Estimating Frequency of Three-Phase Power Systems via Widely-Linear Modeling and Total Least-Squares

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***Abstract*—The frequency of a three-phase power system can be estimated from the parameters of a widely-linear predictive model for the complex-valued $\alpha\beta$ signal of the system. Using the total least-squares (TLS) method, it is possible to estimate the model parameters while compensating for error in both input and output of the model when the voltage readings of the three phases are contaminated with noise. In this paper, we utilize the inverse power method to find a TLS estimate of the parameters of the assumed widely-linear predictive model in an adaptive fashion. Simulation results show that the resultant algorithm, called augmented inverse power iterations (AIPI), outperforms the recently proposed augmented complex Kalman filter (ACKF) and augmented complex extended Kalman filter (ACEKF) algorithms in estimating the frequency of the three-phase power systems. Unlike ACKF and ACEKF, AIPI requires no parameter tuning or prior knowledge of the noise variances. Computational complexity of AIPI is also similar to those of ACKF and ACEKF.**

Keywords*—*adaptive frequency estimation; inverse power iterations; smart grids; total least-squares; widely-linear modeling

I. Introduction

Smart grids enhance the efficiency, reliability, economy, and sustainability of the generation, distribution, and consumption of electricity by collecting and acting on information regarding the behavior of the consumers and suppliers in an automated manner.

System frequency is a vital and sensitive parameter that needs to be continually monitored in the smart grids. To check the health of the power grids and assure reliable measurement of other system parameters such as voltages, currents, and active and reactive powers, accurate power frequency estimation is essential. With the drive of market economy, power systems of future will have to operate much closer to their limits and sustain a perfect balance between generation and load. Deviation of the system frequency from its nominal value consistently represents an imbalance between power generation and load demand. Accordingly, many power-system protection-and-control applications require accurate and fast estimation of the system frequency. An erroneous frequency estimate can result in inadequate load shedding by frequency relays, which in turn may eventually cause a catastrophic grid failure [1].

Frequency estimation of power systems has been investigated for decades generating an ample body of literature, e.g., [2]-[12] and the references therein. Several proposed methods are based on zero-crossing technique [3], phase-locked loop [4], least-squares estimation [5], and extended Kalman filter [6]. Most of these methods use the voltage readings of a single phase of the system. In three-phase systems, none of the phases can faithfully characterize the whole system and its properties. Therefore, a robust frequency estimator should take into account the information of all three phases [7]-[9].

Clark's transform (also known as $\alpha\beta$ transform) gives a single complex-valued signal that can represent a three-phase system [10]. The frequency of a three-phase power system can be estimated assuming a linear predictive model for its complex-valued $\alpha\beta$ signal [11], [12]. However, when the system is unbalanced, for example because of having different phase peak voltages in the aftermath of a voltage sag, the $\alpha\beta$ signal is non-circular (improper) and its real and imaginary parts have different statistical properties [13], [14]. In such cases, the $\alpha\beta$ signal is better described by a widely-linear predictive model rather than a strictly-linear one [15].

In [12], an algorithm for frequency estimation of three-phase power systems utilizing the $\alpha\beta$ signal is developed based on the widely-linear (augmented) complex least mean square (ACLMS) algorithm [16]. In unbalanced situations, this algorithm significantly outperforms its strictly-linear counterpart proposed in [11], which is based on the complex least mean square (CLMS) algorithm [17], while retaining the simplicity and numerical stability of the LMS-type algorithms. However, it assumes a noise-free environment, i.e., where the voltage measurements are exact and error-free. This assumption is often unrealistic since several kinds of error can corrupt the measurements, e.g., sampling, quantization, and instrument errors.

In [18], the widely-linear (augmented) complex-valued Kalman filter (ACKF) and the widely-linear (augmented) complex-valued extended Kalman filter (ACEKF) algorithms [19] are employed to estimate frequency of three-phase power systems when the voltage readings are contaminated with noise. These algorithms perform considerably better that the

ACLMS algorithm. However, they require careful tuning of the covariance matrices of state and observation noises. In practice, such a tuning is not straightforward and can only be realized effectively if the variances of the measurement noises are known *a priori*.

Total least-squares (TLS) is a fitting method that improves accuracy of the least-squares estimation techniques when both the input and output data of a linear system are subject to observational error. TLS finds an estimate for the system parameters that fits the input to the output with minimum perturbation in the data [20].

