The value and credits of n-authors publications

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Abstract

Collaboration among researchers is becoming increasingly common, which raises a large number of scientometrics questions for which there is not a clear and generally accepted answer. For instance, what value should be given to a two-author or three-author publication with respect to a single-author publication? This paper uses axiomatic analysis and proposes a practical method to compute the expected value of an n-authors publication that takes into consideration the added value induced by collaboration in contexts in which there is no prior or ex-ante information about the publication's potential merits or scientific impact. The only information required is the number of authors. We compared the obtained theoretical values with the empirical values based on a large dataset from the Web of Science database. We found that the theoretical values are very close to the empirical values for some disciplines, but not for all. This observation provides support in favor of the method proposed in this paper. We expect that our findings can help researchers and decision-makers to choose more effective and fair counting methods that take into account the benefits of collaboration.

Keywords: Co-authorship; Counting methods; Publication value; Axiomatic analysis; Bibliometrics.
JEL classification: C65, D04.

1. Introduction

Collaboration among researchers is becoming increasingly common, which may reflect the increasing complexity and interdisciplinary content of research (Gazni et al., 2012; Katz and Martin, 1997; Larivière et al., 2015; Persson et al., 2004; Wuchty et al., 2007). In this context, the more researchers are involved in a project, the more difficult it is for third parties (e.g., a reviewers' panel or an evaluation committee) to quantify or observe (even imperfectly) the contribution of each researcher.

The problem is so severe that some authors in the literature propose that publications should unambiguously list the specific contribution of each author (Cronin, 2001; Hu, 2009; Tscharntke et al., 2007).

The increasing collaboration raises a large number of scientometrics questions for which there is not a clear and generally accepted answer. For instance, what value should be given to a two-author or a three-author publication with respect to a single-author publication? Who should be ranked first, an individual with a single-author publication or an individual with two three-author publications? The answer to these questions is crucial because academics and researchers (as well as their institutions) are ranked, rewarded, financed and promoted according to the quantity and quality of their publications. Consequently, we need to develop adequate counting methods that are sufficiently flexible and that can benefit from generalized support.

The problem of how to count publications with several authors has been discussed previously in the literature (Egghe et al., 2000; Lindsey, 1980; Price, 1981; among others). The most common solution is to allocate the credits proportionally to the number of authors (fractional counting). This practice is incomplete because it ignores the potential synergies that result from collaborations, and consequently underestimates the credits of each author and the overall value of the publication. Publications with several authors tend to have more citations than single-author publications (Hsu and Huang, 2011; Onodera and Yoshikane, 2015; among others).[1] Therefore, the value of a publication should increase with the number of authors. This issue has been consistently ignored in the literature.

Another common solution is to allocate the full credit of a publication to every author (full counting). This practice overestimates the credits of each author and the value of the publication—each author is treated as a single author, which leads to serious distortions in comparing individuals with different co-authorship patterns. Moreover, it creates incentives to the addition of "ghost" co-authors, which is not desirable. Thus, as pointed out by Hirsch (2010), we need adequate counting methods that take into consideration the number of authors involved in a publication. This issue is the objective of this paper.

The methods discussed so far are useful when all authors are equally important, as for example, when authors are ordered alphabetically, which is common in mathematics, economics, finance, and high energy physics (Frandsen and Nicolaisen, 2010;

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[1]For instance, Hsu and Huang (2011) approximated the relation between the number of authors and the number of citations by the expression $citations_n = (n/5)^{1/3}$, where n denotes the number of authors.

Hu, 2009; Marušić et al., 2011; Waltman, 2012). However, in other scientific fields, authors are ordered in accordance with their contribution to the publication, with the first author typically being regarded as the most important. In this context, several approaches have been proposed in the literature (Waltman, 2016). One possibility is to allocate the full credit to the first author and/or the corresponding author, which can be more than one, and no credit to the other authors (Egghe et al., 2000; Gauffriau et al., 2007; Huang et al., 2011; Lange, 2001; Van Hooydonk, 1997).

However, in general, all listed authors have contributed to the publication and for that reason should receive some credit, with the most credits being given to the first author, followed by the second author, and so on. Several distributions of credits have been proposed in the literature. These include the arithmetic counting method (Van Hooydonk, 1997), in which credits are linearly distributed in decreasing order among the authors, the geometric counting method (Egghe et al., 2000), in which each author always gets twice the credits of the following author, the harmonic counting method (Hagen, 2008; Sekercioglu, 2008), in which the i-th ranked author receives $1/i$ of the credit received by the first author, and the axiomatic counting method (Stallings et al., 2013), which is conceptually the most similar to that proposed in the present paper. Other counting methods and procedures have been proposed (Abramo et al., 2013; Assimakis and Adam, 2010; Kim and Diesner, 2014; Liu and Fang, 2012; Lukovits and Vinkler, 1995; Trueba and Guerrero, 2004). All these methods are based on some intuitively correct argument, which makes it difficult to compare them or to claim that one method is superior to the others (Kim and Kim, 2015; Xu et al., 2016).

This paper proposes a practical and simple method to compute the expected value of an n-authors publication that takes into consideration the potential added value induced by collaboration in contexts in which there is no prior or ex-ante information about the publication's potential merits or scientific impact. The only information required is the number of authors. The method is neutral to the identity and affiliation of the authors, and it is flexible enough to accommodate information such as the order of authors, the Journal Impact Factor, the number of citations, or to be used in connection with other scientometrics indicators, like the h-index (Hirsch, 2005) and its several variations and extensions that have been proposed in the literature (Bornmann et al., 2011). These aspects make the proposed method extremely practical and useful to deal with real life situations in which consensus is difficult, and distinguishes the present paper from the existing literature.

In this context, we apply a set of principles or axioms that we consider fun-

damental to determine the expected value of an n-authors publication.[2] First, we present two basic and generally accepted inequalities regarding the expected value of n-authors publications. These inequalities will help us to build our argument. Subsequently, we require that the value of a publication is equal to the aggregated sum of all authors' efforts and that successful collaboration must satisfy some minimum amount of aggregate effort. In addition, there is an upper bound on the maximum effort provided by each co-author. In this context, each author expends its effort in the collaboration that returns the largest amount of credits and has no incentives to expend effort in any other collaboration. Lastly, in order to obtain analytical results we consider that every feasible effort is equally likely.

The result is a unique expression for the expected value of n-authors publications that increases (at a decreasing rate) with the number of authors. Some properties of the proposed method are discussed in connection with the existing literature.

Finally, using citation impact data, we contrasted the expected values with empirical bibliometric data from the Web of Science (WoS, Clarivate Analytics). We found that the theoretically expected values are very close to the empirical values. This observation provides strong support in favor of this method of calculating the expected value of n-authors publications described in this paper. Nonetheless, these results also make explicit that this method should not be taken as a universal solution to all disciplines.

This paper is organized as follows: Section 2 presents the main axioms and provides explanatory information, Section 3 presents the theoretical results, Section 4 compares the obtained theoretical results with bibliometric data, and Section 5 discusses the results of this study.

