Cryptanalysis

Cryptographic Hashes

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Cryptographic Hashes

- A cryptographic hash ("CH") is a "one way function," h, from all binary strings (of arbitrary length) into a fixed block of size n (called the size of the hash) with the following properties:
    1. Computing h is relatively cheap.
    2. Given y=h(x) it is infeasible to calculate x. ("One way," "non-invertibility" or "pre-image" resistance). Functions satisfying this condition are called One Way Hash Functions (OWHF)
    3. Given u, it is infeasible to find w such that h(u)=h(w). (weak collision resistance, 2^{nd} pre-image resistance).
    4. It is infeasible to find u, w such that h(u)=h(w). (strong collision resistance). Note 4→3. Functions satisfying this condition are called Collision Resistant Functions (CRFs).

Observations

- Collision Resistance → 2^{nd} pre-image resistance
- Let $f(x)= x^2-1 \pmod p$.
  - $f(x)$ acts like a random function but is not a OWHF since square roots are easy to calculate mod p.
- Let $f(x)= x^2 \pmod{pq}$.
  - $f(x)$ is a OWHF but is neither collision nor 2^{nd} pre-image resistant
- If either $h_1(x)$ or $h_2(x)$ is a CRHF so is $h(x)= h_1(x)||h_2(x)$
- MDC+signature & MAC+unknown Key require all three properties
- Ideal Work Factors:

Type	**Work**	**Property**
OWHF	2^n	Pre-image 2^{nd} Pre-image
CRHF	$2^{n/2}$	Collision
MAC	2^t	Key recovery, computational resistance

One-Way Functions

- Hashes come from two basic classes of one-way functions
  - Mathematical
    - Multiplication: $Z=X \bullet Y$
    - Modular Exponentiation: $Z = Y^X$ *(*mod n) (Chaum vP Hash)
  - Ad-hoc (Symmetric cipher-like constructions)
    - Custom Hash functions (MD4, SHA, MD5, RIPEMD)

Chaum-vanHeijst-Pfitzmann Compression Function

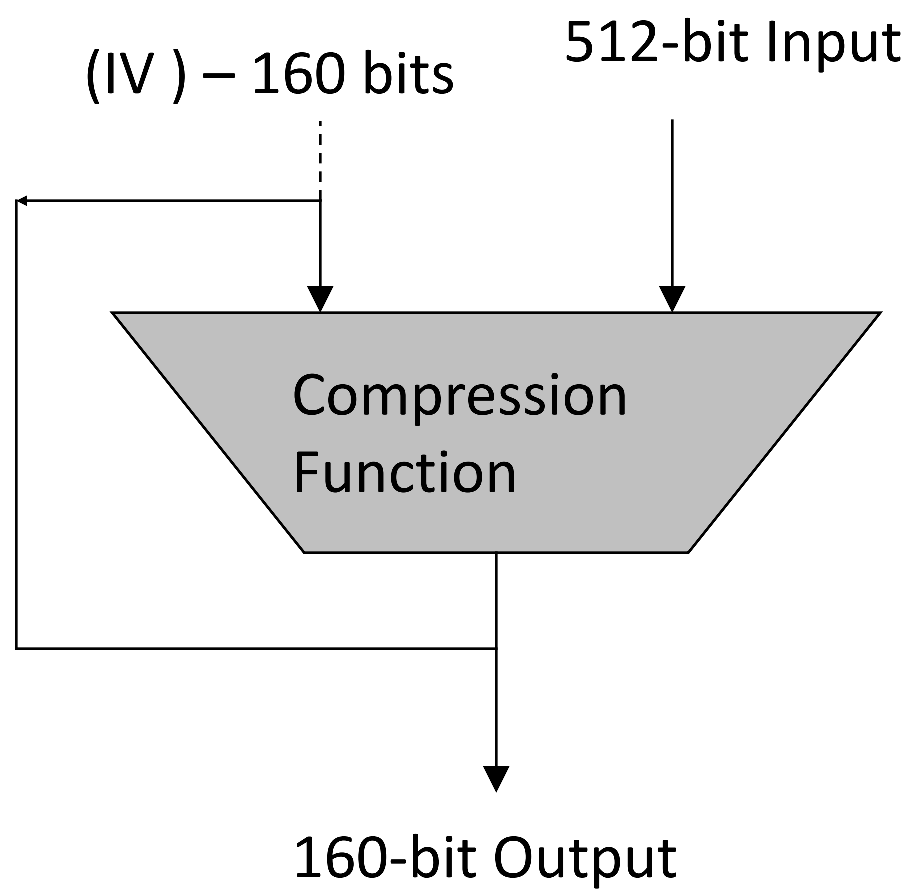
- Suppose p is prime, q=(p-1)/2 is prime, a is a primitive root in F_p, b is another primitive root so a^x=b (mod p) for some unknown x).
- g: $\{1,2,\ldots,q-1\}^2 \rightarrow \{1,2,\ldots,p-1\}$, q=(p-1)/2 by:
  - g(s, t) = $a^s b^t$ (mod p)
- Reduction to discrete log:

  Suppose g(s, t)= g(u, v) can be found. Then $a^s b^t$ (mod p)= $a^u b^v$ (mod p).

  So a^{s-u} (mod p)= b^{v-t} (mod p). Let b= a^x (mod p). Then (s-u)=x(y-t) (mod p-1).

  But p-1= 2q so we can solve for x, thus determining the discrete log of b.

A Cryptographic Hash: SHA-1



SHA-1: State and message schedule

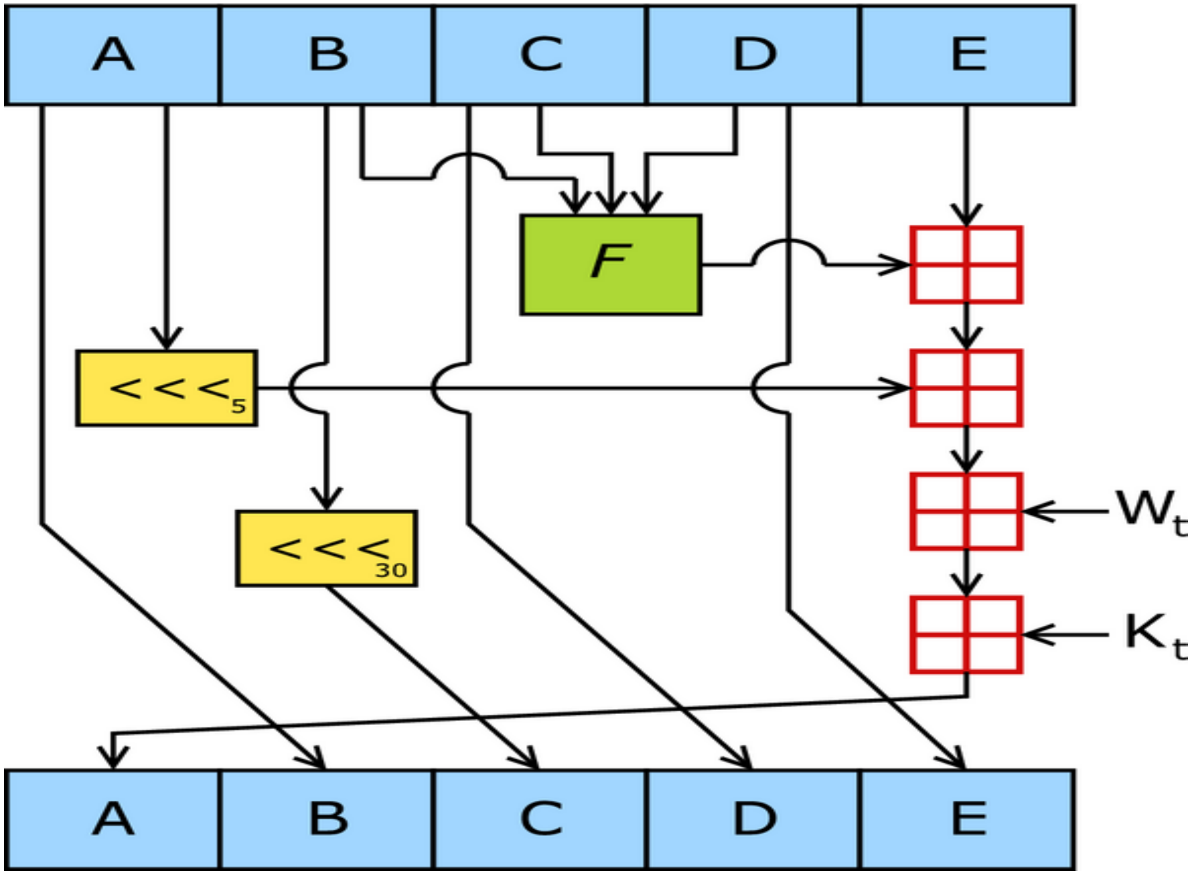
- Compression function takes 160-bit state and 512 bit input and produces new 160 bit state (one Merkle Damgard round)
- 512-bit message input block: 16 32-bit words (M_0, …, M_{15})
- Compression consists of 80 rounds
  - Each round uses one 32 bit word derived from input block
  - Message expansion algorithm produces subsequent rounds
    - $W_t = M_t$, $0 \leq t < 16$
    - $W_t = (W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16}) <<<1$, $16 \leq t < 80$
    - Structure of round is same for all 80 rounds:

      $X = (a<<<5)+f_t(b,c,d)+e+W_t+K_t$

      E= d; d=c; c= b<<<30; b=a; a= x;

      Three f_t functions. First used in rounds 0 through 19, Second used in rounds 20 through 39. Third used in rounds 40-59. First reused in rounds 60-79

SHA-1round



Picture from Wikipedia

A Cryptographic Hash: SHA -1

- Depending on the round, the “function f is one of the following.

  $f(X,Y,Z)= (X\wedge Y)\vee((\neg X\wedge Z)$

  $f(X,Y,Z)= (X\wedge Y)\vee(X\wedge Z)\vee(Y\wedge Z)$

  $f(X,Y,Z)= X\oplus Y\oplus Z$

- Note first two are non-linear. Third is linear and provides diffusion.

