Vortex induced vibrations of long cylindrical structures

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Abstract

We consider the problem of vortex induced vibrations of long cylindrical structures. In particular we calculate the hydroelastic natural modes of the coupled fluid/structure system. For the structural part we use linear theory, since the amplitude of the oscillation is small. For the hydrodynamic forces we use experimental data for forces on harmonically oscillating cyliners. We solve for the normal modes of the coupled fluid/structure system using an iterative procedure. Examples involving constant and shear current are considered. It is shown that the frequencies and mode-shapes of the the coupled fluid/structure system can vary substantially from those of the structure vibrating in still fluid.

1 Introduction

The problem of vortex induced vibrations occurs whenever a cylindrical structure is immersed in a current. Then if the frequency of vortex formation is close to any of the natural frequencies of the structure, synchronization occurs between the wake and the structure, both oscillating at the same frequency.

As experiments show (Staubli, 1983, Gopalkrishnan, 1993), the lift and added mass force on an oscillating cylinder are quite different than those on a stationary cylinder. The lift force in particular, which is the excitation force, rises for small amplitudes from its value on the stationary cylinder, reaches a maximum, and then goes to zero in a linear manner. The lift vanishes for an amplitude over diameter ratio y_m which depends on the reduced frequency, but never exceeds a value of A/d close to one. In fact once y_m is exceeded, the lift becomes negative, i.e. it acts like a damping force. Vortex-induced oscillations are therefore self-limiting. Although the amplitude of vortex-induced vibrations is not high, at most about one diameter of the structure, the persistence of the oscillation often causes failure because of fatigue.

There is a need therefore for predictive tools for calculating the vortex-induced response of long flexible structures. Calculating the response from first principles, i.e. through simultaneous simulation of the Navier-Stokes equations and the equations of structural dynamics is still an impossible task for real systems (it can only be accomplished for simple structures at low Reynolds numbers). On the other hand semi-empirical models have a long history, and one is referred to any of the many excellent review articles on the subject (Sarpkaya, 1979, Bearman, 1984, 1993).

In this paper we investigate the hydroelastic modes of a flexible structure in a current. The method of calculating the modes relies on experiments to determine the hydrodynamic forces, but models the synchronization problem itself from first principles.

2 Formulation

We consider a long cylindrical structure of length l and diameter D in a current of velocity V. The diameter of the structure and the velocity of the current may vary across the length. The velocity is however assumed to be time-invariant.

Vortex induced vibrations have small amplitude: The maximum possible amplitude is equal to about one diameter of the structure. Consequently the structural model can be linearized, and the equation of motion is:

$$m\frac{\partial^2 y}{\partial t^2} + b\frac{\partial y}{\partial t} - \frac{\partial}{\partial x}(T\frac{\partial y}{\partial x}) + \frac{\partial^2}{\partial x^2}(EI\frac{\partial^2 y}{\partial x^2}) = f(x,y,t) \tag{1}$$

where m is the mass per unit length of the structure, b is the structural damping, EI the bending stiffness and f the fluid load per unit length, which is a function of the motion of the structure.

We are interested in the normal modes of the coupled fluid/structure system. The structure oscillates harmonically in time and we set:

$$y = Re[Y \exp(i\omega t)] \tag{2}$$

where Y is complex and $Re[A]$ stands for "real part of A".

When synchronization between the oscillation of the structure and the vortex shedding in the wake occurs, the dominant part of the hydrodynamic force oscillatory at the same frequency as the oscillation. We decompose the hydrodynamic force into one part that is 180^o out of phase with the accelaration (the added mass force) and one part which is in phase with the velocity (the excitation lift force).

$$f = Re[(aC_m(\omega)a\omega^2 Y + iC_L(\omega)qD\frac{Y}{|Y|})\exp(i\omega t)] \tag{3}$$

where a is the potential flow added-mass coefficient of the cross-section i.e. $a = \pi\rho D^2/4$, $q = \frac{1}{2}\rho V^2$, whereas $C_L(\omega, Y)$ and $C_m(\omega, Y)$ are frequency and amplitude dependent non-dimensional coefficients. We assume that the dependence of C_L, C_M on ω and Y is known. Finally the expression $Y/|Y|$ which has magnitude one ensures that the excitation force is in phase with the velocity.

Then equation (1) becomes:

$$\frac{d^2}{dx^2}(EI\frac{d^2Y}{dx^2}) - \frac{d}{dx}(T\frac{dY}{dx}) + (-M\omega^2 + ib\omega)Y = iC_L qD\frac{Y}{|Y|} \tag{4}$$

where M is the total inertia per unit length:

$$M = m + aC_M \tag{5}$$

The displacement Y is subject to the appropriate boundary conditions at the ends of the structure (e.g. pinned, fixed, etc.). Equation (4) with the boundary conditions define a non-linear eigenvalue problem for the frequency ω and the mode Y. We note that the problem is non-linear due to the hydrodynamic coefficients C_L and C_M, which are non-linear functions of the amplitude Y. We also note that unlike the problem of free vibrations of a beam, the modes of the coupled fluid/structure system are in general not orthogonal to each other. Moreover we use onlya finite number of such modes. If Y is a solution of (4) then $Y \exp(i\beta)$ is also a solution with β arbitrary. In other words the solution is unique to within an arbitrary constant phase.

