Electronics of Radio

Notes on David Rutledge's book

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Basic concepts

- Potential difference (V, ϕ): $\phi = \int_a^r E \cdot ds$, energy per charge, 1V = 1 J/s
- Kirkoff 1: $\sum_{loop} V_i = 0$ (Conservation of energy)
- Kirkoff node: $\sum_{node} I_i = 0$ (Conservation of charge)
- V$(t) = V_p \cos(\omega t)\,, \omega = 2\pi f$, I$(t) = I_p \cos(\omega t)\,, \omega = 2\pi f$
- Instantaneous power: $P(t) = V(t)I(t) = V_p I_p\, cos^2(\omega t)$
- Average power: $P_a = \int_0^{1/f} V(t)I(t)dt = V(t)I(t) = \int_0^{2\pi/\omega} V_p I_p\, cos^2(\omega t)dt = \frac{V_p I_p}{2}$
- Band names:

Name	Frequency
VLF	3-30kHz
LW	20-300kHz
MW	300kHz-3MHz
HF	3MHz-30MHz
VHF	30-300MHz

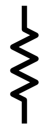
Name	Frequency
UHF	300MHz-1GHz
uW	1-30GHz
milliW	30-300GHz
submilliF	>300GHz

Signals

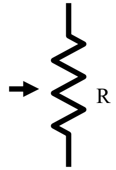
- Gain (G) expressed in decibels: $G = 10\ log_{10}(^{P_{out}}/_{P_{in}})$
- Mixer:
  - $V(t) = \cos(\omega_1 t)\cos(\omega_2 t) = \frac{1}{2}[\cos(\omega_+ t) + \cos(\omega_- t)]$, $\omega_+ = \omega_1 + \omega_2$, $\omega_- = \omega_1 - \omega_2$
- Modulation

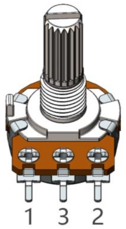
Name	Equation
AM	$V(t) = a(t)\cos(\omega_c t)$
FM	$V(t) = V_c\cos((\omega_c + a(t))t)$
FSK	$V(t) = V_c\cos(\omega_1 t)$, if 1 $V(t) = V_c\cos(\omega_0 t)$, if 0
PSK	$V(t) = +V_p\cos(\omega t)$, if 1 $V(t) = -V_p\cos(\omega t)$, if 0

Resistors, capacitors, inductors




- Resistors
  - Analytic model: $IR = V$
  - Energy dissipated: $E = \int_{t_i}^{t_f} IV\,\mathrm{dt} = \int_{t_i}^{t_f} I^2 R dt$





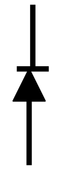
- Capacitors
  - Analytic model: $CV = q, C\frac{dV}{dt} = i$
  - Capacitor Energy stored: $E = \int_{t_i}^{t_f} CV\frac{dV}{dt}\,\mathrm{dt} = \frac{1}{2}CV^2$

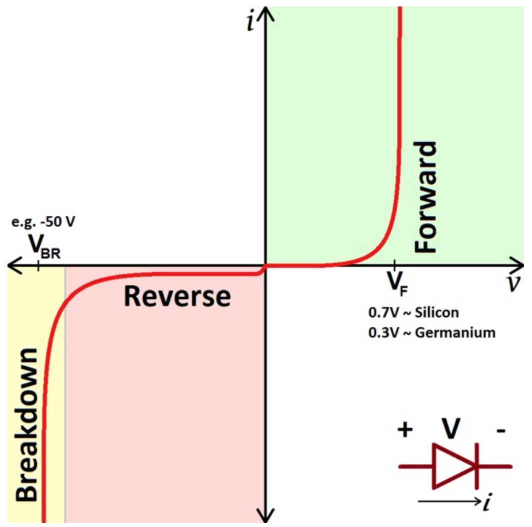
- Inductors
  - Analytic model: $V = L\frac{di}{dt}$
  - Inductor Energy stored: $E = \int_{t_i}^{t_f} IV\,\mathrm{dt} = \int_{t_i}^{t_f} LI\frac{dI}{dt}dt = \frac{1}{2}LI^2$