2. The axioms of an n-authors publication

Suppose that $n = 1, ..., \infty$ authors collaborate on an academic or scientific project. Suppose that each of these authors is equally important, i.e., they are expected to

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[2]Axiomatic approaches have been frequently used to study problems in scientometrics. For instance, in the axiomatic characterization of bibliometric impact indicators, as the h-index (Deineko and Woeginger, 2009; Kongo, 2014; Miroiu, 2013; Quesada, 2009, 2010, 2011b; Woeginger, 2008b,c) or some of its variants (Adachi and Kongo, 2015; Quesada, 2011a; Woeginger, 2008a, 2009), as well as the Euclidian index (Perry and Reny, 2016), and in the axiomatic characterization of rankings of authors and journals derived from bibliometric indicators (Bouyssou and Marchant, 2010, 2011, 2014, 2016; Marchant, 2009).

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provide the same effort and obtain the same credits, which does not mean that they will necessarily do it.

Let v_n denote the associated n-authors publication value and $\overline{v}_n$ denote the associated n-authors publication expected value. We distinguish between the publication value and the publication expected value. The former is unique to each collaboration and unknown to third parties, while the latter is an estimation of this value. The objective of this paper is to approximate the latter value.

Let $\overline{c}_n$ denote the credits awarded to each author, which in the case that all authors are equally important corresponds to the publication expected value divided by the number of authors, i.e., $\overline{c}_n = \overline{v}_n/n$. Let e_{in} denote the effort or contribution of author $i = 1, ..., n$ in the n-authors publication or collaboration. In our context, e_{in} captures simultaneously the quantitative and the qualitative dimensions of effort.

In what follows, we present and discuss two basic inequalities that should be satisfied by the expected value of an n-authors publication. Subsequently, we present a set of axioms that relate effort, value and the publication expected value, and that characterize the approach in this paper.

2.1. *Basic inequalities and discussion*

We start by noting that the addition of more authors should increase the expected value of a publication (Hsu and Huang, 2011; Onodera and Yoshikane, 2015; among others), or at least not decrease it. The interaction of authors with potentially different experiences and knowledge generates positive synergies and the cross-fertilization of ideas. In other words, the following inequality should be always satisfied:

$$\overline{v}_1 \leqslant \overline{v}_2 \leqslant ... \leqslant \overline{v}_n \quad \text{for} \quad n = 1, ..., \infty. \tag{1}$$

The expected value of a publication with n authors must have at least the same value as a publication with $n - 1$ authors, and so on. The aggregate effort of n authors should result in something quantitatively and qualitatively better than the aggregate effort of $n - 1$ authors. The question is how much better?

In this context, if all authors are equally important, i.e., they provide the same expected effort, the publication average value (which is also the credits awarded to each author if all authors are equally important) must decrease with the number of authors. Otherwise, there would be an incentive to add "ghost" co-authors to the collaboration because the publication average value would increase. In other words,

the following inequality should be always satisfied:

$$\frac{\overline{v}_1}{1} \geqslant \frac{\overline{v}_2}{2} \geqslant ... \geqslant \frac{\overline{v}_n}{n} \quad \text{for} \quad n = 1, ..., \infty. \tag{2}$$

Therefore, in the case that all authors are equally important, the credits awarded to each author and the publication average value are the same, i.e., $\overline{c}_n = \overline{v}_n/n$ for $n = 1, ..., \infty$.

In our context, inequalities (1) and (2) are general and intuitive, and they could have been written in the form of axioms.[3] However, in order to avoid dependence issues with the axioms presented below, we will not do it.

Together, inequalities (1) and (2) imply the following bounds on the expected value of an n-authors publication:

$$\overline{v}_{n-1} \leqslant \overline{v}_n \leqslant \frac{n}{n-1}\overline{v}_{n-1} \quad \text{for} \quad n = 2, ..., \infty. \tag{3}$$

In other words, the expected value of an n-authors publication should be in the interval defined by inequality (3).

In this context, the most extreme upper bound in this interval is obtained when $\overline{v}_n = \frac{n}{n-1}\overline{v}_{n-1}$ for $n = 2, ..., \infty$. Consequently, we obtain recursively that $\overline{v}_n = n\overline{v}_1$ and $\overline{c}_n = \overline{v}_1$ for $n = 1, ..., \infty$. The rule that results from this upper bound is often found in practice and is based on awarding the full value of a single-author publication to each of the n authors (full counting). This practice is excessively generous because the value of a single-author publication is multiplied by the number of authors.

Similarly, the most extreme lower bound in this interval is obtained when $\overline{v}_n = \overline{v}_{n-1}$ for $n = 2, ..., \infty$. In this case, we obtain recursively that $\overline{v}_n = \overline{v}_1$ and $\overline{c}_n = \overline{v}_1/n$ for $n = 1, ..., \infty$. The rule that results from this lower bound corresponds to another commonly used practice, which consists in dividing the value of a single-author publication equally among the n authors (fractional counting). This practice is more similar to the one in this paper, but ignores the added value resulting from collaboration and, consequently, underestimates the value of a publication.

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[3]For instance, inequality (1) appears in Stallings et al. (2013) as an axiom. Monotonicity axioms are common in studies characterizing bibliometric impact indicators. For instance, it is common to impose that the number of publications and/or citations should not lower the value of the indicator (Adachi and Kongo, 2015; Deineko and Woeginger, 2009; Kongo, 2014; Quesada, 2009, 2011a,b; Woeginger, 2008a,b,c, 2009).

The bounds found in inequality (3) are frequently seen as insufficient. Consequently, they are unable to provide meaningful and acceptable predictions about the expected value of an n-authors collaboration. In what follows, we present a set of axioms, which will allow us to obtain an analytical expression for the expected value of an n-authors publication. Then, as a corollary to this main result, we will see that the satisfaction of those axioms implies the satisfaction of inequality (3).

2.2.1. **Axioms about aggregated effort**

We start by noting that the value of an n-authors publication (v_n) must take into consideration the effort of each of the n authors (e_{in}), i.e., the value of an n-authors publication must be equal to the aggregated sum of the efforts of the n authors participating on it.

Axiom 1 (aggregate collaboration function). $v_n = e_{1n} + e_{2n} + ... + e_{nn}$ *for* $n = 1, ..., \infty$.

Collaboration in Axiom 1 is expressed in the most neutral and simplest way: as the sum of the n authors' contributions. However, we could have considered other functional forms. For instance, we could have considered some terms that would explicitly capture synergies and positive interaction effects resulting from collaboration between individuals with potentially different experiences and knowledge. Similarly, we could have considered some terms that would explicitly capture the increasing coordination difficulty and the possibility of free-riding, which frequently undermine the processes of collaboration. However, we have no theory to support any such particular functional form. Consequently, the inclusion of such terms would be pretty much ad-hoc and questionable, and would inevitably influence the results in one way or another. For that reason, collaboration in Axiom 1 is expressed in the most neutral and simplest way.

The set of possible effort profiles $(e_{1n}, e_{2n}, ..., e_{nn})$ that can be considered in Axiom 1 is uncountable and very diverse. However, not all these effort profiles can be accepted as valid. Some effort profiles have associated such low levels of aggregate effort that successful collaboration is impossible. Consequently, not all collaborations will necessarily lead to a publication.

In our context, a publication is only possible if the aggregate effort is above some threshold. The following axiom establishes the minimum amount of aggregate effort necessary for an n-authors' collaboration to result in a publication.

