How VIVA Works

M. Triantafyllou

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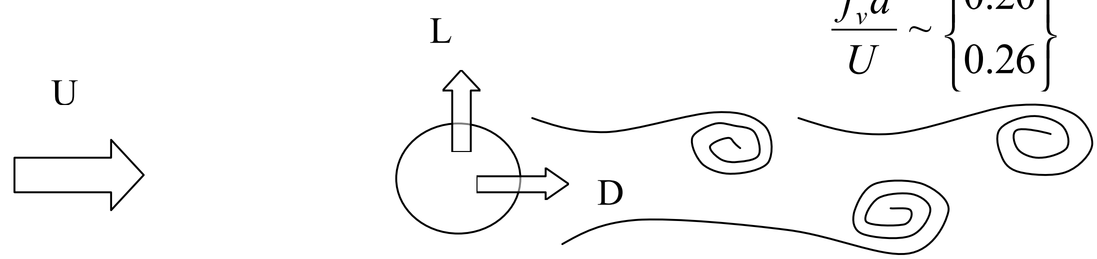
<u>HOW EXISTING VIVA WORKS</u>

$$\frac{f_v d}{U} \sim \begin{Bmatrix} 0.20 \\ 0.26 \end{Bmatrix}$$

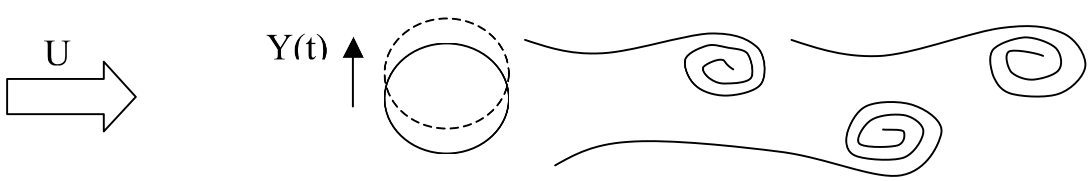


Vortices cause oscillatory forces L, D

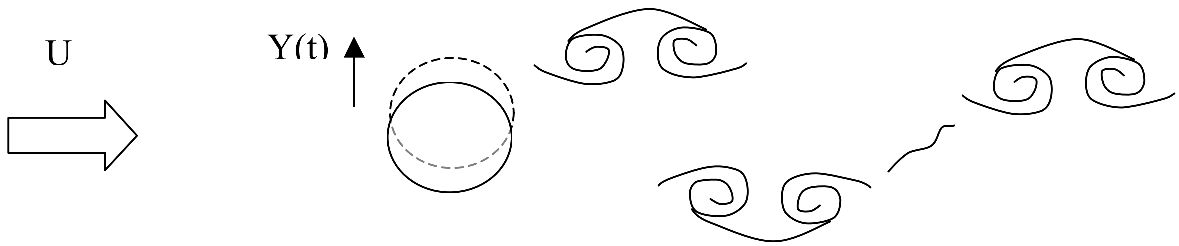
$$L = \frac{1}{2}\rho d U^2 C_L \cos(\omega_v t)$$

$$D = \frac{1}{2}\rho d U^2 \overline{C_D} + \frac{1}{2}\rho d U^2 \widetilde{C}_D \cos(2\omega_v t + \psi)$$

These forces cause the cylinder to oscillate if flexibly attached. Once the cylinder starts moving the <u>forces change</u>. Even the vortical patterns change.



$$y(t) = 0.20 d \cos(\omega t)$$



$$y(t) = 0.60d\cos(\omega t)$$

Lift forces L causes Transverse oscillations

$$y(t) = A\cos\omega t$$

$$\frac{dy}{dt}(t) = -A\omega\sin\omega t$$

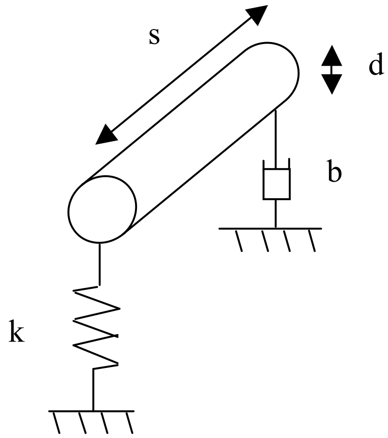
$$\frac{d^2y}{dt^2}(t) = A\omega^2\cos\omega t$$

