Laboratory 2: Modeling and Simulating in SimScape

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1 Assignment 1

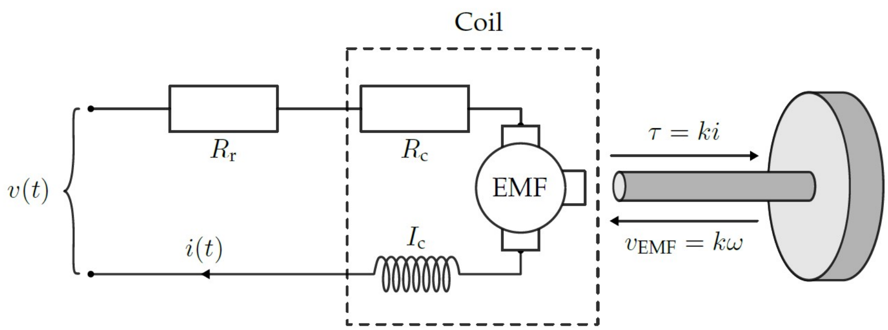

Figure 1: The motor.

1.1 State-space model of the servo motor

If we choose i(t) as our state variable, and applying the Kirchhoff's voltage law gives:

$$v(t) = (R_r + R_c) \cdot i(t) + v_{EMF} + L \cdot \frac{di(t)}{dt}$$

Where $v_{EMF} = k \cdot \omega$. On the other hand the angular velocity of the motor is equal to the total torque divided by the moment of inertia.

$$\frac{d\omega}{dt} = \frac{\tau_{total}}{J} = \frac{k \cdot i(t) - \beta \cdot \omega}{J}$$

Therefore, the state-space model:

$$\frac{di(t)}{dt} = -\frac{R_r + R_c}{L} \cdot i(t) - \frac{k}{L} \cdot \omega + \frac{1}{L} \cdot v(t) \tag{1}$$
$$\frac{d\omega}{dt} = \frac{k}{J} \cdot i(t) - \frac{\beta}{J} \cdot \omega \tag{2}$$
$$\frac{d\theta}{dt} = \omega \tag{3}$$

1.2 Numerical values of the model parameters

From the table of specifications for the motor **M 586 0585**:

- $R_r + R_c$ *(Terminal resistance)*: 1.15 $[\Omega]$
- R_c *(Armature resistance)*: 0.8 $[\Omega]$
- L *(Armature inductance)*: 3.39 $[mH]$
- J *(Rotor moment of inertia)*: $3.88 \cdot 10^{-5}$ $[kg \cdot m^2]$

Regarding damping constant (k), the continuous stall current (i_{csc}) is 3.90 $[A]$. We know that:

$$\begin{aligned} \tau_{el} &= k \cdot i \\ &= \tau_{brake} + \tau_{damp} \end{aligned}$$

$$\begin{aligned} k \cdot i_{csc} &= \tau_{brake} + \beta \cdot \omega \\ \tau_{brake} &= k \cdot i_{csc} - \beta \cdot \omega \end{aligned}$$

$$k = \frac{\tau(\omega = 0)}{i_{csc}} = \frac{0,2\ [N \cdot m]}{3.90\ [A]} = 0,051 \left[\frac{N \cdot m}{A}\right] \tag{4}$$

We know that the torque:

$$\tau = \frac{P_{max}}{\omega}$$

Where the power is the same for each operational point. Then, we can compute β as the slope of the curve:

$$\beta = -\frac{\Delta\tau}{\Delta\omega} = \frac{0.1\ [N \cdot m]}{8000 rpm} = 1,25 \cdot 10^{-5} \left[\frac{N \cdot m}{rpm}\right] \cdot \frac{1}{\frac{2\pi\ [rad]}{60\ [s \cdot rpm]}} = 1,10 \cdot 10^{-7} \left[\frac{N \cdot m \cdot s}{rad}\right] \tag{5}$$

Finally, we have defined the parameters on Matlab.

```matlab
%% Assignment 1

Rc_Rr = 1.15; % (Rc + Rr) Ohm
k = 0.051; % Nm/A
beta = 1.19 * 10^(-7); % Nm/s
L = 3.39*10^(-3); % H
J = 3.88*10^(-5); % kg*m^2
```

Figure 2: Parameters assignment 1.