Axiom 2 (minimum aggregate effort). *Successful collaboration must satisfy:* $e_{1n} + e_{2n} + ... + e_{nn} \geqslant \overline{v}_{n-1}$ *for* $n = 1, ..., \infty$.

An n-authors publication is only possible if the aggregate effort is above the expected value of an $n - 1$-authors publication. In other words, the value of an n-authors publication must exceed the expected value of an $n - 1$-authors publication, and the value of an $n - 1$-authors publication must exceed the expected value of an $n - 2$-authors publication, and so on. The idea is that the consideration of additional co-authors must lead to an increase in value above the expected value of the nearest collaboration with a lower number of co-authors. Otherwise, individuals would have no incentives to consider additional co-authors.

2.2.2. **Axioms about individual effort**

The main difficulty regarding the individual effort provided by each author (e_{in}) is the fact that it is not observable by third parties (e.g., a reviewers' panel or an evaluation committee), even imperfectly—and sometimes not even observable by the other co-authors.

In this context, in order to obtain analytical results and pointwise predictions about the expected value $\overline{v}_n$, the uniform distribution seems to be the most focal and neutral assumption, because it has implicitly an impartial and equal treatment of all possible efforts. This idea becomes even more natural if we have no theory to support any other distribution (the same axiom appears in Stallings et al. (2013)).

Axiom 3 (uniform distribution of effort). e_{in} *is independent and uniformly distributed for* $i = 1, ..., n$ *and* $n = 1, ..., \infty$.

Consequently, in order to obtain the expected value of $\overline{v}_n$, we must build expectations about the effort provided by each author. In this context, Axioms 1 and 3 imply that $\overline{v}_n = E(e_{1n}) + E(e_{2n}) + ... + E(e_{nn})$ for $n = 1, ..., \infty$.

Moreover, in order to allocate the publication credits in the most fair and objective way, the credits awarded to each author in an n-authors publication must be equal to the expected effort provided.[4] In this context, in the case that all authors

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[4]Equity and fairness require an adequate balance between reward and effort. In this context, the sense of fairness depends on the individual comparison between their own balance and the balance of the others, with whom the individual deems to be relevant references (Adams, 1963). In our context, the effort used as reference is the effort provided by the other co-authors, while the reward used as reference is the credits awarded to the other co-authors.

are equally important, they are expected to provide the same level of effort and obtain the same credits, i.e., $E(e_{1n}) = ... = E(e_{nn}) = \overline{e}_n = \overline{c}_n$ and $\overline{e}_n = \overline{c}_n = \overline{v}_n/n$ for $n = 1, ..., \infty$.

In Section 2.2.1, the joint consideration of Axioms 1 and 2 establishes a floating lower bound on the individual effort, but in order to have a well-defined problem, we also need an upper bound on the individual effort.

In this context, we consider that an author is someone with a certain maximum amount of effort to spend in a research collaboration. Therefore, before taking part in a research collaboration, a rational author takes into consideration the expected effort required and the credits obtained in each collaboration, in order to maximize the amount of credits obtained. Consequently, if the author i participates in an n-authors collaboration that means that the maximum amount of available effort that author i has to spend in that collaboration is lower than the expected effort required to participate in an $n-1$-authors collaboration, i.e., $e_{in} \leqslant \overline{e}_{n-1}$ for $i = 1, ..., n$ and $n = 1, ..., \infty$. This behavior is rational because the credits tend to decrease with the number of authors (see inequality (2)). Therefore, author i chooses the research collaboration with the lowest possible number of authors, but in which its effort is expected to be enough. Otherwise, the n-authors collaboration would not be stable, because author i would have an incentive to move its effort to a collaboration with a lower number of authors, but that would return more credits.

Axiom 4 (maximum individual effort). $e_{in} \in [0, \overline{e}_{n-1}]$ *for* $i = 1, ..., n$ *and* $n = 2, ..., \infty$.

This axiom implies that as the number of authors increases, the maximum amount of individual effort spent by each author decreases, which is in line with inequality (2), and the connection between expected effort and reward made after Axiom 3, i.e., $\overline{e}_n = \overline{c}_n = \overline{v}_n/n \leqslant \overline{e}_{n-1} = \overline{c}_{n-1} = \overline{v}_{n-1}/(n-1)$ for $n = 2, ..., \infty$. Otherwise, each author would prefer to spend that same amount of effort in a publication with fewer co-authors that would award more credits. In this context, Axiom 4 can also be seen as an incentive compatible and a collaboration stability condition.

The following example illustrates some of the arguments that have motivated the axioms presented in this paper.

Example 1. *Suppose that $n = 3$ and $\overline{v}_2 = 4/3$ which implies that $\overline{e}_2 = \overline{c}_2 = 2/3$. Therefore, according to Axioms 3 and 4, the individual effort can take any value in the interval $[0, 2/3]$ with equal probability, but conditional on satisfying the minimum*

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aggregate effort condition $e_{13} + e_{23} + e_{33} \geqslant \overline{v}_2 = 4/3$ of Axiom 2. In this context, the effort profiles $(e_{13}, e_{23}, e_{33}) = (0.65, 0.60, 0.10)$ and $(e_{13}, e_{23}, e_{33}) = (0.45, 0.45, 0.45)$, among many other effort profiles, satisfy the minimum aggregate effort condition of Axiom 2 and for that reason are valid effort profiles in the computation of the publication value of Axiom 1. This example considers only two effort profiles, but the set of feasible effort profiles is uncountable and very heterogeneous. It is the consideration of all these feasible effort profiles that allows us to obtain a unique prediction of the n-authors publication expected value. On the other hand, the effort profiles $(e_{13}, e_{23}, e_{33}) = (0.65, 0.20, 0.10)$ and $(e_{13}, e_{23}, e_{33}) = (0.40, 0.40, 0.40)$, among many other effort profiles, do not satisfy the minimum aggregate effort condition of Axiom 2 and for that reason are not valid because they would result in collaboration failures.

3. The value of an *n*-authors publication

In this section, in order to compute the expected value of an *n*-authors publication, we consider all possible effort configurations that simultaneously satisfy Axioms 1-4. These axioms determine how and which effort profiles should be considered in the computation of the expected value of an *n*-authors publication. The following proposition presents the main result in this paper.

Proposition 1. *Suppose that Axioms 1-4 are satisfied. Then, the expected value of an n-authors publication is uniquely given by:*

$$\overline{v}_n = \frac{2n}{n+1}\overline{v}_1, \tag{4}$$

for $n = 1, ..., \infty$, where $\overline{v}_1$ is the value of a single-author publication, and the amount of credits awarded to each author when all authors are equally important is uniquely given by $\overline{c}_n = \overline{v}_n/n$ for $i = 1, ..., n$ and $n = 1, ..., \infty$.

A few comments regarding the meaning and interpretation of the result obtained are as follows.

First, by construction, expression (4) is uniquely characterized by Axioms 1-4.

Second, the expected value and the credits of the *n*-authors publications are normalized with respect to the value of the single-author publication $\overline{v}_1$. Once we know or attribute a value to $\overline{v}_1$, we can compute the value of $\overline{v}_n$ and $\overline{c}_n$. In this context, the value of the single-author publication is the unit of measure.