$$L(t) = \frac{1}{2}\rho dU^2 C_L\cos(\omega t + \phi)$$

$$= \frac{1}{2}\rho dU^2 C_{L\alpha}\cos\omega t - \frac{1}{2}\rho dU^2 C_{LV}\sin\omega t$$

$$C_{L\alpha} = C_L\cos\phi$$

$$C_{LV} = C_L\sin\phi$$



$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = L(t)$$

$$-m\omega^2 A\cos\omega t - bA\omega\sin\omega t + kA\cos\omega t$$
$$= \frac{1}{2}\rho ds U^2 C_{L\alpha}\cos\omega t - \frac{1}{2}\rho ds U^2 C_{LV}\sin\omega t$$

$\Rightarrow$

$$-(m\omega^2 A + \frac{1}{2}\rho s d U^2 C_{L\alpha}) + kA = 0 \qquad (1)$$

$$\frac{1}{2}\rho ds U^2 C_{LV} = bA\omega \qquad (2)$$

Two equations in two unknowns (A,ω)

$$\frac{1}{2}\rho s d U^2 C_{L\alpha} = \frac{\pi}{4}\rho d^2 sA\omega^2 C_m$$

$$\left[m + \left(\frac{\pi}{4}\rho d^2 s\right)C_m\right]\omega^2 = k$$

$$m_a = \left(\frac{\pi}{4}\rho d^2 s\right)C_m$$

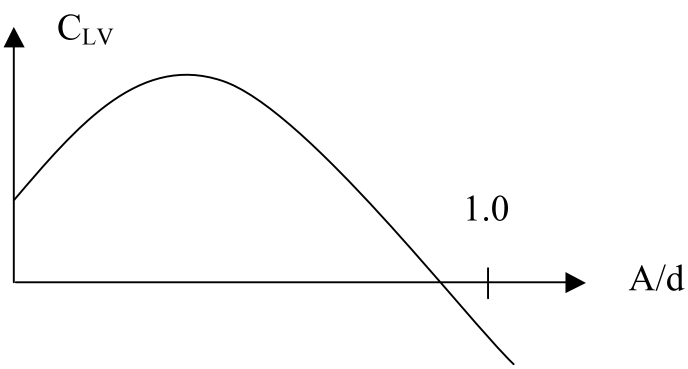
$$\boxed{\omega = \sqrt{\frac{k}{m + m_a}}} \tag{1a}$$

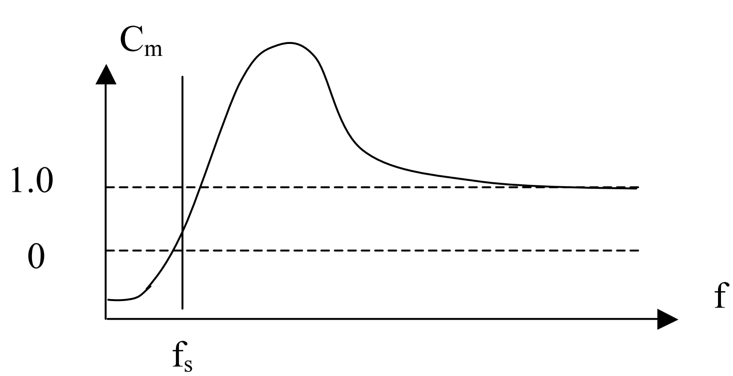
$$\boxed{A = \frac{\left(\frac{1}{2}\rho d s U^2\right) C_{LV}}{b\omega}} \tag{2a}$$