We now derive two interesting integral relations that follow directly from equation (4). We multiply (4) by the complex conjugate of Y, Y^*, and integrate along the length of the structure. Then after integrating by parts and separating real and imaginary parts we obtain:

$$\int_0^l (EI|\frac{d^2Y}{dx^2}|^2 + T|\frac{dY}{dx}|^2)dx = \omega^2 \int_0^l M|Y|^2 dx \tag{6}$$

$$\int_0^l b\omega|Y|^2 dx = \int_0^l C_L q D|Y| dx \tag{7}$$

Equations (6), (7) are energy conservation laws. The first one expresses that there is a potential/kinetic energy balance (including the inertia of the fluid). The second one expresses that the net power transfer from the fluid to the structure (given by the right hand side) is equal to the power dissipated by the structural damping. In the limit of zero structural damping, the net power transfer is equal to zero.

3 Numerical method

We first discretize equation (4) using second order finite differences. The boundary conditions at the ends are taken care of by introducing fictitious points such that the conditions are satisfied. At the end one obtains a set of N non-linear algebraic equations of the form (N is the number of discretization points):

$$a_{i,i-2}Y_{i-2} + a_{i,i-1}Y_{i-1} + a_{i,i}Y_i + a_{i,i+1}Y_{i+1} + a_{i,i+2}Y_{i+2} + (-M_i\omega^2 + ib\omega)Y_i = iC_{Li}q_iD_i\frac{Y_i}{|Y_i|} \quad (8)$$

where subscript i refers to the $i-th$ point.

The system of equations (8) is solved as follows:

For a range of frequencies around the natural frequency of the k-th mode we solve equation (8). As M_i and C_{Li} are functions Y_i, an iterative procedure is followed. Let Y_i^n denote the value of Y at the i-th point at the n-th iteration. The iterations are generated from the following equation:

$$a_{i,i-2}Y_{i-2}^{(n)} + a_{i,i-1}Y_{i-1}^{(n)} + a_{i,i}Y_i^{(n)} + a_{i,i+1}Y_{i+1}^{(n)} + a_{i,i+2}Y_{i+2}^{(n)} + (-M_i^{(n-1)}\omega^2 + ib\omega)Y_i^{(n)} =$$

$$= iC_{Li}^{(n-1)}q_iD_i\frac{\phi_{k,i}}{|\phi_{k,i}|} + iC_{Li}^{(n-1)}q_iD_i(\frac{Y_i^{(n-1)}}{|Y_i^{(n-1)}|} - \frac{\phi_{k,i}}{|\phi_{k,i}|}) \quad (9)$$

where ϕ_k is the k-th mode of the structure vibrating in still water. This procedure requires therefore inverting the matrix at the left-hand side of equation (9) several times. The inversion can be done easily, since the matrix a_{ij} is five-diagonal and can be inverted with only $5N$ operations. To ensure convergence even in the case of zero structural damping, the term Rq_iY_i is added on both sides of equation (8), where R is chosen bigger than the maximum value of the derivative of C_L with respect to the amplitude.

The starting point of the iteration scheme is the natural mode of the structure in still fluid with maximum amplitude equal to one diameter. Iterations stop once the solution for Y has converged to within desired accuracy (for example 1 per cent).

Once the process has been completed the frequency of the hydroelastic mode can be determined from the resonance-like behaviour of the response near that frequency.

Further fine-tuning is then done by repeating the first two steps near resonance to obtain a more accurate value of the frequency. Because of the sensitivity of the lift coefficient on the reduced frequency, it is important that the frequency be determined with high accuracy, much higher in fact than the one required for the amplitude. (Typically 0.1 per cent or less).

A sequence of hydroelastic modes is found. In general, both the frequencies and the mode shapes are different than those of the structure vibrating in still fluid.

4 Choice of the hydrodynamic coefficients C_M and C_L

So far we have simply assumed that we know the C_M and C_L, and we have formulated the method in such a way that it does not depend on the specific form of C_M and C_L (although the results of course do depend on the specific form). For the results presented here we have used a "strip-theory" approach.

