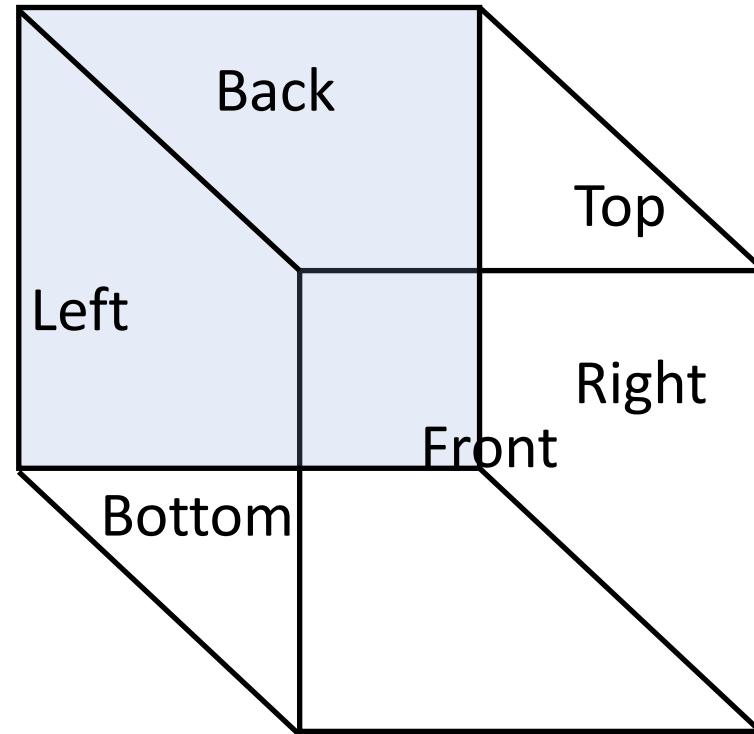


# Cube

Rubik cube labels

**Legend**

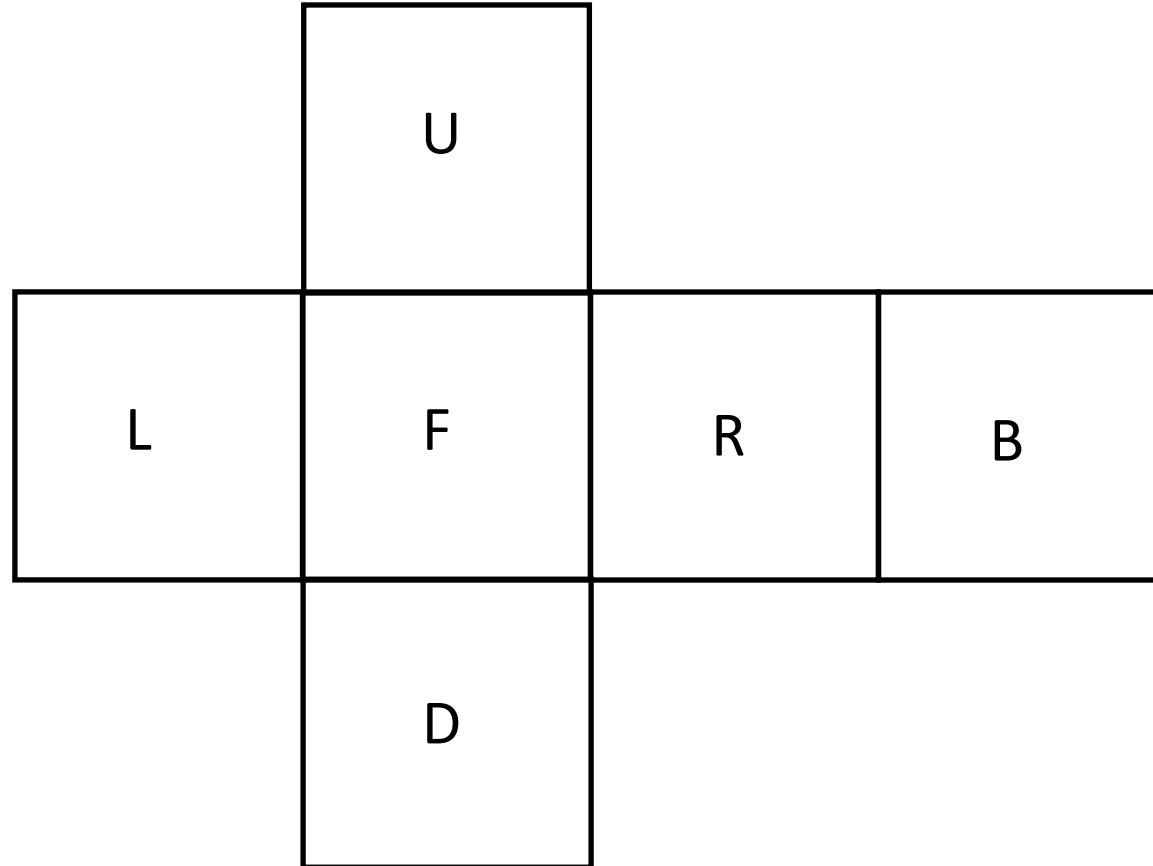
- Front (F)
- Back (B)
- Up (U)
- Down (D)
- Left (L)
- Right (R)