SHA-0/1

Absence of this term is only difference between SHA-0 and SHA-1

$A= 0x67452301$, $B= 0xefcdab89$,
$C= 0x98badcfe$, $D= 0x10325476$
$E= 0xc3d2e1f0$

$F_t(X,Y,Z)= (X \wedge Y) \vee ((\neg X) \wedge Z)$,
 $t= 0,\ldots,19$
$F_t(X,Y,Z)= X \oplus Y \oplus Z$,
 $t= 20,\ldots,39$
$F_t(X,Y,Z)= (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z)$,
 $t= 40,\ldots,59$
$F_t(X,Y,Z)= X \oplus Y \oplus Z$, $t= 60,\ldots,79$

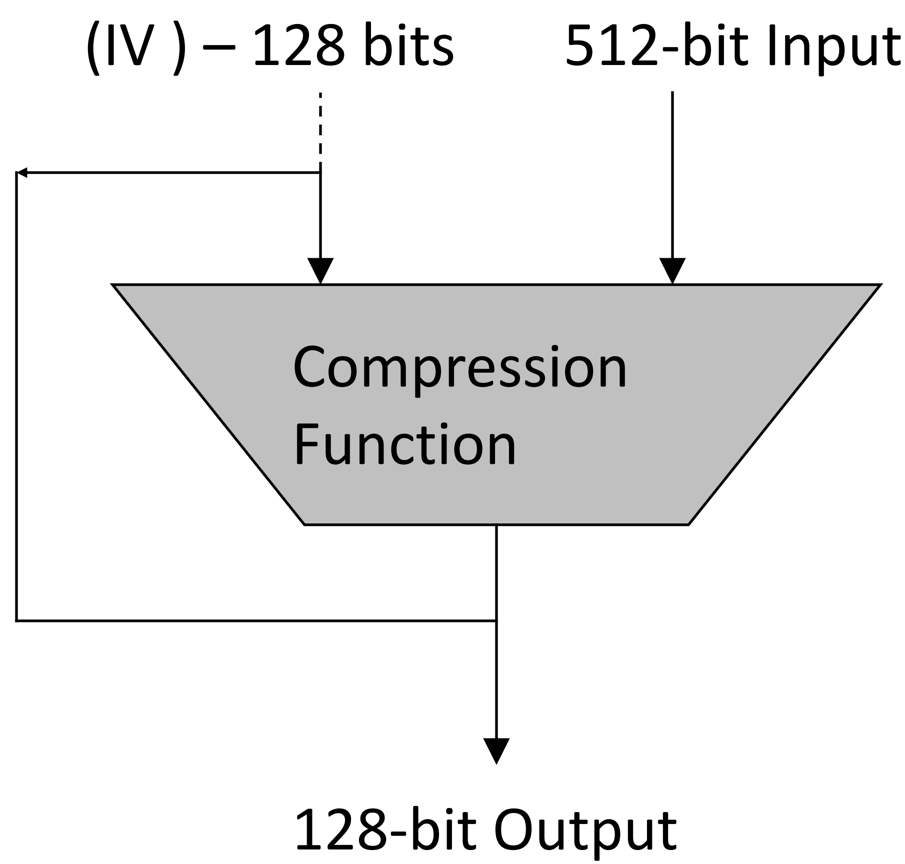
$K_t= 0x5a827999$, $t= 0,\ldots,19$
$K_t= 0x6ed9eba1$, $t=20,\ldots,39$
$K_t= 0x8f1bbcdc$, $t= 40,\ldots,59$
$K_t= 0xca62c1d6$, $t=60,\ldots,79$

Do until no more input blocks {
 If last input block
  Pad to 512 bits by adding 1 then 0s then 64 bits of length.
 M_i= input block(32 bits)
  $i= 0,\ldots,15$
 $W_t= M_t$, $t= 0,\ldots,15$;
 $W_t= (W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16})$ $\boxed{<<<1}$,
  $t= 16,\ldots,79$
 a= A; b= B; c= C; d= D; e= E;
 for(t=0 to 79) {
  $x= (a<<<5)+f_t(b,c,d)+e+W_t+K_t$
  e= d; d=c; c= b<<<30;
  b=a; a= x;
  }
 A+= a; B+=b; C+= c; D+= d; E+= e;
 }
Output (A, B, C, D, E)

MD4

- Invented by Rivest, ca 1990
- Weaknesses found by 1992
  – Rivest proposed improved version (MD5), 1992
  – SHA-0/1, 1993/1995
  – SHA-2, 2001
  – SHA-3, 2012
- Dobbertin found MD4 collision in 1998

A Cryptographic Hash: MD-4



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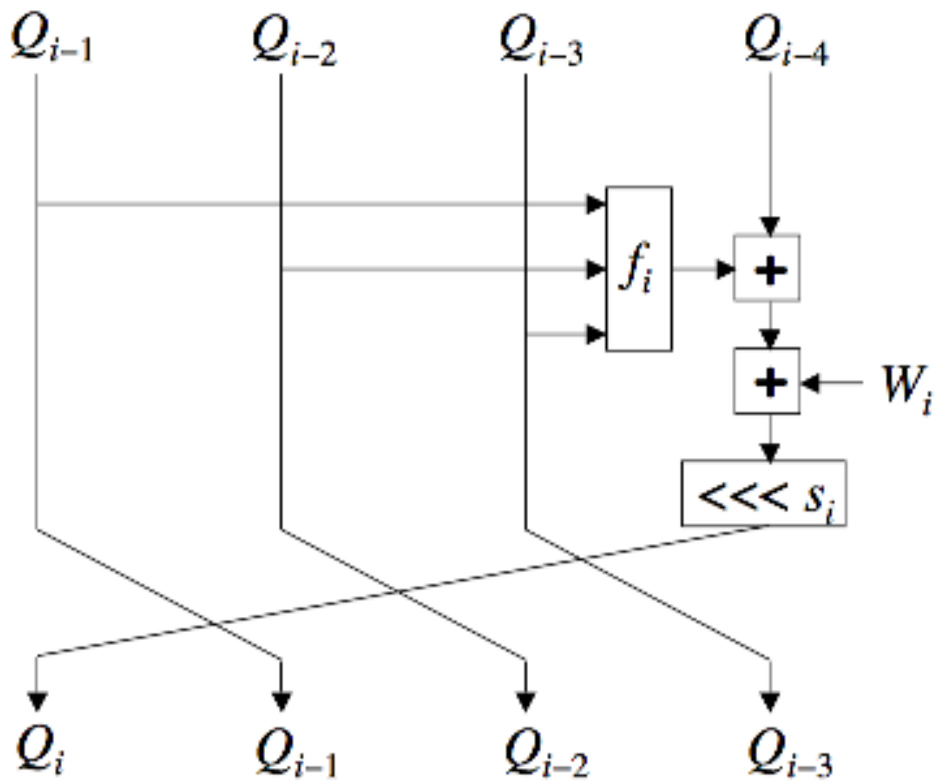
MD4: State and message schedule

- Compression function takes 128-bit state and 512 bit input and produces new 128 bit state (one Merkle Damgard round)
- 512-bit message input block: 16 32-bit words (M_0, …, M_{15})
- Compression consists of 48 rounds
  - Each round uses one 32 bit word derived from input block
  - Message expansion algorithm produces subsequent rounds
    - $W_t = M_{s(t)}$, $0 \le t < 47$
    - Structure of round is same for all 48 rounds, 3 round functions

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
s(t)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
t	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
s(t)	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11	15
t	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
s(t)	0	8	4	12	2	10	6	14	1	9	5	13	3	11	7	15

MD4 round



$f_i(A,B,C)$

$(A \wedge B) \vee (\neg A \wedge C)$, $0 \leq i < 16$

$(A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$, $16 \leq i < 32$

$A \oplus B \oplus C$, $32 \leq i < 48$

K_0 = 0x00000000,
K_1 = 0x5a827999,
K_2 = 0x6ed9eba1.

- Where

$$f_i(A,B,C) = \begin{cases} F(A,B,C) + K_0 & \text{if } 0 \leq i \leq 15 \\ G(A,B,C) + K_1 & \text{if } 16 \leq i \leq 31 \\ H(A,B,C) + K_2 & \text{if } 32 \leq i \leq 47 \end{cases}$$

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MD4 Algorithm

```
// M = (Y_0, Y_1, ..., Y_{N-1}), message to hash, after padding
// Each Y_i is a 32-bit word and N is a multiple of 16
MD4(M)
    // initialize (A, B, C, D) = IV
    (A, B, C, D) = (0x67452301, 0xefcdab89, 0x98badcfe, 0x10325476)
    for i = 0 to N/16 − 1
        // Copy block i into X
        X_j = Y_{16i+j}, for j = 0 to 15
        // Copy X to W
        W_j = X_{σ(j)}, for j = 0 to 47
        // initialize Q
        (Q_{−4}, Q_{−3}, Q_{−2}, Q_{−1}) = (A, D, C, B)
        // Rounds 0, 1 and 2
        Round0(Q, X)
        Round1(Q, X)
        Round2(Q, X)
        // Each addition is modulo 2^32
        (A, B, C, D) = (Q_44 + Q_{−4}, Q_47 + Q_{−1}, Q_46 + Q_{−2}, Q_45 + Q_{−3})
    next i
    return A, B, C, D
end MD4
```

Overview of attack

- Try to find one block collision
- Denote $M = (X_0, X_1, \ldots, X_{15})$
- Define M' by $X'_i = X_i$ for $i \neq 12$ and $X'_{12} = X_{12}+1$
- Word X_{12} only appears in steps 12, 19, 35
  - This provides a "natural" round division of the attack
- We have the freedom to choose $X_0, X_1, \ldots, X_{11}$ at our convenience
- Goal is to find pair M and M′ with $\Delta_{35} = (0,0,0,0)$

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Dobbertin's attack strategy

- Specify a differential condition
- If condition holds, there's a probability of collision---try enough times for overall probability to be high.
- Derive system of nonlinear equations: solution satisfies differential condition
- Find efficient method to solve equations
- Find enough solutions to yield a collision
- Find one-block collision, where $M= (X_0,X_1,\ldots,X_{15})$, $M'= (X'_0,X'_1,\ldots,X'_{15})$
- Difference is subtraction mod 2^{32}
- Blocks differ in only 1 word
  - Difference in that word is exactly 1
- Limits avalanche effect to steps 12 thru19
  - Only 8 of the 48 steps are critical to attack!
  - System of equations applies to these 8 steps

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Notation

- Suppose $(Q_j, Q_{j-1}, Q_{j-2}, Q_{j-3}) = \mathrm{MD4}_{0\ldots j}(IV, M)$ and $(Q'_j, Q'_{j-1}, Q'_{j-2}, Q'_{j-3}) = \mathrm{MD4}_{0\ldots j}(IV, M')$
- Define $\Delta_j = (Q_j - Q'_j,\ Q_{j-1} - Q'_{j-1},\ Q_{j-2} - Q'_{j-2},\ Q_{j-3} - Q'_{j-3})$ where subtraction is modulo 2^{32}
- Let $\pm 2^n$ denote $\pm 2^n \bmod 2^{32}$.
  - 2^{25} = 0x02000000 and -2^5 = 0xffffffe0
- All arithmetic is modulo 2^{32}

Three phases of MD4 attack

1. Show: Δ_{19}= $(2^{25},-2^{5},0,0)$ implies probability at least $1/2^{30}$ that the Δ_{35} condition holds
   - Uses differential cryptanalysis
2. "Backup" to step 12: We can start at step 12 and have Δ_{19} condition hold
   - By solving system of nonlinear equations
3. "Backup" to step 0: And find collision
   - In each phase of attack, some words of M are determined
   - When completed, have M and M′
     - Where $M \neq M'$ but $h(M) = h(M')$
   - Equation solving step is tricky part
     - Nonlinear system of equations
     - Must be able to solve efficiently

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Steps 19 to 35

- Differential phase of the attack
- M and M′ as given above
  - Only differ in word 12
- Assume that $\Delta_{19}= (2^{25},-2^5,0,0)$
  - $G(Q_{19},Q_{18},Q_{17})=$
    $G(Q'_{19},Q'_{18},Q'_{17})$
- Then we compute probabilities of "Δ" conditions at steps 19 thru 35
- Total probability: 2^{-30}, actually 2^{-22}

j	Δ_j				i	s_j	p	Input
	ΔQ_j	ΔQ_{j-1}	ΔQ_{j-2}	ΔQ_{j-3}				
19	2^{25}	-2^5	0	0	*	*	*	*
20	0	2^{25}	-2^5	0	1	3	1	X_1
21	0	0	2^{25}	-2^5	1	5	1/9	X_5
22	-2^{14}	0	0	2^{25}	1	9	1/3	X_9
23	2^6	-2^{14}	0	0	1	13	1/3	X_{13}
24	0	2^6	-2^{14}	0	1	3	1/9	X_2
25	0	0	2^6	-2^{14}	1	5	1/9	X_6
26	-2^{23}	0	0	2^6	1	9	1/3	X_{10}
27	2^{19}	-2^{23}	0	0	1	13	1/3	X_{14}
28	0	2^{19}	-2^{23}	0	1	3	1/9	X_3
29	0	0	2^{19}	-2^{23}	1	5	1/9	X_7
30	-1	0	0	2^{19}	1	9	1/3	X_{11}
31	1	-1	0	0	1	13	1/3	X_{15}
32	0	1	-1	0	2	3	1/3	X_0
33	0	0	1	-1	2	9	1/3	X_8
34	0	0	0	1	2	11	1/3	X_4
35	0	0	0	0	2	15	1	$X_{12}, X_{12}+1$