Credit: Make Electronics

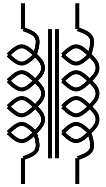
Diodes, transformers

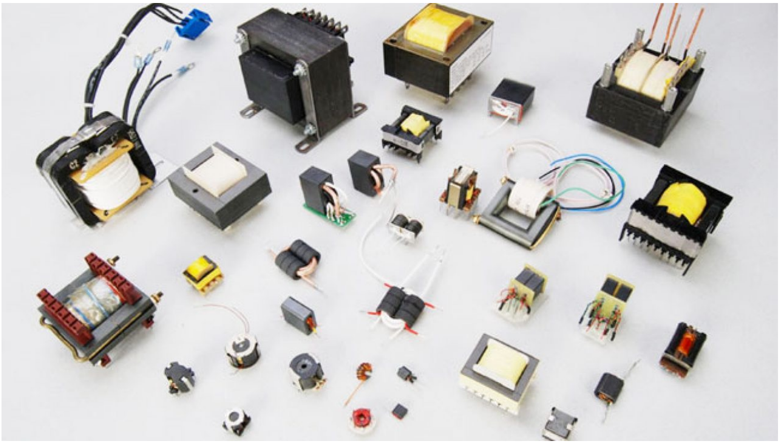
- Diodes
  - Devices that allow current to flow only in one direction
  - Silicon diodes, for example have, essentially infinite resistance if $V_{ac}<0$, that is if the cathode is at a higher potential than the anode and very low resistance if $V_{ac}> .7V$.
  - The cathode is usually labelled with a band
- Transformers
  - AC only: $\frac{N_2}{N_1} = \frac{V_2}{V_1}$



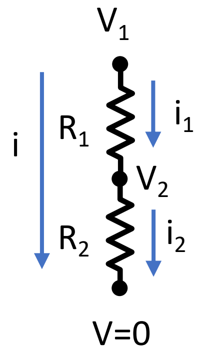


Credit: Make Electronics

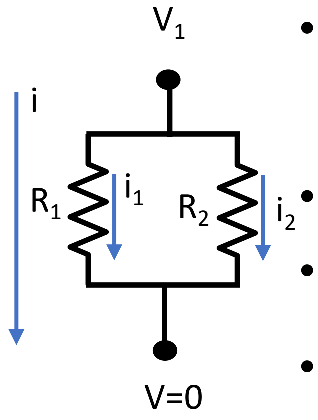




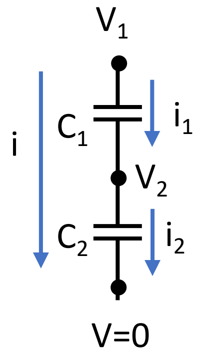
Simple circuit analysis with Kirchhoff



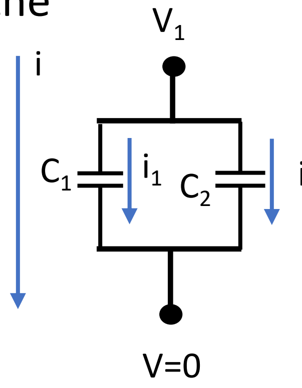
- R_{eq} is the equivalent resistance, replacing the top left circuit with a single resistance.
- By Kirchhoff's node rule, $i_1 = i_2 = i$, so
- $\frac{V_1-V_2}{R_1} = \frac{V_2}{R_2} = \frac{V_1}{R_{eq}}$ thus $\frac{R_1}{R_{eq}} V_1 = V_1 - V_2$ and $\frac{R_2}{R_{eq}} V_1 = V_2$. Adding, we get $\frac{R_1}{R_{eq}} V_1 + \frac{R_2}{R_{eq}} V_1 = V_1$. Dividing by V_1 and solving, we get $R_1 + R_2 = R_{eq}$



- Again let R_{eq} is the equivalent resistance, replacing the bottom left circuit with a single resistance.
- By Kirchhoff's node rule, $i_1 + i_2 = i$, so
- $\frac{V_1}{R_1} + \frac{V_1}{R_2} = \frac{V_1}{R_{eq}}$.
- Solving, we get. $\frac{R_1 R_2}{R_1+R_2} = R_{eq}$

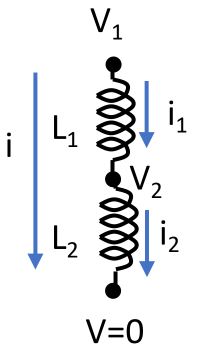


- C_{eq} is the equivalent capacitance, replacing the top right circuit with a single capacitor.
- By Kirchhoff's node rule, $i_1 = i_2 = i$, so
- $C_1 \frac{d\,(V_1-V_2)}{dt} = C_2 \frac{d\,(V_2)}{dt} = C_{eq} \frac{dV_1}{dt}$
- $\frac{C_{eq}}{C_1} \frac{d\,(V_1)}{dt} = \frac{d\,(V_1-V_2)}{dt}$ and $\frac{C_{eq}}{C_2} \frac{d\,(V_1)}{dt} = \frac{d\,(V_2)}{dt}$
- Adding and cancelling the $\frac{d\,(V_1)}{dt}$, we get
- $\frac{C_{eq}}{C_1} + \frac{C_{eq}}{C_2}$ =1 and solving, we get. $\frac{C_1 C_2}{C_1+C_2} = C_{eq}$