Except that $$C_m\left(\frac{A}{d}, \omega\right)$$

$$C_{LV}\left(\frac{A}{d}, \omega\right)$$

SOULUTION IS ITERATIVE AND NEEDS EXPERIMENTAL DATABASE FOR C_m, C_{LV}



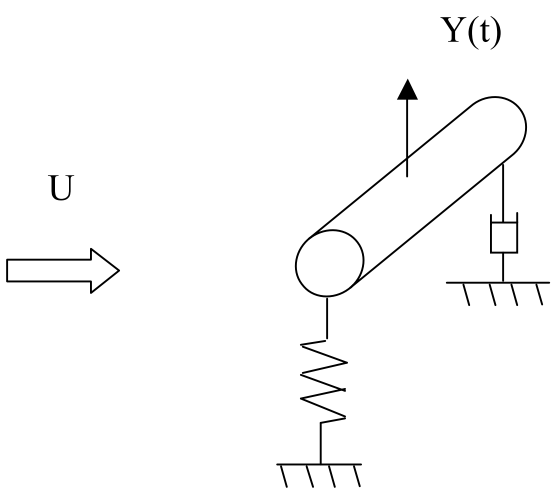


Change in added mass is <u>dramatic</u>

$$\omega = \sqrt{\frac{k}{m + \left(\frac{\pi}{4}\rho d^2 s\right)C_m}} = \sqrt{\frac{k / \left(\frac{\pi}{4}\rho d^2 s\right)}{m^* + C_m}}$$

$$m^* = \frac{m}{\frac{\pi}{4}\rho d^2 s}$$ (mass ratio)

if $m^* < 2$, $m^* + C_m$ varies a lot



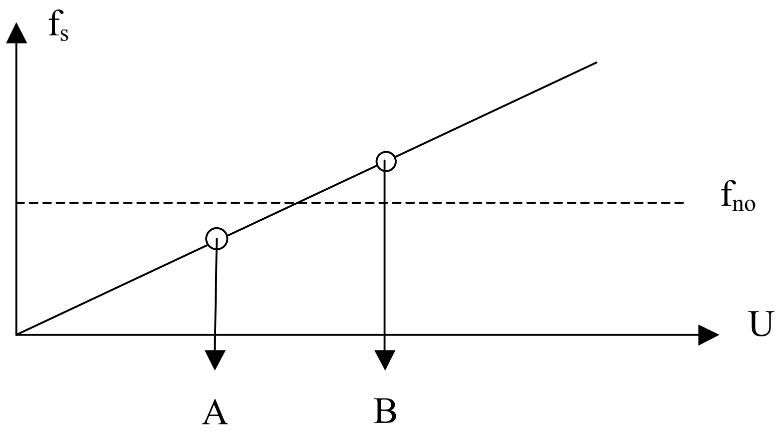
$$f_{no} = \frac{1}{2\pi}\sqrt{\frac{k}{m + m_a}}$$

Mass for C_m=1

Change U, increasing it from 0 up

Frequency of vortex shedding f_s

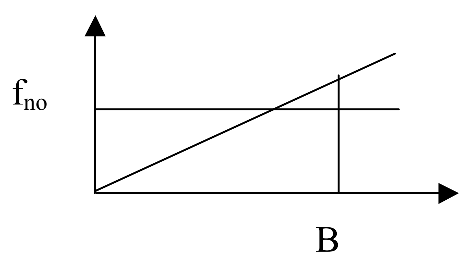
$$f_s = st\frac{U}{d}$$



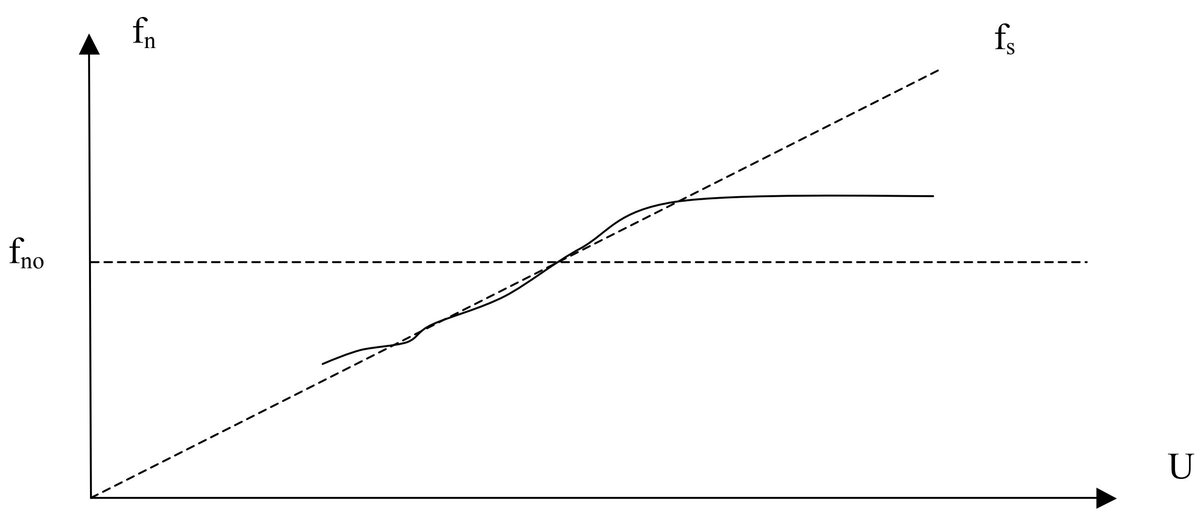
At A $f_{no} > f_s$ so $C_m > 1$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_a}} < f_{no}$$

