TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

Chapter 1

OVERVIEW

It is natural for humans to wonder how the world works. Small children commonly ask a plethora of questions, sometimes pushing their parents to the point of exasperation. I was one such child, always asking questions, and later devouring books and magazines, with a hunger to learn more. My first exposure to particle physics came at age eleven when I discovered my parents' copy of *A Brief History of Time* by Stephen Hawking. The ideas described therein – fundamental particles with exotic properties, the flexible nature of spacetime, what might lie beyond the edges of our understanding – boggled my mind, and I loved it. Over the following years I checked out much of the local library's collection of popular physics books. It is this desire to know more about the fundamental nature of the universe that has brought me to the point of pursuing a doctorate in experimental particle physics.

1.1 Short Overview of Particle Physics

This hunger to figure out how things work has long been a part of human life. One of the most lasting questions has been "What, exactly, are things made of?" The ancient Greeks tried to distill all matter into four elements: fire, earth, water, and air. They also came up with the idea of an "atom," the smallest possible piece of matter, a piece that could be divided no further. Natural philosophers through the ages discovered various chemical elements. However, not until relatively recently was the concept of the atom accepted scientifically. Up until the late 19th century, the issue was still debated. Dmitri Mendeleev's organization of the elements into a table according to the similarity of their properties hinted at an underlying structure. In addition, an atomic theory explained certain features of the behavior of gases, as applied by Ludwig Boltzmann and Amedeo Avogadro.

However, the issue was not fully settled until the beginning of the 20th century. During this period Albert Einstein used an atomic theory of matter to explain the random motion of particles in a fluid (called Brownian motion after Robert Brown, who first observed it), and Jean Perrin verified the theory experimentally. Around this time progress was also being made on investigations into the structure of the atom. J.J. Thomson discovered that the cathode rays he was working with were made up of light, negatively-charged particles, dubbed electrons. He believed that these particles formed atoms, electrons floating in a sea of positive charge so that the overall element was electrically neutral: the so-called "plum pudding model." However, experiments shooting positively-charged alpha particles at a sheet of gold foil demonstrated that the atom included a hard, positively-charged nucleus. This led Ernest Rutherford to propose the planetary model of the atom, in which the positive nucleus is orbited by a cloud of negative electrons. The model did not explain why the electrons did not lose energy and fall into the nucleus, though; this explanation came when Niels Bohr applied a quantized idea of energy to the atom. The quantum theory of light, proposed by Max Planck and Albert Einstein, said that light existed in packets of a given energy, known as quanta. Applied to the atom, it meant that electrons were only permitted to possess energy and angular momentum in discrete steps. They moved between these energy levels by absorbing or emitting light of specific energies. Rutherford later noticed that the atomic masses were roughly integer multiples of the hydrogen mass; he proposed that atomic nuclei were made of positively-charged protons, and later added electrically neutral neutrons to make up the needed leftover mass. Meanwhile, quantum mechanics was soon formulated to describe the properties of particles' behavior in terms of waves of quantized energy. Paul Dirac formulated a quantum theory that was consistent with Einstein's special relativity, which dealt with motion at speeds approaching that of light. In addition, the idea of forces being mediated by the exchange of force particles was gaining traction: quantum electrodynamics formulated the electromagnetic force as the interaction between charged particles and photons, or particles of light. At this point many new particles were being discovered, including the pion, such as the positron and the muon, along with a large number of heavier particles. Quarks were proposed in Murray Gell-Mann's Eightfold Way to explain the many new heavy particles being found. The quark model successfully explained the properties of neutrons and protons: those particles along with some of the new discoveries were "baryons," made up of three quarks, while other new particles were "mesons," made up of a quark and an anti-quark. The force between the quarks was the same one keeping the protons and neutrons together inside the atomic nucleus, mediated by particles called gluons. Since it was strong enough to overcome the repulsion between the positively-charged protons, it was called the strong force. Another force, called the weak force, was known to be responsible for the decay of neutrons into protons. Through the development of modern particle theory, it was predicted that heavy particles mediated this force. The discoveries of these particles, the W and the Z bosons, marked a success for the current theory: it had predicted particles that had previously been undiscovered.

