Entropy and NIST 800-90b

A personal journey

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What are cryptographic random numbers?

- A cryptographic random number consisting of n bits has 2^n possible values.
- All these values should be “equally likely.”
- If you are told k of the bits, you should have no better chance of guessing the remaining bits than $\frac{1}{2^{n-k}}$.
- If you are handed a set of n bits cryptographic random numbers of size N knowing $N-1$ of them should give you no advantage in guessing the remaining one.

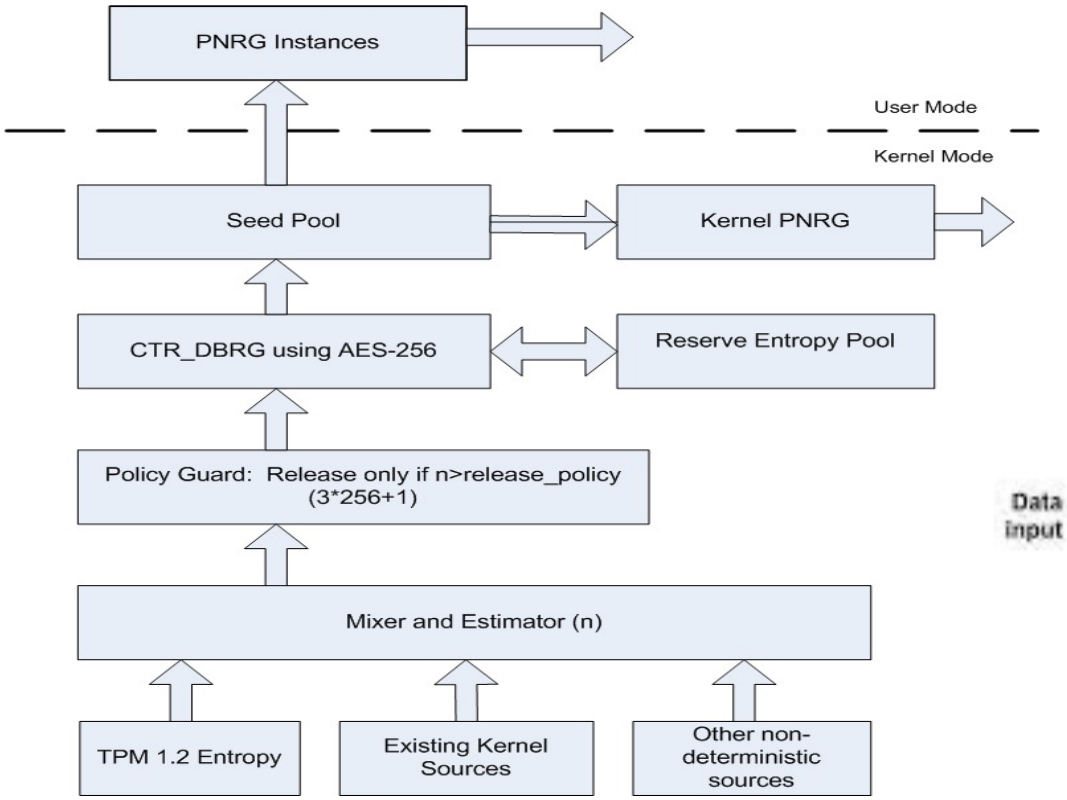
Cryptographic random numbers

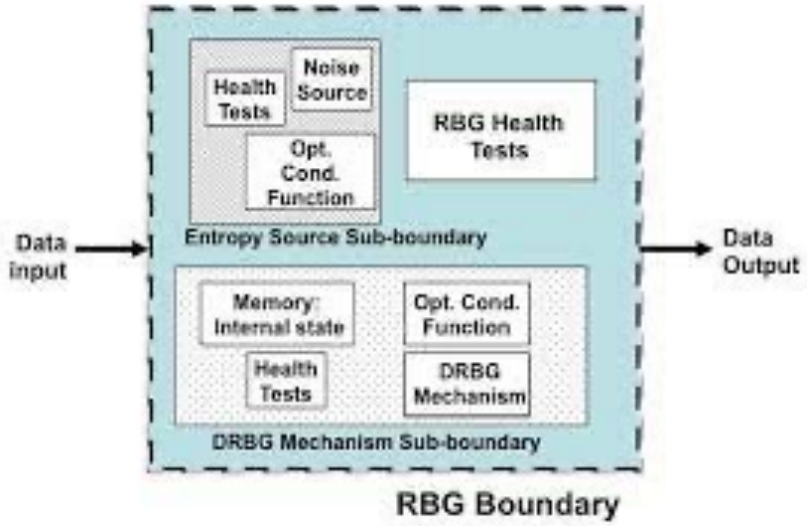
- Critical in cryptographic algorithms and protocols
  - Past weaknesses have been catastrophic.
  - Random number weakness and bad key management are greatest points of attack on cryptographic systems.
- Bad entropy is principal basis for practical attacks
  - Netscape browser attack is famous example.
  - More recent Debian entropy attack (Mind your p's and q's) is another.
  - Hall of fame, epic fails for bad entropy
    - "But the entropy looked random"
    - "No one can predict interrupt arrival times"
    - "No one could guess the sample values, it's too complex"
- Other attacks
  - Intrusion (read privileged entropy pool)
  - Incremental guessing attacks

How can you produce cryptographic random numbers in the real world?

- NIST 800-90C specifies overall design of a cryptographic random number system.
- Components are:
  - Entropy Subsystem including characterized noise source, health tests, entropy conditioning. This is the critical component which prevents adversaries from guessing keys. The output of this system is a seed containing enough “entropy” (more later) to generate keys. The entropy subsystem is specified in NIST 800-90B. This is the hard part.
  - A deterministic random number generator (DRNG). This takes a seed and safely produces a long sequence of cryptographically secure random numbers. This is specified in NIST 800-90A. This is the easy part.