In this paper, we utilize the TLS concept together with a widely-linear predictive model for the $\alpha\beta$ signal to estimate the frequency of a three-phase power system from its noisy phase voltage observations. To find the TLS estimate of the model parameters, we employ the inverse power method [21]. The system frequency is then calculated adaptively using the parameter estimates. Simulation results testify that the new algorithm is superior to the ACKF and ACEKF algorithms in estimating the frequency of three-phase power systems without requiring any prior knowledge about the noises or any parameter tuning.

II. ALGORITHM

The phase voltages of a three-phase power system can be expressed as

$$v_n^a = V_n^a \cos(2\pi f \tau n + \phi) + \eta_n^a$$
$$v_n^b = V_n^b \cos\left(2\pi f \tau n + \phi - \tfrac{2\pi}{3}\right) + \eta_n^b$$
$$v_n^c = V_n^c \cos\left(2\pi f \tau n + \phi + \tfrac{2\pi}{3}\right) + \eta_n^c$$

where V_n^a, V_n^b, and $V_n^c \in \mathbb{R}$ are the peak values, $f \in \mathbb{R}$ is the system frequency, $\tau \in \mathbb{R}$ is the sampling interval, $n \in \mathbb{N}$ is the time index, and $\phi \in \mathbb{R}$ is a constant phase. Moreover, η_n^a, η_n^b, and $\eta_n^c \in \mathbb{R}$ denote the measurement noises.

Let us consider the complex-valued voltage signal

$$v_n = v_n^\alpha + j v_n^\beta$$

where $j = \sqrt{-1}$ and v_n^α and $v_n^\beta \in \mathbb{R}$ are given by Clarke's $\alpha\beta$ transform [10], i.e.,

$$\begin{bmatrix} v_n^\alpha \\ v_n^\beta \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_n^a \\ v_n^b \\ v_n^c \end{bmatrix}.$$

This signal can be used to estimate the system frequency [22]. A *widely-linear* predictive model for v_n is formulated as

$$\tilde{v}_{n-1} h + \tilde{v}_{n-1}^* g = \tilde{v}_n$$

or

$$[\tilde{v}_{n-1}, \tilde{v}_{n-1}^*] \begin{bmatrix} h \\ g \end{bmatrix} = \tilde{v}_n$$

where $\tilde{v}_n$ is the noiseless version of v_n, i.e., without taking into account η_n^a, η_n^b, and η_n^c. In addition, h and $g \in \mathbb{C}$ are the model parameters that we wish to identify and superscript $*$ stands for complex conjugate. It is shown in [12] that, from h and g, the system frequency can be estimated as

$$\hat{f} = \frac{1}{2\pi\tau} \sin^{-1}\left(\sqrt{\Im^2(h) - |g|^2}\right)$$

where $\Im(\cdot)$ and $|\cdot|$ denote the imaginary part and the absolute value, respectively.

In order to identify h and g at the presence of noise, we utilize an adaptive filter whose tap-weights vector, denoted by $\mathbf{w}_n = [w_{1,n}, w_{2,n}]^T \in \mathbb{C}^{2\times 1}$, is taken as an estimate of $[h, g]^T$ at iteration n. In the context of total least-squares (TLS) estimation, $\mathbf{w}_n$ is computed such that it fits the filter input data to the filter output data by incurring minimum perturbation in the data, i.e., it holds that

$$(\mathbf{X}_n + \mathbf{\Delta}_n)\mathbf{w}_n = \mathbf{y}_n + \boldsymbol{\delta}_n$$

where

$$\mathbf{X}_n = \begin{bmatrix} v_0, \dots, v_{n-2}, v_{n-1} \\ v_0^*, \dots, v_{n-2}^*, v_{n-1}^* \end{bmatrix}^T,$$
$$\mathbf{y}_n = [v_1, \dots, v_{n-1}, v_n]^T,$$

and $\mathbf{\Delta}_n \in \mathbb{C}^{n\times 2}$ and $\boldsymbol{\delta}_n \in \mathbb{C}^{n\times 1}$ are the minimum input and output perturbations, respectively. Using the singular value decomposition (SVD) of the augmented data matrix, $[\mathbf{X}_n, \mathbf{y}_n]$, the TLS solution for $\mathbf{w}_n$ is given by [23]