Experiments making all these particle discoveries have progressed from observing cosmic rays in cloud chambers and bubble chambers to observing collisions created in higher- and higher-energy accelerators (hence the current name of the field, "high-energy physics"). Most recently, the last of the six quarks, the top quark, was discovered at the Tevatron accelerator in 1995. All these particles and their interactions are described within the framework of the Standard Model of particle physics; the Standard Model combines the quantum electrodynamics of charged particles and photons, the quark and gluon model, called quantum chromodynamics, and the interactions of the weak force into one framework. The Standard Model will be further explained in in Chapter 2. Currently, the Standard Model is considered to be the primary description of particle physics. However, it predicts one particle that has not yet been observed, the Higgs boson. Finding the Higgs and verifying this last piece of the Standard Model is one of the current goals of particle physics. However, there are observations that the Standard Model cannot explain, such as the fact that neutrinos have mass. The Standard Model is also incomplete in that it does not incorporate the last fundamental force, gravity. Further, some physicists think that various additions to the Standard Model make it more elegant. These new models predict various new particles. Therefore, scientists are also on the lookout for any evidence of particle physics existing outside the Standard Model.

1.2 Introduction to the Z Boson

This analysis studies the production of the Z boson. The Z is one of the particles that mediates the weak force. It was first observed in 1973 in the Gargamelle bubble chamber at CERN when an electron would sometimes apparently start moving on its own. This was understood to be due to a neutrino striking the electron (by their nature, neutrinos rarely interact with matter and so are generally not seen in particle detectors). However, it was not previously known that neutrinos could interact with electrons in this way. This new interaction implied the existence of a new particle, the Z, as had been predicted by electroweak theory. The Z was not observed directly until a decade later, when it was produced in proton collisions in the Super Proton Synchrotron (SPS) accelerator at CERN in 1983.

The Z has a relatively large mass, like the other mediator of the weak force, the W; however, unlike the electrically-charged W, the Z is chargeless. The Z interacts with all matter particles (as opposed to force carriers); it can be formed in a high-energy interaction between two particles that are oppositely-charged but otherwise alike. This includes the quark constituents of protons, which means it can be formed in the proton-proton collisions of the LHC. The Z can also decay into similar pairs of particles, one such possible pair being the electron and the positron; this is the decay studied in this analysis.

The Z has been well-studied in previous experiments, which makes it an ideal initial study for the LHC experiments. Its mass is well-known enough to be used to adjust the experiments' energy scales. In addition, the fact that the Z is produced from the constituents of protons means that it can shed light on the protons' internal structure. The Z also tends to decay in a similar way as the proposed Higgs boson. A particular Z decay signature may look like the Higgs and vice versa. Therefore, in order to look for the Higgs, the decays that look like it must be very well-studied – that way, if the Higgs is seen, it will be known that it is definitely something new.

1.3 Useful Concepts

A few definitions and concepts should be introduced before moving on to more specific explanations of the analysis. In the following pages, much will be described by the word "kinematic" or "kinematics". This refers to the properties of position or motion. So, "kinematic criteria" means selections applied according to quantities such as momentum or angle. Often, a calculation may need to deal with the full range of possible values for multiple quantities, or the "phase space." Phase space refers to the collection of total possible values for a set of quantities. A given system is represented by one set of values out of that collection. For some calculations it is necessary to include contributions from the entire phase space, for example integrating over all possible angles for a particle's direction.

Experimental particle physics is built on "events." An event is the name for is a particle interaction, particularly one that has been captured and recorded by the detector. In general, events caused by the specific process being studied are termed "signal," while any events from other sources are called "background." A significant part of any analysis is designing criteria that select signal events while rejecting background events, so that the analysis focuses only on data containing that process. The number of events for a given signal can be used to calculate quantities of interest, for example how often that process occurs relative to others.

1.4 Anatomy of an Event

A proton-proton interaction begins with beams of protons accelerated in opposite directions. The protons are formed into "bunches," with many, many millions of protons per bunch. These beams of proton bunches are directed to cross at the center of the detector. In an encounter between two protons, the fundamental interaction takes place between individual "partons," which is the collective term for the quarks and gluons that make up the proton. Each "bunch crossing" results in a small number of interactions, the majority of which do not interact strongly enough to be interesting. However, sometimes two partons from separate protons interact very strongly. These are the types of interactions that are typically interesting. An interaction in which the partons interact very energetically is a "hard interaction." Conceptually, the partons "hit each other hard." Hard interactions are typically detected by having end-product particles with a lot of momentum in the transverse direction, perpendicular to the protons' original direction of motion. Only in hard interactions is the original momentum disturbed so much; in "soft," or low-energy, interactions, most of the protons' momentum continues in the same direction, down the beam-pipe.