We assume that at each location C_M and C_L depend on the local properties of the flow only, and more precisely that are the same as those on a two-dimensional cylinder oscillating with the same frequency and amplitude as that of the structure. For a two dimensional cylinder C_M and C_L are functions of the reduced frequency $S = \omega D/(2\pi V)$, the non-dimensional amplitude $\xi = |Y|/D$, and the Reynolds number VD/ν, as it follows readily from non-dimensional analysis. The dependence on the latter is rather weak (at least in the subcritical regime), and can be neglected. For the dependence of C_M and C_L on S and ξ we have used the most extensive set of measurements existing right now, which is that of Gopalkrishnan (1993).

In regions of the structure where there is no current the lift coefficient vanishes $C_L = 0$, and the added mass coefficient is equal to one, whereas there is a quadratic damping force per unit length:

$$F_d = \frac{1}{2} C_d \rho \frac{\partial y}{\partial t} \left| \frac{\partial y}{\partial t} \right| \tag{10}$$

where C_d is the drag coefficient for a circular cylinder.

Consistently with the assumption of harmonic oscillations, we replace the quadratic damping above with the equivalent linear damping. The latter is defined as a linear damping term that dissipates the same power as the quadratic one over one cycle of vibration. The equivalent linear damping therefore is defined as:

$$F_{de} = \frac{1}{2} C_d \rho \psi \omega Y \tag{11}$$

where $\psi = 8/(3\pi)\omega|Y|$.

Before presenting results obtained with the method described in the previous section, we note that equation (4), with the "strip-theory" approximation, accepts an exact solution for the special case where the diameter of the structure and the velocity of the current are constant throughout the length of the structure, and the structure can be assumed to be unbounded. Then if we set for Y:

$$Y = A \exp{(-ikx)}, \tag{12}$$

equation (4) gives:

$$EIk^4 + Tk^2 = M\omega^2 \tag{13}$$

$$b\omega A = C_L q D. \tag{14}$$

Equations (13) and (14) can be solved simultaneously to yield A/D and ω. The procedure of solving the equations is identical to that of finding the amplitude and frequency of a flexibly-mounted cylinder in a current.

The exact solution of equation (12) has phase that varies linearly with x and describes therefore a travelling wave along the structure with celerity ω/k. This solution is of interest only for vary long structures, where end-effects can be neglected, since it obviously can not satisfy any boundary conditions along the x direction.

5 Results

A code has been developed which implements the procedure described before for determining the hydroelastic modes. The code is intended as a tool for modelling actual systems operating in the ocean environment. It is also useful however as a tool for understanding the physics of the problem. In this section we present some sample results which demonstrate some of the subtleties of the fluid/structure interaction problem.

One of the things we are investigating is a comparison between the natural modes of the coupled fluid/structure system and the modes of the structure itself. To make the comparison clearer we have chosen a structure of constant diameter $D = 0.2m$ and length $l = 500m$, simply supported at the ends with constant tension equal to 1000000 N. For such a structure the natural modes of vibration in still fluid are simple sines. The damping was assumed equal to 1.5 percent of the critical damping for the first natural mode of the structure.

