Simple error propagation

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If you have some function of the variables x, y, z, etc.: $f(x, y, z, \dots)$, and you want to find the error in f given measurements of x, y, z, ... with values x_0, y_0, z_0 and errors σ_x, σ_y, σ_z, respectively, then use the following:

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\Big|_{x=x_0}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\Big|_{y=y_0}\right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z}\Big|_{z=z_0}\right)^2 \sigma_z^2 + \cdots \quad (1)$$

It's easy to find the rule now for simple situations, such as $f(x, y) = x \cdot y$:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2} \quad (2)$$

$$= \sqrt{y^2\sigma_x^2 + x^2\sigma_y^2} \quad (3)$$

$$= \sqrt{x^2y^2\left(\frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{x^2}\right)} \quad (4)$$

$$= xy\sqrt{\frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{x^2}} \quad (5)$$

(Of course you can use the forms in Eqn. 3, 4, or 5, whichever is most convenient, since they're all the same.)

Work through the following examples yourselves, and if you have any questions about these, I can answer on Friday before or after recitation.

(1) $f(x, y) = 2x - 3y$

(2) $f(x) = 3x^2$

(3) $f(x) = 1/x$

(3) $f(x, y) = x/y$

So that just leaves propagating error for linear fits (linear regression).