At B $f_{no} < f_s$ so $C_m < 1$



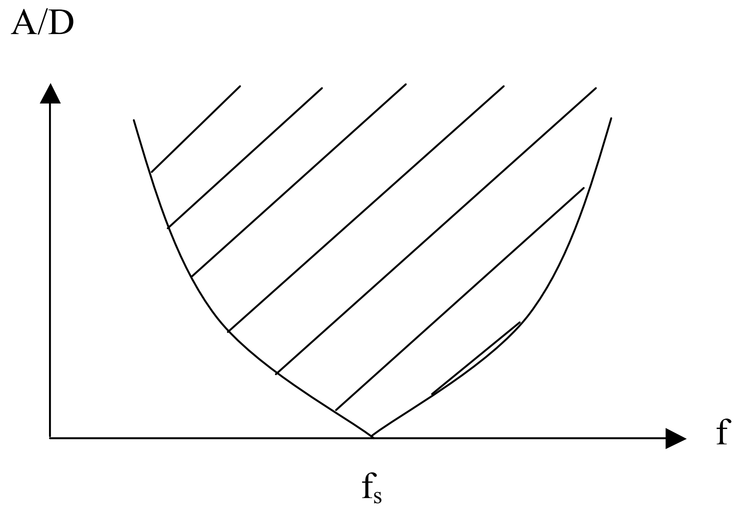
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_a}} > f_{no}$$



CAPTURE REGION

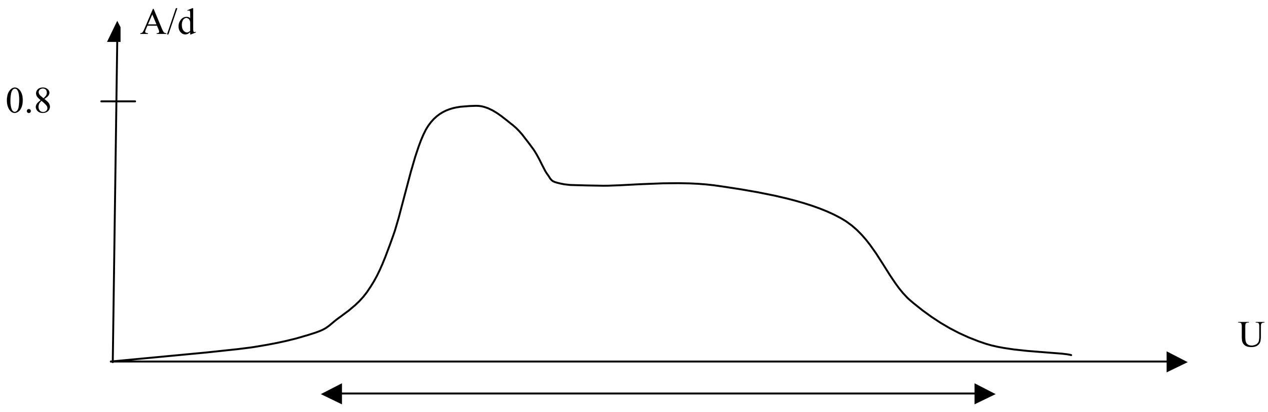
(or <u>lock-in</u>)