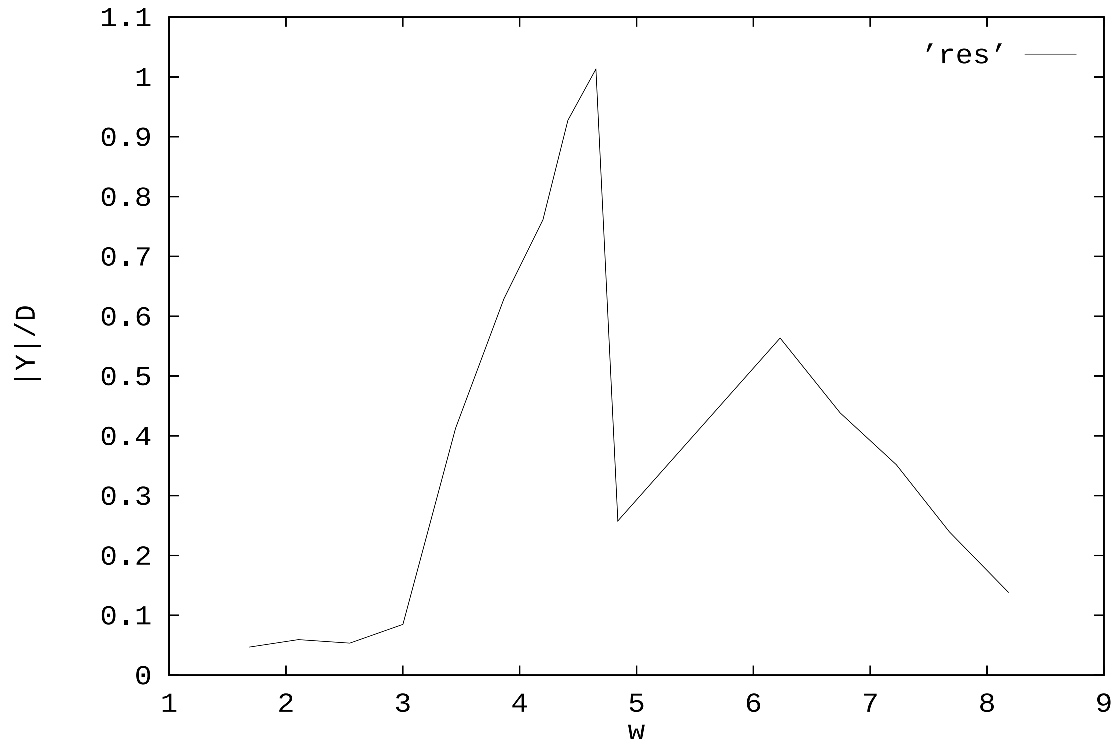

Figure 1: Response curve for constant current.

We first consider the case of uniform current $V = 0.85m/s$ throughout the structure. Several hydroelastic modes (16) are found. The plot of the maximum amplitude of oscillation versus frequency, which we call the response curve, is shown in figure 1. There is a primary peak which corresponds to synchronization of the structure with the vortex shedding in the wake and which results in maximum amplitude of about one diameter. There is also a secondary peak, with much smaller amplitude, which corresponds to superharmonic synchronization, i.e. synchronization of the structure with a frequency equal to twice that of the natural vortex shedding.

The mode shape of the highest amplitude mode is shown in figure 2, and is a perfect sign. The imaginary part of the mode is also shown, and is very small, practically negligible. The plot in figure 2 also demonstrates the robustness of the numerical method, since all peaks in figure are equal, as the symmetry of the problem requires.

We next consider the case of a sheared current. The value at the top is $1.1m/s$, and the velocity at the bottom is $0.6m/s$ (i.e. the mean value is the same as the constant value used before). The response curve in figure 3 is quite different however than the one for constant current in figure 1. The response shows a broader frequency range and two distinct primary

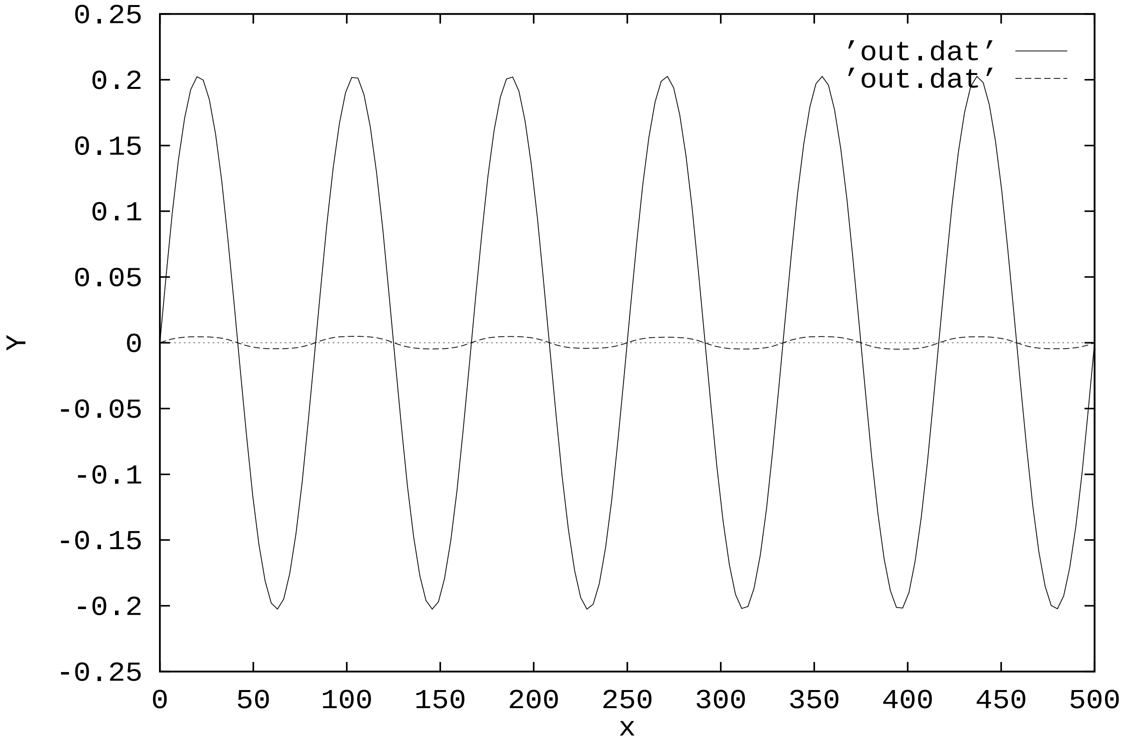

Figure 2: Constant current: Mode-shape for the hydroelastic mode of maximum amplitude, real and imaginary part.