Sample 800-90 RNG System



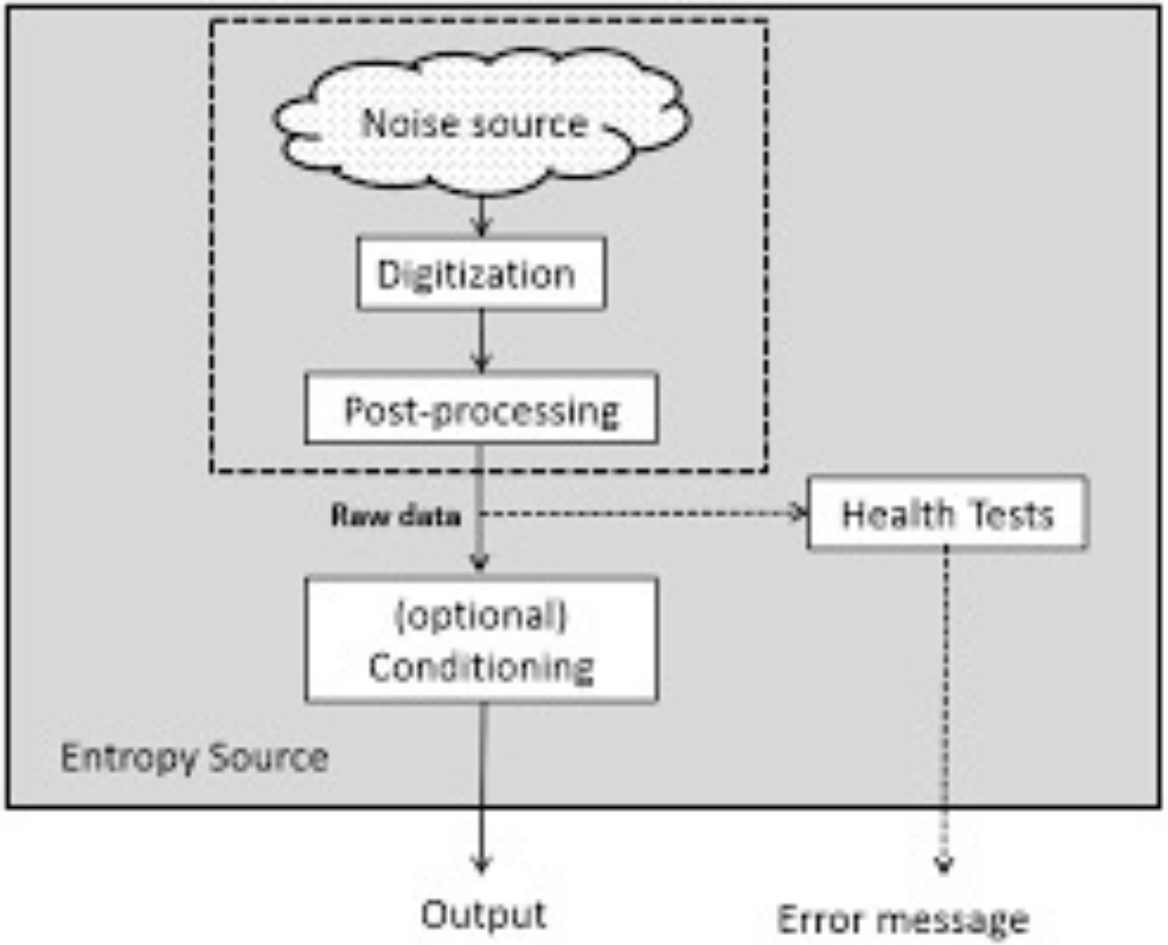


The easy part of cryptographic random number generation

- "Anyone discussing deterministic generation of random number is, strictly speaking, already in a state of sin" – von Neuman.
  - So, a "seed" with full entropy is critical
- Smooths and stretches entropy
- DBRG's can be built using
  - Block ciphers
  - Hash functions
  - Stream ciphers
  - Even public key systems
- Building good, certifiable DRBG's is a "solved problem"
  - Only a few "gotcha's" to be careful about

NIST 800-90B entropy subsystem

- This is the hard part
  - No finite number of statistical tests can “prove” entropy.



What is entropy?

- Entropy is a measure of uncertainty or equivocation. It comes from thermal physics.
  - Entropy is related to how easy it is to "guess" the outcome of an experiment.
  - It is measured in bits (as we'll see). If you have n bits of entropy, you should be able to determine the outcome after 2^n "guesses."
  - In symmetric crypto, for example, if a key has n bits of entropy and you have a solid encryption algorithm, given ciphertext, an adversary should need to try 2^n keys to get the plaintext.
- Caution
  - Entropy is defined with respect to probability distributions. It cannot be calculated using statistical tests.
  - Example probability distribution: A fair coin toss has the distribution $P(X = heads) = \frac{1}{2}$, $P(X = tails) = \frac{1}{2}$.
  - *If you have data from an experiment whose trial outcomes are about half heads and half tails, it does not mean it has the foregoing distribution or the foregoing distribution's entropy.*
  - *Conditioned output can masquerade as entropy rich.*

Shannon's mathematical definition

- Suppose we have an experiment, with a finite set of outcomes, $\Omega = \{x_1, x_2, \dots, x_n\}$ where the outcomes occur with probability $p_1, p_2, \dots, p_n$ respectively. The probability distribution is $P(X = x_i) = p_i$. Note:
  - $\sum_{i=1}^{n} p_i = 1$
  - In general, $p_i \neq p_j$ for $i \neq j$
  - For a probability distribution to be useful, it should be *stationary*, that is, every time you perform an experiment, the probability distribution should be the same. This does *not* mean the outcome of two successive experiments should be the same!
  - These are *very* strong conditions.
- Shannon entropy: $H(X) = -\sum_{i=1}^{n} p_i \lg(p_i)$, $\lg(x) = log_2(x)$

