QUANTUM MECHANICS

de Broglie

for ALL things, light & matter:
$\lambda = h/p$
$\nu = E/h$

Schrödinger Equation

$$\hat{H}\left|\Psi\right\rangle = i\hbar \frac{\partial}{\partial t}\left|\Psi\right\rangle$$

$$\hat{H}\left|\psi(\xi)\phi(t)\right\rangle = E_n \left|\psi(\xi)\right\rangle e^{iE_n t/\hbar}$$

this is non-relativistic
Ψ can be complex-valued
boundary conditions lead to energy quantization
not derived (initially) but fit to reality

Time Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

Quantum Things

- Mass, charge, etc.
- Energy (cf. blackbody radiation, photoelectric effect, photons & other fields)
- Interference – 2 slit experiment
- Tunneling (radioactive decay, electronic devices)
- Zero-point motion, i.e., electron never at 0 energy
- Diffraction in matter

Operators

position: $\langle x \rangle$: $\hat{x} = x$
function of position: $\langle f(\boldsymbol{r}) \rangle$: $\hat{f} = f(\boldsymbol{r})$
velocity: $\langle v \rangle = \frac{d\langle x \rangle}{dt}$: $\hat{\boldsymbol{v}} = \frac{\hbar}{im}\boldsymbol{\nabla}$
Note that this is velocity of expectation, but gives velocity in QM
momentum: $m\frac{d\langle x \rangle}{dt}$: $\hat{\boldsymbol{p}} = \frac{\hbar}{i}\boldsymbol{\nabla}$
energy: $\hat{H} = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V$
exchange: $\hat{P}_{12} \equiv \hat{P}_{12}\left|\Psi(\xi_1,\xi_2)\right\rangle = \left|\Psi(\xi_2,\xi_1)\right\rangle$; eig. val's are $\pm 1 \forall \left|\Psi\right\rangle$ known to man; $\hat{P}_{12}\left(\hat{P}_{12}\left|\Psi(\xi_1,\xi_2)\right\rangle\right) = \left|\Psi(\xi_1,\xi_2)\right\rangle$
parity: $\hat{\Pi} \equiv \hat{\Pi}\left|\Psi(\vec{r})\right\rangle = \left|\Psi(-\vec{r})\right\rangle$; eig. val's are ± 1 if they $\exists$ since $\hat{\Pi}\left(\hat{\Pi}\left|\Psi(\vec{r})\right\rangle\right) = \left|\Psi(\vec{r})\right\rangle$
raising/lowering SHO:
ang. momentum: $\boldsymbol{J} =$

Many-particle systems

General solution

Special case: no external forces

Identical particles

Slater determinant

Pauli exclusion principle

Fermi energy – bosonic system, non-interacting

Fermi energy – fermionic system, non-interacting

Angular momentum and spin

Spin paramagnetic resonance

Rabi frequency
Rotating-wave approximation

Angular momentum / spin operators

Operators

Hydrogen-like atoms

$H^0 = -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{kZe^2}{r}$

$H^{rel} = -\frac{p^4}{8m_e^3c^2}$

$H^{s-o} = \frac{kZe^2}{2m_e^2c^2}\frac{1}{r^3}\vec{s}\cdot\vec{L}$

$H^{hf} = \frac{Ze^2}{4\pi\varepsilon_0}\frac{g_N}{4M_N m_e c^2}\left(\frac{3\vec{r}(\vec{r}\cdot\vec{I})}{r^5} + \frac{8\pi}{3}\vec{I}\delta(\vec{r})\right)\cdot(\vec{L}+2\vec{s})$