# Cube

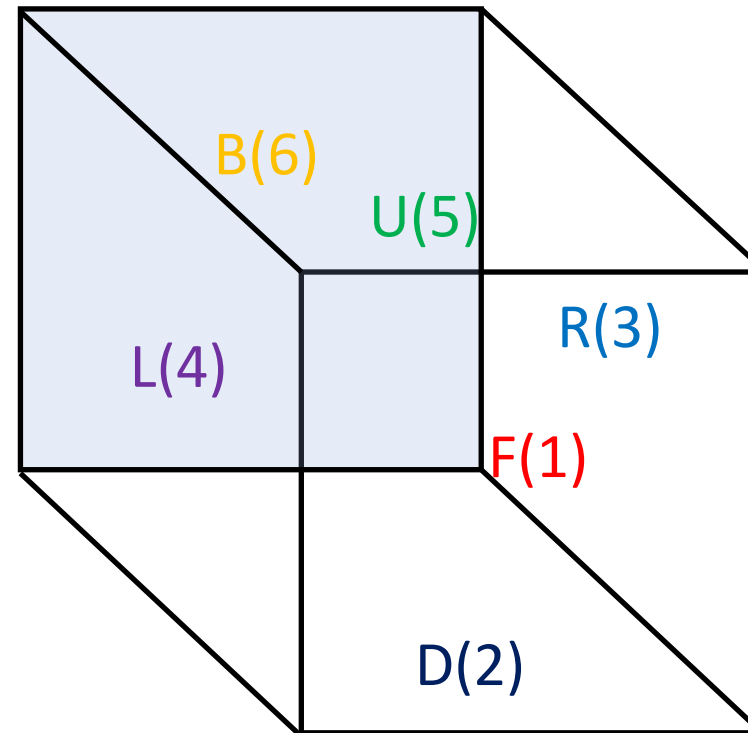
## Legend

- Front (F)
- Back (B)
- Up (U)
- Down (D)
- Left (L)
- Right (R)