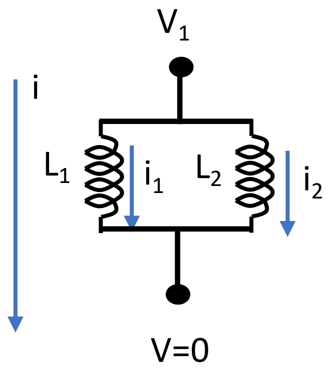


- C_{eq} is the equivalent capacitance, replacing the bottom right circuit with a single capacitor.
- By Kirchhoff's node rule, $i_1 + i_2 = i$
- $C_{eq} \frac{dV_1}{dt} = C_1 \frac{dV_1}{dt} + C_2 \frac{dV_1}{dt}$, so
- $C_{eq} = C_1 + C_2$

Simple circuit analysis with Kirchhoff

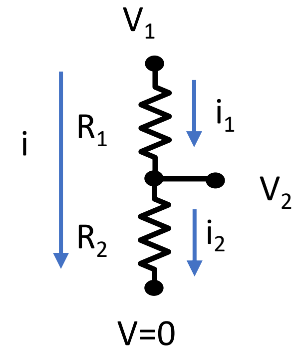


- Let L_{eq} be the equivalent inductance, replacing the top left circuit with a single inductor.
- By Kirchhoff's node rule, $i_1 = i_2 = i$, so
- $L_{eq}\frac{di}{dt} = V_1$, $L_1\frac{di_1}{dt} = V_1 - V_2$, $L_1\frac{di_2}{dt} = V_2$
- $V_1 = L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$ and
- $L_{eq} = L_1 + L_2$

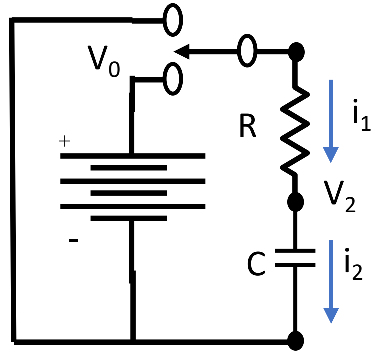


- Let L_{eq} be the equivalent inductance, replacing the bottom left circuit with a $\frac{di}{dt} = \frac{V_1}{L_{eq}}, \frac{di_1}{dt} = \frac{V_1}{L_1}, \frac{di_2}{dt} = \frac{V_1}{L_2},$
- single inductor.
- By Kirchhoff's node rule, $i_1 + i_2 = i$, so
- $\frac{V_1}{L_{eq}} = \frac{V_1}{L_1} + \frac{V_1}{L_2}$ and
- $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

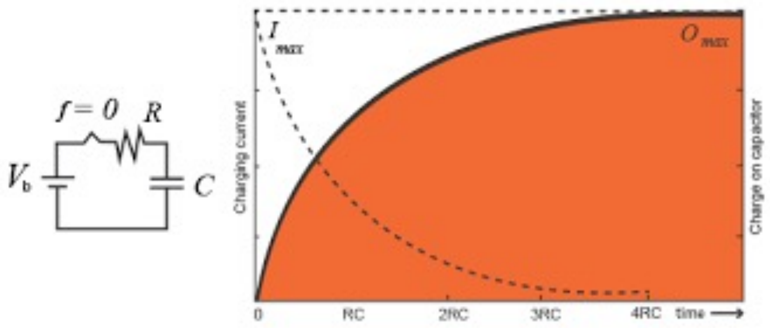
- The circuit on the right, is useful and is called a *voltage divider*.
- $i = i_1 = i_2$ so $\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2}$, $V_1 - V_2 = \frac{R_1}{R_2}V_2$
- Thus, $V_1 = (1 + \frac{R_1}{R_2})V_2$ and so
- $V_2 = \frac{R_2}{R_1 + R_2}V_1$