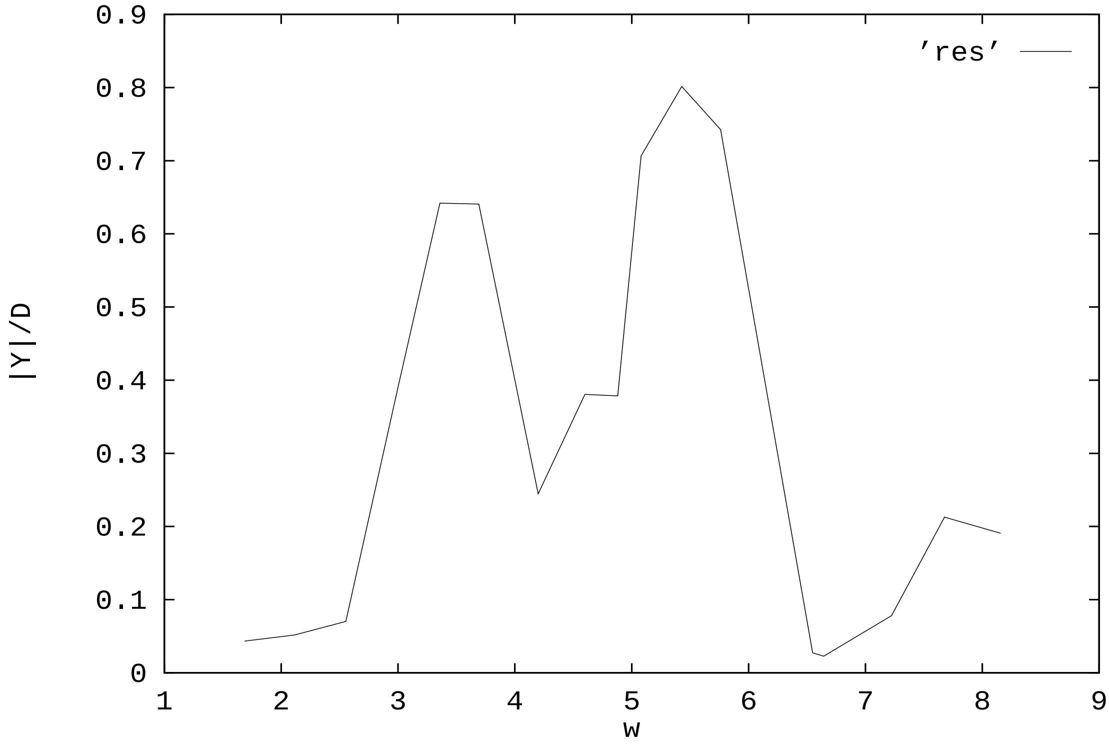

Figure 3: Response curve for sheared current.

peaks, one at lower frequencies, the 8th mode, and one at higher frequencies, the 14th mode. (There is also a secondary peak corresponding to superharmonic synchronization).

The two peaks correspond to different type of modes, as shown in figures 4, and 5. The low frequency mode has high amplitude at the bottom of the structure whereas the high frequency mode has the maximum amplitude at the upper part of the structure. This behaviour can easily be understood by looking at the synchronization frequency: For each mode, the maximum amplitude occurs where the local reduced frequency is equal to approximately 0.17. For mode I this occurs in the region of low current velocity, while for the second mode occurs in the region of high current velocities. In other words the 1st peak corresponds to a mode that draws its energy from the region of low velocities of the current, whereas the 2nd peak corresponds to a mode that draws its energy from the region of high velocity of the current. From the fatigue analysis point-of-view the second mode is the more significant, since it has both higher frequency and amplitude, but also smaller wavelength.

The fact that both modes are substantially different than the modes of the structure in still fluid (which are perfect sines) is clear from figures 4 and 5. It can be better seen if we project the two modes into the set of normal modes of the structure (i.e. take the discrete