$\boxed{f_V = f}$

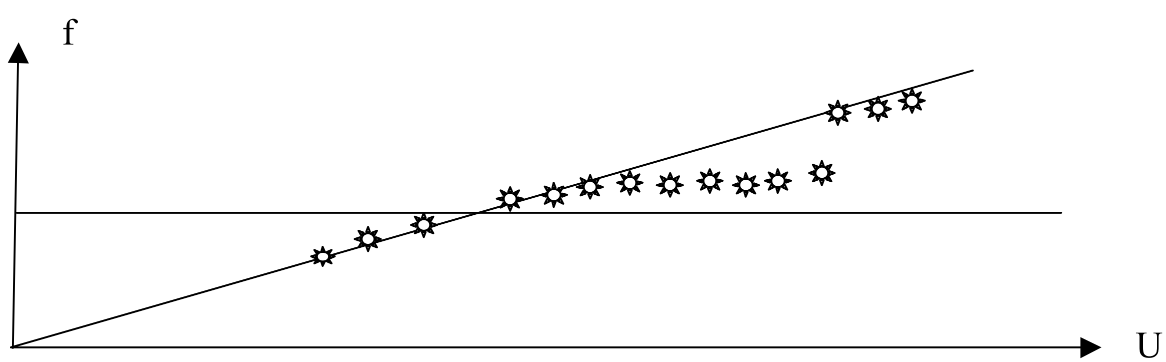


NATURAL FREQUENCY OF SYSTEM CHANGES WITH U

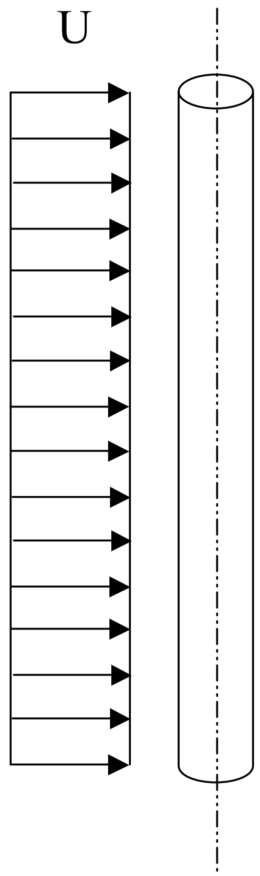
DEPENDING ON A, ω VORTICES MAY SHED AT *f* NOT f_s



Response is much wider then resonance



FLEXIBLE BEAM



$$m\frac{\partial^2 y}{\partial t^2} + b\frac{\partial y}{\partial t} - \frac{\partial^2}{\partial x^2}\left\{EI\frac{\partial^2 y}{\partial x^2}\right\}$$

$$+\frac{\partial}{\partial x}\left\{T\frac{\partial y}{\partial x}\right\} = L(x,t)$$

LIFT FORCE VARIES WITH X

Vortex shedding pattern can vary with x

$\theta_n(x)$ is a natural mode with $C_m = 1$, at frequency ω_n

$$-(m+\hat{m}_a)\omega_n{}^2\theta_n - \frac{d^2}{dx^2}\left\{EI\frac{d^2\theta_n}{dx^2}\right\} + \frac{d}{dx}\left\{T\frac{d\theta}{dx}\right\} = 0$$

$\hat{m}_a$ for $C_m = 1$

Set $y(x,t) = \theta_n(x)A\cos\omega t$

Assuming $\omega \cong \omega_n$

$$m\frac{\partial^2 y}{dt^2}+b\frac{\partial y}{\partial t}+\left\{-\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y}{\partial x^2}\right]\right\}+\frac{\partial}{\partial x}\left[T\frac{\partial y}{\partial x}\right]$$

$$=m\frac{\partial^2 y}{dt^2}+b\frac{\partial y}{\partial t}+A\cos\omega t\left\{-\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2\theta_n}{\partial x^2}\right]\right\}+\frac{\partial}{\partial x}\left[T\frac{\partial\theta_n}{\partial x}\right]$$

$$=m\frac{\partial^2 y}{dt^2}+b\frac{\partial y}{\partial t}+A\cos\omega t\left\{+\left(m+\hat{m}_a\right)\omega_n^{\ 2}\theta_n\right\}$$

$$=L(t)=-m_a\frac{\partial^2 y}{\partial t^2}-\frac{1}{2}\rho dU^2 C_{LV}\sin\omega t$$

$$\left\{-\left(m+m_a\right)\omega^2+\left(m+\hat{m}_a\right)\omega_n^{\ 2}\right\}=0$$

$$b\omega A\theta_n(x)=\frac{1}{2}\rho dU^2 C_{LV} \qquad ?$$

$$\int_0^L\left(b\frac{\partial y}{\partial t}\right)\frac{\partial y}{\partial t}dx=\int_0^L\left(\frac{1}{2}\rho dU^2 C_{LV}\right)\frac{\partial y}{\partial t}dx$$

$$\boxed{A=\frac{\int_0^L\frac{1}{2}\rho dU^2 C_{LV}\left|\theta_n(x)\right|dx}{\int_0^L b\omega\theta_n^{\ 2}(x)dx}}$$