Third, the results obtained must be interpreted in expected terms. Clearly, in reality, not all *n*-authors publications are worth the same. Some publications may

be worth n times more than a single-author publication because of strong synergies and other interaction effects. In the other extreme, some other publications may be worth as much as a single-author publication because of free-riding and coordination problems. In between, we may have single-authored publication that are worth more than multi-authored publications, and so on.[5]
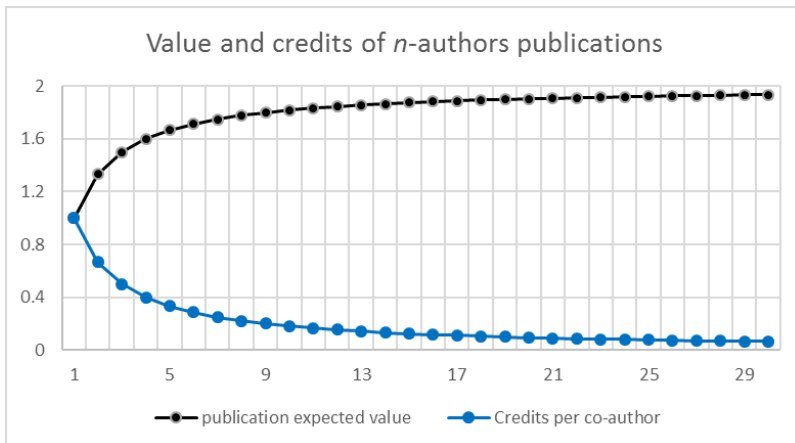

Figure 1: Equaly important authors - credits awarded to each author and the expected value of the n-authors publication for $n = 1, \ldots, 30$ (unit of measure $\overline{v}_1 = \overline{c}_1 = 1$).

Fourth, note that $\overline{v}_n$ converges to $2\overline{v}_1$ as $n \to \infty$. In other words, regardless of the number of co-authors, the expected value of an n-authors publication cannot be larger than twice the expected value of a single-author publication (see Figure 1). As we will see below, this observation may not be empirically supported for all disciplines. However, we point out that publications with large number of co-authors tend to be specific and very particular (Cronin, 2001). For instance, Xu et al. (2016) point out that most credit allocation methods fail when there are many co-authors and that we need specific methods to deal with hyper-authorship, which is understood as ten or more co-authors (Liu and Fang, 2012; Tscharntke et al., 2007).

Fifth, as mentioned before, we do not need to explicitly impose inequalities (1) and (2) in order for expression (4) in Proposition 1 to satisfy the bounds implied by those inequalities (see inequality (3)). The following result formalizes this observa-

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[5]This type of uncertainty and lack of information is the root of the problem of computing each author's contribution, and the reason why several authors in the literature suggest that publications should unambiguously list the contribution of each author (Cronin, 2001; Hu, 2009; Tscharntke et al., 2007).

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
$\overline{c}_n$	1.000	$0.66\overline{6}$	0.500	0.400	$0.33\overline{3}$	0.286	0.250	0.222	0.200
$\overline{v}_n$	1.000	$1.33\overline{3}$	1.50	1.60	$1.66\overline{6}$	1.714	1.750	1.778	1.800

Table 1: Equaly important authors - credits awarded to each author and the expected value of the n-authors publication (unit of measure $\overline{v}_1 = \overline{c}_1 = 1$).

tion.

Corollary 1. *Expression (4) satisfies inequalities (1) and (2).*

Table 1 shows the obtained expected value and the credits awarded to each author in the case that all authors are equally important. In the case of single-author publications, the expected value and the credits awarded have the same value, which is normalized to the unit, i.e., $\overline{v}_1 = \overline{c}_1 = 1$.

Note that as the number of authors and thus the expected value of the publication increases, the credits awarded to each author do not. The expected value increases monotonically with the number of authors, but at a decreasing rate (i.e., $\overline{v}_n$ is concave in the number of authors), which implies that the marginal contribution of a new author becomes less and less significant. Consequently, the credits awarded to each author decrease.

The expected value and credits of an n-authors publication in Proposition 1 have some interesting properties. For instance, if all authors are equally important, then our results state that in scientific terms "two 2-authors publications are worth more credits than one single-author publication (i.e., $0.66\overline{6} + 0.66\overline{6} > 1$)", and "two 3-authors publications are worth the same credits as one single-author publication (i.e., $0.500 + 0.500 = 1$)", and so on. Note also that in our context, the overall expected value of a 2- and a 3-authors publication are worth 33% and 50% more than a single-author publication, respectively. The exact relations are shown in Table 1.

3.1. The case of n ordered authors

Finally, in the case that authors' importance is determined by the order in which their names appear, we can also apply the publication expected value obtained in Proposition 1, but with the credits distributed among the ordered co-authors according to one of the methods proposed in the literature (Abramo et al., 2013; Assimakis and Adam, 2010; Egghe et al., 2000; Hagen, 2008; Kim and Diesner, 2014; Liu and

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	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$\overline{c}_{1n}$	$1.000(100)$	$1.000(75.0)$	$0.91\overline{6}(61.\overline{1})$	$0.8\overline{3}3(52.0)$	$0.76\overline{1}(45.\overline{6})$	$0.700(40.8)$
$\overline{c}_{2n}$		$0.3\overline{3}3(25.0)$	$0.41\overline{6}(27.\overline{7})$	$0.4\overline{3}3(27.1)$	$0.42\overline{7}(25.\overline{6})$	$0.414(24.2)$
$\overline{c}_{3n}$			$0.16\overline{6}(11.\overline{1})$	$0.2\overline{3}3(14.6)$	$0.26\overline{1}(15.\overline{6})$	$0.271(15.8)$
$\overline{c}_{4n}$				$0.100(06.3)$	$0.150(9.0)$	$0.176(10.3)$
$\overline{c}_{5n}$					$0.06\overline{6}(4.0)$	$0.105(06.\overline{1})$
$\overline{c}_{6n}$						$0.048(02.\overline{7})$
$\overline{v}_n$	1.000	$1.33\overline{3}$	1.500	1.600	$1.66\overline{6}$	1.714

Table 2: Ordered authors - credits awarded to each author (the values inside the brackets are the percentages over the total) and the expected value of the n-authors publication (unit of measure $\overline{v}_1 = \overline{c}_1 = 1$).

Fang, 2012; Lukovits and Vinkler, 1995; Sekercioglu, 2008; Stallings et al., 2013; Trueba and Guerrero, 2004; Van Hooydonk, 1997).

Table 2 shows the credits awarded to each author ($\overline{c}_{in}$) in an ordered n-authors publication for $n = 1, ..., 6$, according to the method proposed by Stallings et al. (2013). As in this paper, the Stallings et al. (2013) approach also follows an axiomatic approach. Their approach is very practical and can deal with a large variety of particular cases in terms of authors' ordering. Osório (2018) presents a detailed study on the properties and limitations of this and other counting methods.

4. Comparison of theoretical (expected) and empirical values

In this section, we study whether the expected values in Table 1 make sense. In particular, how do the theoretical values correlate with empirical values? In this context, we study whether the theoretical values are in agreement with empirical values based on a large dataset with bibliometric data. In order to achieve this objective, we use citations as a measure of "value", because citations are usually applied to assess the usefulness and the value of publications for other researchers (Bornmann, 2017).