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Computing p

- Consider Δ_{35}
- Suppose j = 34 holds: Then Δ_{34}= (0,0,0,1) and

$$
\begin{aligned}
Q_{35} &= (Q_{31} + H(Q_{34}, Q_{33}, Q_{32}) + X_{12} + K_2) \lll 15 \\
&= ((Q'_{31} + 1) + H(Q'_{34}, Q'_{33}, Q'_{32}) + X_{12} + K_2) \lll 15 \\
&= (Q'_{31} + H(Q'_{34}, Q'_{33}, Q'_{32}) + (X_{12} + 1) + K_2) \lll 15 \\
&= Q'_{35}
\end{aligned}
$$

- Implies Δ_{35}= (0,0,0,0) with probability 1
  - As summarized in j = 35 row of table

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Steps 12 to 19

- Analyze steps 12 to 19, find conditions that ensure $\Delta_{19} = (2^{25}, -2^5, 0, 0)$
  - $G(Q_{19},Q_{18},Q_{17}) = G(Q'_{19},Q'_{18},Q'_{17})$, as required in differential phase
- Step 12 to 19—equation solving phase
- This is most complex part of attack
  - Last phase, steps 0 to 11, is easy

j	i	s_j	M Input	M' Input
12	0	3	X_{12}	$X_{12}+1$
13	0	7	X_{13}	X_{13}
14	0	11	X_{14}	X_{14}
15	0	19	X_{15}	X_{15}
16	1	3	X_0	X_0
17	1	5	X_4	X_4
18	1	9	X_8	X_8
19	1	13	X_{12}	$X_{12}+1$

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Steps 12 to 19

- To apply differential phase, must have $\Delta_{19} = (2^{25}, -2^{5}, 0, 0)$

  $Q_{19} = Q'_{19} + 2^{25}$

  $Q_{18} + 2^{5} = Q'_{18}$

  $Q_{17} = Q'_{17}$

  $Q_{16} = Q'_{16}$

- At step 12 we have

  $Q_{12} = (Q_8 + F(Q_{11}, Q_{10}, Q_9) + X_{12}) \lll 3$

  $Q'_{12} = (Q'_8 + F(Q'_{11}, Q'_{10}, Q'_9) + X'_{12}) \lll 3$

- Since $X'_{12} = X_{12} + 1$ and $(Q_8, Q_9, Q_{10}, Q_{11}) = (Q'_8, Q'_9, Q'_{10}, Q'_{11})$, $(Q'_{12} \lll 29) - (Q_{12} \lll 29) = 1$

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Equations for 12 to 19

- Similar analysis for remaining steps yields system of equations:

$$
\begin{aligned}
1 &= (Q'_{12} \lll 29) - (Q_{12} \lll 29) \\
F(Q'_{12}, Q_{11}, Q_{10}) - F(Q_{12}, Q_{11}, Q_{10}) &= (Q'_{13} \lll 25) - (Q_{13} \lll 25) \\
F(Q'_{13}, Q'_{12}, Q_{11}) - F(Q_{13}, Q_{12}, Q_{11}) &= (Q'_{14} \lll 21) - (Q_{14} \lll 21) \\
F(Q'_{14}, Q'_{13}, Q'_{12}) - F(Q_{14}, Q_{13}, Q_{12}) &= (Q'_{15} \lll 13) - (Q_{15} \lll 13) \\
G(Q'_{15}, Q'_{14}, Q'_{13}) - G(Q_{15}, Q_{14}, Q_{13}) &= Q_{12} - Q'_{12} \\
G(Q_{16}, Q'_{15}, Q'_{14}) - G(Q_{16}, Q_{15}, Q_{14}) &= Q_{13} - Q'_{13} \\
G(Q_{17}, Q_{16}, Q'_{15}) - G(Q_{17}, Q_{16}, Q_{15}) &= Q_{14} - Q'_{14} + (Q'_{18} \lll 23) \\
&\quad - (Q_{18} \lll 23) \\
G(Q'_{18}, Q_{17}, Q_{16}) - G(Q_{18}, Q_{17}, Q_{16}) &= Q_{15} - Q'_{15} + (Q'_{19} \lll 19) \\
&\quad - (Q_{19} \lll 19) - 1
\end{aligned}
$$

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Solving the equations

- To solve this system must find
  $(Q_{10}, Q_{11}, Q_{12}, Q_{13}, Q_{14}, Q_{15}, Q_{16}, Q_{17}, Q_{18}, Q_{19}, Q'_{12}, Q'_{13}, Q'_{14}, Q'_{15})$
  so that all equations hold.
- Since there are 14 variables and 8 equations, we have wiggle room
- Given such a solution, we determine X_j for j = 13, 14, 15, 0, 4, 8, 12
  so that we begin at step 12 and arrive at step 19 with Δ_{19} condition satisfied
- This phase reduces to solving (nonlinear) system of equations
- Can manipulate the equations so that
  - Choose $(Q_{14},Q_{15},Q_{16},Q_{17},Q_{18},Q_{19})$ arbitrary
  - Which determines $(Q_{10},Q_{13},Q'_{13},Q'_{14},Q'_{15})$

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Conditions for solution

- Three conditions must be satisfied:

$$G(Q_{15}, Q_{14}, Q_{13}) - G(Q'_{15}, Q'_{14}, Q'_{13}) = 1$$
$$F(Q'_{14}, Q'_{13}, 0) - F(Q_{14}, Q_{13}, -1) - (Q'_{15} \lll 13) + (Q_{15} \lll 13) = 0.$$
$$G(Q_{19}, Q_{18}, Q_{17}) = G(Q'_{19}, Q'_{18}, Q_{17})$$

- First 2 are “check” equations
  - Third is “admissible” condition
- Naïve algorithm: choose six Q_j, yields five Q_j, Q'_j until 3 equations satisfied
- How much work is this?

Message conditions for equations

- Using this we can solve for seven message words:
  - X_{13}= anything
  - $X_{14}= (Q_{14}{<}{<}{<}21)-Q_{10}-F(Q_{13}, Q_{12}, Q_{11})$
  - $X_{15}= (Q_{15}{<}{<}{<}21)-Q_{11}-F(Q_{14}, Q_{13}, Q_{12})$
  - $X_0= (Q_{16}{<}{<}{<}21)-Q_{12}-G(Q_{15}, Q_{14}, Q_{13}) - K_1$
  - $X_4= (Q_{17}{<}{<}{<}21)-Q_{13}-G(Q_{16}, Q_{15}, Q_{14}) - K_1$
  - $X_8= (Q_{18}{<}{<}{<}21)-Q_{14}-G(Q_{17}, Q_{16}, Q_{15}) - K_1$
  - $X_{12}= (Q_{19}{<}{<}{<}21)-Q_{15}-G(Q_{18}, Q_{17}, Q_{16}) - K_1$

Solution

- Choose Q_{12}= -1, Q_{12}'=0, Q_{11}=0. Then
  - $Q_{15}' = Q_{15} - G(Q_{18}', Q_{17}, Q_{16}) + G(Q_{18}, Q_{17}, Q_{16}) + (Q_{19}' \lll 19) - (Q_{19} \lll 19) - 1$
  - $Q_{14}' = Q_{14} - G(Q_{18}', Q_{17}, Q_{16}) + G(Q_{18}, Q_{17}, Q_{16}) + (Q_{18}' \lll 23) - (Q_{19} \lll 23)$
  - $Q_{13} = (Q_{14} \lll 21) - (Q_{14}' \lll 21)$
  - $Q_{13}' = Q_{13} - G(Q_{16}, Q_{15}', Q_{14}') + G(Q_{16}, Q_{15}, Q_{14})$
  - $Q_{10} = (Q_{13}' \lll 25) - (Q_{13} \lll 25)$
  - $F(Q_{18}', Q_{17}, Q_{16}) - F(Q_{18}, Q_{17}, Q_{16}) = (Q_{15}' \lll 13) - (Q_{15} \lll 13)$
  - $G(Q_{18}', Q_{17}, Q_{16}) - G(Q_{18}, Q_{17}, Q_{16}) = Q_{12} - Q_{11}'$
- Choose Q_{14}, …, Q_{19} arbitrarily and solve for Q_{10}, Q_{13}, Q_{13}', Q_{14}', Q_{15}'
  - $G(Q_{15}, Q_{14}, Q_{13}) - G(Q_{15}', Q_{14}', Q_{13}') = 1$
  - $F(Q_{14}', Q_{13}', 0) - F(Q_{14}, Q_{13,}, -1) = 0$
  - $G(Q_{19}', Q_{18}', Q_{17}) = G(Q_{19}, Q_{18,}, Q_{17})$

Continuous Approximation

- Each equation holds with probability $1/2^{32}$
- Appears that 2^{96} iterations required
  - Since three 32-bit check equations
  - Birthday attack on MD4 is only 2^{64} work!
- Solution
  - A “continuous approximation”
  - Small changes, converge to a solution

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Approximation technique

- Generate random Q_i values until first check equation is satisfied
  - Random one-bit modifications to Q_i
  - Save if 1st check equation still holds **and** 2nd check equation is "closer" to holding
  - Else try different random modifications
- Modifications converge to solution
  - Then 2 check equations satisfied
  - Repeat until admissible condition holds

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Steps 0 to 11

- At this point, we have (Q_8,Q_9,Q_{10},Q_{11}) and $MD4_{12\ldots47}(Q_8,Q_9,Q_{10},Q_{11},X) = MD4_{12\ldots47}(Q_8,Q_9,Q_{10},Q_{11},X')$
- To finish, we must have $MD4_{0\ldots11}(IV,X) = MD4_{0\ldots11}(IV,X') = (Q_8,Q_9,Q_{10},Q_{11})$
- Recall, X_{12} is only difference between M, M′
- Also, X_{12} first appears in step 12
- Have already found X_j for j= 0,4,8,12,13,14,15
- Free to choose X_j for j= 1,2,3,5,6,7,9,10,11 so that $MD4_{0\ldots11}$ equation holds easily!

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Recap

- Attack proceeds as follows…
  1. Steps 12 to 19: Find (Q_8,Q_9,Q_{10},Q_{11}) and X_j for j= 0,4,8,12,13,14,15
  2. Steps 0 to 11: Find X_j for remaining j
  3. Steps 19 to 35: Check $\Delta_{35}= (0,0,0,0)$
     - If so, have found a collision!
     - If not, go to 2.