Oftentimes the two partons exchange a particle, such as a photon or gluon. In other cases another particle is produced, such as a Z boson; this is the scenario studied in this analysis. Particles formed in this way are typically heavy and therefore short-lived, decaying into lighter, more stable particles. This analysis studies the Z's decay into two electrons, which are the lightest charged particles and therefore (to conserve charge) do not decay. These decay products are what fly "out" into the detector with some energy and direction, and it is these quantities that are measured by the detector.

However, additional processes may contribute to the signature left in the detector. One of the initial partons or final decay products may radiate an additional particle which then ends up in the detector. This is known as "initial-state radiation" (ISR) and "final-state radiation" (FSR) respectively. In addition, there are two ways that particles unrelated to the hard interaction may show up in the detector: underlying event and pileup. The "underlying event" refers to the interactions taking place between the proton remnants, the parts of the proton "left over" from the hard interaction. "Pileup" refers to any interactions that happen between other protons in the bunch. Statistically, at most only one hard interaction happens in any given bunch crossing; the rest are soft. Both underlying event and pileup can contribute energy deposits that are recorded as part of the event. Typically they are both fairly low-energy and therefore easily filtered out as background.

Fully-decayed, end-product particles leave a signature in the detector by interacting with its material. Some detector material emits light when particles pass through; in these materials, the amount of light is proportional to the energy of the particle. Other parts of the detector rely on an electric signal generated as the particle passes by. These particle interactions with the detector are collectively known as "hits." In the process of reconstruction, hits are linked together to follow the path of the particle that caused them. For example, a series of hits in the tracker could show the trajectory of a charged particle as it passed through. The curvature of the trajectory could be measured and turned into a value for the momentum, knowing the value of the magnetic field. This track could then be linked with a significant emission of light in the calorimeter. If the energy from the light matches the momentum from the track, this object is reconstructed as an electron. A rough, on-the-fly reconstruction process takes place in real time to decide whether or not to save the event, called triggering. Saved events undergo a more thorough reconstruction of their detector signals later, since this more thorough version takes too long to do in real time. However, not all objects reconstructed as a given particle are actually due to a real particle of that type passing through the detector. Some reconstructed objects are "fakes." These fakes are typically selected out by more stringent criteria on various particle properties. However, this selection is done at the analysis level, not during reconstruction.

1.5 Measurement of Cross Section

The cross section, the quantity being calculated in this analysis, is a measure of "how often things happen." However, it is not measured as a rate (occurrences per unit time); instead it is measured as a cross-sectional area and represents the probability of the given interaction occurring. An analogy may be made to trying to hit a target with a tennis ball. The bigger the target is, the more likely you will be able to hit it. The sizes of both objects matter: a trajectory that would cause a tennis ball to just miss the target would cause a basketball to clip the edge, solely because of the basketball's larger size. Therefore "cross section" may be more precisely defined as the effective cross-sectional area of the target, taking into account the sizes of both the target and the projectile. When both the target and projectile are particles, they may interact "at a distance," that is, without "touching" each other, through the fundamental forces. (For example, electrons are thought to be point particles and therefore have no spatial extent, but they still attract and repel other particles through the electromagnetic force.) This interaction-at-a-distance increases the effective cross section. An interaction involving two particles that interact with each other very strongly has a large cross section. The cross section depends on the strength of the force between them.

According to the definition, "interaction cross section" applies only to the particles doing the colliding. However, a cross section is usually associated with the entire process, such as (in this case) $q\bar{q} \rightarrow Z/\gamma^* \rightarrow e^+e^-$. Here, the number calculated as the cross section takes into account only those events where a Z/γ^* is formed, as well as the fact that only some of those events decay into electrons. The fraction of events that decay to a certain final state is known as the branching ratio, abbreviated BR. When it is not explicitly mentioned along with the cross section for a given final state, it is understood to be included. Adding up the cross sections for all of these possible interactions would give the full proton-proton interaction cross section.