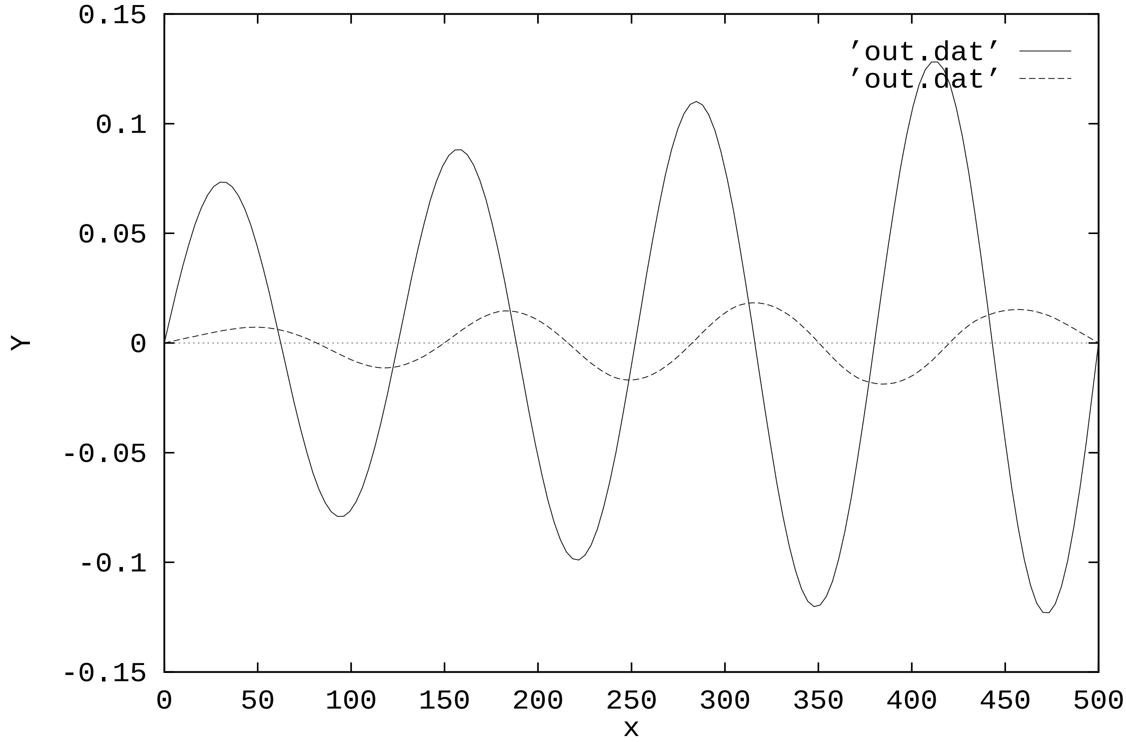

Figure 4: Shear current: Mode-shape for the hydroelastic mode of maximum amplitude with the lower frequency, real and imaginary part.

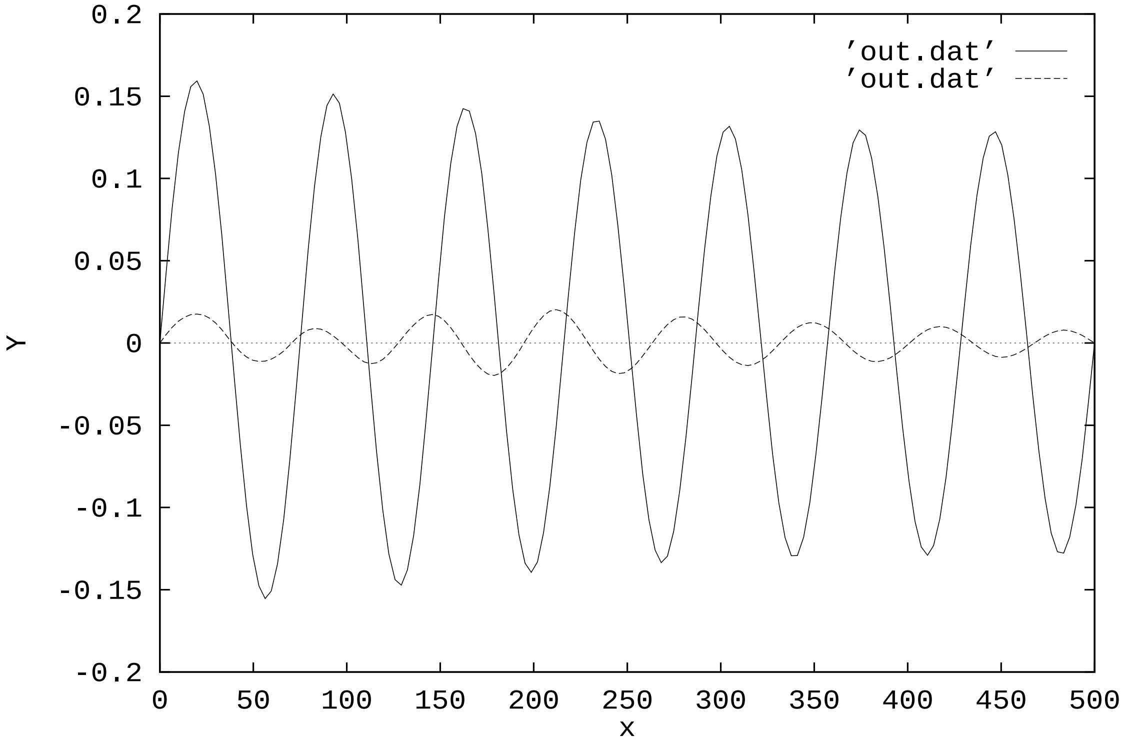

Figure 5: Shear current: Mode-shape for the hydroelastic mode of maximum amplitude with the higher frequency, real and imaginary part.