1.2.1 Block diagram - Simulink

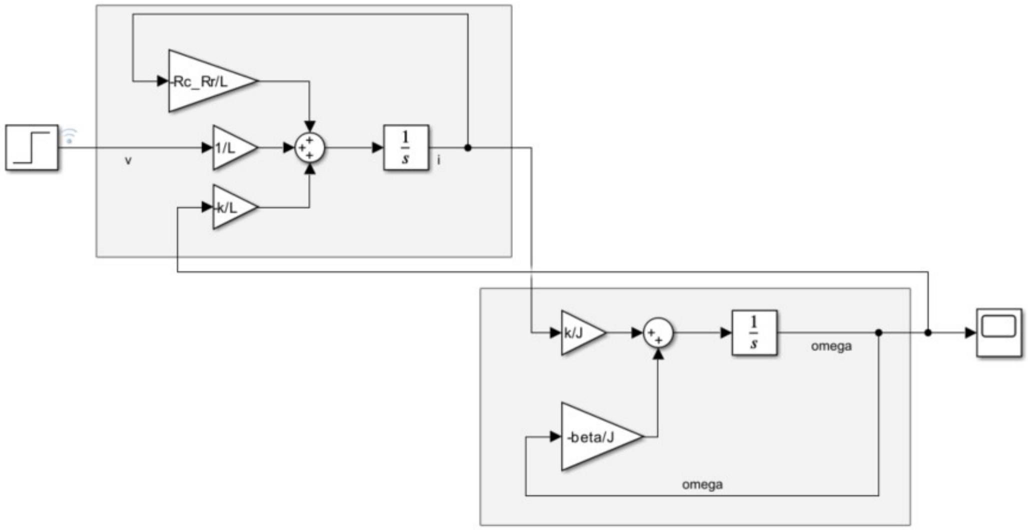

Figure 3: Simulink.

1.2.2 Object oriented model - SimScape

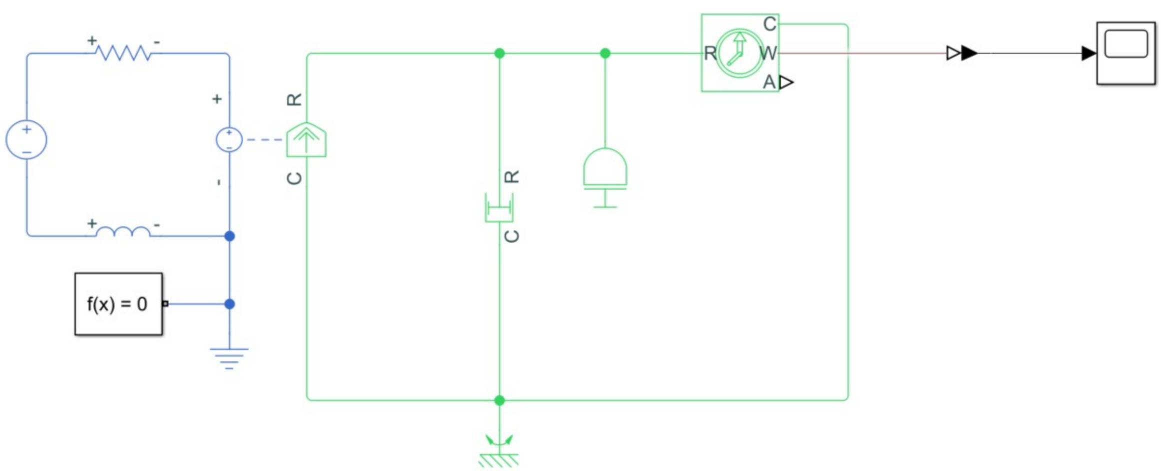

Figure 4: SimScape.

Do they match?

