FATIGUE CALCULATION IN VIVA

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1 OVERVIEW OF METHODOLOGY

The current calculation of fatigue life in VIVA is based on the following assumptions:

1. The response may consist of a single-frequency or multi-frequency response. When the response in multi-frequency, it is assumed that each frequency peak has independent probability distribution than the other peaks.
2. The response consists of a single of several narrow-band spectral peaks, each Gaussian distributed.
3. There is a single-slope $S-N$ curve describing the fatigue properties of the riser.
4. Miner's rule applies to the calculation of fatigue.

2 THEORY

2.1 Single-frequency narrow band response

Consider the riser stress response, $s(t)$, at a particular location along the riser to have a single narrow-band spectral peak, which is Gaussian distributed.

The fatigue curve is assumed to have a single slope and to be described by the equation:

$$N = (A/S)^b \tag{1}$$

where N is the number of cycles to fatigue, S is the stress amplitude of a sinusoidal stress signal, and A and b are constants. Note that VIVA allows the input of the constants of this curve in different form – see the manual for more information, since the specific form does not affect the theory.

Using Miner's rule for a stationary and ergodic, Gaussian distributed, narrow-band signal $s(t)$, the fatigue damage D over a time period T is calculated as:

$$D = \frac{T}{T_o \beta} \int_0^\infty \frac{y^{a+1}}{M_o} \exp(-\frac{y^2}{2M_o}) dy \tag{2}$$

where T_o is the average upcrossing period, M_o denotes the area under the (narrow-band) spectrum of the stress (i.e., the square of the root-mean-square, which is equal to the variance), and β is simply:

$$\beta = A^b \tag{3}$$

The integral can be calculated exactly as:

$$D = \frac{T}{T_o \beta} (2M_o)^{a/2} \Gamma(1 + \frac{a}{2}) \tag{4}$$

where $\Gamma(x)$ is the Gamma function of x.

2.2 Multi-frequency response

When the response consists of n narrow-band peaks, where each peak is characterized by a mean period $T_{o,j}$ and spectral area $M_{o,j}$, with $j = 1, 2, ...n$, the damage is simply the sum of the damages, i.e.:

$$D = \frac{T}{\beta} \Gamma(1 + \frac{a}{2}) \sum_1^n \frac{1}{T_{o,j}} (2M_{o,j})^{a/2} \tag{5}$$

3 Fatigue Life Calculation

The fatigue life is calculated by assuming that the damage is equal to 1, $D = 1$, and solving for the fatigue life T. For example, equation (4) gives:

$$T = \frac{\beta}{\Gamma(1 + \frac{a}{2})} \frac{1}{\sum_1^n \frac{1}{T_{o,j}} (2M_{o,j})^{a/2}} \tag{6}$$

4 Implementation in VIVA

VIVA calculates a multi-frequency response, each component with an amplitude A_j and frequency f_j, $j = 1, 2, ...n$. Also, the bending moment and stress for each mode is calculated and the multi-frequency stress is calculated at each point of the riser.

Next, it is assumed that each frequency component of the stress has a narrow band spectral peak with variance $M_{o,j} = A_j^2/2$ and average frequency $T_{o,j} = 1/f_j$.

The calculation uses the theory outlined above.

4.1 Imported Modes

In the newest version of VIVA the user can import the natural frequencies, modal shapes and curvature shapes, instead of having VIVA calculate the modes and stresses.

VIVA iterates only for the amplitude of each mode and the calculates the stress from the imported curvature modes directly. Finally, the fatigue life is calculated assuming each mode to be narrow-banded, and Gaussian distributed, and uses the methodology above.

5 Influence the High Stress Harmonics on Fatigue

The newest version of VIVA incorporates high stress harmonics. These harmonics are assumed to be narrow band and independently distributed from the other peaks, so the calculation proceeds as with the multi-frequency response.