The cross section σ is related to the event count n by

$$n = \sigma \times \mathcal{L}$$

$\mathcal{L}$ in this equation is the "luminosity," a measure of the number of proton collisions. Two types of luminosity are spoken of in particle physics: instantaneous luminosity and integrated luminosity. Instantaneous luminosity, denoted here by L, refers to the rate of potential proton interactions. Specifically, it is defined as the number of protons passing through a given area in a given amount of time. If the same number of protons are squeezed into a smaller area, the instantaneous luminosity is higher; the protons in the colliding beams are more likely to interact. This is analogous to trying to walk through a room with many other people in it; you are much more likely to bump into people in a small, crowded room than a large room in which the people are much more spread out. Integrated luminosity, or $\mathcal{L}$ here, is the instantaneous luminosity accumulated over a definite period of time, effectively the total number of protons that have passed through a unit of area. It is a measure of the total amount of data gathered. The luminosity in the above equation refers to the integrated luminosity. Essentially, the equation says that the total amount of data (the luminosity) times the likelihood a specific interaction will happen (the cross section) gives you the total number of events of that type that have happened (the event count, n). In practice this formula must be modified to account for the natural limitations of the detector: Two new factors, A and ϵ, must be introduced. A, the "acceptance," accounts for the fact that the detector cannot be designed in a way that allows it to see every possible particle. In particular, any particles that continue in the direction of the proton beam will be lost – the presence of the beam pipe precludes detector material in that direction. The value of the acceptance is the fraction of the events that can theoretically be seen by the detector, often defined in terms of a region of kinematic phase space: "above energy X and within angular constraints Y." For simplicity's sake, these constraints are typically chosen to include only regions of the detector and particle energies for which the particles of interest can be well-reconstructed. The "efficiency," ϵ, takes into account the fact that even for particles that leave a definite signature in the detector, they may still not be recorded as such. High efficiency is a primary goal of any experiment, but the sources of inefficiency can never be completely eliminated. The efficiency is often broken down into efficiencies of the individual steps:

$$\epsilon = \epsilon_{trig} \times \epsilon_{reco} \times \epsilon_{sel}$$

The individual efficiencies correspond to triggering, reconstruction, and selection. The boundary between what is considered for the acceptance versus for the efficiency is somewhat fluid: failing to record a particle with a very low energy may be considered an issue of acceptance (because the detector is not designed for such small energies) or a matter of efficiency (because though the particle left a signature in the detector, it was not reconstructed). It is merely a matter of definition, and in the end result makes no difference.

The inclusion of acceptance and efficiency turn the previous expression of event count and cross section into

$$n = \sigma \times \mathcal{L} \times \epsilon \times A$$

The number of events actually seen (n) is reduced by the factors accounting for the detector limitations. In order to calculate the cross section σ (including the branching ratio) of the $q\bar{q} \to Z/\gamma^* \to e^+e^-$ interaction, which is the quantity this analysis aims to get, the equation must be rearranged into

$$\sigma_Z \times \text{ BR } (Z \to ee) = \frac{n_{Z\to ee}}{\mathcal{L} \times \epsilon \times A}$$

Now $\sigma \times$ BR is expressed in terms of other quantities which can be measured or calculated. This equation is the heart of this analysis. The "meat" of the analysis is then measuring the quantities that combine to make up the cross section, and putting it all together.

1.6 Reference Frame and Coordinates

For some high-energy physics experiments, there is a choice of several reference frames that can be used for measurement, each with their own benefits. However, in the case of proton-proton collisions, the logical reference frame for measurements is the frame at rest with respect to the detector, the "lab frame," because it is the same as the proton-proton center of momentum frame. Therefore all measurements are done in the lab frame.

A particular set of coordinates is often used to describe the direction of outgoing or intermediate particles. The direction of the incoming particles defines an axis, the beam axis. The azimuthal angle, measured around this axis and analogous to longitude on the earth's surface, is labeled ϕ. The angle along the axis, the polar angle (analogous to latitude), is labeled by θ. However, it is much more common to measure this angle in terms of pseudorapidity, η, which is given as a function of θ:

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

η is preferred because of its relation to another quantity, rapidity, y, which is a function of the particle's energy and momentum:

$$y = \frac{1}{2}\ln\left(\frac{E + p_L}{E - p_L}\right)$$

where p_L is the longitudinal component of the particle's momentum (the component parallel to the beam axis). Rapidity is useful when dealing with relativistic speeds, such as those at which the particles typically travel. When viewed from separate reference frames that are moving at relativistic speeds with respect to each other, a particle's speed, direction, and energy appear to be different. However, rapidity has special status in that the difference in rapidity between two particles does not change when the reference frame is changed. A particle's rapidity can be approximated by the pseudorapidity if it is very light or traveling very fast (pseudorapidity is identical to rapidity if the particle has zero mass, in which case it is also moving at the speed of light). In addition, outgoing particles tend to be distributed uniformly in terms of rapidity, as opposed to the polar angle θ, which does not enjoy the same uniform distribution. Since rapidity itself is not a measure of angle, pseudorapidity is used as the coordinate. Rapidity is calculated in cases where the particle of interest may be heavy and may not have a high speed, for example the Z boson examined in this analysis.

1.7 General Analysis Tools

A few concepts are common to particle physics analyses and should be mentioned in general.

One of the primary tools in an analysis is a type of data graph known as a "histogram." A histogram is a graphical representation of a data set, used often in statistical analysis. In general, each point in the data set has a value, and the points are grouped into intervals or "bins" according to that value. These histograms show distributions of a particular quantity, for example an electron's energy. Having more events than expected in a certain bin or a series of bins can signal some new physics process. The benefit of grouping the data into bins is that the bins can be widened or narrowed, according to how many events are being studied. For very few events, wide bins are used in order to get a statistically significant number of events in each bin. For very many events, when statistical significance is not as much of an issue, narrow bins are preferable: the shape of the distribution can be seen in more detail. In general, any distribution mentioned means a histogram has been used.

One particularly useful tool in particle physics is the ability to simulate events. The current machinery (described in detail in Chapter 4) encompasses what is known about particles and how they interact. Comparing distributions from the simulated data, what is expected, to those from real data, what actually happened, can aid in the discovery of new physics: something that was not expected will not be in the simulation. Some new physics scenarios are included in various simulation programs, so any potential discovery can be checked against those predictions to see if they agree.

However, the simulations may not be theoretically complete, or may include bugs that noticeably affect the output. The term "data-driven" is used to describe an analysis method that uses only real data, taken with the detector, instead of relying on simulated data. The purpose of using a data-driven method is to eliminate the possibility of a physics result being affected by an error or inaccuracy in the simulation itself. Since some quantities, such as efficiency, are much easier to measure using simulated data, devising data-driven methods to measure those quantities is an important and potentially challenging part of analysis.

1.7.1 Software

This analysis made use of many software packages to perform specific functions within the process of analyzing the data. Most of the software will be explained in the chapter to which each piece is particularly relevant. However, two programs in particular served many purposes throughout the whole of the analysis and should therefore be mentioned separately. The CMS Software (CMSSW) framework is the collaboration-developed software used for many aspects of CMS operation and use. It encompasses data-taking, reconstruction, simulation, and analysis. ROOT is a general data analysis package, written in C++, and developed and maintained at CERN. It is widely used in the field of experimental particle physics. ROOT implements many tools and functions necessary to various types of analysis, such as histograms, fitting, and statistics. CMSSW can interface with ROOT, providing access to ROOT's capabilities within a CMSSW analysis setting.

Chapter 2

THEORY INTRODUCTION

2.1 The Standard Model

The Standard Model describes the current understanding of how fundamental particles interact. There are three different types of particles: quarks and leptons, which make up matter, and bosons, which transmit forces. Quarks and leptons are collectively called fermions; two fermions cannot exist in the same state as each other. Bosons do not have this constraint; many bosons can be in the same state. The particles are shown in Figure 2.1. The first three columns show the first three generations of the quarks and leptons, while the last column shows the bosons. Each fermion generation contains two quarks, a neutrino, and a charged lepton. In general, the generations get progressively heavier. The electron is lighter than the muon, which is lighter than the τ. The same relation exists between the quark generations; the top quark is the heaviest known particle in the Standard Model. Normal, everyday matter is made up solely of particles from the first generation. All normal matter is made of atoms, which in turn consist of a nucleus and electrons surrounding the nucleus. Atomic nuclei are made of protons and neutrons, which are made of up and down quarks: the proton by uud and the neutron by udd.

2.2 quarks

There are six "flavors" of quarks, often represented by their first letters: up, down, charm, strange, top, and bottom. The first quark in each pair has charge $+\frac{2}{3}$, while the second has charge $-\frac{1}{3}$. These charges are given in terms of the electron charge; though the charges are fractional, the quarks are always arranged into composite particles that have integer multiples of the electron charge.