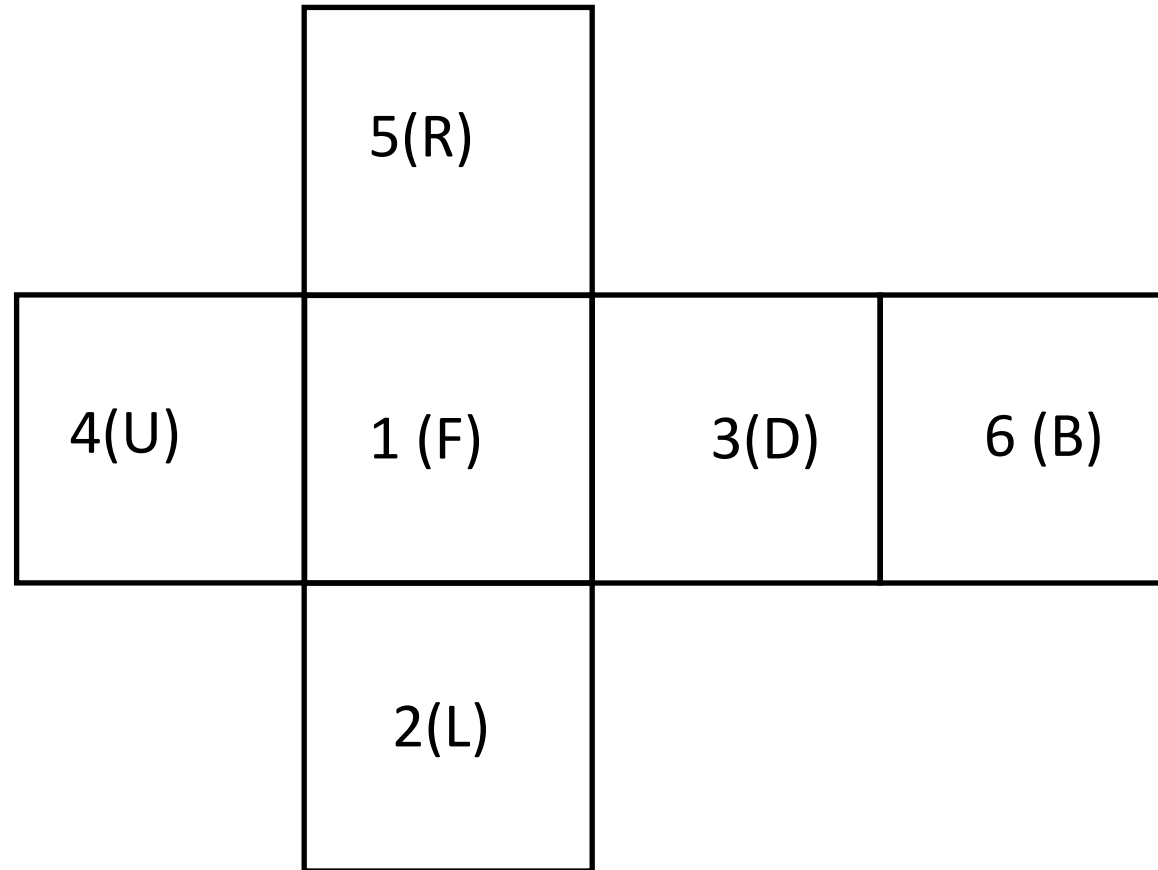


# Cube

Standard dice order  
Opposite sides add to 7



# Cube



# Face permutations of cube

FBUDLR 162534 -----	Element -----	Order -----	FBUDLR 162534 -----	Element -----	Order -----
162534	(1)	1	432516	(1463)	4
164325	(2453)	4	436125	(145) (263)	3
165243	(25) (34)	2	435261	(14) (25) (36)	2
163452	(2354)	4	431652	(142) (356)	3
251643	(12) (56) (34)	2	521634	(1562)	4
253416	(123) (465)	3	524316	(153) (246)	3
256134	(1265)	4	526143	(15) (26) (34)	2
254361	(124) (365)	3	523461	(154) (236)	3
342561	(1364)	4	613425	(16) (23) (45)	2
341625	(132) (456)	3	615234	(16) (25)	2
345216	(13) (25) (46)	2	614352	(16) (24) (35)	2
346152	(135) (264)	3	612543	(16) (34)	2

# Face permutations of cube

FBLRDU	Element	Order	FBLRDU	Element	Order
-----	-----	-----	-----	-----	-----
FBLRDU	1	1	UDLRFB	(FUBD)	4
FBUDLR	(LURD)	4	UDBFLR	(FUR) (LBD)	3
FBRLUD	(LR) (DU)	2	UDRLBF	(FU) (LR) (DB)	2
FBDURL	(LDRU)	4	UDFBRL	(FUL) (DRB)	3
LRFBUD	(FL) (RB) (DU)	2	RLFBDU	(FRBL)	4
LRDUFB	(FLD) (UBR)	3	RLUDFB	(FRD) (LUB)	3
LRBF3U	(FLBR)	4	RLBFUD	(FR) (LB) (DU)	2
LRUDBF	(FLU) (DBR)	3	RLDUBF	(FRU) (LDB)	3
DULRBF	(FDBU)	4	BFDULR	(FB) (LD) (UR)	2
DUFBLR	(FDL) (URB)	3	BFRLDU	(FB) (LR)	2
DURLFB	(FD) (LR) (UB)	2	BFUDRL	(FB) (LU) (DR)	2
DUBFRL	(FDR) (LBU)	3	BFLRUD	(FB) (DU)	2

# Subgroups

- $G = \{(1), (2453), (25)(34), (2354), (12)(34)(56), (123)(465), (1265), (124)(365), (1364), (132)(456), (13)(25)(46), (135)(264), (1463), (145)(263), (14)(25)(36), (142)(356), (1562), (153)(245), (15)(26)(34), (154)(236), (16)(23)(45), (16)(24)(35), (16)(25), (16)(34)\}$
- $H = \{(1), (25)(34), (123)(465), (124)(365), (132)(465), (135)(264), (145)(263), (142)(356), (153)(245), (154)(236), (16)(25), (16)(34)\}$
- $V = \{(1), (25)(34), (16)(25), (16)(34)\}$
- $H \triangleleft G, V \triangleleft G$
- $C_1 = \{(1), (2453), (25)(34), (2354)\}$
- $C_2 = \{(1), (123)(465), (132)(456)\}$
- $N_1 = N(C_1) = \{(1), (2453), (25)(34), (2354), (16)(25), (16)(34), (16)(24)(35), (16)(23)(45)\}$
- $D_1 = \langle (15)(26)(34), (14)(25)(36) \rangle = \{(1), (15)(26)(34), (14)(25)(36), (123)(465), (132)(456), (16)(24)(35)\}$
- $VC_2 = H, VC_1 = N_1, N_1 \in S_2(G), C_2 \in S_3(G)$
- $|C((12)(34)(56))| = 12, |C((25)(34))| = 8, |C((123)(465))| = 6$
- ccls:  $(4,4)_3, (3,2,2,2)_2$
- Subgroup orders: 1, 2, 3, 4, 6, 8, 12
- Element orders: 1 (1), 2 (9), 3 (8), 4 (6)
- $G=S_4, H=A_4$