Some entropy source calculations

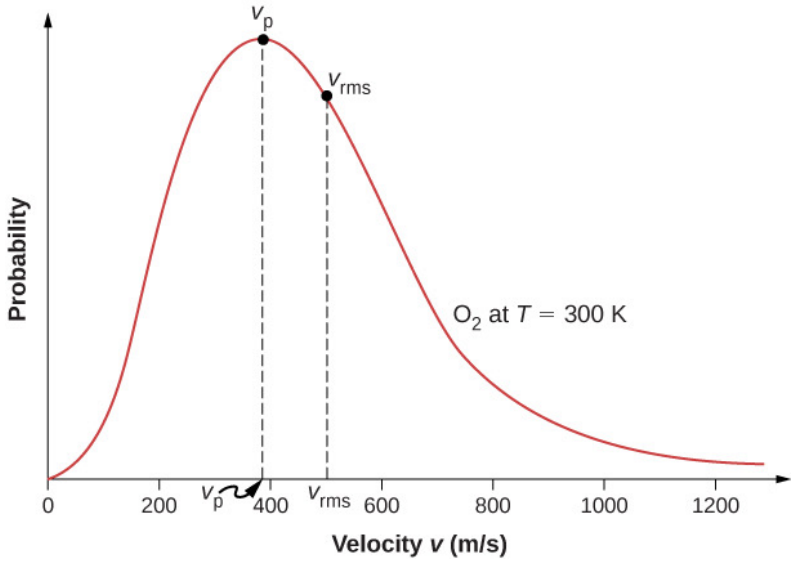
- Fair coin toss
  - $P(X = 1) = \frac{1}{2}\ (heads), P(X = 0) = \frac{1}{2}\ (tails)$
  - $H(X) = -\left(\frac{1}{2}\lg\left(\frac{1}{2}\right) + \frac{1}{2}\lg\left(\frac{1}{2}\right)\right) = 1.$
  - Note: A fair coin is unbiased: $p_{heads} = p_{tails}$
- Biased (but independent) coin tosses
  - $\Pr(x = 1) = \frac{1}{4}, \Pr(x = 0) = \frac{3}{4}$
  - $H(X) = -\left(\frac{1}{4}\lg\left(\frac{1}{4}\right) + \frac{3}{4}\lg\left(\frac{3}{4}\right)\right) = .85\ bits$
  - A "conditioner," like a hash function, can take biased noise samples and "even them."

Other measures of entropy

- Renyi entropy: $H_\alpha(X) = \frac{1}{1-\alpha} \sum_{i=1}^{n} {p_i}^\alpha$
  - Useful when calculating collision properties, usually $\alpha = 2$
- "Min" entropy: $H_{min}(X) = -\lg(max_i\ p_i)$
- $H_{shannon}(X) \geq H_{renyi}(X) \geq H_{min}(X)$, they are equal for a flat distribution.
- NIST focuses on H_{min}. Here's why:
  - Suppose we have the distribution, $P(X = x_1) = \frac{1}{2}$, $P\left(X = x_j\right) = \frac{1}{2(n-1)},\ j = 2,3, \dots, n$
  - An optimal adversarial strategy is to guess x_1 all the time, thus succeeding half the time.
  - $H_{shannon}(X) = -\left(\frac{1}{2}\lg\left(\frac{1}{2}\right) + \frac{n-1}{2(n-1)}\lg\left(\frac{1}{2(n-1)}\right)\right) \approx \frac{1}{2} + \frac{\lg(2n)}{2}$, for large n. This gives a distortedly pessimistic measure of an attacker's chance of succeeding.

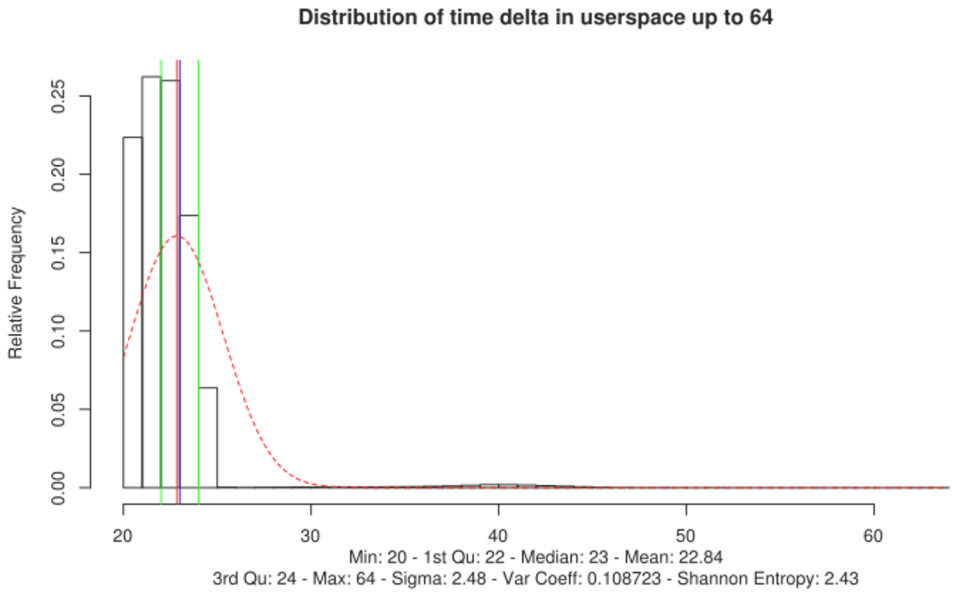
HW sources of entropy (God)

- Hardware
  - Thermodynamics (Johnson noise, …)
  - Oscillator jitter
  - Unsynchronized ring oscillators (Intel's HW RNG is based on this)
  - Noisy diodes
  - Radioactive decay
  - "Open pins" on Raspberry Pi's
  - Coin tosses (with a fair coin)
- Finding the probability distribution is easy: ask a physicist
- Easy to implement in microelectronics



SW sources of entropy (the devil)

- Software sources have been pseudo-science based
- Here is a list (Red is bad. Why? Don't know distribution, also entropy starvation, non-stationarity. Green is good but new.)
  - Disk arm speed variation
  - Process id, thread id (predictable)
  - Interrupt arrival time
  - Ticks since boot
  - Cursor, mouse
  - New: execution jitter
- Finding the probability distribution is hard or impossible except for jitter then you can ask a cryptographer



NIST 800-90B evolution

- 2008: Basic structure: We know it's hard. Document it.
- 2012: We're worried about entropy, here are a bunch of tests to run
  - Justification for software entropy is ad hoc or non-existent: "interrupt arrival times are impossible to guess." (wrong).
  - Use HW if you can: Intel's Ivy bridge RNG (launched 2012)
- 2016: People who don't have a good probability model for their noise sources, don't have entropy.
  - Let's use hardware as a model, hardware sources have distributions
  - Health tests are important because there can be failures
  - Should software entropy have more lax standards? [No!]
- 2018: No, seriously, you have to justify entropy estimators even for a software noise source, so you need source probability models.
  - Here are more tests (restart) so it's harder to cheat especially at boot
  - New software techniques arise (jitter)
  - Linux and some BSD entropy is justified
  - By the way, future standard will be stricter [2021]

A new hope

- Jitter execution entropy
  - High quality and relatively easy (i.e.- possible) to analyze.
  - Adopted by Linux, some BSD's and Apple plus others.
  - Prediction: Eventually everyone will adopt it.