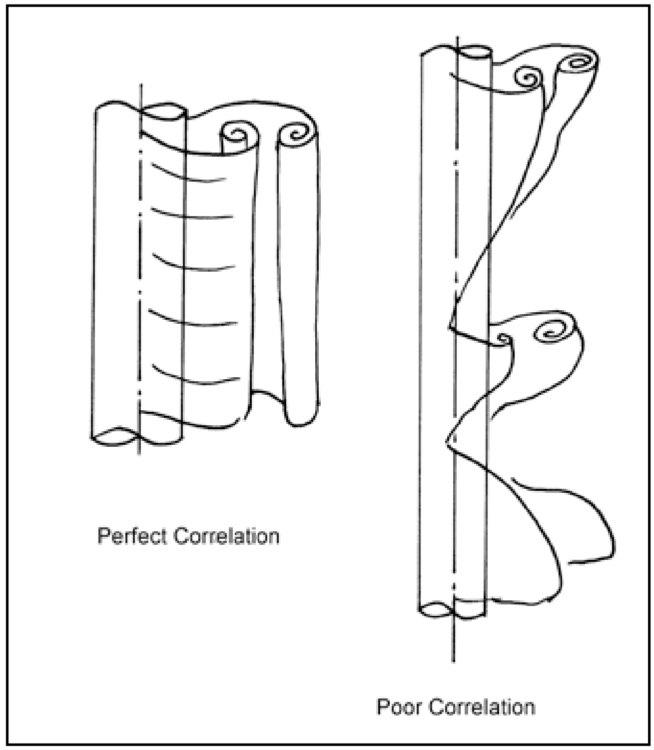
This only works if <u>current uniform</u>

STILL THERE ARE MAJOR ISSUES:

- Are C_{LV}, C_m same as for rigid cylinders?
- Are vortices shed coherently or not?

ANSWERS:

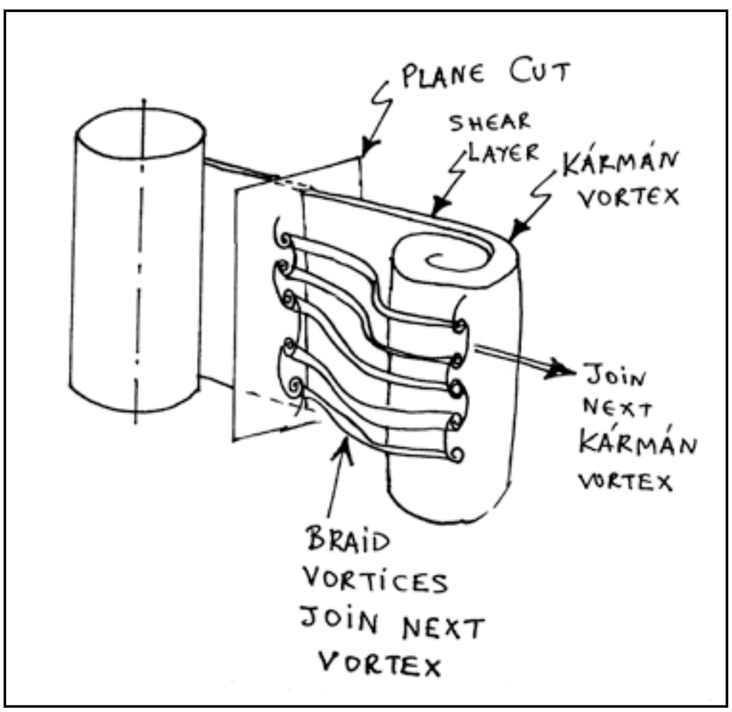
- C_{LV}, C_m are well approximates by rigid cylinder data
- Vortices are shed coherently but not necessarily over the entire length