2.3 leptons

The leptons consist of the electron, muon, and tau (τ), each with charge -1, and their corresponding neutrinos, the electron neutrino, muon neutrino, and τ neutrino, which are chargeless. Until recently it was believed that all the neutrinos are massless. However, recent experiments indicate that neutrinos do have a small mass.

2.4 bosons

The bosons in the last column transmit the forces by which particles interact. The massless photon (γ) transmits the electromagnetic force; this particle is more familiar in its everyday context as the particle that makes up light. The gluon (g), also massless, transmits the strong force, which holds protons and neutrons together (like glue, hence the name). The relatively heavy Z and W bosons transmit the weak force, which causes nuclear decay. Each force has an intrinsic strength, represented by its "coupling constant." The strong force's coupling constant (α_S) is larger than that of the electromagnetic force (α); the weak force's (G_F) is weaker.

2.5 antiparticles

In general, each particle has an antiparticle partner, often denoted by a bar ($\bar{u}$), of the opposite charge and quantum numbers; quantum numbers are used to describe the properties of the particles and corresponding interactions. For instance, all leptons have a non-zero "lepton number;" if the initial state of a set of particles has a total lepton number of 1, then any interaction between those particles will result in a final state with the same lepton number. Lepton number is always conserved in this way, as are charge and other quantum numbers. The antiparticle of the electron (e^-) has the opposite charge and lepton number and is called the positron (e^+). Neutrinos, being chargeless, have antineutrinos with opposite lepton number. Quarks and gluons have another quantum "number" known as color. The three colors are red, green, and blue, with the corresponding "anti-colors" being anti-red, anti-green, and anti-blue. Color is unique in that all observed states of quarks and gluons are "colorless." In two-quark states, a color and its anti-color "combine" to make a

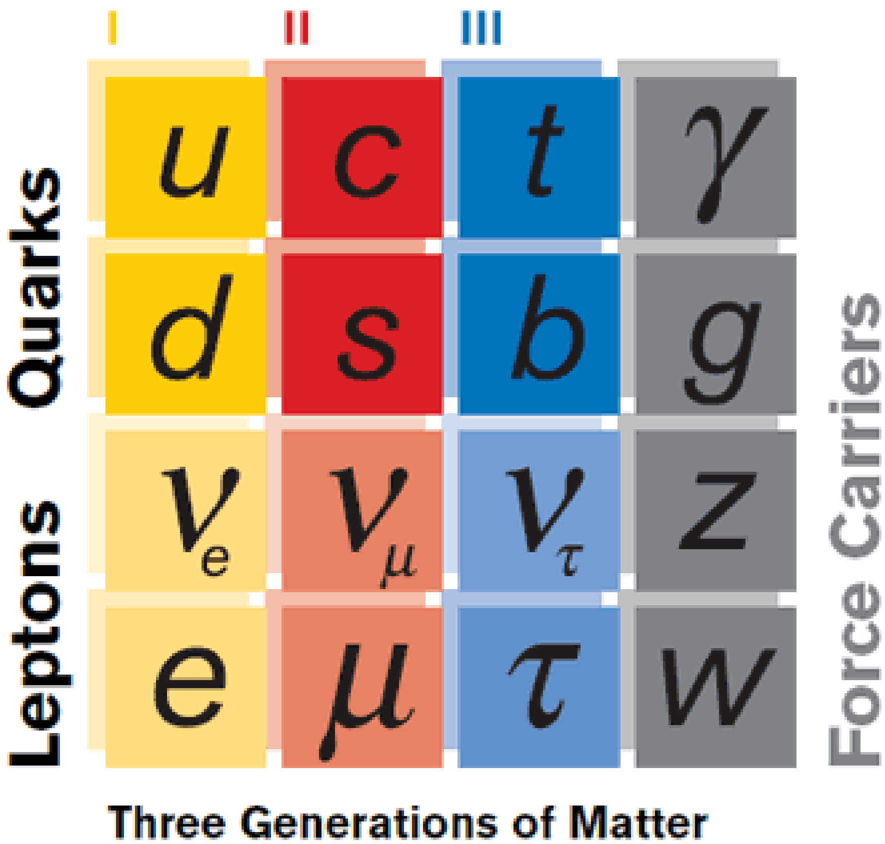

Figure 2.1: Particles of the standard model that are known to exist. The Higgs boson is also predicted as part of the Standard Model, but has not yet been discovered.

colorless state. Three-quark states are also colorless, but through a different combination: the three quarks are colored red, green, and blue (or the corresponding anticolors), which "combine" (like colored light) to form a state with no color In practice, antiparticles are often called by their particle names; since it is known that a neutral Z must decay to one positive and one negative particle, no distinction is made between "positron" and "electron" – both are called "electrons."