1.3 Simulation

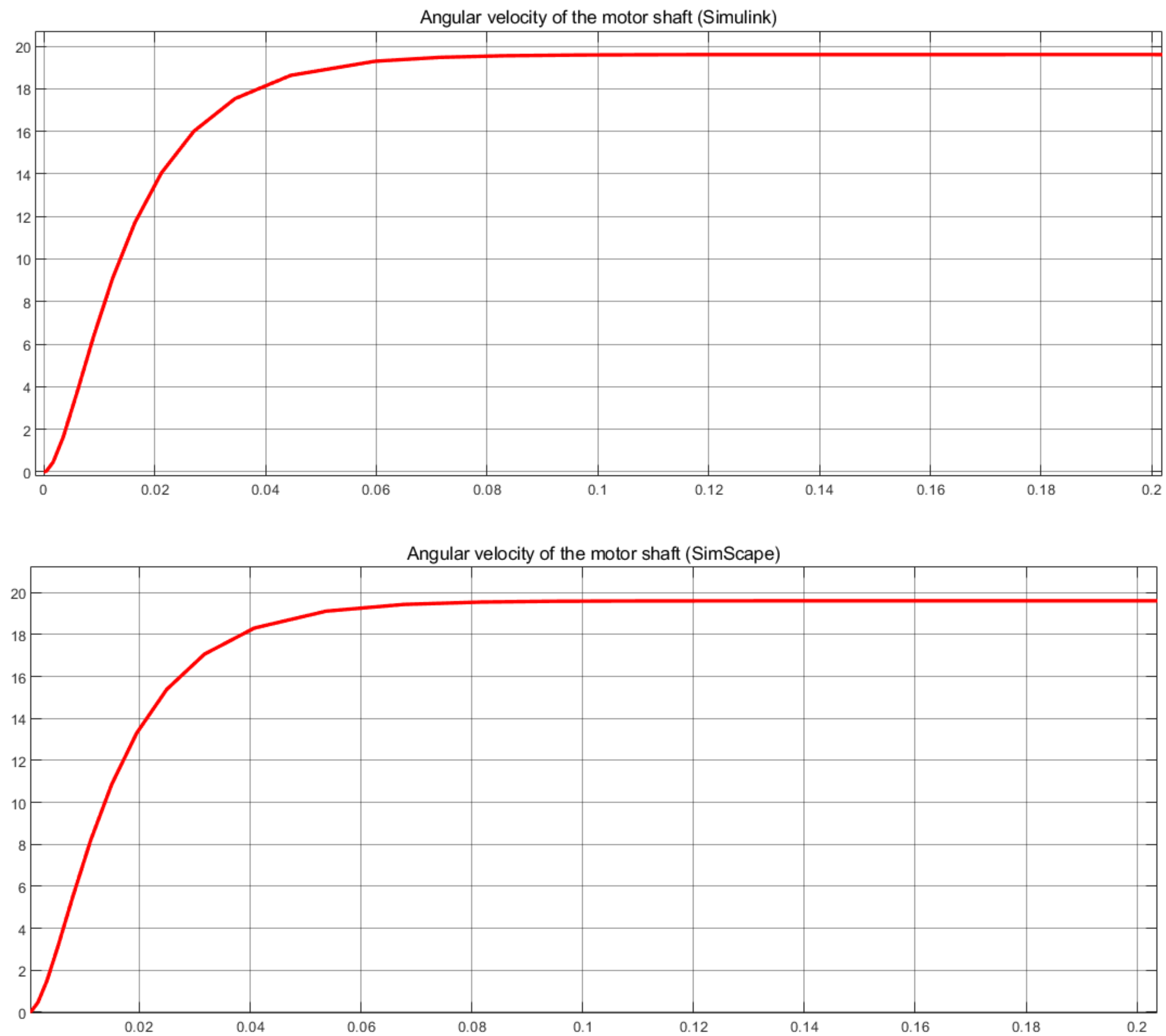

Figure 5: Simulations with Simulink and SimScape.

1.4 Differences between object-oriented and block diagram based modelling

Discuss the differences between object-oriented and block diagram based modeling. What are the benefits, and what are the drawbacks?

1.4.1 Object oriented modelling

Benefits:

- Easier to implement.
- Graphical representation is easy.

Drawbacks:

- Not a component for everything. Thus, new components must be added.

1.4.2 Block diagram based modelling

Benefits:

- Does not depend on interconnecting components.
- Modelling more flexible.

Drawbacks:

- Not always possible.
- Hard to implement.
  - Adding additional components add state-space variables which change the model
  - Hard to build reusable components.
- Graphical representation hard to understand.