# Dodecahedron

Opposite sides add to 13

Face adjacency

1: 2,a,5,6,4

2: 1,4,8,7,a

3: 4,6,b,c,8

4: 1,6,3,8,2

5: 1,a,9,b,6

6: 1,5,b,3,4

7: 2,8,c,9,a

8: 2,4,3,c,7

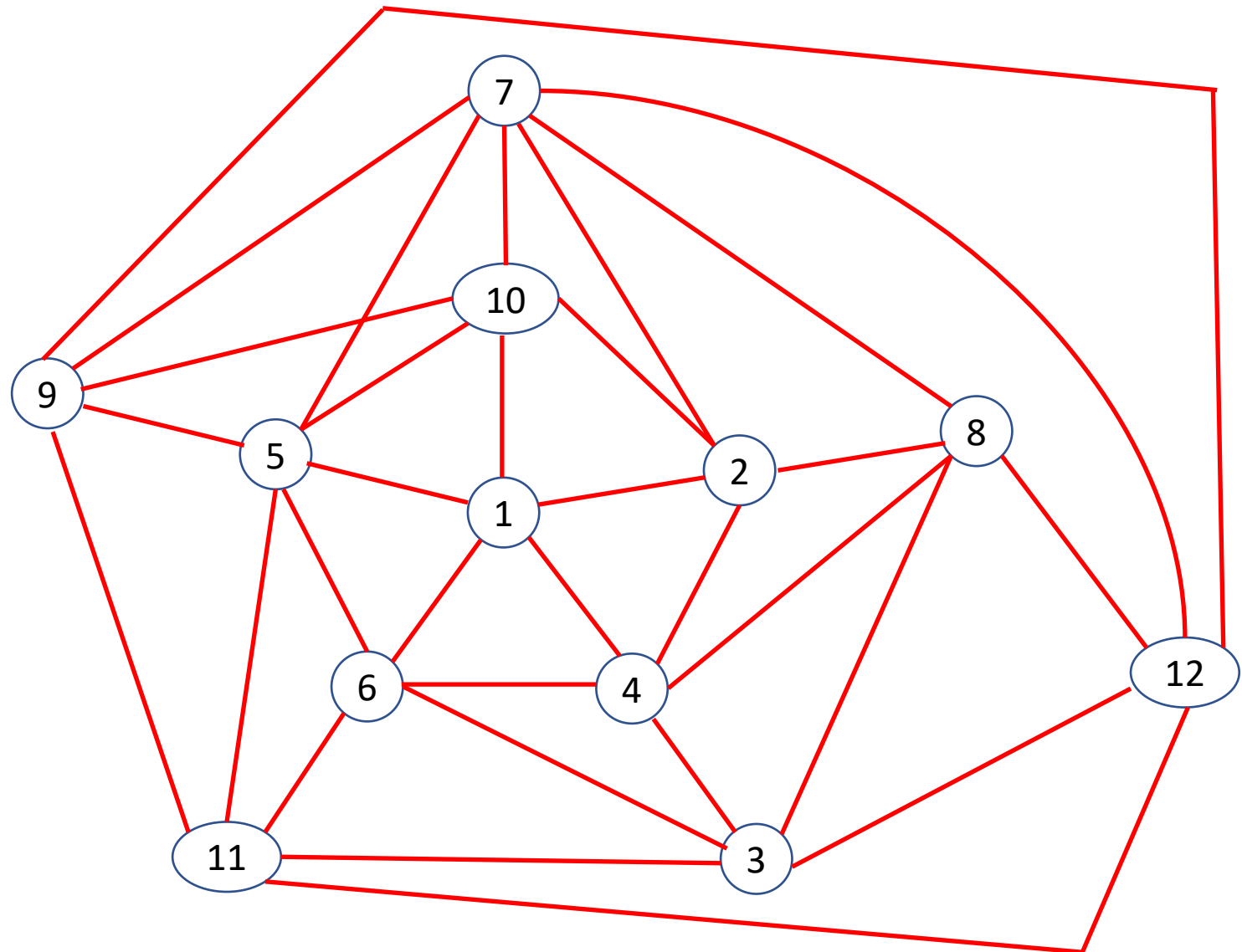
9: 5,a,7,c,b

a: 1,2,7,9,5

b: 3,6,5,9,c

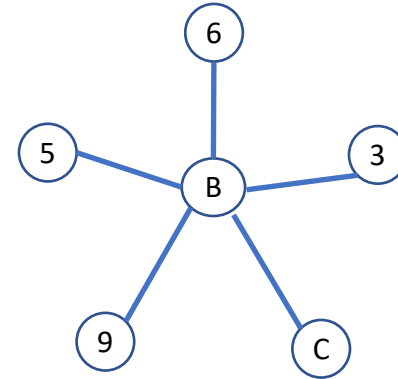
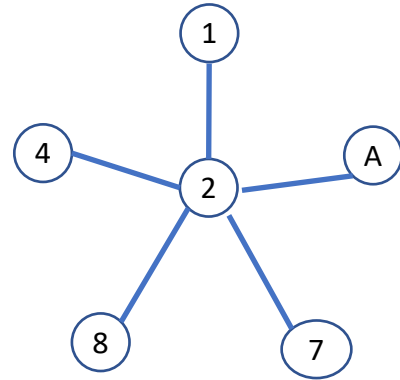
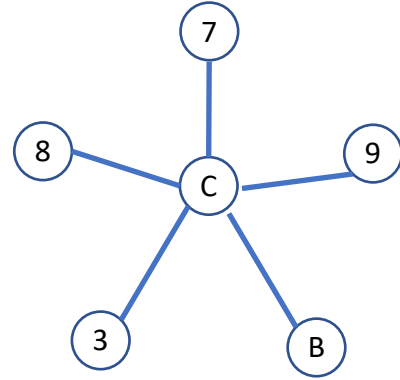
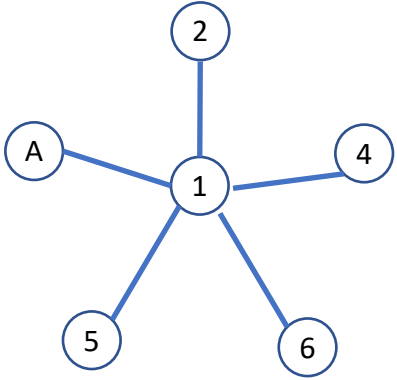
c: 3,b,9,7,8