- History
  - B. Sunar, W. J. Martin, D. R. Stinson, *A Provably Secure True Random Number Generator with Built-in Tolerance to Active Attacks* IEEE. Mostly HW focused.
  - Stinson, part 2: What about software based on predicting execution time on modern processor?
  - Works on small processors too: Keaton Mowery, Michael Wei, David Kohlbrenner, Hovav Shacham, and Steven Swanson, *Welcome to the Entropics: Boot-Time Entropy in Embedded Devices.*
  - Mueller, *CPU Time Jitter Based Non-Physical True Random Number Generator.*
  - There's lots more

How does Jitter execution work?

- Collect Entropy

  for (i= 0 to n-1)

  Get real time clock (t_{start})

  Execute standard code block

  Get real time clock (t_{end})

  $\Delta_i = t_{end} - t_{start}$
- The Δ_i's (usually one byte per sample as specified by NIST 800-90B) are the noise source for constructing a seed.
- Why is there uncertainty in the Δ_i?
  - Answer: Thank you ARM, Intel, RISC-V and IBM

Why is there uncertainty in the Δ_i?

- CPU instruction pipelines fill level affects execution time of an instruction. These pipelines add to execution jitter.
- The CPU clock cycle is different than the memory bus clock speed. Wait states for the synchronization of memory access adds to time variances (this also reflects hardware variability effects).
- The CPU frequency scaling alters the processing speed of instructions.
- The CPU power management may disable CPU features.
- Instruction and data caches
  - Tests showed that before the caches are filled, the time deltas are bigger by a factor of two to three.
- CPU topology and caches used jointly by multiple CPUs affect execution time.

But wait, there's more

- CPU frequency scaling depending on the work-load.
- Branch prediction units
- TLB hits and misses
- Kernel locks
- Moving processes from one CPU to another
- Hardware interrupts can occur regardless what the operating system was doing in the meanwhile. [*This is not the same as interrupt arrival time.*]
- Large memory segments whose access times vary due to the physical distance from the CPU.
- Aren't these variations predictable?
  - Amazingly, no!

Why is Jitter execution entropy “good”

- Can be modelled
  - Jitter execution depends only on “core” hardware: CPU’s, memory system, interconnect.
  - Component probability models are relatively simple (like HW): normal distributions around an average performance (memory, interconnect, speculation and prediction).
  - Some sources of variation derived from physical phase jitter where there are existing models. Like hardware in this respect.
  - Easy to pick good, short blocks to measure on any CPU, each CPU varies only slightly in measurable entropy even as architectures change.
  - All the models are stationary. You can validate stationarity with chi squared tests.
  - Basically, doesn’t depend on activity
    - Important for “boot entropy,” where critical machine keys are derived.
  - Naturally “unobservable” by adversary
- Gives very high entropy rates
- Doesn’t need to be in “critical sections.” Works well in kernel and user mode. Works well on all CPU’s, under all workloads.

Why are other sources (say interrupt arrival time) “bad”

- Interrupt arrival modelling difficult (actually impossible, I think)
  - Presumptive distribution is Poisson arrival which is complicated and depends critically on stable average arrival time.
  - Definitely not stationary.
  - Depends on analysis of potentially huge number of devices.
  - Terrible during boot.
  - Past attacks exploited “observable” interrupt artifacts by adversary.
  - Dependent on workloads and their imposed interrupt activity so accurate estimate would be painfully complex even with a few devices.
  - Gives low entropy rates (even “guessed” rates) so more intermediate processing. It takes much longer to “reseed.”
  - Requires ongoing care to measure interrupt times correctly (If you measure arrival time in the wrong place the entropy is greatly reduced).
- Kernel mode only
- I don’t know of any credible analysis of interrupt noise and I’ve looked hard.

Three weekends and a NIST reading

- I built a (basically) fully NIST compliant RNG in my existing open-source crypto project (which I use for teaching) with justified HW and SW entropy in about three weekends.
  - Used standard NIST 800-90A certified SHA-256 hash-df based DBRG.
  - Used Intel analyzed HW noise source (Noise justification: Rachael J Parker, *Justification for Metastability Based Nondeterministic Random Bit Generator*, Intel and previous papers by *Kocher, Cox, Walker, Gueron, Brickell, et al*).
  - Developed SW Jitter based noise source (Noise justification: You'll see.)
  - Implemented full health and restart tests.
  - Both HW and SW entropy qualified.
  - All in user mode (kernel version is almost identical).

RNG

jlm@New-MacBook-Pro cryptobin % ./test_full_rng.exe --print_all=true

Hardware test
HW noise : 2efccc36185532aecc5714c76a328d2e55bb5b6868cf6ee28ce7c0dd9178c43e
dffda69eb78b66ec
Entropy from pool: 300 bits.
seed: 2efccc36185532aecc5714c76a328d2e55bb5b6868cf6ee28ce7c0dd9178c43e
dffda69eb78b66ec

Hardware derived random numbers:
random number 0: 38c4606144d00c417be407466237b3e08b8f08f81fd98b8a94dd54d74f914fed
random number 1: 74283705ff3ad67c00d029ba8344cc9d014ef98a58edc9c93259cc8cf042cf37
random number 2: 272f7b36a8694254368ca0f534ee45fd56756ef7e90bd49a3d35e87019f9ee68
random number 3: d81302e1a5d30dc7735e62cc2c790ab3595c5cce34665c590556293f1f61e826
random number 4: c804ada32bbe35d75a16cde90ec044d1f09350a82b8355ff50a7ef00f3a90ec3
random number 5: c3b090ddf0c9d6dcd54a3d25ac9ba24813df5dd9119f683cff52c0a4487db23a
random number 6: d25fa7625b34646d2d5402e3b8f65dff2fb806c4a9cfa978303b27a133fd9dbe
random number 7: 5baa492e703d63b3074e40cbb041723203659f42951ce70db1d083dcb25777a6
random number 8: 88210cb9e669ecf79dd13091c4da7de0beb60f553dd6bf6ad39360c587d2370a
random number 9: 7943f321315f440803a377e9ddce3696b4f30515b05510e8f368c15837c4b63d

