Pre-algebra problems

November 10, 2018

1. Solve for x in the following:

$$5\left(\frac{x^2}{3} - \frac{3}{4}\right) = \frac{35}{12}$$

2. Peter has more than 150 books, but less than 200 books. Of these, 20% are novels, and 1/7 are collections of poems. How many books does Peter have?

3. Simplify the following expression

$$\frac{1}{\frac{1}{x} + \frac{1}{y}}$$

such that you end up with a single fraction where neither the numerator nor the denominator has a fraction in it.

4. (Note that the following passage is basically copied from https://en.wikipedia.org/wiki/Thin_lens but I've chosen a different sign convention, so equations won't look *exactly* the same as there.)

The focal length of a lens f is useful for determining how a lens will form images (how "large" they appear, and how far away an object must be for it to be in focus based on how far away you are from the lens).

The focal length of a lens is determined by the lens's shape (geometry) and the material of which it is made (described by the material's index of refraction, n). The equation for this in "geometric optics" (things get more complicated, but save that for college) is given by the **lensmaker's equation**:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)d}{nR_1R_2}\right]$$

f is the focal length, n is the index of refraction of the lens's material, R_1 is the radius of curvature of what we'll call the "front" surface of the lens, R_2 is the radius of curvature of the "back" surface of the lens, and d is the thickness of the lens.

In the case that the thickness of the lens d is "very small" compared to the radii of curvature, then $d/(R_1R_2)$ is going to be very small compared to either $1/R_1$ or $1/R_2$, and the above equation looks like

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} - \text{relatively tiny number}\right]$$

and so you can get relatively accurate results by dropping the last term. This gives you what's called the **thin lens equation**:

$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

where $\approx$ means "approximately equal to," but for simplicity you can just use the equals sign (=) from here on out.

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

Now you have a simple equation you can work with. You'll note that the equation is written in terms of $\frac{1}{f}$ (called the "reciprocal of f") rather than f probably because it's easier to see how to isolate R_1 and R_2 in this form than if you were to solve for f (and reduce/simplify the resulting equation).

Sometimes writing an equation in terms of the reciprocal (or some other expression) of the variable you "care about" makes it easier to work with than fully solving for the variable. When to leave an equation in one form not fully-solved for a variable depends on the situation, and you will get a better sense of when that's appropriate as you work more with equations.

Anyway, there's still value in solving for f; do so, and also solve in terms of each of the following. Try to reduce the expressions as much as possible, e.g. using how you simplifieid the expression in Problem 3.

(a) f
(b) $1/R_1$
(c) R_1
(d) $1/R_2$
(e) R_2
(f) n

Hint: In solving for the reciprocal of R_1 (and likewise for the reciprocal of R_2), it may help to substitute a new letter for the reciprocal and work with that, just like you did for x^2 in Problem 1 (e.g., define $a_1 \equiv 1/R_1$, solve for a_1, then substitute $1/R_1$ back in when you're done).

Some following questions will involve working more with the thin lens equation, so keep your solutions around for those questions.