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Meaningful Collision

- Different contracts, same hash value

****

CONTRACT

At the price of $176,495 Alf Blowfish sells his house to Ann Bonidea …

****

CONTRACT

At the price of $276,495 Alf Blowfish sells his house to Ann Bonidea …

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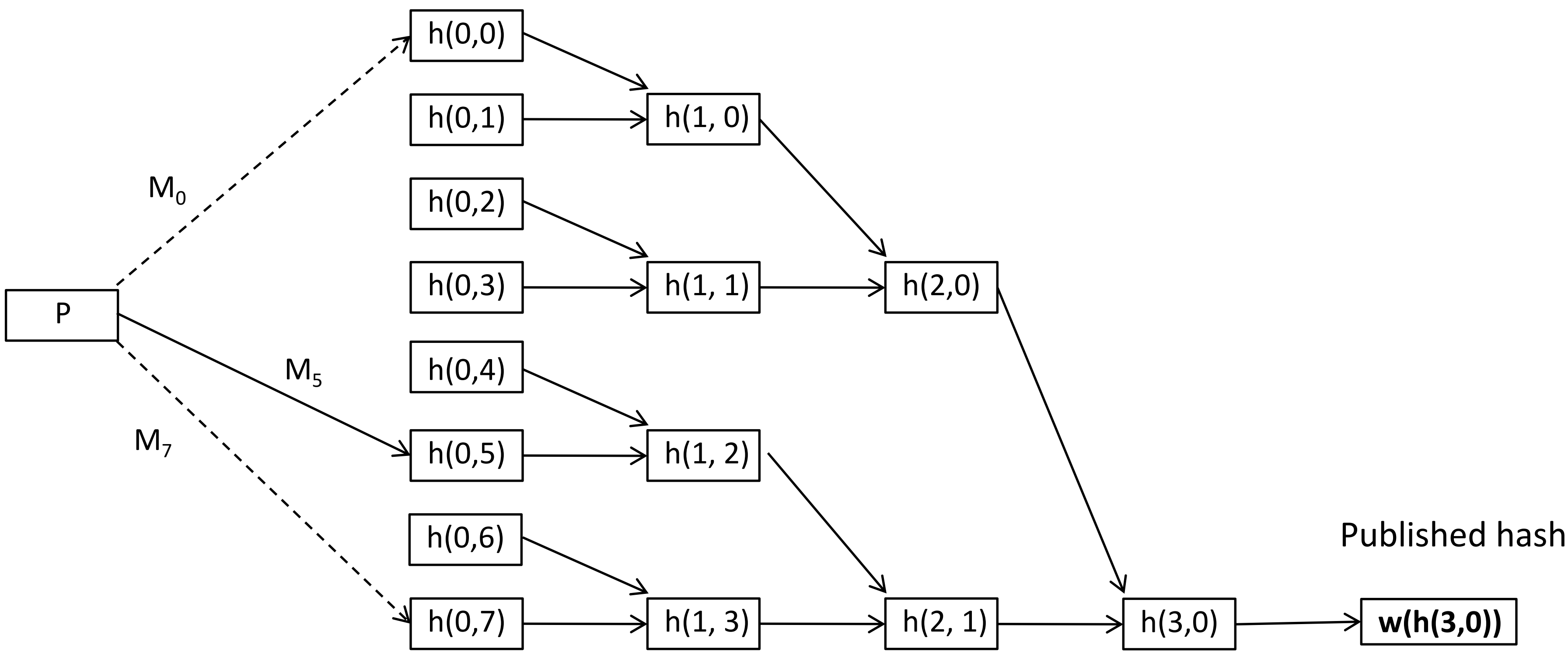
Nostradamus (“herding") attack

- Let h be a Merkle-Damgard hash with compression function f and initial value IV . Goal is to hash a prefix value (P) quickly by appending random suffixes (S).
- Procedure
  - Phase 1: Pick k, generate K= 2^k random d_{0i} from each pair of the values $f(IV||d_{i,i+1})$ and two messages $M_{0,j}$; $M_{1,j}$ which collide under f. Call this value $d_{1,j}$ this takes effort $2^{n/2}$ for each pair. Do this (colliding $d_{i,j}$; $d_{i+1,j}$ under $M_{i,j}$;$M_{i+1,j}$ to produce $d_{i,j+1}$ until you reach $d_{K,0}$). This is the diamond.
  - Publish y = $w(d_{K,0})$ where w is the final transformation in the hash as the hash [i.e. - claim y = h(P||S)].

Diamond structure



Nostradamus (“herding") attack

- The cost of phase 1 is $(2^k - 1)2^{n/2}$.
- In phase 2, guess S’ and compute T= f(IV||P||S’).
- Keep guessing until T is one of the d_{ij}. Once you get a collision, follow a path through the M_{ij} to $d_{K,0}$. Append these M_{ij} to P||S’ and apply w to get right hash.
- Total cost: $W= 2^{n-k-1}+2^{n/2+k/2}+k2^{n/2+1}$. $k=(n-5)/3$ is a good choice. For 160 bit hash, k=52.

Cryptographic Hashes and Performance

Hash Name	Block Size	Relative Speed
MD4	128	1
MD5	128	.68
RIPEMD-128	128	.39
SHA-1	160	.28
RIPEMD-160	160	.24

What to take home

- Symmetric ciphers and hashes provide key ingredients for “distributed security”
  - Fast data transformation to provide confidentiality
  - Integrity
  - Public key crypto provides critical third component (trust negotiation, key distribution)
- It’s important to know properties of cryptographic primitives and how likely possible attacks are, etc.
  - Most modern ciphers are designed so that knowing output of n-1 messages provides no useful information about n^{th} message.
  - This has an effect on some modes of operation.

Jesse Walker, Ph.D.

The Early Years

- Historical Context
- Rabin’s Hash Function
- Davies-Meyer
- MDC2

Historical Context

- Computer scientists introduced **hash functions** to create a compact table index optimizing search

- Requirement: a hash function $H : \mathit{Objects} \rightarrow \mathit{Indices}$ acts like a **random mapping**
  - Minimize probability that $H(m) = H(m')$ when $m \neq m'$

    $m_1 \ldots m_k \leftarrow m;\ h_0 \leftarrow 0$

    do $j = 1$ **to** $k \Rightarrow h_j \leftarrow f(h_{j-1}, m_j)$ **od**

    return h_k
  - f usually chosen to be a number theoretic mixer, e.g., $f(h,m) = (h + am + b)$ **mod** c for primes a, b, c

Digital Signatures

- In 1978 M. Rabin wanted to create a digital signature scheme
- Rabin needed something like a hash function to “compress” the message into a fixed sized “index”
- Requirements:
  - Act like a random mapping
  - **Collision resistance**: it is hard find two documents with same hash or **digest**
  - **2nd pre-image resistance**: given a hash of a document, it is hard to find a second document with same hash
  - **Pre-image resistance**: given a hash value, it is to find a document that produces that hash

Rabin's Hash Function

- Rabin realized *DES*, being a strong pseudo-random mixer, can replace the non-cryptographic f in conventional hash function designs

  $RabinHash\ (h_0, m)$

  $\quad m_1 \ldots m_k \leftarrow m$

  $\quad \mathbf{do}\ j = 1\ \mathbf{to}\ k \Rightarrow h_j \leftarrow DES(m_j, h_{j-1})\ \mathbf{od}$

  $\quad \mathbf{return}\ (h_0, h_m)$

- Must return (h_0, h_m) instead of h_m to obtain collision resistance

  $RabinHash\ (h_0, m_1\ m_2) = DES\ (m_2, DES\ (m_1, h_0)) = DES\ (m_2, h_1) = RabinHash\ (h_1, m_2)$

- Lesson 1: The initial value h_0 must be fixed to obtain collision resistance

Birthday Problems

- The standard **Birthday Problem**:
  - Given q people who live on a planet with an n day year, what is the probability two share a birthday?
  - Answer: Assuming birthdays are uniformly distributed, approximately $q^2/2n$ if $q \leq n^{1/2}$

- The Birthday Problem for two sets:
  - Given a population of q_1 boys and q_2 girls who live on a planet with an n day year, what is the probability a boy and girl share a birthday?
  - Answer: When $q = q_1 = q_2$, assuming birthdays are uniformly distributed, approximately q^2/n if $q \leq n^{1/2}$

Attacking Rabin Hash

Coppersmith: To find a 2^{nd} pre-image for $RabinHash\ (h_0, m_1\, m_2)$:

- Let $h_2 = RabinHash\ (h_0, m_1\, m_2)$ Then compute

  do $j = 1$ **to** $2^{32} \Rightarrow s_j \leftarrow_{\$} \{0,1\}^{56};\ u_j \leftarrow DES\ (s_j, h_0)$ **od**

  do $j = 1$ **to** $2^{32} \Rightarrow t_j \leftarrow_{\$} \{0,1\}^{56};\ v_j \leftarrow DES^{-1}\ (t_j, h_2)$ **od**

- By the Birthday problem for two lists the probability that j, j' exists with $u_j = v_{j'}$ is approximately $(2^{32})(2^{32})/2^{64} = 1$

- Then $RabinHash\ (h_0, s_j\, t_{j'}) = DES\ (t_{j'}, DES\ (s_j, h_0)) = DES\ (t_{j'}, u_j) = DES\ (t_{j'}, v_{j'}) = h_2 = RabinHash\ (h_0, m_1\, m_2)$

Discussion

- Collision resistance implies 2^{nd} pre-image resistance, because if we produce a 2^{nd} pre-image then we also produce a collision
- Exercise: modify the attack to produce pre-images

- Lesson 2: We must somehow neutralize the decryption function to build successful hash functions from block ciphers

- Lesson 3: Hash functions are attacked by multi-block messages, which enables various forms of the Birthday problem to govern their security

Neutralizing Decryption

- In the early 1980s Davies and Meyer observed that $(h, m) \rightarrow DES\ (m, h) \oplus h$ is one-way
  - Given h' it is hard to find m and h such that

$$h' = DES\ (m, h) \oplus h$$

- The **Davies-Meyer construction** replaces DES in the Rabin hash function:

$DaviesMeyerHash\ (m)$

$m_1 \ldots m_k \leftarrow m;\ h_0 \leftarrow iv$

do $j = 1$ **to** $k \Rightarrow h_j \leftarrow DES(m_j, h_{j-1}) \oplus h_{j-1}$ **od**

return h_m

- Does this work?

The Ideal Cipher Model

- Davies and Meyer reasoned as if DES were an **ideal cipher** *E*.
  - For each "key" m, $DES(m, \cdot)$ acts like a random permutation of 64 bits strings $\{0,1\}^{64}$

- It is easy to reason about an ideal cipher *E*:
  - $\Pr[E(m, h) \oplus h = h'] = \Pr[E(m, h) = h' \oplus h] = \Pr[E(m, h) = h''] = 1/2^n$ (pre-image resistance)
  - Also easy to show $\Pr[E(m, h) \oplus h = E(m', h') \oplus h'] = 2^{-n/2}$ (collision resistance) in the ideal cipher model

- Lesson 4. Nearly all hash function rationales or "security proofs" rely on the ideal cipher model
- Lesson 5. The digest size must be at least twice the block size of the underlying block cipher

2^{nd}-Preimages with Davies-Meyer Compression Functions

- It is easy to find **fixed points** for the Davies-Meyer construction

$$E\,(m, h) \oplus h = h \Leftrightarrow E\,(m, h) = 0 \Leftrightarrow h = E^{-1}\,(m, 0)$$

- The Attack: Given a message $m = m_1 \ldots m_k$ compute $h = \mathit{DaviesMeyerHash}\,(m)$ (with E replacing DES) and

  do $j = 1$ **to** $2^{n/2} \Rightarrow s_j \leftarrow_\$ \{0,1\}^n;\ u_j = E\,(s_j, iv) \oplus iv$ **od**

  do $j = 1$ **to** $2^{n/2} \Rightarrow t_{j'} \leftarrow_\$ \{0,1\}^n;\ v_{j'} = E^{-1}\,(t_{j'}, 0) \oplus iv$ **od**

- By the Birthday problem for two lists with high probability there are j, j' with $u_j = v_{j'}$
- Then $\mathit{DaviesMeyerHash}\,(iv, m) = \mathit{DaviesMeyerHash}\,(iv, s_j\, t_{j'}) = \mathit{DaviesMeyerHash}\,(iv, s_j\, t_{j'}\, t_{j'}) = \mathit{DaviesMeyerHash}\,(iv, s_j\, t_{j'}\, t_{j'}\, t_{j'}) = \ldots$

- Conclusion: With Davies-Meyer 2^{nd} pre-image resistance is no more expensive than collision resistance

MDC2: Widening the Block Size

- The Davies-Meyer enhancement can only provide collision resistance to $O(2^{64/2}) = O(2^{32})$ DES operations
- In 1987 IBM proposed MDC2 to obtain $O(2^{64})$ collision resistance

MDC2 (*m*)