$|G| = 60$ ,  $G = A_5$

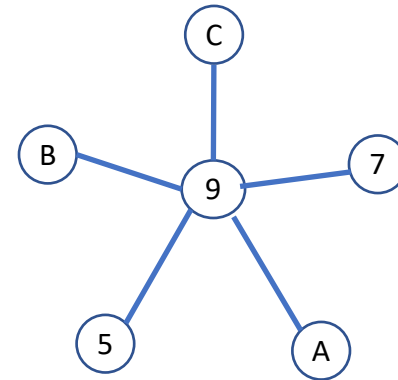
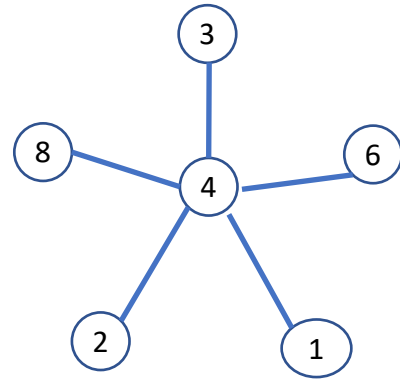
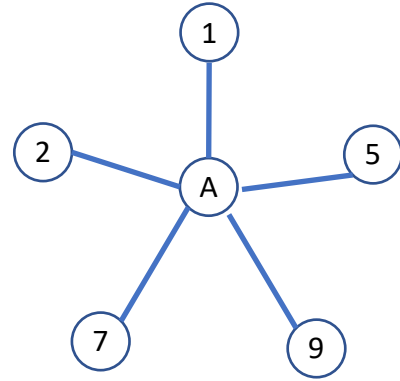
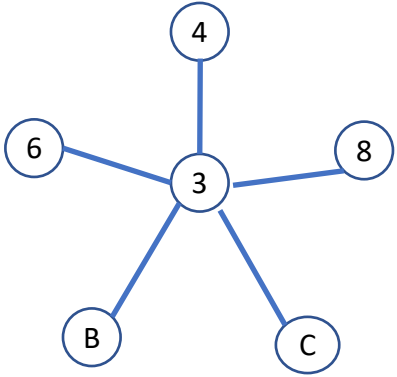




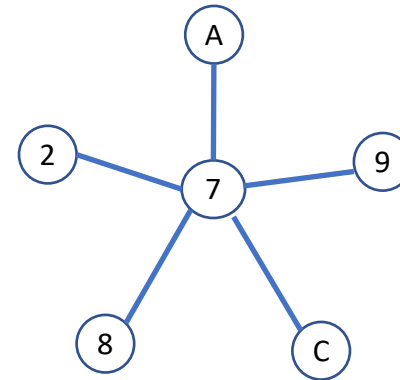
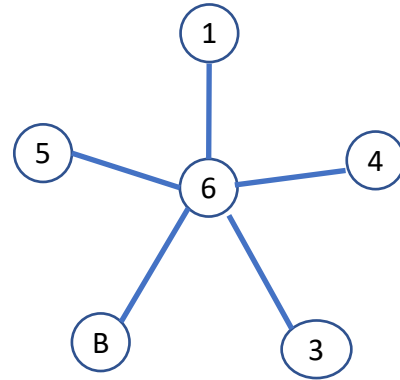
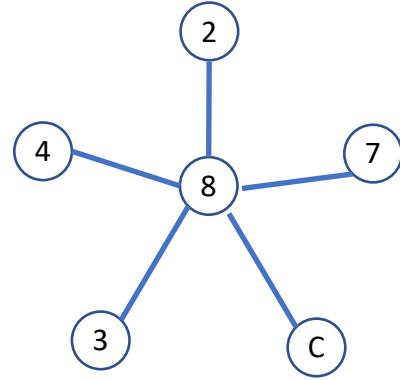
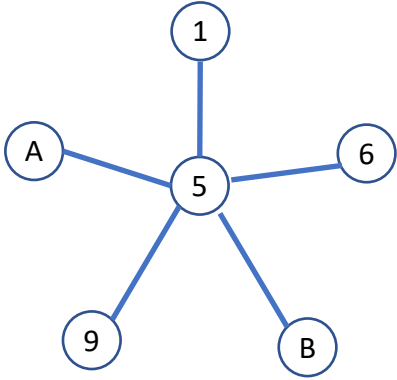
# Dodecahedron



# Dodecahedron



# Dodecahedron



# Dodecahedron Elements

123456789abc 123456789abc	Multiplier: (2a564) (79b38), (254a6) (7b893), (26a45) (7398b), (465a2) (83b97), (1) (1)
123456789abc 219a876534cb	Multiplier: (1487a) (563c9), (18a47) (5396c), (174a8) (5c693), (a7841) (9c365), (1) (12) (39) (4a) (58) (67) (bc)
123456789abc 3478bc12569a	Multiplier: (846bc) (21597), (86c4b) (25719), (8b4c6) (29175), (cb648) (79512), (1) (137) (248) (5b9) (6ca)
123456789abc 435621cb78a9	Multiplier: (5a7cb) (36128), (57bac) (31862), (5cab7) (32681), (bc7a5) (82163), (1) (146) (235) (7c9) (8ba)
123456789abc 51369b247ac8	Multiplier: (1a9b6) (427c3), (196ab) (4732c), (1ba69) (4c237), (6b9a1) (3c724), (1) (15972) (46bc8)
123456789abc 6184b3a295c7	Multiplier: (15b34) (2a9c8), (1b453) (298ac), (1354b) (2ca89), (43b51) (8c9a2), (1) (16382) (5bc7a)

# Dodecahedron Elements

123456789abc  
7ab98c154236

Multiplier: (15b34) (2a9c8), (1b453) (298ac), (1354b) (2ca89), (43b51) (8c9a2), (1)  
(17) (2a) (3b) (49) (58) (6c)