CORRELATION LENGTH:

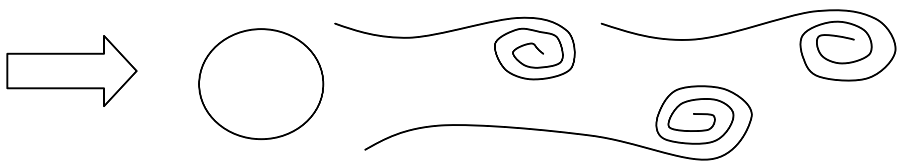
Depends on:

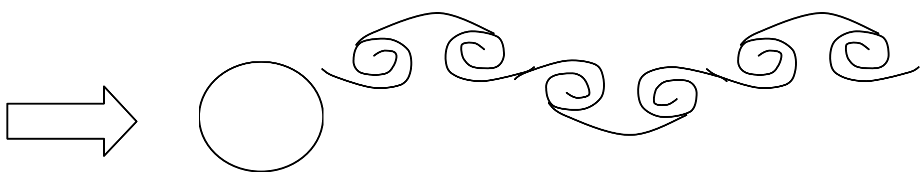
- geometry
- shear
- motion of the riser

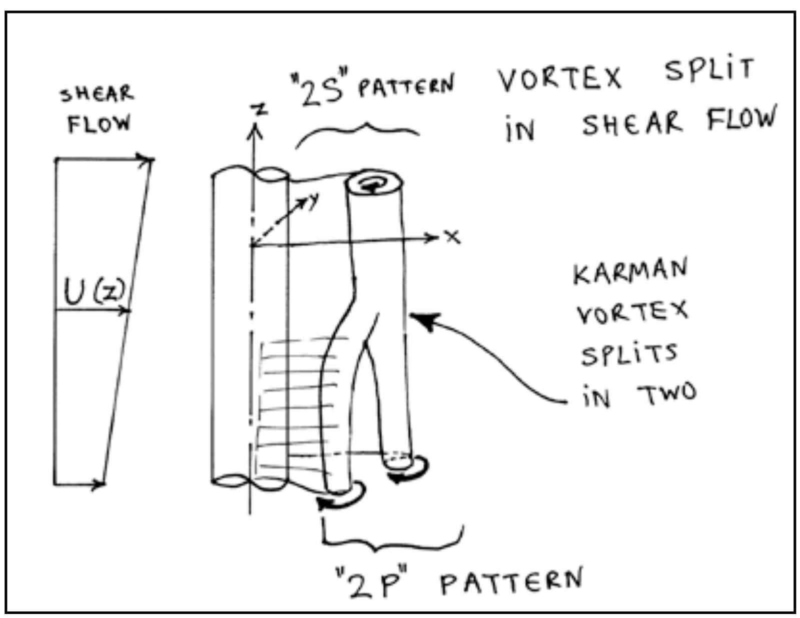


<u>"RIB" Vortices</u>

VORTEX "SPLITS"

