Solucions Problemes Tema 3

1.

(a) $-2z^4 + z^3 + 3z$, ROC:$\mathbb{C} - \{\infty\}$

(b) $3 + 3z + z^3 - 2z^4$, ROC:$\mathbb{C} - \{\infty\}$

(c) $3z^{-1} + z - 2z^2$, ROC:$\mathbb{C} - \{0, \infty\}$

(d) $3 + z^{-1} - z^{-2}$, ROC:$\mathbb{C} - \{0\}$

2.

(a) $\dfrac{1}{1 - z^{-1}}$, ROC:$|z| > 1$

(b) $\dfrac{1}{(1 - z^{-1})^2}$, ROC:$|z| > 1$

(c) $\dfrac{1}{1 - az^{-1}} + \dfrac{1}{1 - a^{-1}z^{-1}}$, ROC: $\begin{cases} |z| > 1/|a| & \text{si } |a| \geq 1 \\ |z| > |a| & \text{si } |a| < 1 \end{cases}$

(d) $\dfrac{1}{1 - az^{-1}} + \dfrac{1}{1 - a^{-1}z^{-1}}$, ROC: $\begin{cases} \emptyset & \text{si } |a| \geq 1 \\ |a| < |z| < 1/|a| & \text{si } |a| < 1 \end{cases}$

(e) $\dfrac{1 - \cos \omega z^{-1}}{1 - 2\cos \omega z^{-1} + z^{-2}}$, ROC:$|z| > 1$

(f) $\dfrac{\cos \omega z^{-3} + \cos \omega z^{-1} - 2z^{-2}}{(1 - 2\cos \omega z^{-1} + z^{-2})^2}$, ROC:$|z| > 1$

(g) $\dfrac{1 - a\cos \omega z^{-1}}{1 - 2a\cos \omega z^{-1} + a^2 z^{-2}}$, ROC:$|z| > |a|$

(h) $-z\dfrac{a\cos \omega z^{-2}(1 - 2a\cos \omega z^{-1} + a^2 z^{-2}) - (1 - a\cos \omega z^{-1})(2a\cos \omega z^{-2} - 2a^2 z^{-3})}{(1 - 2a\cos \omega z^{-1} + a^2 z^{-2})^2}$, ROC:$|z| > |a|$

(i) $\dfrac{z^{-1}(1 + az^{-1})}{(1 - az^{-1})^3} + \dfrac{z^{-1}}{(1 - az^{-1})^2}$, ROC:$|z| > |a|$

3.

(a) =(b) $\dfrac{1}{1 - \frac{1}{3}z^{-1}} - \dfrac{1}{1 - 2z^{-1}}$, ROC:$\frac{1}{3} < |z| < 2$

5.

(a) $\dfrac{X(z)}{1 - z^{-1}}$

(b) $\dfrac{X(z^{1/2}) + X(-z^{1/2})}{2}$

(c) $X(z^2)$