RNG

Software test
SW noise: 8604aa5c9683f61c5765fb4bf610ff4026afcde5
SW noise: d24e2528dc24a560b5ecdde9f941ff555ba18758
SW noise: 3758ba50858922f3bd919b14b0da4375e3281129
…10 more
Entropy from pool: 259 bits.
seed: 56580a6f33dffb67271838514b72609d292960989636c17b593018764b2c3caf36ebf42e

Software derived random numbers:
random number 0: 5704f41ef758d66ef82483217c9291fd79f5aa18b5bd1dc114049df5e81c1d34
random number 1: c1ae285d23a91a399f5e2c095a769c4195f4c3753a4aca0d78b7d8197378c672
random number 2: b860d249bbddcaf87666815de2def42d8e73c6de798b667d68f55068a5b79d7c
random number 3: 4370488c2a28ed85161cd216b3caf2caae2705e9b893c9e082666a4c93622337
random number 4: 21f3a1438634aa05d4071617ce420786aa574ab4c4258fbbc8d975b2f6c205bb
random number 5: 10e1fc08d6df99515727b7d68c973deee25b52a28304d29db85aec71735939dc
random number 6: d9cd2731623a3ee43ba2652bb07e41d00c153d72ab814a24df29883f5c234fb4
random number 7: 18b9e00e397c7ef8c05f61078ce1884c30f3b74e61286574abac5d01991a8c6b
random number 8: f2d71f29ee1aeec8109da2a3e83a1aa9f1ded83bb573441c7202e6892381106e
random number 9: cb467402c79b90e02959d35e0ffe39cf0c2b4da75583f30b2921c0b4073fcc5b

Software jitter sources in my code

- Used five different blocks for jitter including two from Mueller
- For each 8-bit (as required by NIST) noise sample, the estimated entropy varied from 1.8 bit/sample to over 6 bits/sample (!) over the jitter blocks
  - Jitter gives amazingly high rate independent of other activity Can't be tampered with by adversary
- Most important: Can be analyzed and justified.
  - Isn't this just an academic concern?
  - No!

Simple Jitter Block 0

Simple jitter test 0, cpc: 1999533010

largest: 63, smallest: 31, non-zero: 1000, mean: 37.204, adjusted mean: 37.141

44 bins, lower 30, upper: 42, bins from 30 to 42 selected:

0000 0005 0015 0059 0078 0181 0055 0146 0080 0217 0034 0094 0033

Samples: 1000, num loops: 5, expected bin: 37.097, deviation: 32.243

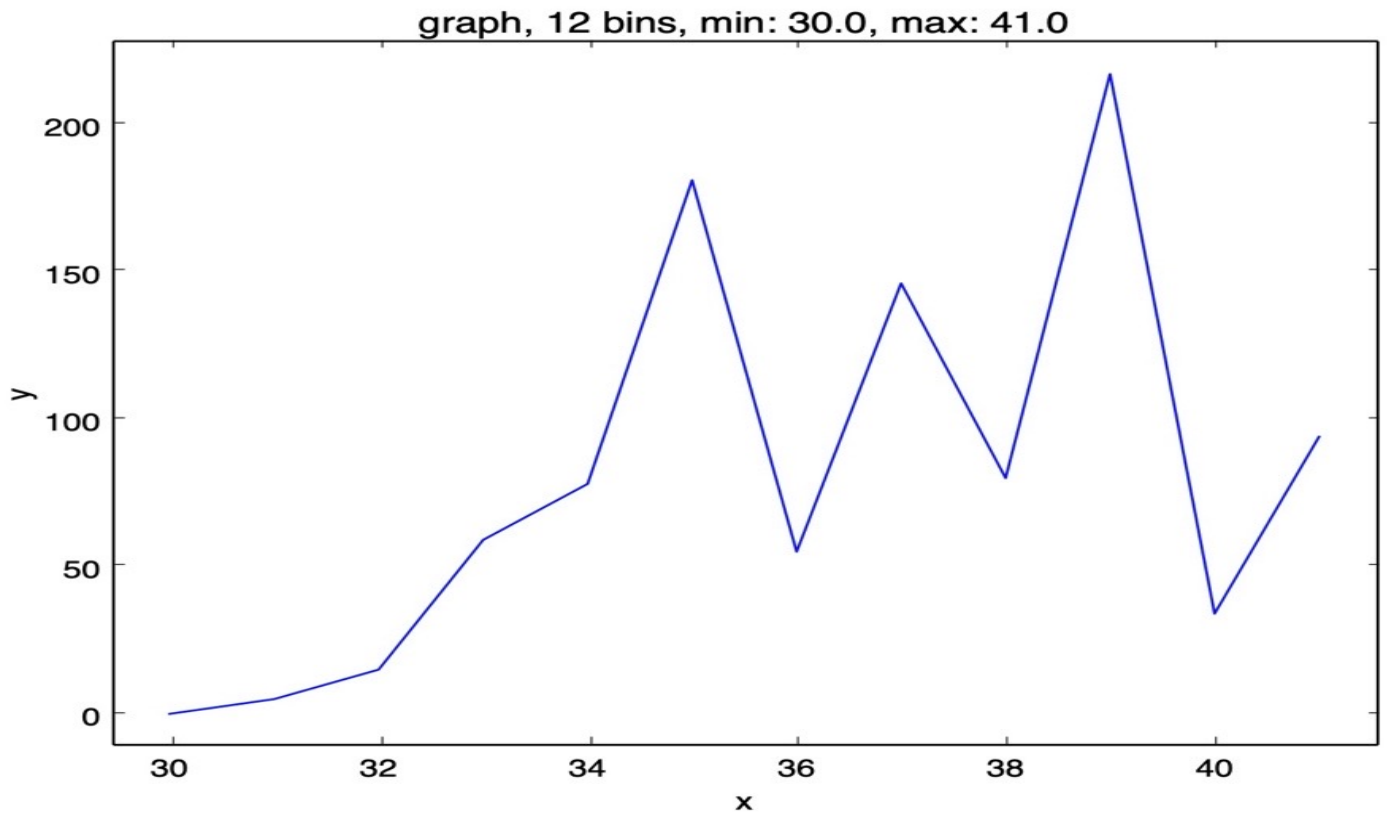
probabilities:

31,0.005 32,0.015 33,0.059 34,0.078 35,0.181 36,0.055 37,0.146

38,0.080 39,0.217 40,0.034 41,0.094

Shannon entropy: 3.168, renyi entropy: 2.927, min entropy: 2.204

Simple Jitter Block 0



Simple Jitter Block 1

Simple jitter test 1, cpc: 2003954612

largest: 125, smallest: 32, non-zero: 1000, mean: 39.565, adjusted mean: 39.440

47 bins, lower 31, upper: 45, bins from 31 to 45 selected:

0000 0001 0010 0024 0045 0073 0090 0038 0286 0094 0110 0073 0104 0036 0013

Samples: 1000, num loops: 5, Expected bin: 39.286, deviation: 33.994

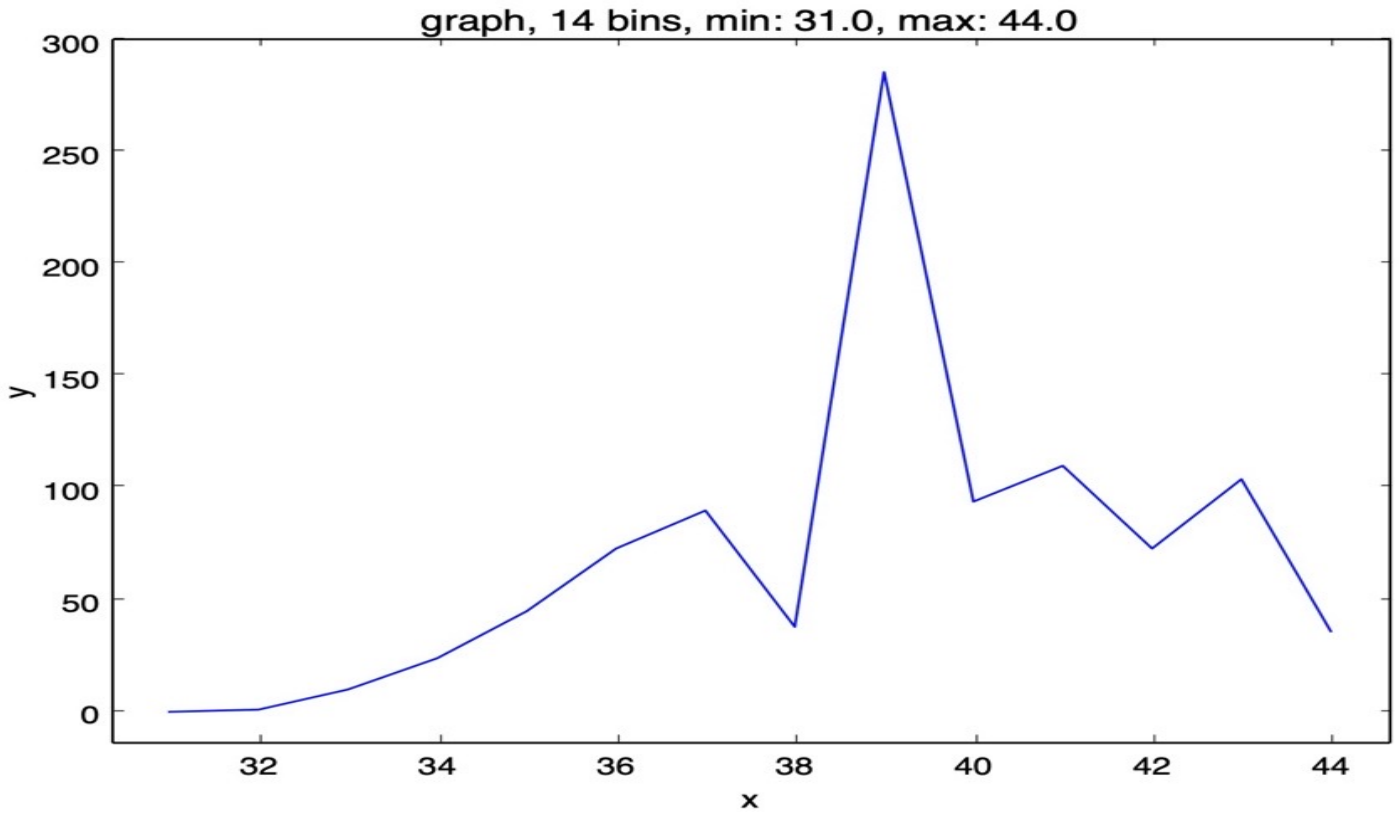
probabilities:

32,0.001 33,0.010 34,0.024 35,0.045 36,0.073 37,0.090 38,0.038

39,0.286 40,0.094 41,0.110 42,0.073 43,0.104 44,0.036

Shannon entropy: 3.231, renyi entropy: 2.858, min entropy: 1.806

Simple Jitter Block 1



Simple Jitter Block 2

Simple jitter test 2, cpc: 2003743142

largest: 165, smallest: 49, non-zero: 1000, mean: 64.158, adjusted mean: 63.993

79 bins, lower 50, upper: 73, bins from 50 to 73 selected:

0002 0000 0012 0010 0032 0009 0046 0082 0050 0079 0088 0103 0031 0067 0079 0100 0031 0043 0022 0028 0025 0014 0003 0007