123456789abc  
82973c1a64b5

Multiplier: (1a9b6) (427c3), (196ab) (4732c), (1ba69) (4c237), (6b9a1) (3c724), (1)  
(18a47) (396c5)

123456789abc  
9c275a386b14

Multiplier: (5a7cb) (36128), (57bac) (31862), (5cab7) (32681), (bc7a5) (82163), (1)  
(196ab) (2c473)

123456789abc  
a1b5794682c3

Multiplier: (846bc) (21597), (86c4b) (25719), (8b4c6) (29175), (cb648) (79512), (1)  
(1a2) (3bc) (457) (698)

123456789abc  
b6839c14a572

Multiplier: (1487a) (563c9), (18a47) (5396c), (174a8) (5c693), (a7841) (9c365), (1)  
(1b7) (26c) (384) (59a)

123456789abc  
c7593b2a4861

Multiplier: (2a564) (79b38), (254a6) (7b893), (26a45) (7398b), (465a2) (83b97), (1)  
(1c) (27) (35) (6b) (49) (8a)

# Dodecahedron Element List

(1)

(2a564) (79b38)

(254a6) (7b893)

(26a45) (7398b)

(465a2) (83b97)

(12) (39) (4a) (58) (67) (bc)

(12) (39) (4a) (58) (67) (bc) (1487a) (563c9) = (124) (357) (6a8) (9cb)

(12) (39) (4a) (58) (67) (bc) (18a47) (5396c) = (28361) (5a7cb)

(12) (39) (4a) (58) (67) (bc) (174a8) (5c693) = (12795) (48cb6)

(12) (39) (4a) (58) (67) (bc) (a7841) (9c365) = (12a) (3cb) (475) (689)

(137) (248) (5b9) (6ca)

(137) (248) (5b9) (6ca) (846bc) (21597) = (13268) (5cab7)

(137) (248) (5b9) (6ca) (86c4b) (25719) = (13) (2b) (46) (58) (79) (ac)

(137) (248) (5b9) (6ca) (8b4c6) (29175) = (1354b) (2ca89)

(137) (248) (5b9) (6ca) (cb648) (79512) = (139) (287) (4ca) (56b)

# Dodecahedron Element List

(146) (235) (7c9) (8ba)

(146) (235) (7c9) (8ba) (5a7cb) (36128) = (14) (26) (3a) (58) (7b) (9c)

(146) (235) (7c9) (8ba) (57bac) (31862) = (142) (375) (68a) (9bc)

(146) (235) (7c9) (8ba) (5cab7) (32681) = (1487a) (3c956)

(146) (235) (7c9) (8ba) (bc7a5) (82163) = (143b5) (28c9a)

(16382) (5bc7a)

(16382) (5bc7a) (1a9b6) (427c3) = (2a564) (79b38)

(16382) (5bc7a) (196ab) (4732c) = (1a5) (296) (38c) (47b)

(16382) (5bc7a) (1ba69) (4c237) = (19) (2b) (38) (4c) (5a) (67)

(16382) (5bc7a) (6b9a1) (3c724) = (1b7) (26c) (384) (59a)

(15972) (46bc8)

(15972) (46bc8) (1a9b6) (427c3) = (15b34) (2a9c8)

(15972) (46bc8) (196ab) (4732c) = (156) (293) (4ab) (7c8)

(15972) (46bc8) (1ba69) (4c237) =a (15) (2b) (37) (49) (6a) (8c)

(15972) (46bc8) (6b9a1) (3c724) = (15a) (269) (3c8) (467)

# Dodecahedron Element List

(17) (2a) (3b) (49) (58) (6c)

(17) (2a) (3b) (49) (58) (6c) (15b34) (2a9c8) = (17529) (4c68b)

(17) (2a) (3b) (49) (58) (6c) (1b453) (298ac) = (17b) (2c6) (348) (5a9)

(17) (2a) (3b) (49) (58) (6c) (1354b) (2ca89) = (173) (284) (59b) (6ac)

(17) (2a) (3b) (49) (58) (6c) (43b51) (8c9a2) = (174a8) (35c69)

(18a47) (396c5)

(18a47) (396c5) (1a9b6) (427c3) = (189) (27a) (3b6) (4c5)

(18a47) (396c5) (196ab) (4732c) = (18b) (2c5) (364) (79a)

(18a47) (396c5) (1ba69) (4c237) = (18623) (57bac)

(18a47) (396c5) (6b9a1) (3c724) = (18) (24) (3a) (5c) (67) (b9)

(196ab) (2c473)

(196ab) (2c473) (5a7cb) (36128) = (19) (2b) (38) (4c) (5a) (67)

(196ab) (2c473) (57bac) (31862) = (19257) (4b86c)

(196ab) (2c473) (5cab7) (32681) = (198) (2a7) (36b) (45c)

(196ab) (2c473) (bc7a5) (82163) = (193) (278) (4ac) (5b6)



# Dodecahedron Element List

(1a2) (3bc) (457) (698)

(1a2) (3bc) (457) (698) (846bc) (21597) = (1a) (25) (3c) (49) (67) (8b)

(1a2) (3bc) (457) (698) (86c4b) (25719) = (1a5) (296) (38c) (47b)

(1a2) (3bc) (457) (698) (8b4c6) (29175) = (1a9b6) (27c34)