$$\mathbf{w}_n = -\frac{[z_{1,n}, z_{2,n}]^T}{z_{3,n}}$$

where $\mathbf{z}_n = [z_{1,n}, z_{2,n}, z_{3,n}]^T$ is the right singular vector corresponding to the smallest singular value of $[\mathbf{X}_n, \mathbf{y}_n]$ or the eigenvector corresponding to the smallest eigenvalue of

$$\mathbf{\Psi}_n = \begin{bmatrix} \mathbf{X}_n^H \\ \mathbf{y}_n^H \end{bmatrix} [\mathbf{X}_n, \mathbf{y}_n].$$

An estimate of $\mathbf{z}_n$ can be adaptively found by executing an iteration of the inverse power method [21] at each time instant:

$$\mathbf{z}_n = \mathbf{\Psi}_n^{-1} \mathbf{z}_{n-1}. \tag{1}$$

The matrix $\mathbf{\Psi}_n$ may be recursively updated as

$$\mathbf{\Psi}_n = \mathbf{\Psi}_{n-1} + \begin{bmatrix} v_{n-1}^* \\ v_{n-1} \\ v_n^* \end{bmatrix} [v_{n-1}, v_{n-1}^*, v_n]. \tag{2}$$

Applying the matrix inversion lemma [21] to (2) yields

$$\mathbf{\Psi}_n^{-1} = \mathbf{\Psi}_{n-1}^{-1} - \frac{\mathbf{q}_n}{1 + [v_{n-1}, v_{n-1}^*, v_n]\mathbf{q}_n} \mathbf{q}_n^H$$

where

$$\mathbf{q}_n = \mathbf{\Psi}_{n-1}^{-1} \begin{bmatrix} v_{n-1}^* \\ v_{n-1} \\ v_n^* \end{bmatrix}.$$

Dividing both sides of (1) by $-z_{3,n-1}$ gives

$$\frac{\mathbf{z}_n}{-z_{3,n-1}} = \mathbf{\Psi}_n^{-1} \begin{bmatrix} \mathbf{w}_{n-1} \\ -1 \end{bmatrix}.$$

If we define $\mathbf{u}_n = [u_{1,n}, u_{2,n}, u_{3,n}]^T \in \mathbb{C}^{3\times 1}$ as

$$\mathbf{u}_n = \frac{\mathbf{z}_n}{-z_{3,n-1}},$$

TABLE I

FREQUENCY ESTIMATION USING THE AIPI ALGORITHM

Initialization:

$\boldsymbol{\Psi}_0^{-1} = \delta^{-1}\mathbf{I}_3$, $\delta > 0$ is a small number and $\mathbf{I}_3$ is the 3×3 identity matrix

$\mathbf{w}_0 = \begin{bmatrix}1\\0\end{bmatrix}$

At each iteration $n = 1,2,\ldots$:

$$\mathbf{q}_n = \boldsymbol{\Psi}_{n-1}^{-1}\begin{bmatrix}v_{n-1}^*\\v_{n-1}\\v_n^*\end{bmatrix}$$

$$\boldsymbol{\Psi}_n^{-1} = \boldsymbol{\Psi}_{n-1}^{-1} - \frac{\mathbf{q}_n}{1 + [v_{n-1}, v_{n-1}^*, v_n]\mathbf{q}_n}\mathbf{q}_n^H$$

$$\mathbf{u}_n = \boldsymbol{\Psi}_n^{-1}\begin{bmatrix}\mathbf{w}_{n-1}\\-1\end{bmatrix}$$

$$\mathbf{w}_n = -\frac{[u_{1,n}, u_{2,n}]^T}{u_{3,n}}$$

$$\hat{f}_n = \frac{1}{2\pi\tau}\sin^{-1}\left(\sqrt{\Im^2(w_{1,n}) - |w_{2,n}|^2}\right)$$

we will have

$$\mathbf{u}_n = \boldsymbol{\Psi}_n^{-1}\begin{bmatrix}\mathbf{w}_{n-1}\\-1\end{bmatrix}$$

$$\mathbf{w}_n = -\frac{[u_{1,n}, u_{2,n}]^T}{u_{3,n}}.$$

Accordingly, the system frequency is adaptively estimated as

$$\hat{f}_n = \frac{1}{2\pi\tau}\sin^{-1}\left(\sqrt{\Im^2(w_{1,n}) - |w_{2,n}|^2}\right).$$

We call the resultant algorithm *augmented inverse power iterations* (AIPI) and summarize it in Table I. We also present the computational complexity of frequency estimation using this algorithm and the ACKF and ACEKF algorithms of [18] in Table II.