Which one do you prefer in this case? Why?
In this case is better the object oriented modelling because it's easier to implement.

1.5 Motor driven by an ideal current source

Now, assume that the servo motor is driven by an ideal current source, and remove any inductor. To model these changes, would you use Simulink or SimScape? Why?

Now, we would use SimScape. Because the inductance L can just be removed whereas we need a new state-space model in Simulink. Therefore, the current $i(t)$ is not a state-space variable anymore.

2 Assignment 2

As in the previous case, we have defined the parameters on Matlab.

```matlab
%% Assignment 2

% Wheel
d_wheel = 20; % diameter of the wheel (mm)
r_wheel = (d_wheel/2)*10^(-3); % radius of the wheel (m)
density = 2.7*10^(-3); % Aluminiumm density (kg/m^3)
thickness = 10*10^(-3); % Thickness of the wheel (m)
m_wheel = ((pi*r_wheel)^2)*thickness*density; % mass of the wheel (kg)
J_wheel = (1/2)*m_wheel*(r_wheel^2); % Inertia of the wheel (kg*m^2)

kr = 75; % Spring constant (N/m)
b = 25; % Friction coefficient (Ns/m)
m = 2; % mass (kg)
```

Figure 6: Parameters assignment 2.

2.0.1 Object oriented model - SimScape

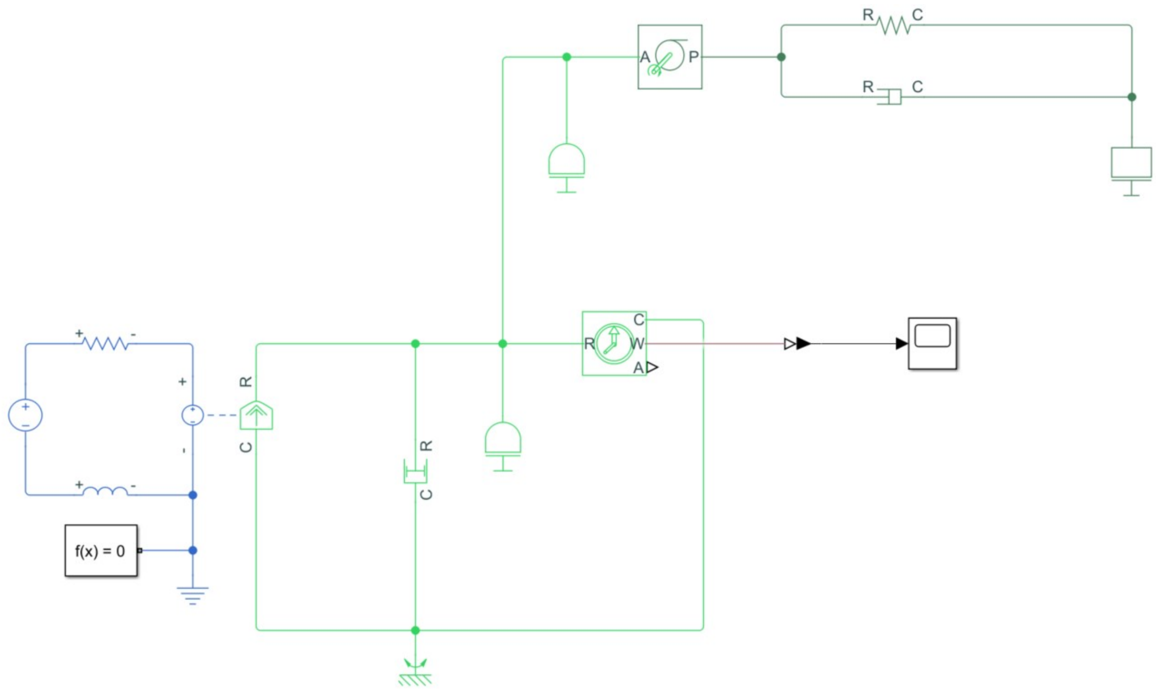

Figure 7: SimScape.

2.0.2 Simulation

Simulation results for the velocity of the load in SimScape.

2.1 Variation in the friction coefficients or spring constants

Is the system sensitive to variations in the friction coefficients or spring constants? Which system properties change when the friction coefficients or spring constants are varied?