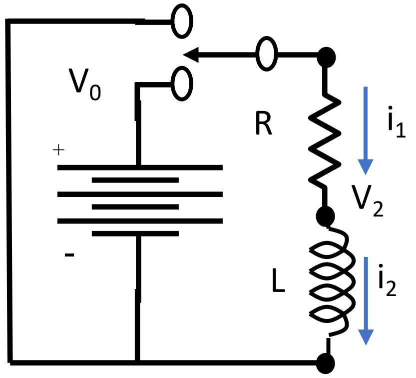


RC/RL circuit analysis with Kirchhoff

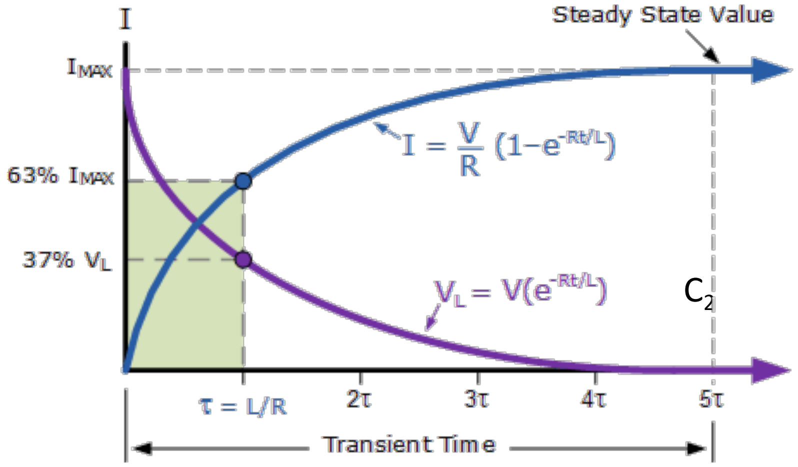


- RC behavior: charging
  - $V_0 - V_2 = i_1 R = V_R, i_1 = \frac{V_R}{R}$
  - $i_2 = C\frac{dV_2}{dt}, V_C = V_2$
  - $i_1 = i_2, V_C = V_0 - V_R$
  - $\frac{V_R}{R} = C\frac{dV_C}{dt}$, RC $\frac{dV_C}{dt} = V_0 - V_C$, or RC $\frac{dV_C}{dt} + V_C = V_0$
  - Solution is $V_C = V_0 - V_0 e^{-\frac{t}{RC}}$





- RL behavior: charging
  - $V_0 - V_2 = i_1 R = V_R$
  - $V_L = V_2 = L\frac{di_2}{dt}$
  - $i_1 = i_2, V_R = V_0 - V_L$, so L $\frac{d}{dt}\frac{V_0 - V_L}{R} = V_L$
  - $\frac{L}{R}\frac{d\,V_L}{dt} + V_L = 0$
  - Solution is $V_L = V_0 e^{-\frac{Rt}{L}}$



Phasors

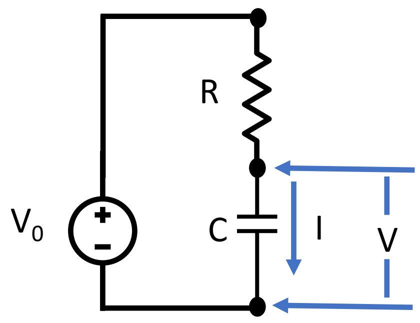
- $V(t) = RI(t)$
- $V(t) = L\dot{I}(t)$
- $I(t) = C\dot{V}(t)$
- Suppose $V(t) = Acos(\omega t + \theta)$ and $\mathrm{I}(t) = Bcos(\omega t + \phi)$. If $\phi > \theta$, we say the current leads the voltage.
- $V(t) = Re(e^{j(\omega t+\theta)})$, and $\mathrm{I}(t) = Re(e^{j(\omega t+\phi)})$
- Now define $V = Ae^{j\theta}$ and $I = Be^{j\phi}$, so $|V| = A$, $|I| = B$, $\angle V = \theta$, and $\angle I = \phi$. V and I are called phasors and do not include time. Note that $V(t) = Re(Ve^{j\omega t})$ and $\mathrm{I}(t) = Re\left(Ie^{j\omega t}\right)$.
- Note that $I = CVj\omega$, for a capacitor and $V = LIj\omega$, for an inductor