III. SIMULATIONS

We consider a three-phase power system where $f = 50$ Hz, $\tau = 2$ ms, and the noises (η_n^a, η_n^b, and η_n^c) are zero-mean white Gaussian with variance σ_η^2. In Fig. 1, we depict the learning curves, i.e., $E[\hat{f}_n]$ versus time, for the AIPI, ACKF, and ACEKF algorithms when $\sigma_\eta^2 = 0.01$ and $\hat{f}_0 = 0$. In Figs. 2 and 3, we plot the steady-state bias and mean-square error, defined as $|E[\hat{f}_\infty] - f|$ and $E[(\hat{f}_\infty - f)^2]$, respectively, against σ_η^2. We evaluate the expectations by taking the ensemble average over 10^3 independent trials and the steady-state values by averaging over last 10^3 of 40×10^3 iterations. We also adjust the noise covariance matrices of the ACKF and ACEKF algorithms to achieve their best performance at each scenario. In Fig. 1, the system is balanced. In Fig. 2, it has experienced a Type-C voltage sag where there is 30% voltage drop and 15.6° phase offset in phases a and b with respect to the balanced state. In Fig. 3, a short-circuit between phase a and the ground, resulting in $V_n^a = 0$, has made the system highly unbalanced. Figs. 2 and 3 show that the AIPI algorithm outperforms the ACKF and ACEKF algorithms. Moreover, the increase in unbalancedness of the system, i.e., non-circularity of v_n, widens the performance gap between the AIPI algorithm and the Kalman-filter-based ones.

TABLE II

COMPUTATIONAL COMPLEXITY OF FREQUENCY ESTIMATION USING THE AIPI, ACKF, AND ACEKF ALGORITHMS IN TERMS OF NUMBER OF REQUIRED ARITHMETIC OPERATIONS PER ITERATION

	$\times$	$+$	$/$	$\surd$	$\sin^{-1}$
AIPI	35	25	2	1	1
ACKF	31	21	1	1	1
ACEKF	38	36	1	1	1

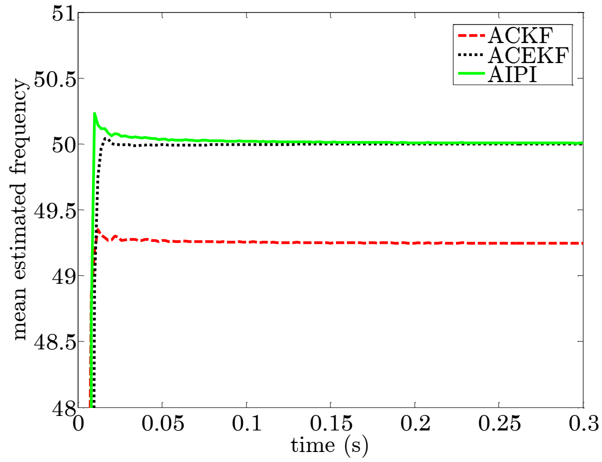

Fig. 1. The learning curves of different algorithms when the system is balanced and $\sigma_\eta^2 = 0.01$.

IV. CONCLUSION

We have developed an adaptive frequency estimation algorithm for three-phase power systems by assuming a widely-linear predictive model for the system's $\alpha\beta$ complex-valued signal and finding a recursive total least-squares estimate for the model parameters via the inverse power method. As verified by the simulation results, the proposed algorithm outperforms the augmented complex Kalman filter (ACKF) and the augmented complex extended Kalman filter (ACEKF) in frequency estimation of the unbalanced three-phase power systems while having similar computational complexity. The proposed algorithm also precludes the inconvenience of adjusting the noise covariance matrices encountered in the Kalman-filter-based algorithms.