2.6 higgs

Not shown in Figure 2.1 is the Higgs boson, which is predicted by the Standard Model to give mass to the other particles. The Higgs has not yet been discovered; finding it (if it exists) is one of the goals of the LHC.

2.7 gravity

The Standard Model does not include the final fundamental force, gravity. Gravity is the weakest of the forces and does not measurably affect interactions between fundamental particles. The reason that it is the only one observable on a cosmological scale is because its "charge," mass, is cumulative; there is no anti-mass to counter it the way a negatively-charge electron and a positively-charged proton combine to make an electrically neutral hydrogen atom. Gravity has been described on a macroscopic scale, but so far no theory has fully united gravity with the other fundamental forces. Such a "theory of everything" is one of the goals that theoretical physics pursues.

TABLE OF MASSES

2.8 forces and groups?

gauge groups, little intro to group theory

A "group" is a set of elements with specific properties. In particular, any transformation of an element in the group results in another element in the group.

A "Lie" group is one in which any transformation can be done in very small steps, each of which results in an element still in the group. The different fundamental forces are described by different Lie groups.

which particles feel which forces?

Each of the three forces in the Standard Model is described by such a Lie group. Each group is represented by a set of square matrices of some dimension, given by the number in parentheses. The matrices used for these representations are all unitary (hence the common "U" designation). "Unitary" means that the matrix's inverse is equal to its complex conjugate transpose. In other words, if you flip the matrix's elements about its diagonal and replace each imaginary unit i with $-i$, then multiply it by the original matrix, the result is the identity matrix.

2.9 EM: U(1)

The electromagnetic force is described by the $U(1)$ group. Essentially, this describes rotation about an axis. The single parameter (an angle) requires only one matrix, corresponding to a single force carrier, the photon. The electromagnetic force causes interactions between photons and any particles that have electric charge. This includes quarks, leptons, and the charged weak bosons.

2.10 Weak: SU(2)

The weak force is represented by the $SU(2)$ group. The "S" addition in the name stands for "special" and means that each matrix's trace (the sum of elements along the matrix's diagonal) is zero. $SU(2)$ represents rotations in three dimensions using three matrices, and therefore has three parameters corresponding to the single angle of $U(1)$. These give rise to the three carriers of the weak force, the two oppositely-charged W's and the Z. The weak force acts on any particles that have the "weak version" of electrical charge, namely all fermions. In particular, the weak force is the only way neutrinos interact.

2.11 QCD: SU(3)

Finally, the strong force is represented by $SU(3)$, which does not have an easy analogy to rotations like the previous two groups. It uses eight matrices, which correspond to the eight different gluon states that mediate the force. The three dimensions of the group relate to the three "color charges," red, green, and blue. Strong force interactions happen between all particles that carry this color charge, namely quarks, but also gluons themselves. This so-called self-interaction on the part of the gluons causes the strong force to increase with distance. At short distances, the quarks making up another particle essentially act free; only when they get further apart do they feel the force keeping them in their bound state. This situation is related to the fact that quarks are never observed alone, only within these bound states. An interaction that is energetic enough to cause quarks to separate is also energetic enough to create more quarks with which the original quarks form new bound states.

2.12 PDFs

Since quarks do not exist in an isolated state outside hadrons, any interaction between them must also take into account the structure of the protons. The quark structure of protons is given by a set of functions called "parton distribution functions" or PDFs:

$$f_i(x)$$

These functions represent the probability of finding a given quark flavor i inside the proton with a given momentum. The quark's momentum itself is not used; rather, the fraction of the proton's momentum that the quark carries, or x. Since no quark can have momentum greater than that of the proton itself, the value of x ranges from 0 to 1. A proton is said to consist of two up quarks and a down quark; however, these represent the "valence" quarks only. A "sea" of quarks is also present, formed from gluons and each existing only momentarily, illustrated in Figure **??**. Therefore the PDFs represent all quark flavors, not just u and d.