$m_1 \ldots m_k \leftarrow m;\ h_0 \leftarrow iv$

do $j = 1$ **to** $k \Rightarrow$

$h_{left}\ h_{right} \leftarrow h_{j-1}$

$d \leftarrow DES(h_{left}, m_j) \oplus m_j$

$e \leftarrow DES(h_{right}, m_j) \oplus m_j$

$h_j \leftarrow d_{left}\ e_{right}\ e_{left}\ d_{right}$

od

return h_m

Discussion

- The construction $(h,m) \to DES\,(h, m) \oplus m$ offers the same collision and pre-image bounds as Davies-Meyer
  - Nearly an identical argument in the ideal cipher model
  - This is the **Matyas-Meyer-Oseas construction**
- Swapping the left and right digest halves is essential for security
  - Collisions could be found in $2^{32} + 2^{32} = 2^{33}$ instead of 2^{64} DES operations, because without the swap the digest is just the concatenation of digests from two independent hashes
- Steinberger proved MDC2 is collision resistant in 2007

Length Problems 1

- Let $m \in (\{0,1\}^{56})^+$, i.e., m is a string whose bit length is a multiple of 56
- For any string n it is easy to verify *hash* (m n) = *hash* (*hash* (m) n) for each of the hash constructions we have considered
  - This is called a **length extension attack**
  - Length extension attacks succeed even if the attacker never sees m
- Length extension attacks indicate something is still missing from our construction

Length Problems 2

- Suppose the message digest of a hash function is n bits wide
- Consider the message $m = m_1 \ldots m_k$ for $k \geq 2^{n/2}$
- By the standard birthday problem there is at probability of at least 0.5 that at least two messages in $\{m_1, m_1 m_2, m_1 m_2 m_3, \ldots, m_1 \ldots m_k\}$ collide.
- Lesson 6. To achieve collision resistance the length of all the combined inputs to a hash function must be less than $2^{n/2}$ bits

Early Years Summary

- The Davies-Meyer hash is too weak for practical applications
  - Collisions found in 2^{32} DES operations
- The MDC2 hash is too expensive for practical use
  - 1 DES operation $\approx$ 500 cycles; 1 MDC2 operation $\approx$ 1000 cycles = 125 cycles ***per byte***
- There is something wrong in the way early hash functions deal with the length of their inputs
- Question: Even though the inner loop is collision/pre-image/2nd pre-image resistant, why do we believe the hash function is?

Revolution

- At Crypto 1989 Merkle and Damgård published papers revolutionizing hash function design
- Replace the *DES* construction by a clean **compression function** abstraction $compress : \{0,1\}^s \times \{0,1\}^n \to \{0,1\}^n$ operating on s bit message blocks and an n bit **chaining variable**
- Define a padding scheme to block length extension attacks
- Because it blocks length extension attacks, the padding scheme extends compression function's collision resistance to the entire hash function

MD-Hash (m)

$m' \leftarrow pad\,(m); m_1 \ldots m_k \leftarrow m'; h_0 \leftarrow iv$

do $j = 1$ **to** $k \Rightarrow h_j \leftarrow compress(h_{j-1}, m_j)$ **od**

return h_m

Merkle-Damgård Padding

- If the compression function *compress* operates on s bit message blocks and n bit chaining variables then

  pad (m)

  $l \leftarrow |m|$ -- find m's length in bits

  $t \leftarrow s - (l \ \mathbf{mod}\ s) - n/2 - 1$ -- compute number of 0

  if $t < 0 \Rightarrow t \leftarrow s + t$ **fi** -- bits needed

  $m' \leftarrow m\ 1\ 0^t\ \langle l \rangle_{n/2}$ -- append a 1 bit, t 0 bits, and l

  return m' -- encoded as an $n/2$ bit integer

- Key property: *pad* (m) gives the number of bits l of m

- This scheme makes it unambiguous where the message m ends and where the padding ends

Collision Resistance

- Why does collision resistance of *compress* imply collision resistance of *md-hash*?
- Suppose we can easily find $m \neq m'$ with *md-hash* (m) = *md-hash* (m')
- Two cases: *md-hash* (m) = *md-hash* (m') with $|m| = |m'|$, $m = m_1 m_2 \ldots m_k$, $m' = m_1' m_2' \ldots m_i'$
- Case 1: Since $|m| = |m'|$, we know $k = i$ and the last block (of padding) is the same ($m_k = m_i$). There must be some $1 \leq j < k$ such that *compress* (h_{j-1}, m_j) = *compress* (h_{j-1}', m_j') but $m_j \neq m_j'$. This contradicts the assumption it is hard to find collisions for *compress*
- Case 2: Since $|m| \neq |m'|$ we know that the final (padding) blocks $m_k \neq m_i'$ and *compress* (h_{k-1}, m_k) = *compress* (h_{i-1}', m_i'), a contradiction since it is hard to find collisions for *compress*

Example: SHA-1

algorithm SHA-1 (M)
  return sha-md (5a827999 || 6ed9eba1 || 8f1bbcdc || ca62c1dc, M)

algorithm sha-md (K, M)
  M := pad (M)
  parse M **into 512-bit blocks** $M_1 \ldots M_k$
  IV := 67452301 || efcdab89 || 98badcfe || 10325476 || c3d2e1f0
  do $i = 1$ **to** $k \Rightarrow IV :=$ sha-compress (K, IV, M_i) **od**
  return IV

Merkle-Damgård construction

algorithm sha-compress (K, IV, M)
  parse K **into 32-bit blocks** $K_1 \ldots K_4$ **and** IV **into** $IV_1 \ldots IV_5$
  parse M **into 32-bit blocks** $W_1 \ldots W_{16}$
  do $i = 17$ **to** $80 \Rightarrow W_i :=$ LROT ($1, W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16}$) **od**
  $A := IV_1, B := IV_2, C := IV_3, D := IV_4, E := IV_5$
  do $i = 1$ **to** $20 \Rightarrow L_i := K_1, L_{i+20} := K_2, L_{i+40} := K_3, L_{i+60} := K_4$ **od**
  do $i = 1$ **to** $80 \Rightarrow$
    if $1 \le i \le 20 \Rightarrow f := (B \wedge C) \vee ((\neg B) \wedge D)$
    else if $41 \le i \le 80 \Rightarrow f := (B \wedge C) \vee (B \wedge D) \vee (C \wedge D)$
    else $f := B \oplus C \oplus D$ **fi**
    $t :=$ LROT ($5, A$) $+ f + E + W_i + L_i$
    $E := D, D := C, C :=$ LROT ($30, B$), $B := A, A := t$
  od
  $IV_1 := IV_1 + A, IV_2 := IV_2 + B, IV_3 := IV_3 + C, IV_4 := IV_4 + D, IV_5 := IV_5 + E$
  return $IV_1 \,||\, IV_2 \,||\, IV_3 \,||\, IV_4 \,||\, IV_5$

Block cipher key schedule

Block cipher

Davies-Meyer feed-forward

Structural Problems

- Second pre-image attacks
- Random Mapping properties
- Multi-block Differential Attacks

Joux's Multi-collision Attack

- Let *compress* : $\{0,1\}^s \times \{0,1\}^n \rightarrow \{0,1\}^n$ be a collision resistant compression function and $m_1\, m_2$ be a $2s$ bit message

- By assumption we can find $m_1' \neq m_1$ such that *compress* (iv, m_1) = *compress* (iv, m_1') in $2^{s/2}$ operations
- Similarly we can find $m_2' \neq m_2$ such that *compress* (iv, m_2) = *compress* (iv, m_2') in $2^{s/2}$ operations
- Therefore $m_1\, m_2'$, $m_1'\, m_2$, and $m_1'\, m_2'$ are **three** 2nd preimages of $m_1\, m_2$ under md-hash that we have found in $2^{s/2} + 2^{s/2} = 2^{s/2+1}$ operations instead of 2^s

- Clearly the attack can be extended to k block messages to find $2^k–1$ 2nd pre-images in time $k2^{s/2}$ instead of 2^s

- Conclusion: 2nd pre-image resistance from the Merkle-Damgård construction is no stronger than collision resistance

The Random Mapping Property

- A **random oracle** is a public random mapping
  - A random oracle returns a fixed length random string in response to any input
- It is widely assumed in practice that hash functions behave like random oracles
- Let $m = m_1\ m_2$. Then it is easy to see that

$$md\text{-}hash(m_1\ pad(m_1)\ m_2) = md\text{-}hash(md\text{-}hash(m_1)\ m_2)$$

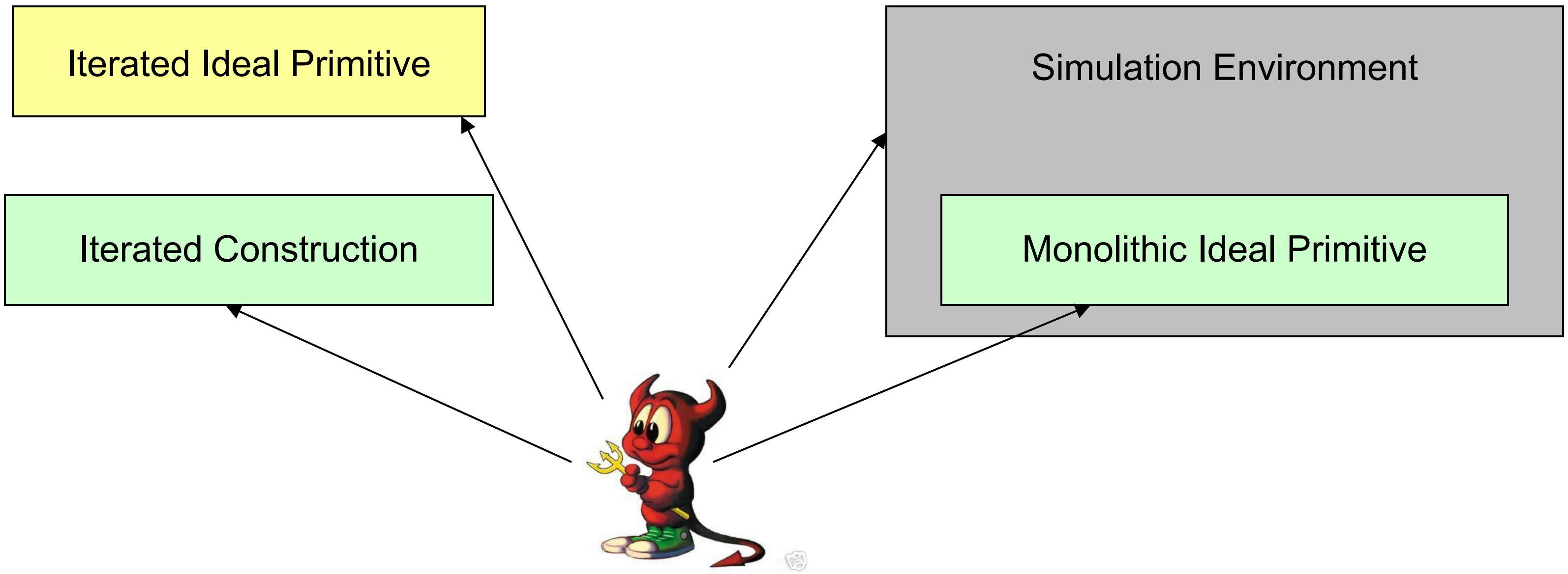
- If *hash* acted like a random oracle, then *hash* (*m pad (m) n*) and *hash* (*hash* (*m*) *n*) should assume independent values
- This makes Merkle-Damgård hash functions hard to use in practice
  - We don't know that constructions using Merkle-Damgård hash functions deliver the security claimed
- Merkle-Damgård hash functions leak that they are iterative constructions

