May 2, 2011 Justin Lanfranchi Page 1 MATH & PHYSICS EQUATION SHEET

QUANTUM MECHANICS

de Broglie

for ALL things, light & matter:

$$\lambda = h/p$$

$$v = E/h$$

Schrödinger Equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

this is non-relativistic Ψ can be complex-valued

boundary conditions lead to energy quantization not derived (initially) but fit to reality

Time Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

Quantum Things

- Mass, charge, etc.
 Energy (cf. blackbody radiation, photoelectric effect, photons & other fields)
- · Interference 2 slit experiment

Tunneling (radioactive decay, electronic devices) Zero-point motion, i.e., electron never at 0 energy Diffraction in matter

Operators

position,
$$\langle x \rangle$$
: $\hat{x} = x$ a function of position, $\langle f(\boldsymbol{r}) \rangle$: $\hat{f} = f(\boldsymbol{r})$ velocity, $\langle v \rangle = \frac{d\langle x \rangle}{dt}$: $\hat{v} = \frac{\hbar}{lm} \nabla$
Note that this is velocity of expectation, but gives velocity in QM momentum, $m \frac{d\langle x \rangle}{dt}$: $\hat{p} = \frac{\hbar}{l} \nabla$ energy: $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V$

Hydrogen-like atoms

$$\begin{split} H^0 &= -\frac{\hbar^2}{2m_e} \nabla^2 - \frac{kZe^2}{r} \\ H^{rel} &= -\frac{\rho^4}{8m_e^2c^2} \\ H^{s-o} &= \frac{kZe^2}{2m_e^2c^2} \frac{1}{f} \vec{s} \cdot \vec{L} \\ H^{hf} &= \frac{Ze^2}{4\pi\epsilon_0} \frac{g_N}{4M_N m_e c^2} \left(\frac{3\vec{r}(\vec{r}\vec{I})}{r^5} + \frac{8\pi}{3} \vec{I} \delta(\vec{r}) \right) \cdot (\vec{L} + 2\vec{s}) \end{split}$$