(1a2) (3bc) (457) (698) (cb648) (79512) = (1a784) (3659c)

(1b7) (26c) (384) (59a)

(1b7) (26c) (384) (59a) (1487a) (563c9) = (1ba69) (2374c)

(1b7) (26c) (384) (59a) (18a47) (5396c) = (1b) (2c) (3a) (49) (56) (78)

(1b7) (26c) (384) (59a) (174a8) (5c693) = (1b453) (298ac)

(1b7) (26c) (384) (59a) (a7841) (9c365) = (1b8) (25c) (346) (7a9)

(1c) (27) (35) (6b) (49) (8a)

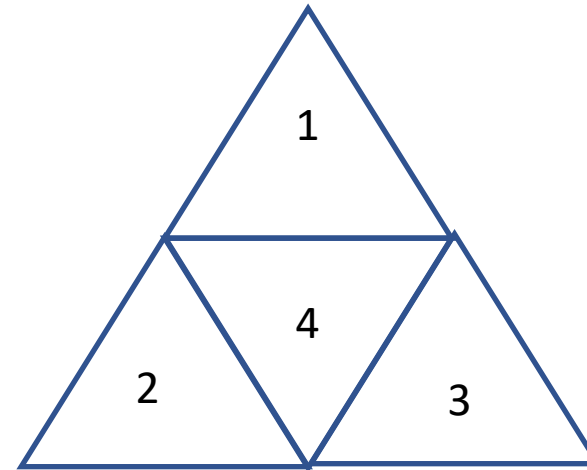
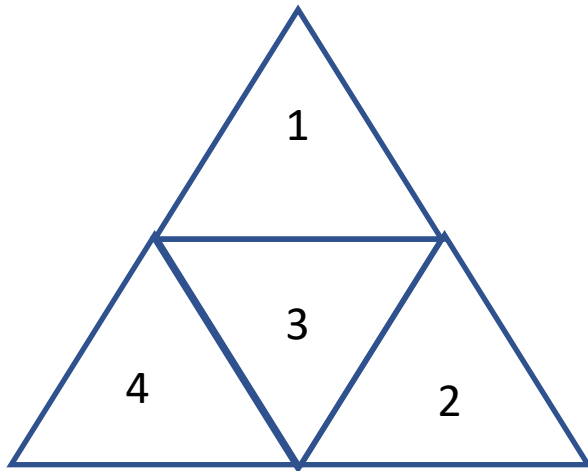
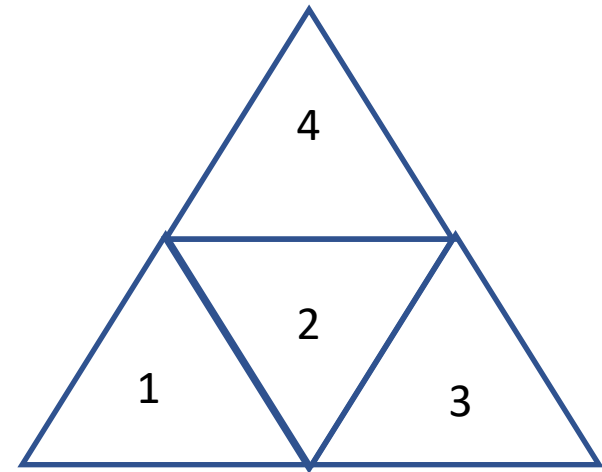
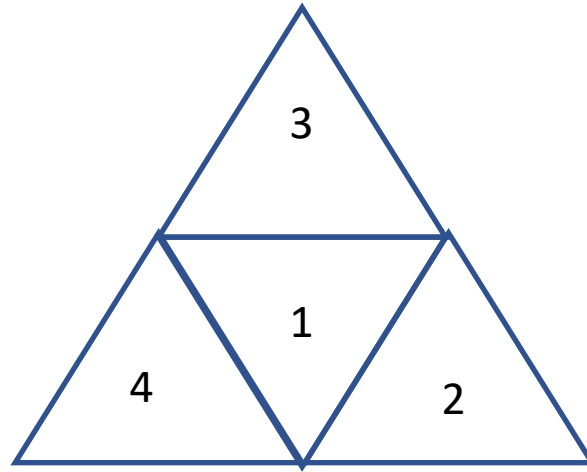
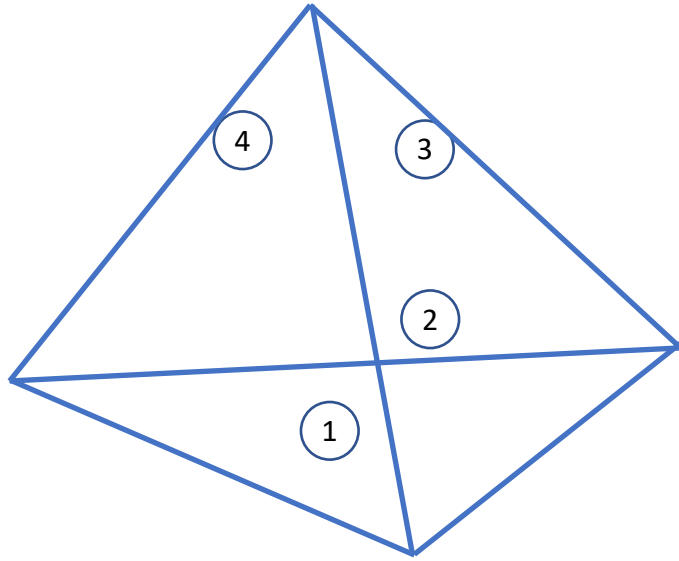
(1c) (27) (35) (6b) (49) (8a) (2a564) (79b38) = (1c) (29) (36) (4b) (58) (7a)