4.1. *Data*

The bibliometric data used in this paper are from an in-house database developed and maintained by the Max Planck Digital Library (MPDL, Munich) and derived from the Science Citation Index Expanded (SCI-E), the Social Sciences Citation Index (SSCI), and the Arts and Humanities Citation Index (AHCI) prepared by

Clarivate Analytics (see https://clarivate.com), formerly the IP & Science business of Thomson Reuters.

The in-house database includes the number of authors for each paper since 1980. In this study, we considered only papers with the document type "article" to avoid distortion of the results by the use of papers with different document types. We included all articles published between 2000 and 2014; more recent years have been excluded because the window for the citation metrics becomes too small. Citations are counted in the in-house database from publication year until the end of 2016.

Two citation-based indicators are considered in this study to empirically assess the value of the n-authors publications. Citations are one of the most frequently used metrics in research evaluations which reflect the impact of publications (as one part of quality) (Martin and Irvine, 1983). The empirical analysis is based on articles from different fields. Since researchers in different fields have different publication and citation cultures, it is standard in bibliometrics to apply field-normalized citations scores for the impact comparison of papers from different fields. In the following, we use two different field-normalized indicators which are standard in bibliometrics to cross-check the results (Bornmann and Marx, 2015):

For the normalized citation score (NCS), the citation counts of an article are divided by the expected citation impact. For the calculation of the expected impact, the average citation rate is calculated based on all articles which have been published in the same year and field as the focal article. To aggregate the citation impact of more than one paper (e.g., all papers with one author), the arithmetic average was used. Percentiles offer an alternative to the mean-based quotient NCS. A percentile is a value below which a certain proportion of publications fall. For the calculation of percentiles, all papers in a field and publication year were ranked in decreasing order by their number of citations. Then, the percentiles have been calculated according to the formula $(i - 0.5)/n \times 100$—whereby i is the rank number and n the number of papers in the set (Hazen, 1914). The percentiles for a set of papers (e.g., all papers with one author) have been aggregated by using the median.

As field classification scheme for normalizing impact, the WoS subject categories have been used for both indicators in this study. These subject categories are based on sets of journals that publish papers in similar research areas.

4.2. Results

Figure 2 shows the comparison of the theoretically derived publication values for different numbers of authors with the empirically derived field-normalized citation scores (NCS and percentiles). The figure does not visualize the field-normalized citation scores, but relative values, which show how the citation impact varies with
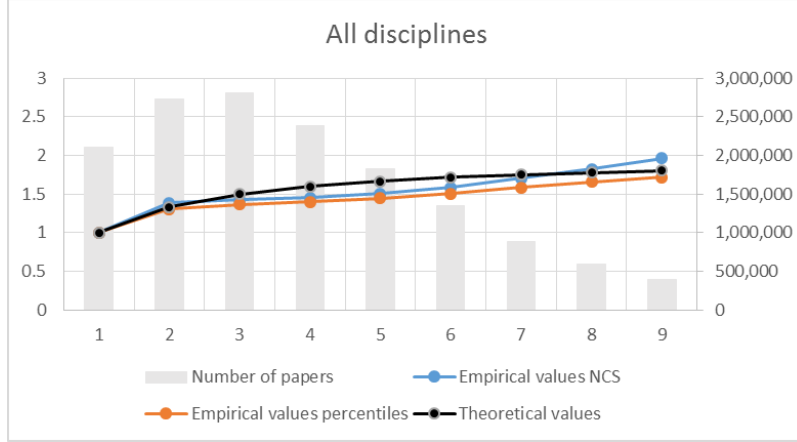
Figure 2: Comparison of the theoretically derived expected values for different numbers of authors with the empirically derived field-normalized citation scores (NCS and percentiles). The figure also shows the number of papers with different numbers of authors.

different numbers of authors. For example, the visualized NCS value (1.38) for two authors has been calculated by multiplying the (empirical) NCS=0.895 (for two authors) with $v_1 = 1$ (see Table 1) and dividing the product by the empirical NCS score for one author (NCS=0.647).

As the results in Figure 2 over all disciplines show, the empirical and theoretical values are in close agreement. Since the WoS database is mostly based on natural sciences publications, the results in Figure 2 are especially driven by these publications. Other disciplines have not only different publication and citation cultures, but also different treatments of authorship positions.

Thus, we produced further results for six broad disciplines: (1) natural sciences, (2) engineering and technology, (3) medical and health sciences, (4) agricultural sciences, (5) social sciences, and (6) humanities. The broad disciplines are aggregated paper sets of WoS subject categories based on the OECD category scheme (major codes). The results for the different disciplines are shown in Figure 3. As expected, the results for the natural sciences are similar to the results in Figure 2. Since Figure 3 also includes the number of papers with the different numbers of authors, it is clearly visible that the WoS database is mainly based on papers from the natural sciences.

The number of papers for the different numbers of authors reveals that the authorship cultures are different among the disciplines. In natural sciences, engineering and technology, medical and health sciences, as well as agricultural sciences, we see
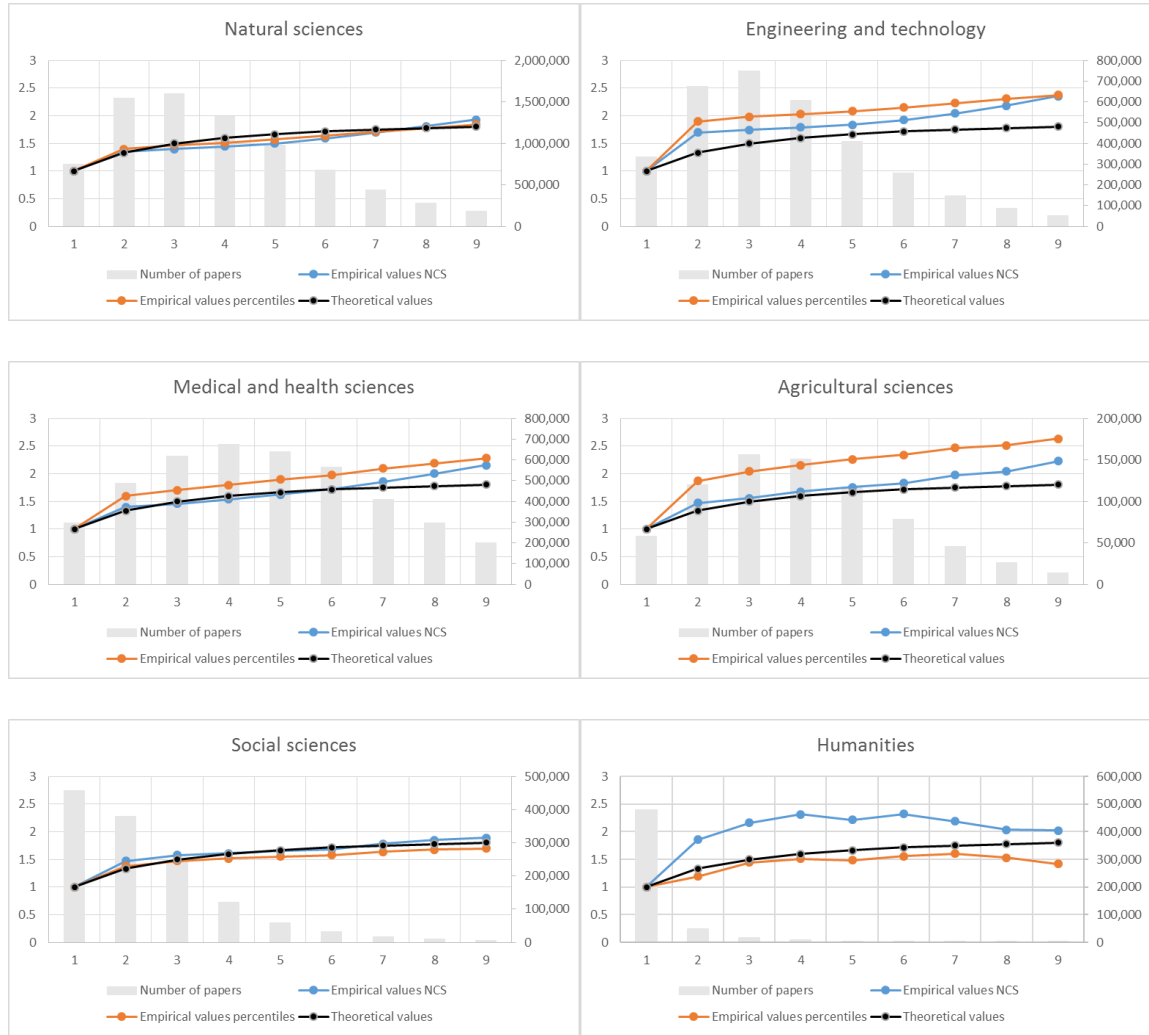
Figure 3: Comparison of the theoretically derived publication values for different numbers of authors with the empirically derived field-normalized citation scores (NCS and percentiles) for six broad disciplines. The figure also shows the number of papers with different numbers of authors.