QUARK MODEL PICTURE

PDFs cannot be calculated from first principles and must be measured by fits to experimental data. The currently-distributed PDF sets account for data from all major past experiments. However, there is still some uncertainty, and precision measurements of Z production at the LHC can improve the community's knowledge of PDFs.

2.13 ewk: SU(2)xU(1)

The forces here have been given in terms of mathematical groups. However, the individual groups by themselves do not necessarily represent the reality of particles. It is known experimentally that the bosons carrying the weak force have mass. In order for the theory to correctly predict the fact those particles should be massive, the electromagnetic and weak interactions had to be combined into a single framework, $SU(2) \times U(1)$, using a specific method. The integrated $SU(2) \times U(1)$ group allows for the collective four force-carrying bosons (photon, positively- and negatively-charged W's, and Z) to be described as compositions of the same underlying states. The elecroweak interaction has its own set of quantum numbers: "weak isospin," and "hypercharge," which combines weak isospin with electric charge. T_3 is the third component of the isospin and relates how to the fermions are arranged within the standard model generations (refer to Figure 2.1). Each generation consists of two pairs, one pair of quarks and one pair of leptons. The placement of each particle within its pair reflects how it interacts with the weak force. For the top particle in each pair, the hypercharge is $+\frac{1}{2}$; for the bottom particle, it is $-\frac{1}{2}$. (Note: this only applies to particles whose spin is "left-handed," or oriented to spin left with respect to its direction of motion. Particles with "right-handed" spin have zero weak isospin; right-handed neutrinos do not interact in the Standard Model. This is equivalent with the assertion that neutrinos must be moving at the speed of light and therefore do not have mass. If neutrinos had mass, they would be moving slower than the speed of light and there could be a reference frame – where the observer moves faster than the neutrino – in which the neutrino would actually be left-handed. Because recent observations have indicated neutrinos do in fact have mass, this aspect of the Standard Model is inaccurate.)

2.14 spont symm break

The fact that the three weak bosons are massive, though, comes from applying "spontaneous symmetry breaking." This method uses the fact that the groups mentioned above have underlying symmetries: that is, they contain parameters that do not affect the physical description in any way. They are symmetric with respect to variations in these parameters. However, the values of these parameters can be chosen in a strategic way and the result manipulated to give masses to the weak bosons. (A consequence of this "strategic choice" is the appearance of another massive particle in the theory: the Higgs boson. The Higgs boson is therefore predicted by the theoretical mechanism used to account for massive weak bosons. Time – and collision data – will tell if the theory in this form is correct.)

2.15 matrix elements/feynman rules???

These matrices act on the set of initial states and result in the set of final states. Each element in the matrix, mapping a single initial state to a single other final state, gives the amplitude with which the transformation happens. Essentially, squaring the matrix element gives the relative probability of that particular transformation. Matrices are often used to represent transitions from one quantum mechanical state to another.

This is used in particle physics as such: a transformation from an initial state, in our case two quarks from colliding protons, to a final state, an electron and a positron, is represented by a Feynman diagram, shown in Figure 2.2. The interaction can be thought to progress to the right. Each straight line represents a fermion, either the quark or the electron. The wavy line represents a boson, the Z. (Gluons are represented by curly lines.) The intersections between particle lines, or vertices, show interactions between the given particles. The matrix element corresponding to this transformation can be written down by attributing factors to the various elements of the Feynman diagram: the particle lines and the vertices. In this way, the diagrams are not just useful conceptual pictures but also powerful calculational tools.

2.16 pQCD: LO, NLO, all that

However, in real life a given interaction is never represented by just one diagram. For each diagram, such as Figure 2.2, there is an infinite number of more-complicated diagrams that represent the same initial and final states. A few examples are illustrated in Figure **??**.

FIGURE WITH MORE FEYNMAN DIAGRAMS

Fortunately, the more complicated a diagram, the smaller its contribution. Each vertex in the diagram contributes one factor of the relevant coupling constant. Since the coupling constants are small, a two-vertex diagram contributes much more than a three-vertex diagram, which contributes more than a four-vertex diagram, etc. However, the corrections from the higher-vertex (or "higher-order") diagrams make noticeable contributions to the overall result, and therefore they should be taken into account as far as possible. Calculating the contributions from successively higher-order diagrams can be difficult, though, and