Random Oracles

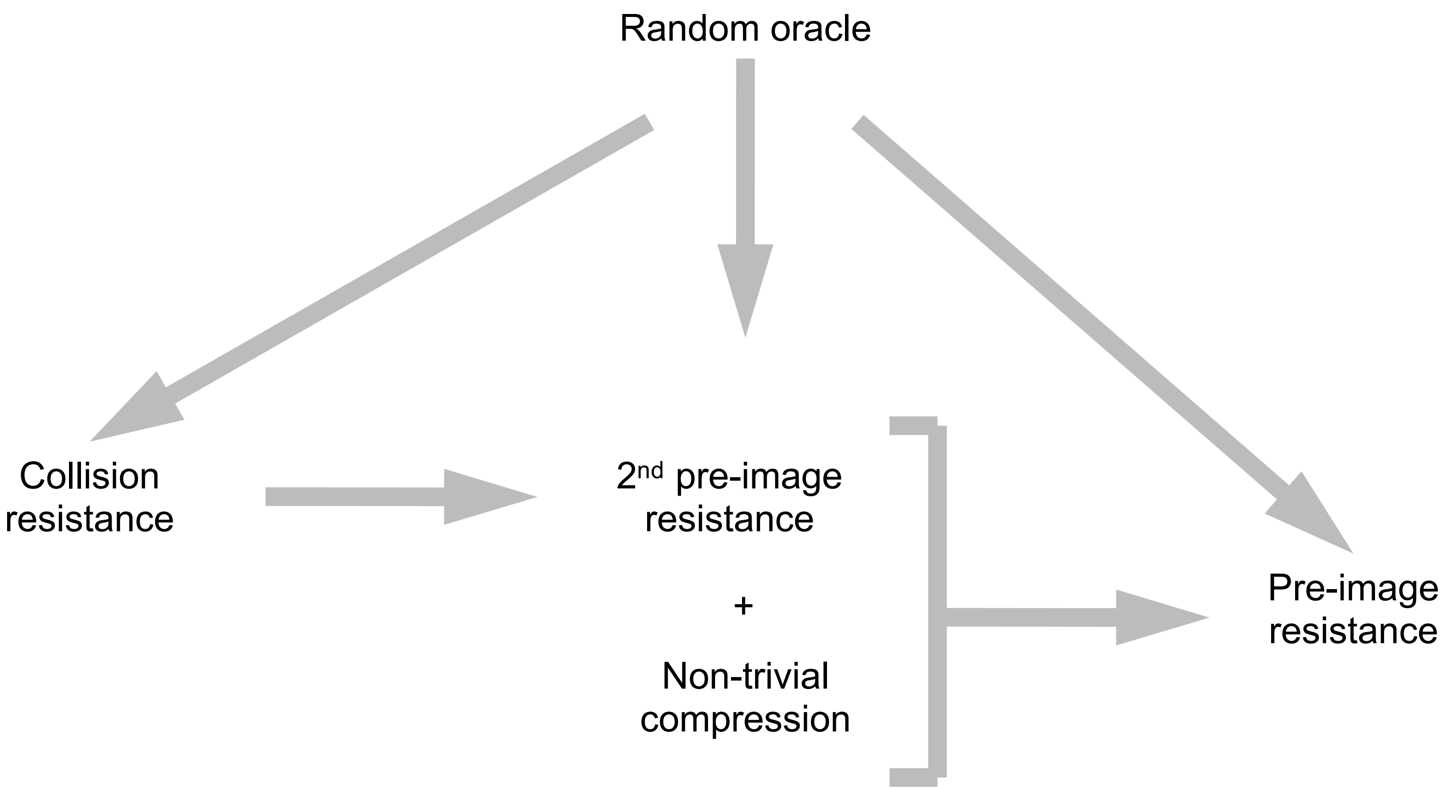
- D. Simon showed that random oracles cannot be instantiated
  - Random oracles assume an infinite world, so can always be distinguished from real-word constructions
- Maurer introduced the notion of **indifferentiability** to replace the notion of distinguishability when reasoning about hash functions
- Collision resistance is not enough; hash functions should be indifferentiable from random oracles

Indifferentiability

- Question: When can an iterated construction replace a monolithic construction?
- Answer: When for every adversary a simulation environment exists wherein the adversary cannot distinguish the real construction from the monolithic construction operating in the simulation



Relationships



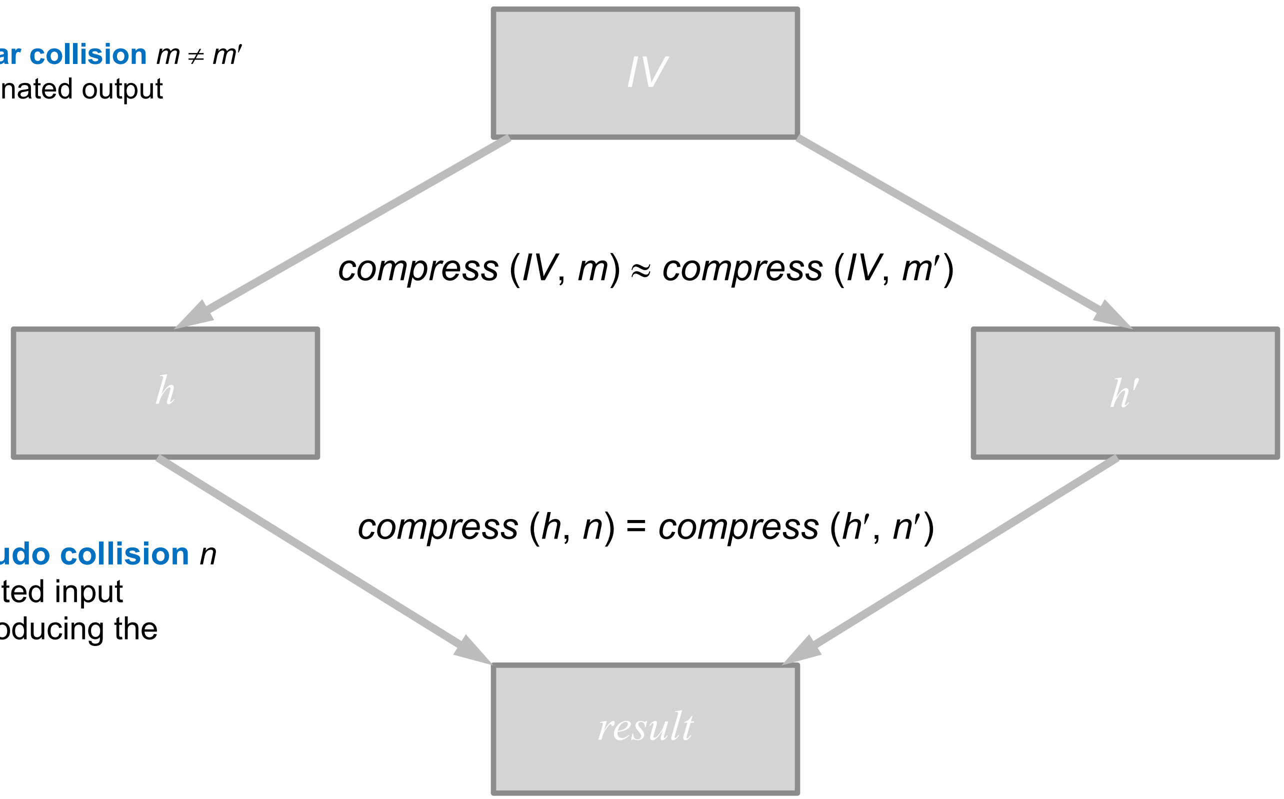
Multi-block Differential Attacks

- Differential cryptanalysis was introduced to study block ciphers
- Given a key K and X, X' with difference $X \oplus X'$, what is the difference $E(K,X) \oplus E(K,X')$?
- This often yields useful information about K and deep insight into E's structure
- Since compression functions for Merkle-Damgård hashing are based on block ciphers, there should be some way to extend differential cryptanalysis to hashing
  - Since hashing is multi-block, we need some way to extend differential cryptanalysis to multi-block attacks

The Multi-Block Technique

Step 1. Find a **near collision** $m \neq m'$ producing a designated output difference $h \approx h'$

Step 2. Find a **pseudo collision** $n \neq n'$ from a designated input difference $h \approx h'$ producing the same result



$m\ n$ and $m'\ n'$ are colliding messages when successful

Wang's Attack

- In 2004 Xiayuan Wang applied the multi-block technique to break the collision resistance of MD4, MD5, and Ripe-MD
  - In 2009 their attack was extended to forge the certificate of real CA that supported MD5
- In 2005 Wang and colleagues used the technique to defeat the collision resistance of SHA-1
  - They showed a collision could be found at cost 2^{62} instead of 2^{80} operations
- These attacks caused deep trauma and introspection in the crypto community
  - "Do we know what a hash function is?"

What Went Wrong?

algorithm SHA-1 (M)

  return sha-md (5a827999 || 6ed9eba1 || 8f1bbcdc || ca62c1dc, M)

algorithm sha-md (K, M)

  M := pad (M)

  parse M **into 512-bit blocks** $M_1 \ldots M_k$

  IV := 67452301 || efcdab89 || 98badcfe || 10325476 || c3d2e1f0

  do $i = 1$ **to** $k \Rightarrow IV :=$ sha-comp ($K. IV, M$) **od**

  return IV

algorithm sha-comp (K, IV, M)

  parse K **into 32-bit blocks** $K_1 \ldots K_4$ **and** IV **into** $IV_1 \ldots IV_5$

  parse M **into 32-bit blocks** $W_1 \ldots W_{16}$

  do $i = 17$ **to** $80 \Rightarrow W_i :=$ LROT ($1, W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16}$) **od**

  $A := IV_1, B := IV_2, C := IV_3, D := IV_4, E := IV_5$

  do $i = 1$ **to** $20 \Rightarrow L_i := K_1, L_{i+20} := K_2, L_{i+40} := K_3, L_{i+60} := K_4$ **od**

  do $i = 1$ **to** $80 \Rightarrow$

    if $1 \le i \le 20 \Rightarrow f := (B \wedge C) \vee ((\neg B) \wedge D)$

    else if $41 \le i \le 80 \Rightarrow f := (B \wedge C) \vee (B \wedge D) \vee (C \wedge D)$

    else $f := B \oplus C \oplus D$ **fi**

    $t :=$ LROT ($5, A) + f + E + W_i + L_i$

    $E := D, D := C, C :=$ LROT ($30, B$), $B := A, A := t$

  od

  $IV_1 := IV_1 + A, IV_2 := IV_2 + B, IV_3 := IV_3 + C, IV_4 := IV_4 + D, IV_5 := IV_5 + E$

  return $IV_1 \,||\, IV_2 \,||\, IV_3 \,||\, IV_4 \,||\, IV_5$

Key schedule doesn't resist related key attacks or compensate for cipher's poor diffusion

Poor diffusion

Discussion

- Davies-Meyer elevates the importance of related key attacks in block cipher designs, because the attacker has control over differences between encryption key
  - The block being hashed is the encryption key
  - The attacks exploit the fact that making small changes in one block can be canceled by a later block
- We have learned that hash functions and block ciphers are attacked in similar ways
  - No longer surprising, given how hash function have been built
- All of the state-of-the-art design techniques for design and validation of block ciphers should be applied to hash function designs
  - e.g., show that every input bit flows to every output bit after a few rounds

The Merkle-Damgård Years

- Merkle-Damgård theory finally puts collision resistance, 2nd pre-image resistance, and pre-image resistance on a firm foundation
- Merkle-Damgård 2nd pre-image is much weaker than anticipated
- Merkle-Damgård hash functions do not act like random oracles
  - So we don’t know many of our constructions are safe
- The Multi-block technique appears to threaten Merkle-Damgård designs

SHA-3 and Modern Hash Function Construction

- The SHA-3 competition
- HAIFA
- Domain Switching
- The Sponge Construction
- And the winner is . . .