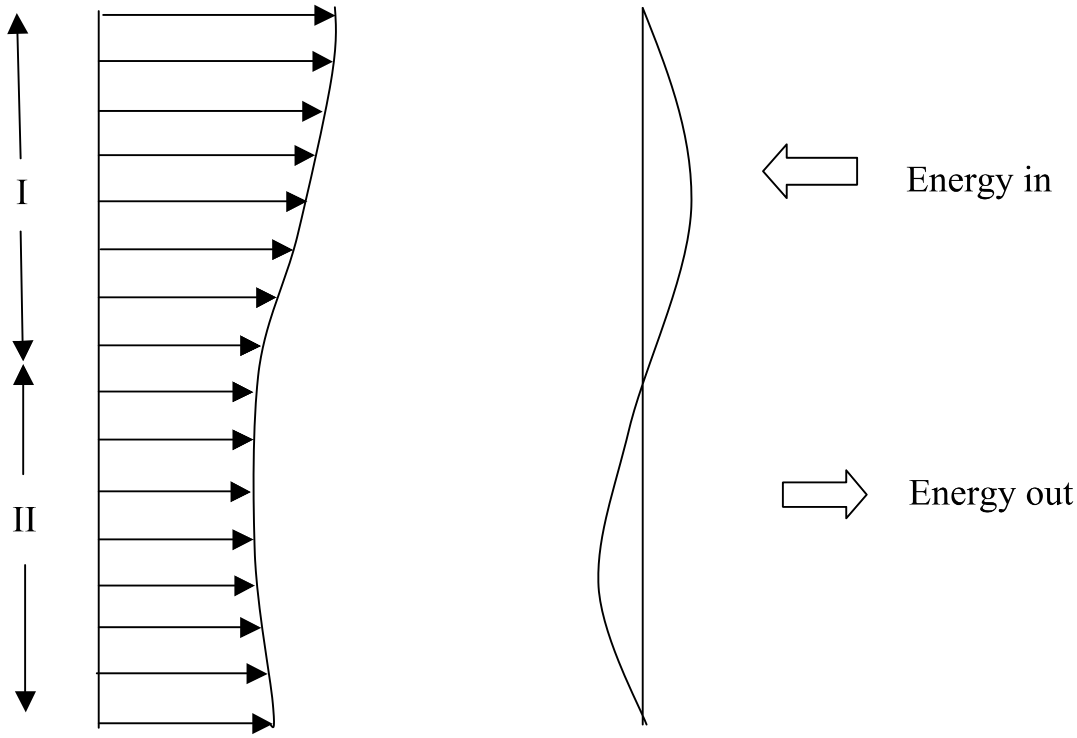






Forces are affected!

SHEAR CURRENT



LIFT FORCE $\sim \frac{1}{2}\rho U^2$

HIGH-SPEED PART DOMINATES

IMPOSING $$\omega \sim 2\pi(St)\frac{U}{d}$$

C_{LV} in I is <u>positive</u>
C_{LV} in II is <u>negative</u>

$$\boxed{\text{NEED COMPLEX MODES}}$$

COMPLEX MODES

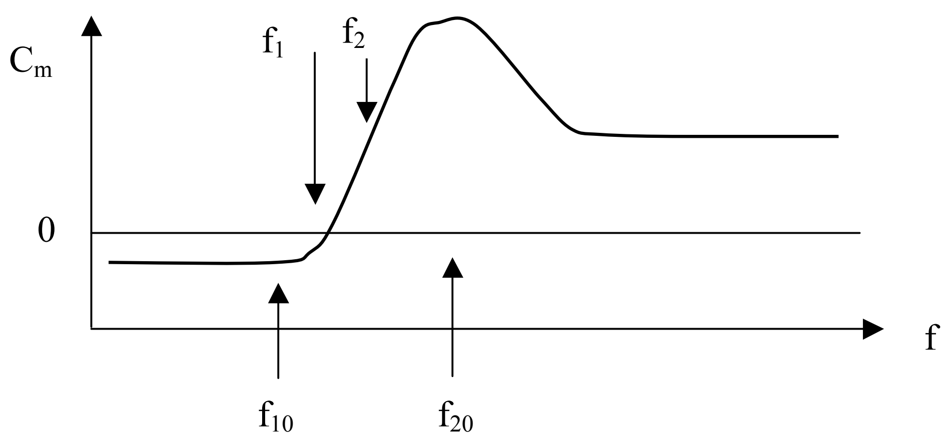
Instead of: $$y(x,t) = A\theta_n(x)\cos\omega_n t$$

Use: $$y(x,t) = A\theta_n(x)\cos[\omega_n t + \psi(x)]$$
$$= \mathrm{Re}\left\{\theta_n(x)e^{i[\omega_n t + \psi(x)]}\right\}$$
$$= \mathrm{Re}\left\{\hat{\theta}_n(x)e^{i\omega_n t}\right\}$$

$$\hat{\theta}_n(x) = \theta_n(x)e^{i\psi(x)}$$

THERE ARE NO NODES SO ENERGY CAN TRAVEL UP AND DOWN THE RISER.

MULTI-FREQUENCY AND MODE SWITCHING RESPONSE



f_{10}, f_{20} are ‘original’ frequencies

In reality f_{10} will increase because $C_m < 1$ and f_{20} will decrease because $C_m > 1$.

THEN f_1, f_2 can get very close -

- MODE SWITCHING
- MULTI-FREQUENCY

Response can be:

- Beating oscillations
- Mode Switching
- Truly Multi-Frequency