Useful Equalities and Definitions

A list of useful mathematical definitions for algorithm analysis and computer science in general. Much of this was copied with minor modification from:

- Cormen, Thomas H., et. al.. *Introduction to Algorithms*. Second Edition. MIT Press. Cambridge, MA. 2001.

Floors and Ceilings

Definition 1. *For real number x, the **floor** of x, $\lfloor x \rfloor$, is the greatest integer less than x.*

Definition 2. *For real number x, the **ceiling** of x, $\lceil x \rceil$, is the least integer greater than x.*

For any real number x,

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1 \tag{1}$$

For any integer n,

$$\lceil n/2 \rceil + \lfloor n/2 \rfloor = n \tag{2}$$

For any real $n \geq 0$ and integers $a, b > 0$:

$$\lceil \lceil n/a \rceil / b \rceil = \lceil n/ab \rceil \tag{3}$$

$$\lfloor \lfloor n/a \rfloor / b \rfloor = \lfloor n/ab \rfloor \tag{4}$$

$$\lceil a/b \rceil \leq (a + (b-1))/b \tag{5}$$

$$\lfloor a/b \rfloor \geq (a - (b-1))/b \tag{6}$$

Polynomials

Definition 3. *Given integer $d > 1$, a **Polynomial in n of degree** d is a function $p(n)$,*

$$p(n) = \sum_{i=0}^{d} a_i n^i \tag{7}$$

*where $a_0, a_1, \ldots, a_d$ are the **coefficients** and $a_d \neq 0$.*

Exponentials

For all real $a > 0, m, n$:

$$a^0 = 1 \tag{8}$$
$$a^1 = a \tag{9}$$
$$a^{-1} = \frac{1}{a} \tag{10}$$
$$(a^m)^n = a^{mn} \tag{11}$$
$$(a^m)^n = (a^n)^m \tag{12}$$
$$a^m a^n = a^{m+n} \tag{13}$$

Logarithms

Definition 4. *Where $b^y = x$, $\log_b x = y$.*

The following is only notation,

$$\log_b^k n = (\log_b n)^k$$

For all $a > 0$,$b > 0$, $c > 0$, n,

$$a = b^{\log_b a} \tag{14}$$
$$\log_c(ab) = \log_c a + \log_c b \tag{15}$$
$$\log_b a^n = n \log_b a \tag{16}$$
$$\log_b a = \frac{\log_c a}{\log_c b} \tag{17}$$
$$\log_b \frac{1}{a} = -\log_b a \tag{18}$$
$$\log_b a = \frac{1}{\log_a b} \tag{19}$$
$$a^{\log_b c} = c^{\log_b a} \tag{20}$$

Roots

Definition 5. *Where $r^n = x$, then the n^{th} root of x is r and is denoted by $\sqrt[y]{x} = x^{\frac{1}{y}} = r$*

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \tag{21}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \tag{22}$$
$$\sqrt[n]{a^m} = (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}} \tag{23}$$