The SHA-3 Competition

- NIST adopted the SHA-2 family in 2003
  - Block sizes of 224, 256, 384, and 512 bits to address Moore's Law
- Design of SHA-2 family very similar to that for SHA-1
  - Is SHA-2 vulnerable to Wang's attack? No, but this was not established until after SHA-3 competition was under way
- Due to similarity of SHA-2 family to SHA-1, consensus was we need a new hash algorithm design
- Crypto community's BKM for designing new algorithms: hold a contest
- NIST published RFP January 7, 2007 announcing competition
- Submissions due October 31, 2007, with 64 designs received

The SHA-3 Competition

- NIST accepted 51 of the 64 submissions into Round 1
- Extensive cryptanalysis of all designs by the international community
  - All designs independently analyzed by multiple parties
  - Majority of designs broken
- Extensive performance data collected at the e-BACS site
- NIST selected 14 designs for Round 2 in July 2009
- NIST selected 5 finalist algorithms in December 2010

Round 2 Candidates and Finalists

Candidate	Designer Origins	Design Type
BLAKE		**ARX, HAIFA**
Blue Midnight Wish		ARX, MD + FT
CubeHash		ARX, MD + FT
ECHO		AES, HAIFA
Fugue		AES, MD + FT
Grøstl		**AES, MD + FT**
Hamsi		s-box, MD + FT
JH		**s-box, Sponge**
Keccak		**s-box, Sponge**
Luffa		s-box, MD + FT
Shabal		Mix, MD
SHAvite-3		AES, HAIFA
SIMD		Mix, MD + FT
Skein		**ARX, MD(+ FT)**

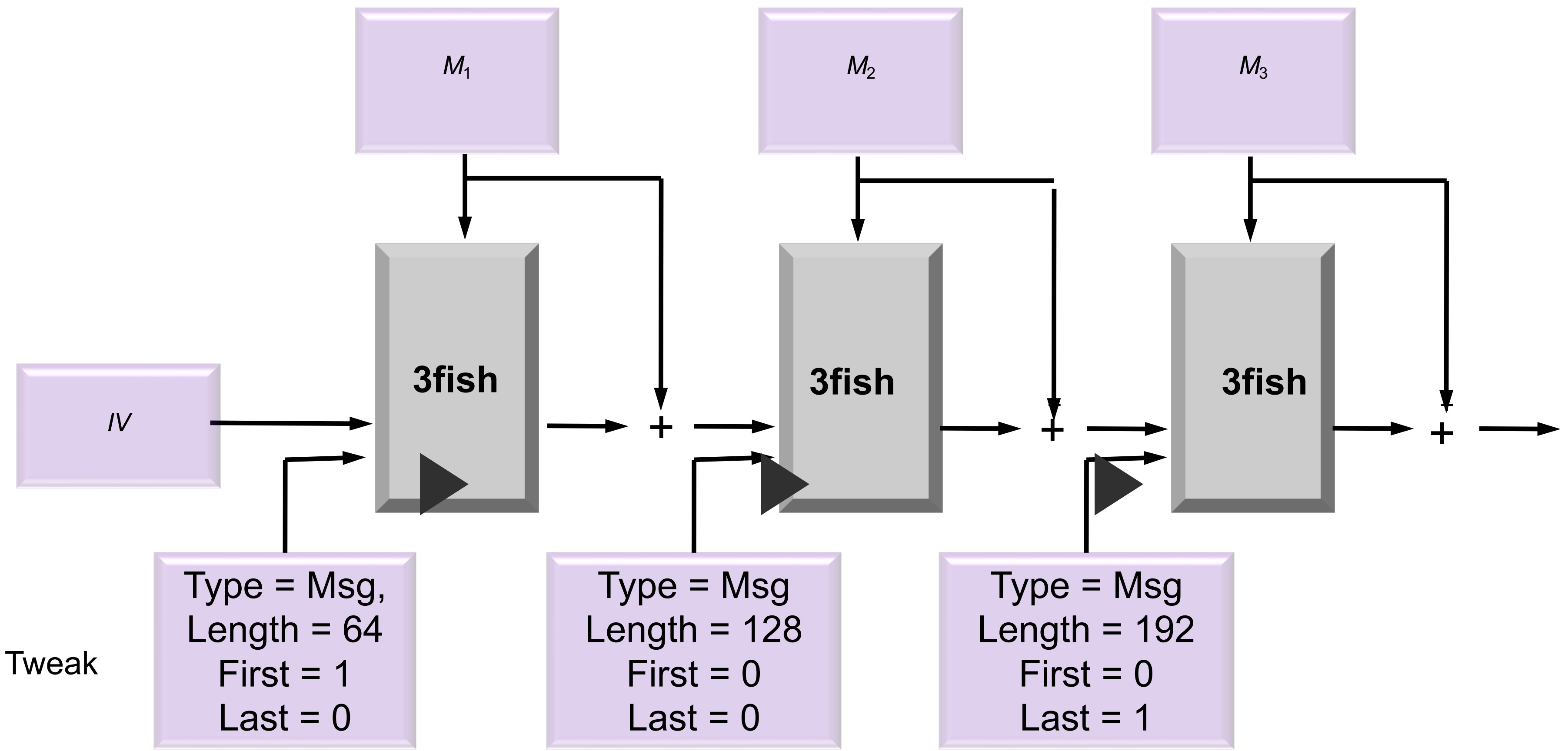
Addressing Merkle-Damgård Weaknesses

- 3 Approaches proposed
  - The HAIFA construction
  - Domain switching (aka "Final Transform")
  - The Sponge construction
- HAIFA and domain switching patch Merkle-Damgård, while a sponge is something entirely new
- All five finalists employ one or more of these approaches
- All five finalists appear to have comparable security levels
  - Significantly better safety margins than SHA-2
  - All are indifferentiable from random oracles

HAIFA Construction

- Developed by Biham and Dunkleman
- Idea: hash each message block through the compression function with the number of bits hashed so far and an optional salt
- Intuition: This makes each compression function invocation independent
- Theoretical foundation:
  - The mapping $m \rightarrow (0, m_1)\ (s, m_2)\ (2s, m_3)\ .\ .\ .\ ((k\!-\!1)s, m_k)$ is a **prefix-free encoding** of m
  - Coron et al proved that the Merkle-Damgård hash of a prefix-free encoded message is indifferentiable from a random oracle

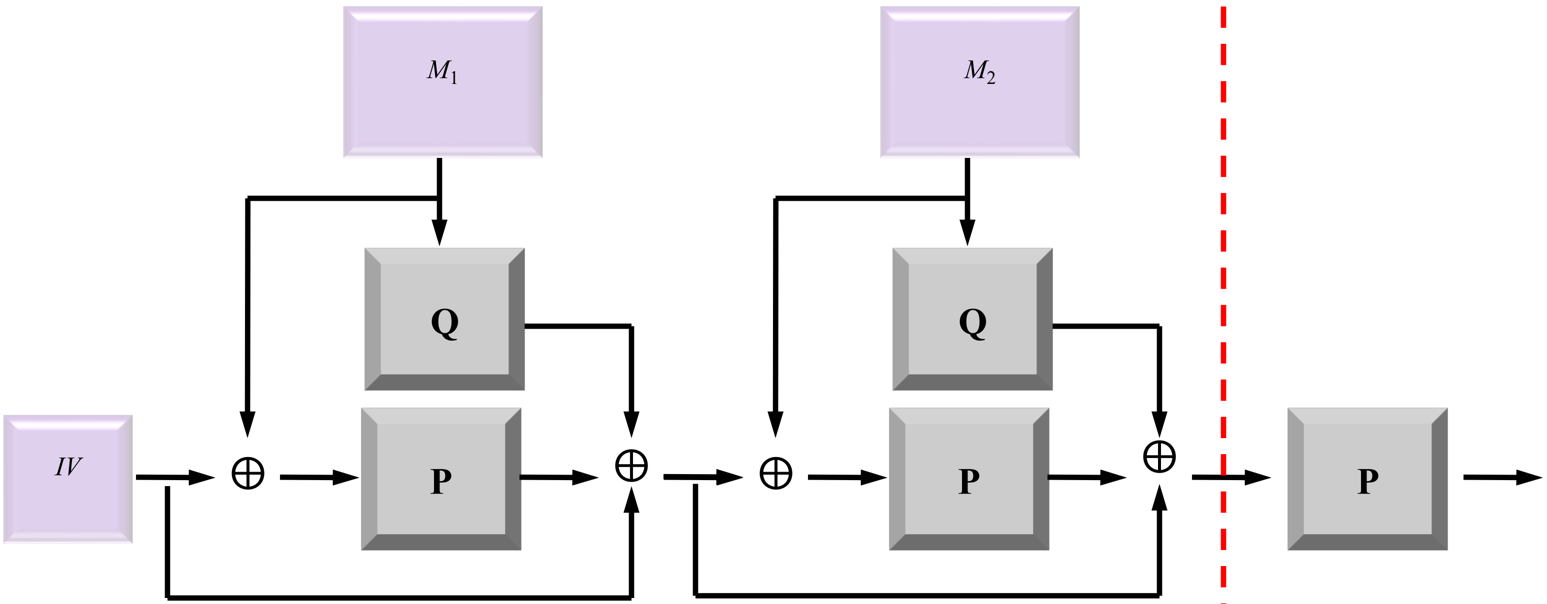
HAIFA Example: Skein's UBI Construction



Domain Switching

- Developed by Bellare and Ristenpart
- Idea: Rehash the output from Merkle-Damgård under an independent compression function
- Intuition: Hide the iterative structure with an independent hash (“domain switch”)
- Theoretical foundation:
  - If the compression function acts like a random oracle, then so is a Merkle-Damgård digest after being post-processed in this way

Domain Switching Example: Grøstl



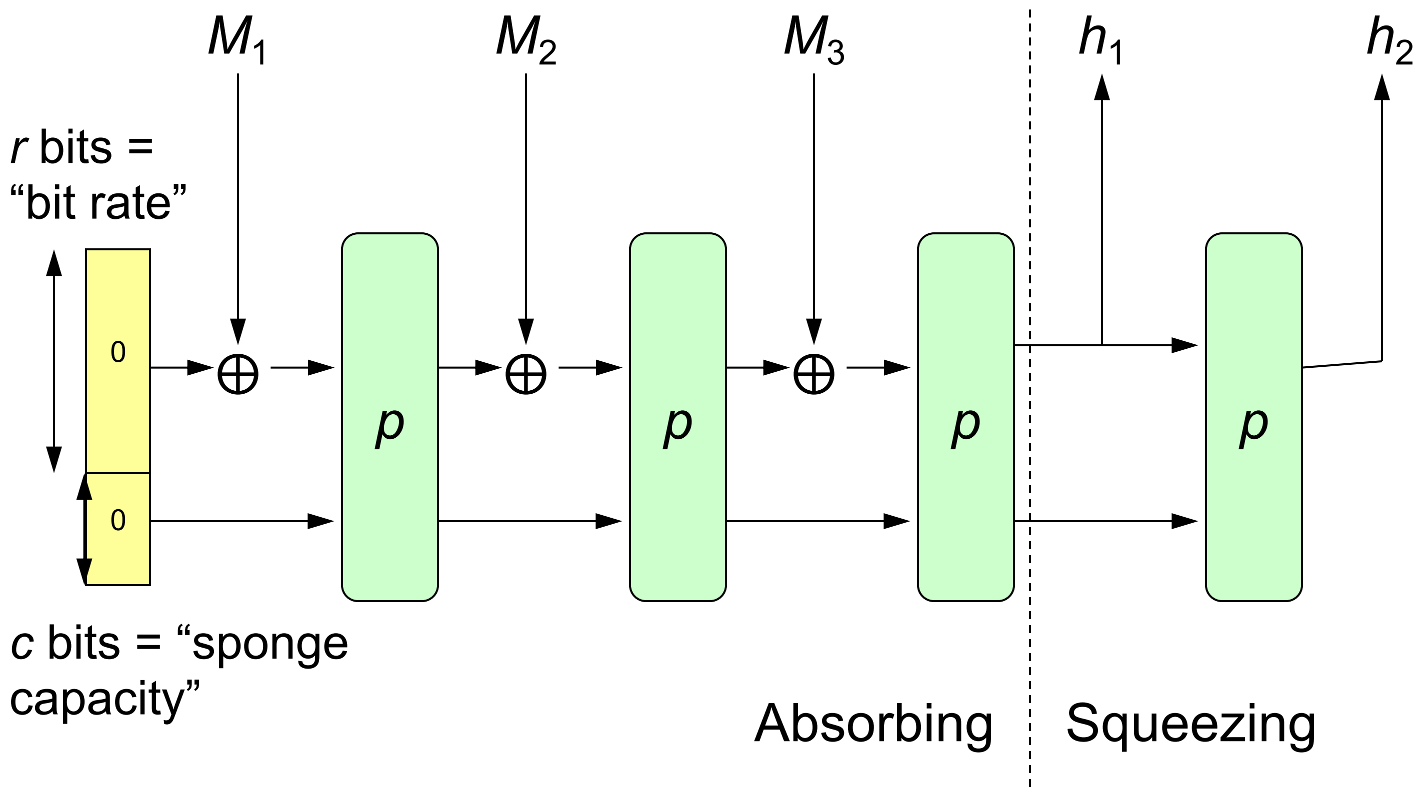
Message hashed in 1st domain

Digest rehashed in a 2nd domain

The Sponge Construction

- Developed by Bertoni, Daemen, Peeters, and Van Assche
- Idea: We don't know the right design criteria except that a hash function act like a random oracle, so make the design act as much like a random oracle as possible
- Intuition: A permutation with a large state space, only some of which can be updated by the environment, acts like a random oracle
- Theoretical foundation:
  - Can prove a sponge is indifferentiable from a random oracle

The Sponge Construction



p = permutation of $\{0,1\}^{c+r}$

And the Winner is . . .

