynomials. Here, since we only have samples from target measure, we use those samples to compute an inverse transformation to the reference that pushes forward the target to the reference through convex optimization. We invert the inverse map to get a direct approximate transport map that pushes forward the reference to the target. The samples generated from this approximate target are compared with the actual target.



5.Results





After completing the computations using the three different models, we aimed to compare each one and attempt to quantify the quality of their results. In hope to correct the GFS to better capture the local temperature complexity, we applied our three obtained statistical relationship to the GFS data. We first re-plotted the initial time series of the observed temperature versus the prediction, however, this time we included 95% credible intervals that show how each method captures uncertainty. In the below examples, we applied our algorithm to the temperature prediction for the 72 hours forecast at the Fairbanks weather station. The colored interval shows the 95 percent credible for the respective methods. Some conclusion from the modeling results includes :

- The linear method is too simple to capture the relationship of our data.
- Compared value of log likelihood for transport maps and normalizing flows, observed that transport maps shows a more accurate trend.

After correcting the existing weather forecast, we generated a probability prediction for ice-fog events at Fairbanks station across the winter season. It is interesting to know that linear method tends to show a higher probability distribution than the other two methods, showing a higher degree of belief of ice fog events on certain days. As we keep in mind that in predicting catastrophic events such as ice-fog, it would be more beneficial to over predict rather than under predict. Also note that real time observation for ice-fog events are scattered and rather not trust worthy. As a result, we are unable to compare our model to real time observations like we did for our temperature predictions.

6. References

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2. Marzouk, Y., Moselhy, T., Parno, M., and Spantini, A. An introduction to sampling via measure transport. arXiv:1602.05023, 2016.