peaks at around two or three authors. In the social sciences and humanities, the publication with only one author is the most frequent publication type (especially in the humanities).

The comparison of the empirical with the theoretical values in Figure 3 indicates that the highest agreement is obtained in natural and social sciences (although the social sciences show a different authorship pattern than the natural sciences). In medical and health sciences, the NCS values are close to the theoretical values, but the percentiles differ. For engineering and technology as well as agricultural sciences, the percentile values in particular differ from the theoretical values. The greatest difference between empirical evidence and theory can be seen for the humanities. We also produced the results for the OECD minor codes to present more detailed field-specific results. The results can be found in Appendix B. The more detailed results confirm the results from the higher aggregation level (the major codes).

To sum up, the empirical results indicate that the theoretical values are close to the empirical values—if citation impact is used to assess the value of publications. However, there are differences between the disciplines: while the natural and social sciences are close to the expectations, the humanities show some differences. The other disciplines demonstrate reasonably good agreement between theoretical and empirical results.

5. Conclusion

Multidisciplinary scientific collaboration is increasing (Gazni et al., 2012; Katz and Martin, 1997; Larivière et al., 2015; Persson et al., 2004; Wuchty et al., 2007). However, it is extremely difficult for third parties (e.g., a panel or an evaluation committee) to quantify the contribution of each author. In this context, we should be able to find ways to account and distinguish between publications with different number of authors in order to obtain evaluations that are more suitable.

This paper attempts to quantify the expected value of an n-authors publication. Publications with several authors tend to have more impact (e.g., in terms of citations) than single-author publications, maybe because of the possibility of synergies and cross-fertilization of ideas (Hsu and Huang, 2011; Onodera and Yoshikane, 2015; among others). However, the strength of this effect is limited because of coordination difficulties, free-riding, and other forms of opportunistic behavior. A greater number of authors may lead to larger aggregate efforts, but not necessarily to larger individual efforts, which tends to decrease with the number of authors.

In this paper, we propose a set of principles and axioms that we consider fundamental to determine the expected value of an n-authors publication. The result is

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a unique measure of the expected value and credits of n-authors publications. The expected value of the publication increases monotonically with the number of authors, but at a decreasing rate because the marginal contribution of new co-authors becomes less and less significant.

Using a comprehensive set of bibliometric data, we found that the theoretically obtained expected values and patterns are close to the empirical values for some disciplines. These results provide support in favor of the method proposed in this paper. However, these results also make explicit that this method should not be taken as a universal solution, which can be applied indiscriminately to quantify the expected value of n-authors publications in all disciplines.

The proposed approach follows a set of principles or axioms that we consider fundamental to derive the expected value of n-authors publications. However, this approach or any other approach obtained according to any other principles or axioms will always be a subject of discussion. As a rule it is difficult to agree on one particular method (Waltman, 2016). There are several reasons for this lack of agreement. First, the expected value of the publication and the associated counting method play a crucial role in academics' and scientists' lives. Second, judgements and evaluations of qualitative issues like those concerning scientific publications are always subjective and open to debate, which creates consensus difficulties and allows the coexistence of different approaches. However, our proposed solution is practical and has the advantage of not requiring information other than the number of authors.

Finally, this study suggests a relation between the expected value of the publication and the number of authors; an aspect that has been discussed, but not formally addressed in the literature (Hsu and Huang, 2011; Onodera and Yoshikane, 2015; among others). In this context, we expect that our findings can help researchers and decision-makers to choose and implement more effective and fair counting methods that take into account the benefits of collaboration.

Expanded (SCI-E), Social Sciences Citation Index (SSCI), Arts and Humanities Citation Index (AHCI) prepared by Clarivate Analytics (see https://clarivate.com), formerly the IP & Science business of Thomson Reuters (Philadelphia, Pennsylvania, USA).

Appendix A. Proofs of the Propositions

Proof of Proposition 1 and Corollary 1. In order to show Proposition 1, we aggregate the mathematical implications of Axioms 1-4. This construction will lead to expression (4). Axioms 1 and 3 establish that the value of a n-authors collaboration is the sum of the n authors' expected efforts, i.e., $\overline{v}_n = \sum_{i=1}^{n} E(e_{in})$ for $n = 1, ..., \infty$. In addition, the independent and uniform assumption of Axiom 3 implies the continuous probability density function (PDF) $f(e_{in}) = 1/\overline{c}_{n-1}$ for $i = 1, ..., n$ and $n = 1, ..., \infty$, which in the case of n independent random variables implies the joint probability density function $\prod_{i=1}^{n} f(e_{in})$ for $n = 1, ..., \infty$. Axiom 2 requires that the aggregated effort of an n-authors collaboration must be above some threshold, i.e., $\sum_{i=1}^{n} e_{in} \geqslant \overline{v}_{n-1}$ for $n = 2, ..., \infty$. Finally, Axiom 4 establishes that in order for an n-authors collaboration to be stable and each author to have incentives to participate in it, the individual effort must also satisfy $e_{in} \in [0, \overline{e}_{n-1}]$ for $i = 1, ..., n$ and $n = 2, ..., \infty$, where $\overline{e}_{n-1} = \overline{c}_{n-1} = \overline{v}_{n-1}/(n-1)$. The consideration of Axioms 1-4 implies that the value of an n-authors collaboration is by construction uniquely given by the following conditional expectation:

$$
\begin{aligned}
\overline{v}_n &= E\Big(\sum_{i=1}^{n} e_{in} | H\Big) = E\Big(\mathbf{1}_H \sum_{i=1}^{n} e_{in}\Big)/P(H) \\
&= \int_{(e_{1n},e_{2n},...,e_{nn})\in[0,\overline{v}_{n-1}/(n-1)]^n} \sum_{i=1}^{n} e_{in} dP(e_{1n}, e_{2n}, ..., e_{nn} | H), \qquad \text{(A.1)}
\end{aligned}
$$