- **Keccak**

- Keccak was designed by Guido Bertoni, Joan Daemen, Michael Peeters, Gilles Van Assche
  - Joan Daemen and Vincent Rijman designed AES

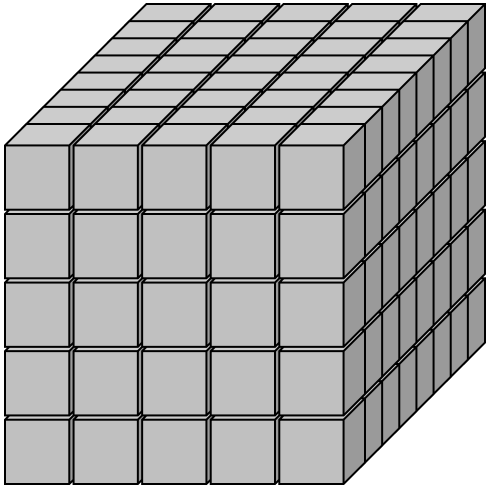
- NIST announced the SHA-3 winner on October 2, 2012
  - AES winner announced on October 2, 2000

- NIST indicated design diversity drove their choice
  - SHA-2, BLAKE, Grøstl, Skein are Merkle-Damgård based

High Level Design

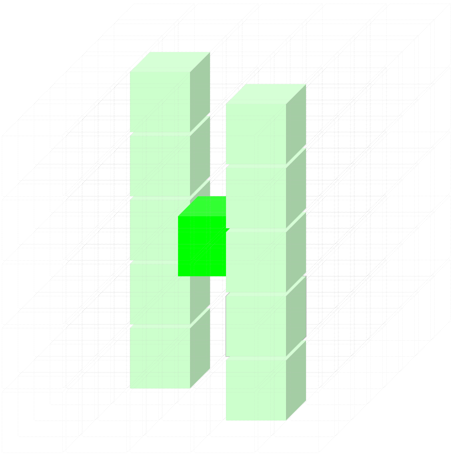
- Keccak uses a 24 round permutation in the sponge construction
- Keccak's permuation is called Keccak-*f* and parameterized by rate *r* and capacity *c*
  - $r+c = 1600 = 25 \times 64$
    - Keccak-512: $r = 512$, $c = 1088 \Rightarrow$ faster with 2^{544} security bound
    - Keccak-256: $r = 256$, $c = 1344 \Rightarrow$ slower with 2^{672} security bound
- Design goal: Keccak-*f* has no exploitable properties
- Keccak-*f*'s design based on the **wide-trail** design strategy
  - Spread a round's non-linear across across the entire round using well-chosen linear transformations to get provable resistance to linear and differential cryptanalysis
- Keccak-*f* round: $\iota \circ \chi \circ \pi \circ \rho \circ \theta(\mathit{state}) = \iota(\chi(\pi(\rho(\theta(\mathit{state})))))$

Keccak State



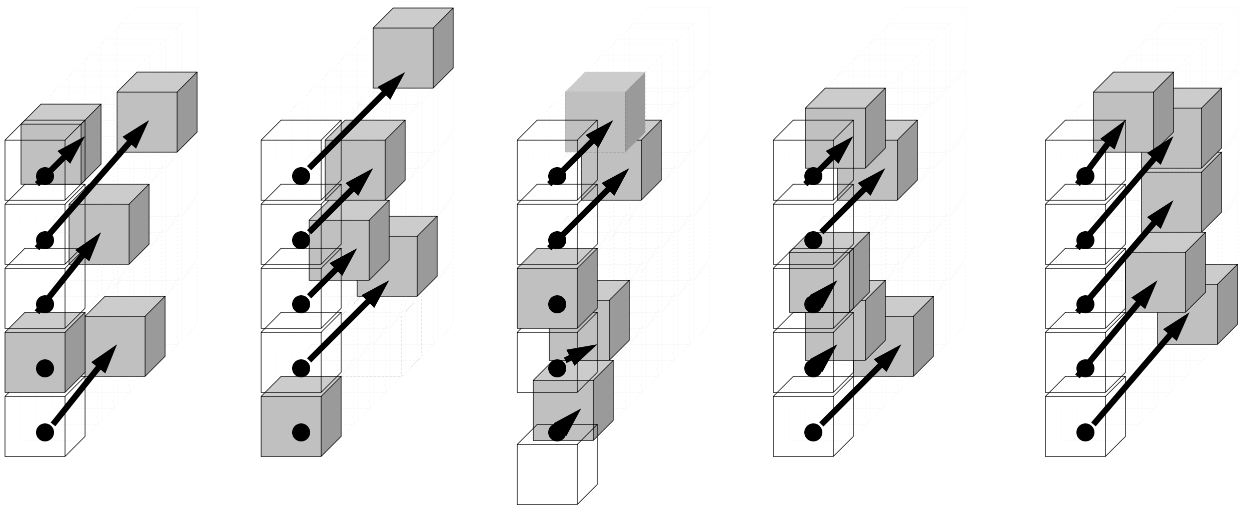
- Keccak represents its 1600 bit state as a $5 \times 5 \times 64$ bit cube

The Keecak θ Function



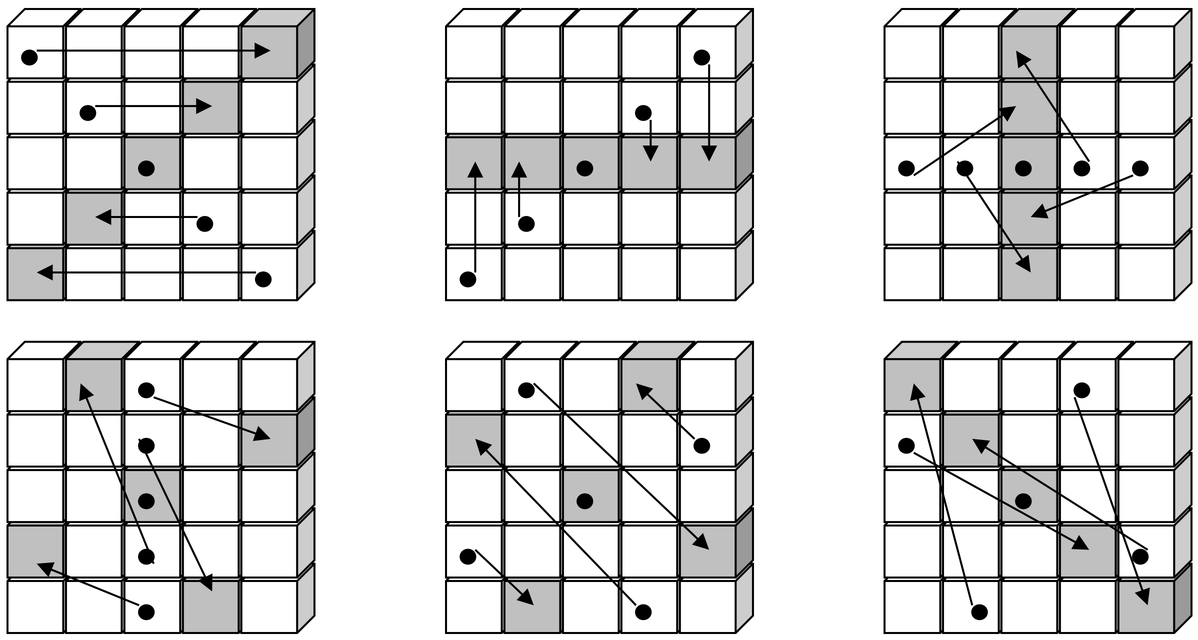
- $\iota \circ \chi \circ \pi \circ \rho \circ \boldsymbol{\theta}$(*state*) = 1 of 24 Keccak-*f* rounds
- θ provides diffusion – each bit affects 11 adjacent bits
- θ implemented by 50 XORs and 5 rotations

The Keccak ρ Function



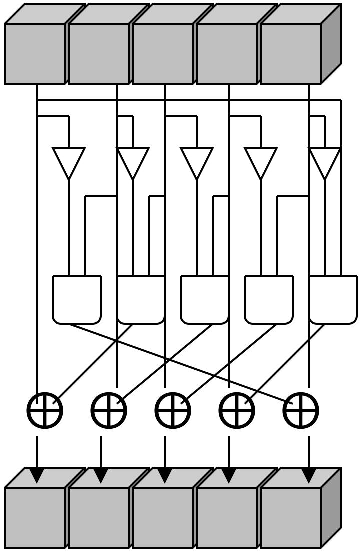
- $\iota \circ \chi \circ \pi \circ \boldsymbol{\rho} \circ \theta(\mathit{state}) =$ 1 of 24 Keccak-f rounds
- ρ provides inter-slice dispersion by moving 25 bits of a slice to 25 different slices
- Implemented by 24 rotations

The Keccak π Function



- $\iota \circ \chi \circ \boldsymbol{\pi} \circ \rho \circ \theta(state) =$ 1 of 24 Keccak-f rounds
- π distributes horizontal/vertical alignment using a period 24 cycle about a fixed origin
- Implemented as a linear mapping of GF(5) $\times$ GF(5)

The Keccak χ Function



- $\iota \circ \boldsymbol{\chi} \circ \pi \circ \rho \circ \theta(\mathit{state}) = 1$ of 24 Keccak-f rounds
- χ provides non-linearity
- Note it is a Feistel construction

The Keccak ι Function

- $\boldsymbol{\iota} \circ \chi \circ \pi \circ \rho \circ \theta$(*state*) = 1 of 24 Keccak-*f* rounds
- ι breaks symmetry, to
  - Defend against slide attacks
  - Reduce the effectiveness of cross-round attacks
- Implemented by adding a round constant to state

SHA-3 Summary

- All of the SHA-3 finalists offer excellent security
- Design diversity drove NIST's selection of Keccak as the SHA-3 winner
- Keccak is indifferentiable from a random oracle, and so meets any conceivable hash function requirement

Key Takeaways

- Cryptographic hash function design has deep roots in conventional computer science, but only received a firm foundation with Merkle-Damgård
- Identifying the right problems to solve has been a treacherous adventure
- New hash function designs should strive to construct random oracles
- Keccak is a worthy winner of the SHA-3 competition

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