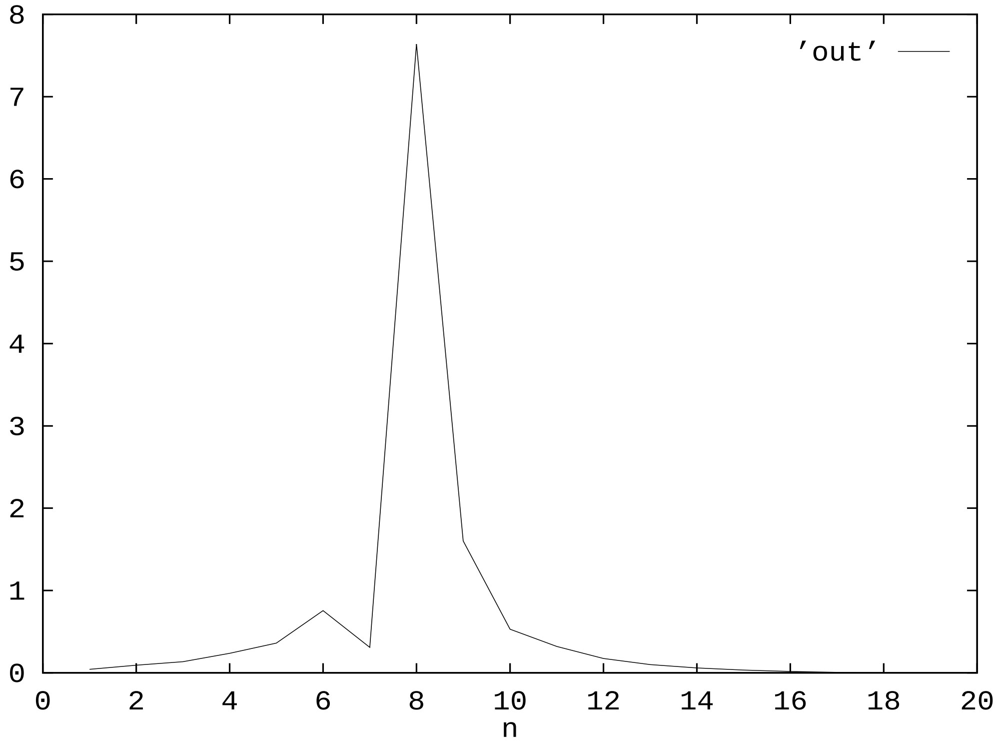

Figure 6: Fourier coefficients for the 8-th mode.

sine-Fourier transform of the shapes in figures 4 and 5. The results are shown in figures 6 and 7 respectively, where the absolute value of the Fourier coefficient is shown as a function of the mode number. Both modes contain a range of modes of the structure around the mode that is synchronized.

We finally consider the case of a strong current ($V = 1.5m/s$) concentrated on the first $100m$ of the structure. Much fewer modes are excited, and the maximum amplitude mode is shown in figure 8. The real part of the mode obtains its highest value at the top where the current is present and is then reduced downwards. The imaginary part of the mode obtains its highest value in the region where the current ends, and then also decays downwards.

6 Conclusions

We have presented a procedure for calculating the natural frequencies and modes of the coupled fluid/structure system. The natural frequencies and mode-shapes are in general different than those of the free vibrations of the structure with the only exception that of a uniform current in a uniform current. Even in uniform current however a non-uniform