for $n = 2, ..., \infty$, where the expectation is taken with respect to the joint density function of the n-dimensional vector of efforts $(e_{1n}, e_{2n}, ..., e_{nn}) \in [0, \overline{c}_{n-1}]^n$, and conditional on the event $H = \{\sum_{i=1}^{n} e_{in} \geqslant \overline{v}_{n-1}\}$, where $P(H)$ denotes the probability of the event H, and $\mathbf{1}_H$ is an indicator function that takes the value 1 when the event H occurs, and 0 otherwise. Consequently, the integral in expression (A.1) can be rewritten in the following equivalent way:

$$
\overline{v}_n = \frac{\int_0^{\overline{c}_{n-1}} f(e_{1n}) ... \int_0^{\overline{c}_{n-1}} f(e_{nn}) \mathbf{1}_H(\sum_{i=1}^{n} e_{in}) de_{nn} ... de_{1n}}{\int_0^{\overline{c}_{n-1}} f(e_{1n}) ... \int_0^{\overline{c}_{n-1}} f(e_{nn}) \mathbf{1}_H de_{nn} ... de_{1n}}, \qquad \text{(A.2)}
$$

for $n = 2, ..., \infty$. Note also that since $f(e_{in}) = 1/\overline{c}_{n-1}$ is constant and independent of e_{in}, we can trivially cancel the numerator by the denominator. However, in order to solve this integral analytically, the indicator function $\mathbf{1}_H$ in expression (A.2) must be passed to the integration limits. In this context, we must rewrite expression (A.2) in the following equivalent way, in which author 1 provides more effort than author 2,

and so on in decreasing order until author n, i.e.:

$$\bar{v}_n = \frac{n! \int_{\frac{\bar{c}_{n-1}}{\bar{v}_{n-1}}}^{\bar{c}_{n-1}} \int_{\frac{e_{1n}}{\bar{v}_{n-1}-e_{1n}}}^{e_{1n}} \cdots \int_{\frac{e_{n-1n}}{\bar{v}_{n-1}-e_{1n}\cdots-e_{n-1n}}}^{e_{n-1n}} (\sum_{i=1}^n e_{in}) de_{nn}...de_{2n}de_{1n}}{n! \int_{\frac{\bar{c}_{n-1}}{\bar{v}_{n-1}}}^{\bar{c}_{n-1}} \int_{\frac{e_{1n}}{\bar{v}_{n-1}-e_{1n}}}^{e_{1n}} \cdots \int_{\frac{e_{n-1n}}{\bar{v}_{n-1}-e_{1n}\cdots-e_{n-1n}}}^{e_{n-1n}} de_{nn}...de_{2n}de_{1n}}, \tag{A.3}$$

for $n = 2, ..., \infty$, where $n!$ is multiplying the numerator and the denominator to denote that each of the n authors must be in each of the $n!$ possible effort ordered permutations in order for expression (A.3) to be equivalent to expression (A.2). The integration lower bound guarantees that condition H is satisfied at all steps of integration. For instance, for e_{nn} we must have $e_{nn} \geqslant \frac{\bar{v}_{n-1}-e_{1n}-e_{2n}...-e_{n-1n}}{1}$, while for e_{n-1n} we must have $e_{n-1n} \geqslant \frac{\bar{v}_{n-1}-e_{1n}-e_{2n}...-e_{n-2n}}{2}$, and so on. For instance, given the efforts provided by author 1 until author $n-2$ (i.e., $e_{1n}, e_{2n}, ..., e_{n-2n}$), and since author $n-1$ cannot provide less effort than the last author n, the minimum effort of author $n-1$ occurs when author $n-1$ and author n divide equally the effort that is still left in order to satisfy condition H, i.e., at $e_{n-1n} = \frac{\bar{v}_{n-1}-e_{1n}-e_{2n}...-e_{n-2n}}{2}$. After this transformation, the integrals in the numerator and denominator of expression (A.3) can be solved analytically. After some algebra, we obtain that:

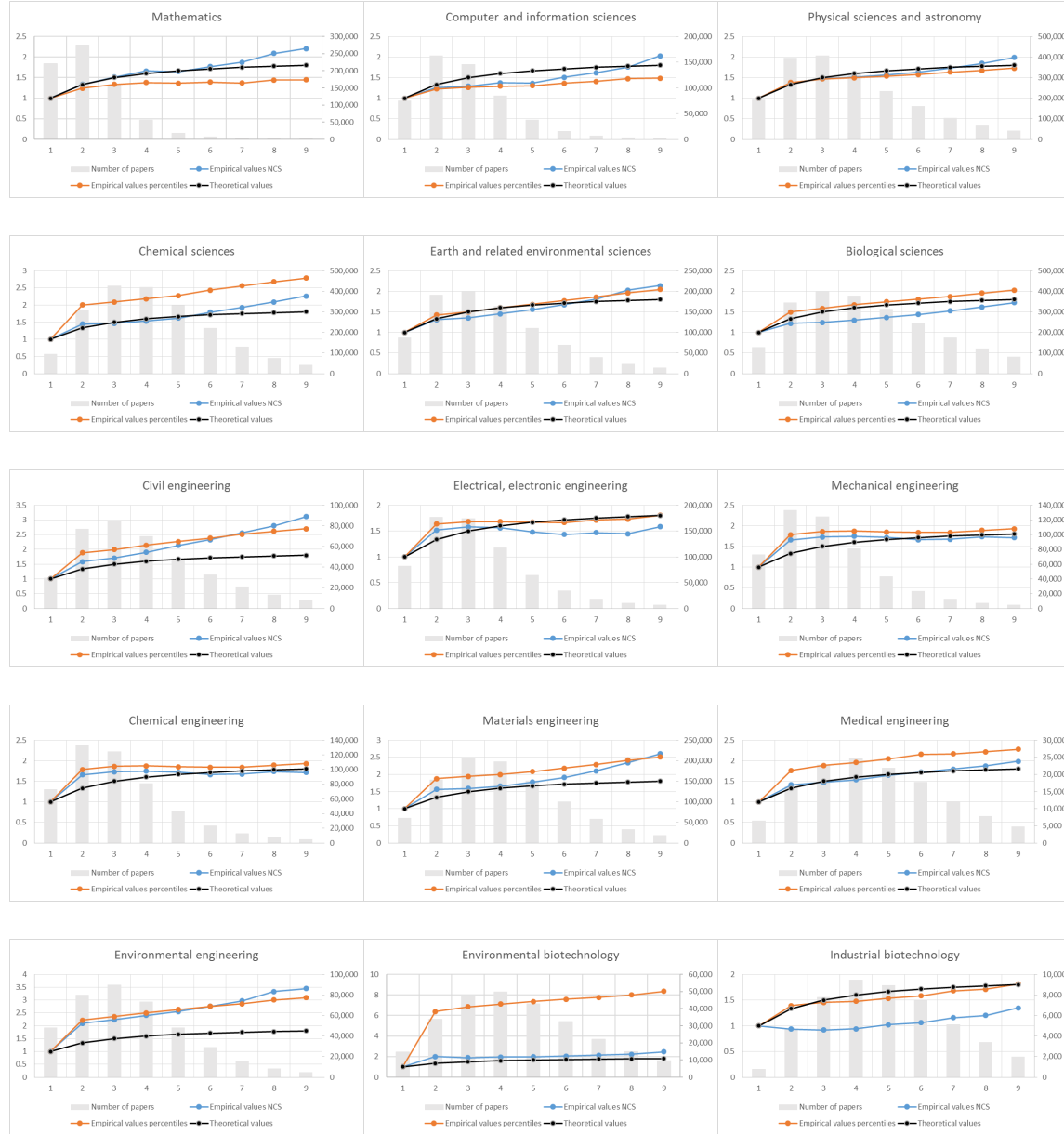$$\bar{v}_n = \frac{n^2}{(n-1)(n+1)}\bar{v}_{n-1}, \tag{A.4}$$