(1c) (27) (35) (6b) (49) (8a) (254a6) (7b893) = (1c) (2b) (34) (57) (68) (9a)

(1c) (27) (35) (6b) (49) (8a) (26a45) (7398b) = (1c) (23) (48) (59) (67) (ab)

(1c) (27) (35) (6b) (49) (8a) (465a2) (83b97) = (1c) (28) (3a) (47) (5b) (69)

# Tetrahedron



# Tetrahedron elements

$(1), (234), (243)$

$(12)(34), (12)(34)(134) = (123), (12)(34)(143) = (124)$

$(13)(24), (13)(24)(142) = (134), (13)(24)(124) = (132)$

$(14)(23), (14)(23)(123) = (142), (14)(23)(132) = (143)$

$(1), (234), (243), (12)(34), (123), (124),$

$(13)(24), (134), (132), (14)(23), (142), (143)$

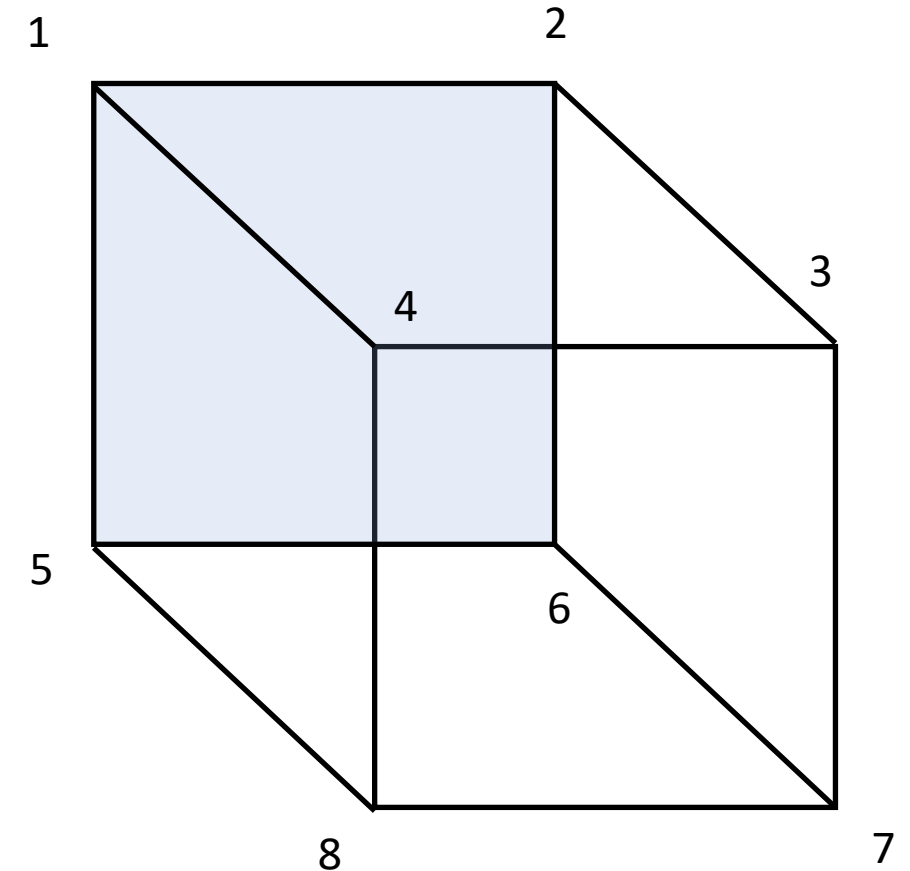
# Cube

## Group actions on vertices

- |                      |                     |
|----------------------|---------------------|
| 1. (1)               | 1. (17)(34)(48)(56) |
| 2. (1234)(5678)      | 2. (12)(35)(46)(78) |
| 3. (1432)(5876)      | 3. (14)(28)(35)(67) |
| 4. (1562)(3487)      | 4. (15)(46)(28)(37) |
| 5. (1265)(4378)      | 5. (245)(386)       |
| 6. (1485)(2376)      | 6. (254)(368)       |
| 7. (1584)(2673)      | 7. (168)(427)       |
| 8. (13)(24)(57)(68)  | 8. (186)(247)       |
| 9. (16)(25)(38)(47)  | 9. (136)(475)       |
| 10. (18)(45)(27)(36) | 10. (163)(457)      |
| 11. (17)(23)(46)(58) | 11. (138)(275)      |
| 12. (17)(48)(35)(26) | 12. (183)(257)      |

$$(1562)(3487)(13)(24)(57)(68) = (17)(23)(46)(58)$$

$$(1485)(2376)(13)(24)(57)(68) = (12)(35)(46)(78)$$



$$P_G(x_1, x_2, x_3, x_4) = 24^{-1}[x_1^8 + 9x_2^4 + 8x_3^2x_1^2 + 6x_4^2]$$

$$P_G(2, 2, 2, 2) = 24^{-1}[256 + 144 + 128 + 24] = 23$$