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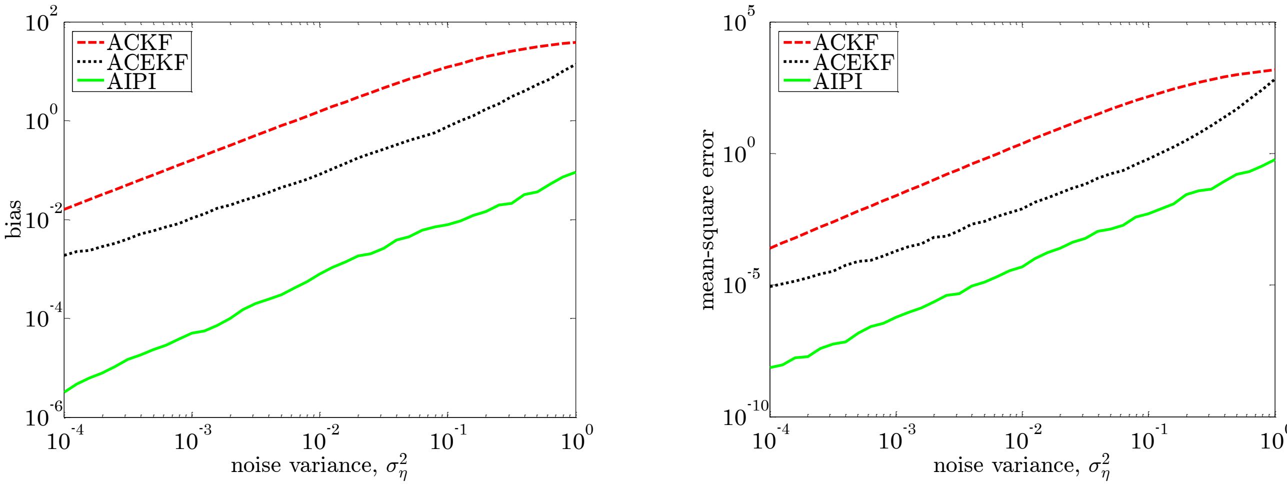
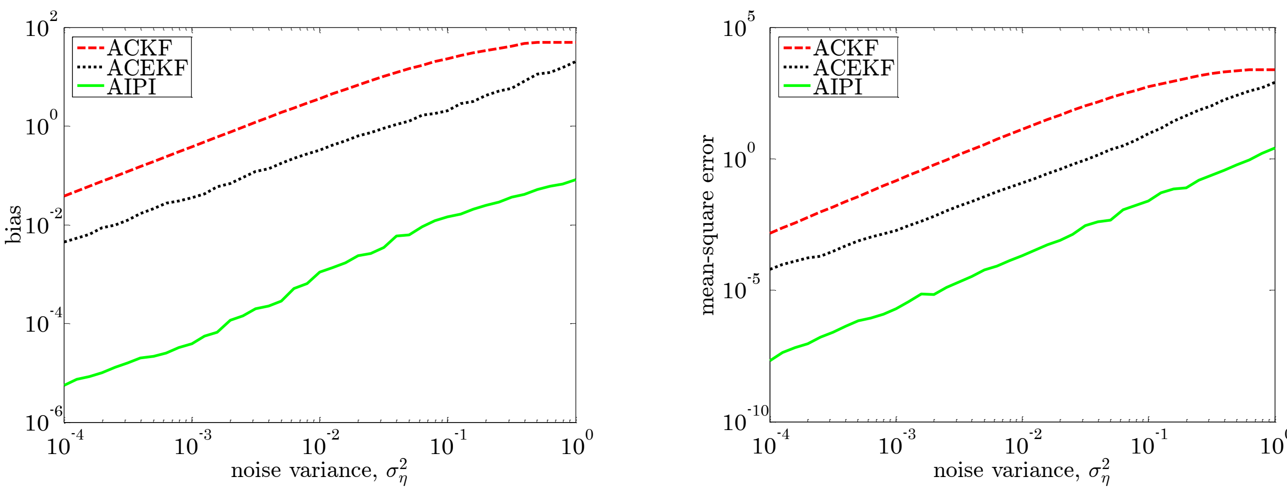


(a) (b)

Fig. 2. The steady-state bias (a) and mean-square error (b) of different algorithms when the system is unbalanced due to a type-C voltage sag.



(a) (b)

Fig. 3. The steady-state bias (a) and mean-square error (b) of different algorithms when the system is unbalanced due to a phase-to-ground fault.

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