for $n = 2, ..., \infty$, where we have made use of the fact that $\bar{c}_{n-1} = \bar{v}_{n-1}/(n-1)$. Therefore, if we know the value of $\bar{v}_1$, we can recursively obtain the expression (4) of Proposition 1. Thus, by simply dividing $\bar{v}_n$ by the number of authors, we obtain that the credits $\bar{c}_n$ awarded to each author when all authors are equally important are given by:
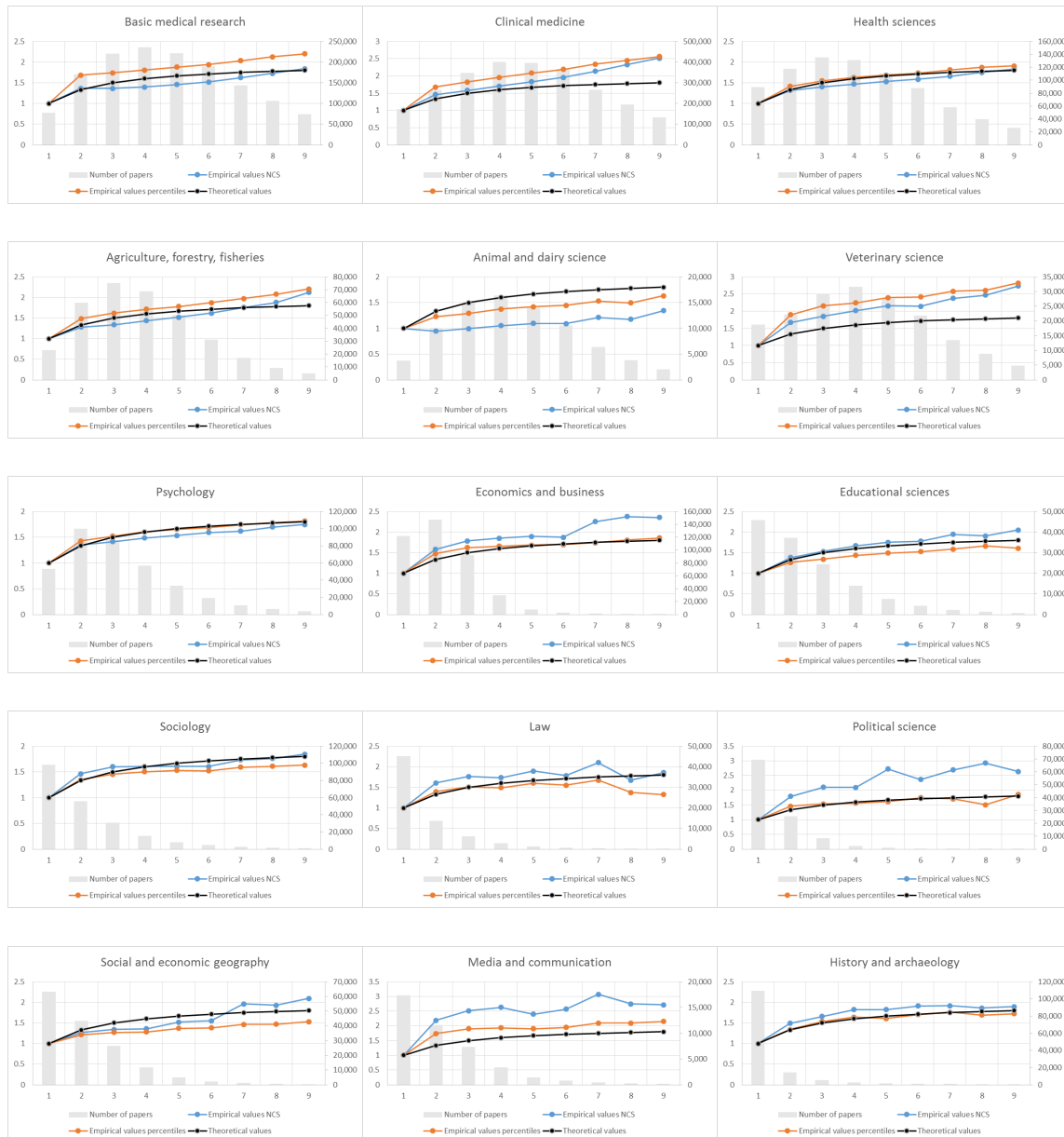
$$\bar{c}_n = \frac{2(n!)^2}{n(n-1)!(n+1)!}\bar{v}_1 = \frac{2}{n+1}\bar{v}_1,$$

for $n = 1, ..., \infty$, where "!" denotes the factorial symbol.

In order to show Corollary 1, it is enough to show that expression (A.4) falls inside the bounds defined by inequality (3), which is implied by inequalities (1) and (2). The lower bound is satisfied if $\bar{v}_n \geqslant \bar{v}_{n-1}$, i.e., if $n^2 \geqslant n^2 - 1$ which is always true. The upper bound is satisfied if $\bar{v}_n \leqslant n\bar{v}_{n-1}/(n-1)$, i.e., if $n^2(n-1) \leqslant n(n^2-1)$ which is also always true. ■

Appendix B. Field-specific results (OECD minor codes)

Basic medical research · Clinical medicine · Health sciences · Agriculture, forestry, fisheries · Animal and dairy science · Veterinary science · Psychology · Economics and business · Educational sciences · Sociology · Law · Political science · Social and economic geography · Media and communication · History and archaeology

Number of papers · Empirical values NCS · Empirical values percentiles · Theoretical values
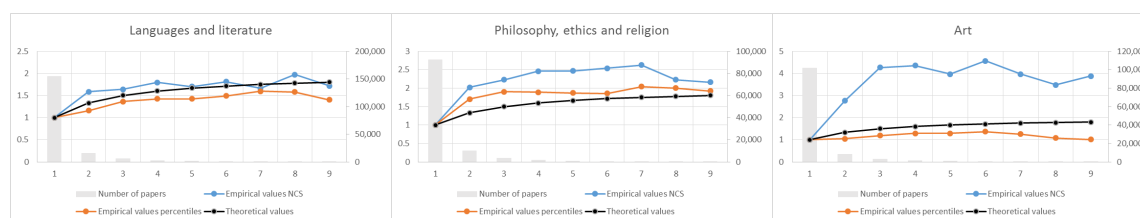
Figure B.6: Comparison of the theoretically derived publication values for different numbers of authors with the empirically derived field-normalized citation scores (NCS and percentiles) for 33 disciplines. The figures also show the number of papers with different numbers of authors.

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