Experiment 1
Stern-Gerlach Experiment

1 Introduction

The **Stern-Gerlach** experiment of 1922 is one of the classic experiments demonstrating the quantized nature of physical observables. The details (as always) are somewhat technical, involving the deflection of silver atoms. But fundamentally, the quantized nature of the electron's spin degree of freedom is illuminated by applying a spatially varying magnetic field to a beam of (effectively) spin-$\frac{1}{2}$ degrees of freedom–the simplest nontrivial quantum system.

In this numerical experiment, we employ some basic aspects of quantum mechanics to simulate the results of this experiment with a variety of tunable parameters. A user-friendly JAVA simulation[1] of this type serves as a guide for our rudimentary simulator.

2 Spin-$\frac{1}{2}$ states and measurement

The most general quantum two-level system can be represented by the state

$$\begin{aligned} |\psi\rangle &= \alpha\,|+\rangle + \beta\,|-\rangle \\ &\doteq \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \end{aligned}$$

where the representation by a simple, column vector in the last line is particularly useful for numerical calculations. Here the basis "kets" $|\pm\rangle$ represent states of definite spin projection along the z axis. A spin measurement along *any* axis will always return $\pm\frac{\hbar}{2}$, and any direction can be used to define a complete set of basis vectors.

Normalization $\langle\psi|\psi\rangle = 1$ requires $|\alpha|^2 + |\beta|^2 = 1$, but there are no other restrictions. It is thus possible to generate a "random" quantum state with no preferred orientation by appropriate randomization of the coefficients α and β. A simple way of getting the job done is to select a random amplitude $0 \le r_i \le 1$ and random phase $0 \le \phi_i \le 2\pi$ for $i = 1, 2$ and set $z_1 = r_1 e^{i\phi_1}$, $z_2 = r_2 e^{i\phi_2}$. Then we can set

$$\alpha = \frac{z_1}{\sqrt{r_1^2 + r_2^2}}, \qquad \beta = \frac{z_2}{\sqrt{r_1^2 + r_2^2}},$$

to obtain a normalized state. Given a state and an observable A, a **measurement** of A returns one of the eigenvalues of the Hermitian operator $\hat{A}$ which represents the observable. For a two-level system, the operator $\hat{A}$ can be represented by a 2×2 matrix,

$$\hat{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \tag{1}$$

[1] http://physics.oregonstate.edu/~mcintyre/ph425/spins/index.html

The eigenvalues a_n and eigenvectors $|a_n\rangle$ of $\hat{A}$ are defined by $\hat{A}\,|a_n\rangle = a_n\,|a_n\rangle$. The randomness enters through the probabilistic statement that individual measurements are not deterministic but governed by a probability $\mathcal{P}_{a_n}$ which gives only the probability of obtaining a_n as

$$\mathcal{P}_{a_n} = \left|\langle a_n|\psi\rangle\right|^2 . \tag{2}$$

After measuring a_n, the state of the system is "projected" to the eigenstate $|a_n\rangle$. The entire process is encoded into the pair of functions `newstate(n)` and `measureop(v,op,n)`. A random state of size n ($n = 2$ for our purposes) is defined by calling `newstate(n)`. Given an operator of appropriate size, the function `measureop(v,op,n)` takes the state `v` and performs a measurement of `op`, returning the value obtained and the new state of the system.