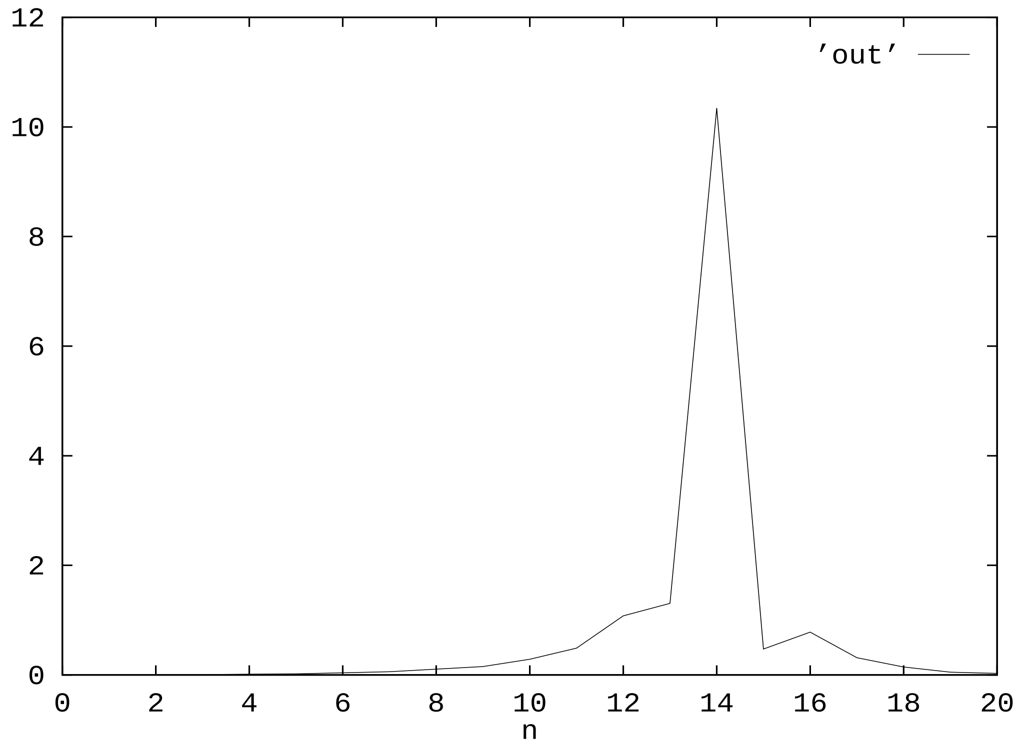

Figure 7: Fourier coefficients for the 14-th mode.

structure will also develop different mode-shapes because the added mass coefficient will be different across the structure as a result of the different reduced frequency across the length.

An interesting finding, that has not been reported before, is that shear current excites different types of modes: Those at lower frequencies which synchronize with the low velocity in the current, those at higher frequencies which synchronize with the high velocity in the current, but also modes that synchronize with intermediate values of the velocity. Each mode obtains its maximum value at the region where it synchronizes with the current.

Several hydroelastic modes can be excited simultaneously in real situations, and there is clearly a need for experimental data on lift and added mass forces under multi-frequency excitation conditions.

7 References

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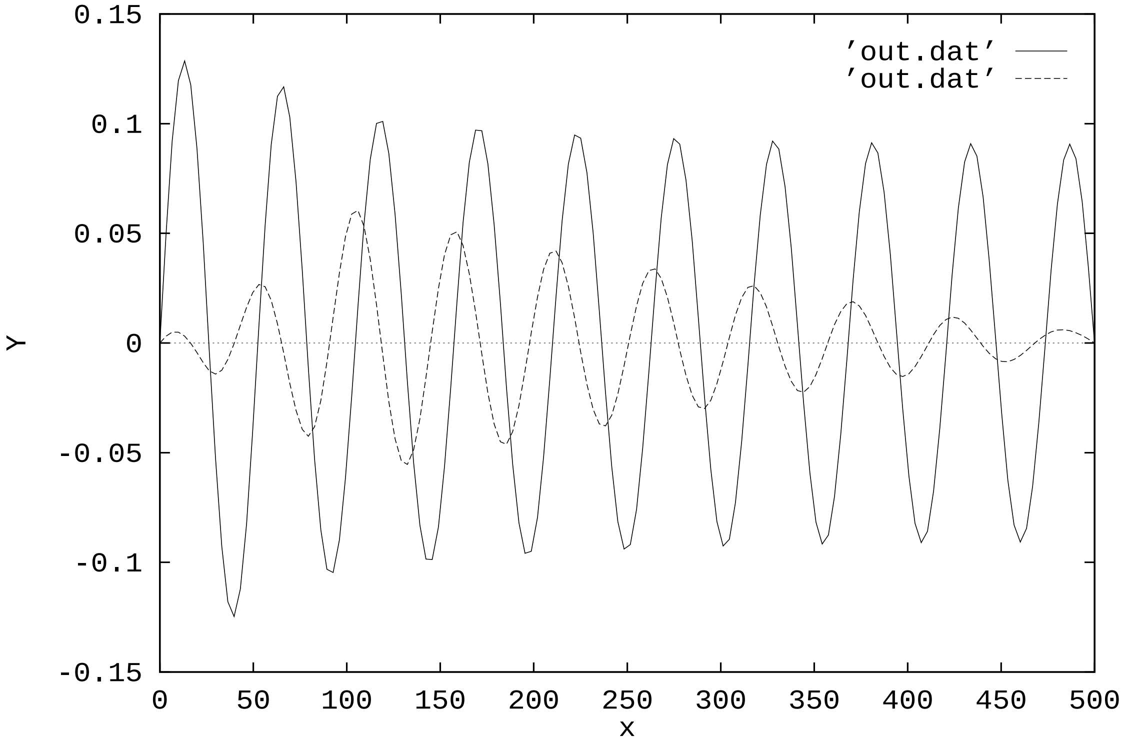

Figure 8: Localized current: Mode-shape for the hydroelastic mode of maximum amplitude, real and imaginary part.

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