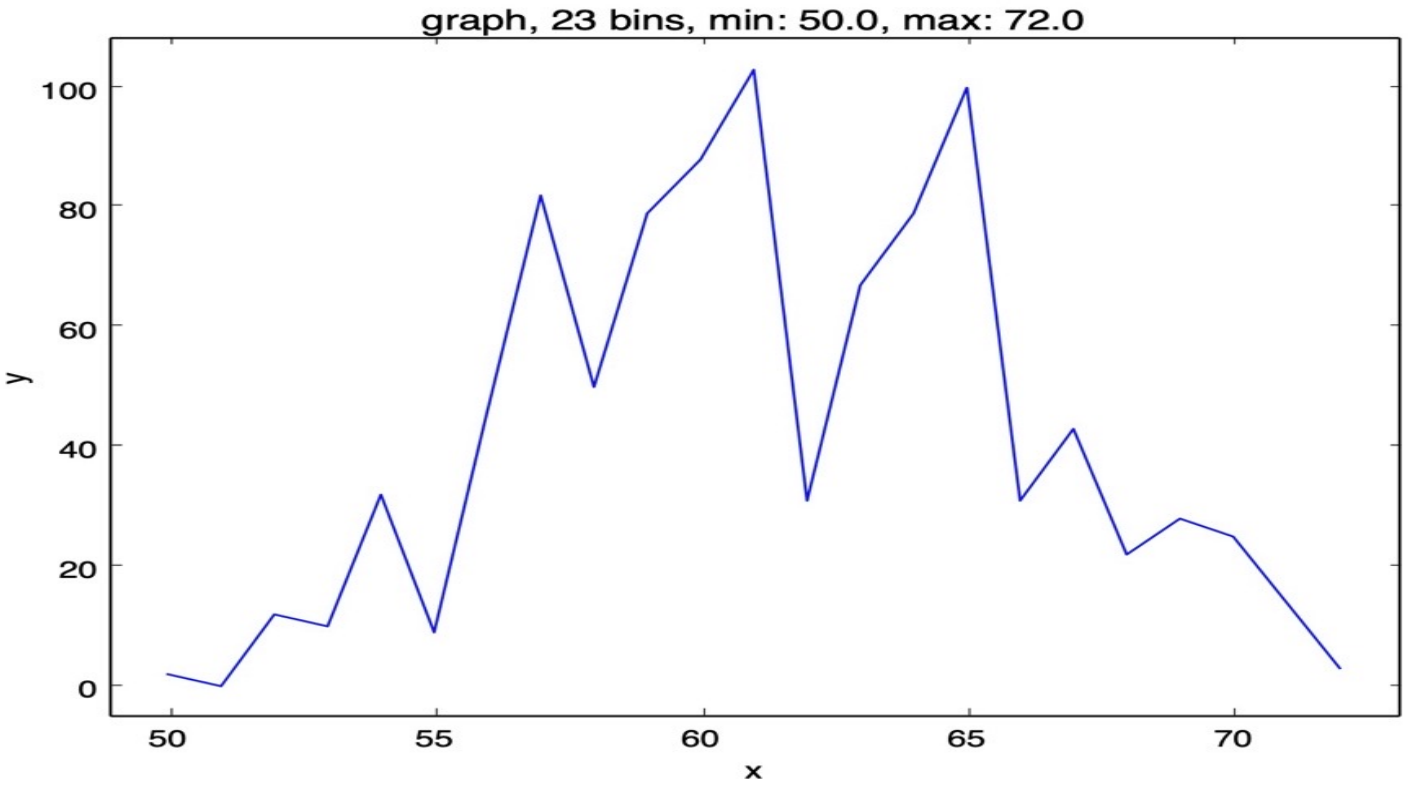
Samples: 1000, num loops: 5, Expected bin: 59.797, deviation: 54.901

probabilities:

50,0.002 52,0.012 53,0.010 54,0.032 55,0.009 56,0.046 57,0.082 …

Shannon entropy: 4.034, renyi entropy: 3.967, min entropy: 3.279

Simple Jitter Block 2



Memory Jitter

Memory jitter test, cpc: 1998883639

largest: 255, smallest: 2, non-zero: 1000, mean: 71.828, adjusted mean: 71.573

141 bins, lower 30, upper: 71, bins from 30 to 71 selected:

0004 0003 0018 0010 0015 0006 0017 0011 0020 0028 0014 0022 …

Samples: 1000, num loops: 5, Expected bin: 50.529, deviation: 46.755

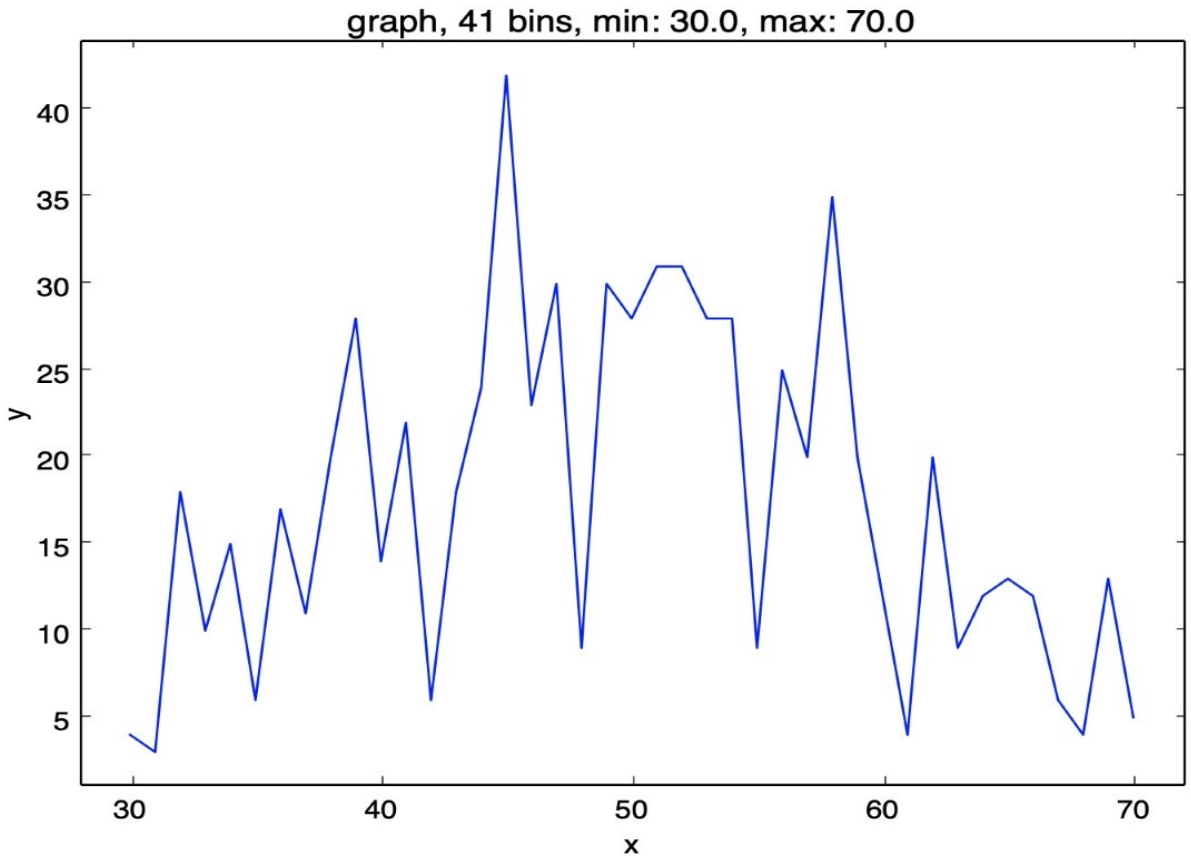
probabilities:

30,0.004; 31,0.003; 32,0.018; 33,0.010; 34,0.015; 35,0.006; …

…

Shannon entropy: 5.476, renyi entropy: 5.873, min entropy: 4.573

Memory Jitter



Hash based execution jitter

Hash jitter test, cpc: 2003971500

largest: 255, smallest: 0, non-zero: 1000, mean: 132.188, adjusted mean: 131.933

264 bins, lower 66, upper: 237, bins from 66 to 237 selected:

…

Samples: 1000, num loops: 5, Expected bin: 132.188, deviation: 131.502

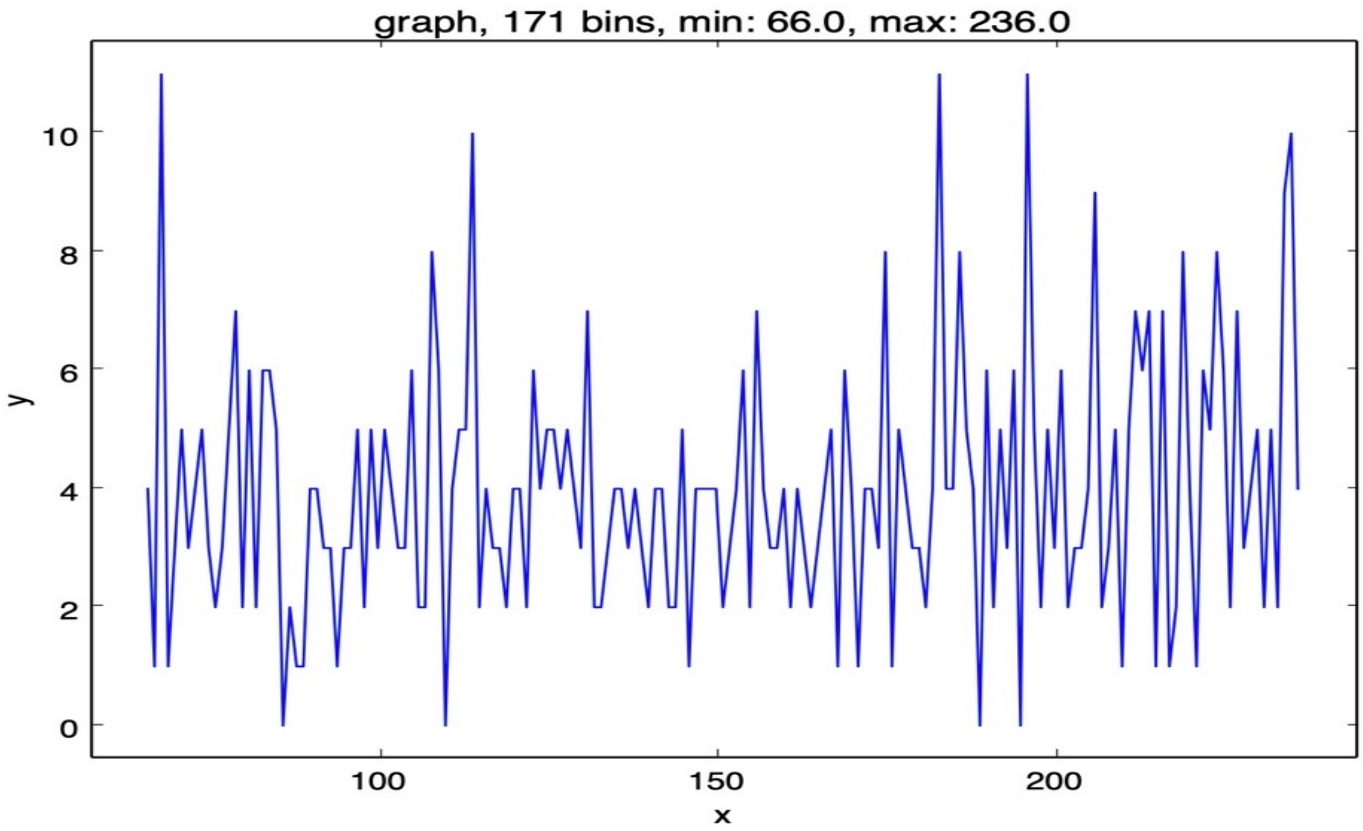
probabilities:

66,.004 67,.001 68,.011 69,.001 70,.003 71,.005 72,.003 73,.004

…

Shannon entropy: 7.791, renyi entropy: 7.637, min entropy: 6.506

Hash based execution jitter



Future work

- Work out hardware model in detail for individual sources of entropy (memory jitter, …).
- Combine complicated jitter sources to obtain different distributions (the “mixture problem”). Done accurately, this is interesting research!
- The level of proof in this presentation is informal or “heuristic,” which is all NIST demands for now.
- NIST will increase level of rigor required in future standards. These arguments can be developed to justify a full stochastic model NSA would be proud of.

Conclusion

- **NIST was completely right!**
  - I apologize for earlier skepticism
- New standard greatly improves security (if you follow it)
  - In cryptography, don't trust anything you can't quantifiably analyze --- you're only fooling yourself.
  - In cryptography, sometimes there are good surprises (jitter).
- Benefits from Jitter
  - No entropy starvation at boot.
  - Defense in depth (qualified HW and SW entropy).
  - Simpler than interrupts and less performance impact
  - Works on all devices (even embedded devices).
  - Widely accepted despite initial skepticism: Linux, (some) BSD versions and Apple (during boot).
  - You can stop drinking the kool-aid: Jettison previous software entropy "pseudo-science."