Circuit analysis and impedance

- Impedance unifies the "simple" ohms law with capacitance and inductance.
- $Z = R$, for resistors, $Z = j\omega L$, for inductors and $Z = \frac{1}{j\omega C}$, for capacitors.
- In general, $Z = R + jX$ and all the ohm like laws hold for resistors, capacitors and inductors .
  - $Z_{eq} = Z_1 + Z_2$ for two components with impedance Z_1, Z_2 connected in series
  - $Z_{eq} = \frac{Z_1 2}{Z_1 + Z_2}$ for two components with impedance Z_1, Z_2 connected in parallel
- For example, for a resistor and capacitor in series has impedance $Z = R + \frac{1}{j\omega C}$

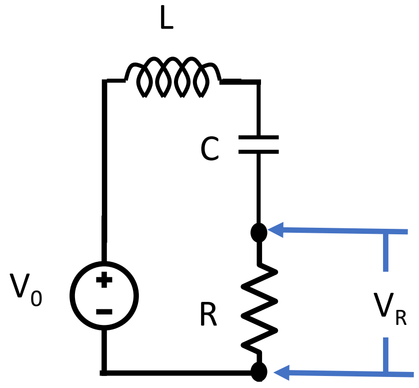
Phasors, impedance and power

- For the circuit on the right, $Z = R + \frac{1}{j\omega C}$ is the impedance for the resistor and capacitor in series.
- The phasor $I = \frac{V_0}{Z}$ and the phasor $V = \frac{I}{j\omega C} = \frac{V_0}{1+j\omega RC}$
- Further, $|I| = \frac{V_0}{|Z|}$, $\angle I = \angle \frac{V_0}{|Z|}$ and $|V| = \frac{|I|}{|j\omega C|} = |\frac{V_0}{1+j\omega RC}|$
- For phasors V, I, define the complex power as $P = \frac{V\bar{I}}{2} = Z\frac{I\bar{I}}{2} = \mathrm{R}\frac{|I|^2}{2} + j\mathrm{X}\frac{|I|^2}{2}$; the first term is the real power, the second is called the *reactive* power.
- The average power is $P_a = Re(P) = Re(\frac{V\bar{I}}{2})$. We define the reactive power as $P_r = Im(P)$.
- $P_r = \omega(E_L - E_C)$, where $E_L$and E_C are respectively, the energy stored in the inductor and capacitor respectively.



Q and phasors

- Consider the series resonance on the right. $Z_{LCR} = R + j\left(\omega L - \frac{1}{\omega C}\right)$
- The phasor, $I = \frac{V_0}{Z_{LCR}}$, and the phasor $V_R = \frac{V_0}{Z_{LCR}} Z_R$, where $Z_R = R$.
- So $V_R = \frac{RC\omega V_0}{RC\omega + j(LC\omega^2 - 1)}$.
- $|V_R|$ is maximum when $\omega^2 LC = 1$. Put $\omega_0 = \frac{1}{\sqrt{LC}}$. When $\omega = \omega_0$, $|V_R| = V_R = V_0$.
- $|V_R| = \frac{V_0}{\sqrt{2}}$, when $X = R$. Note that the power through R when $X = R$ is half the power through R when $X = 0$ or $\omega = \omega_0$.
- Let the frequencies where $R = \pm X$ be denoted ω_u and ω_l, where $\omega_u > \omega_l$.
- We define $Q = \frac{X}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$.
- Solving for ω_u and ω_l, we get $\frac{L\omega_u}{\omega_0} - \frac{\omega_0}{C\omega_u} = R$ and $\frac{L\omega_l}{\omega_0} - \frac{\omega_0}{C\omega_l} = -R$, or, in terms of Q,
- $\frac{\omega_u}{\omega_0} - \frac{\omega_0}{\omega_u} = \frac{1}{Q}$ and $\frac{\omega_l}{\omega_0} - \frac{\omega_0}{\omega_l} = -\frac{1}{Q}$. In fact, $\omega_0 = \sqrt{\omega_u \omega_l}$, and so $\frac{\omega_u}{\omega_0} - \frac{\omega_l}{\omega_0} = \frac{1}{Q}$.
- Thus $Q = \frac{\omega_0}{\omega_u - \omega_l} = \frac{\omega_0}{\Delta\omega}$
- From the definition of P_a, earlier, $Q = \omega_0 \frac{E}{P_a}$, where E is the total energy stored in *L* and *C*, which is in turn the peak E_L and peak E_C at resonance.



Phasors, impedance and power

- For the circuit on the right, $Z = R + \frac{1}{j\omega C}$ is the impedance for the resistor and capacitor in series.
- The phasor $I = \frac{V_0}{Z}$ and the phasor $V = \frac{I}{j\omega C} = \frac{V_0}{1+j\omega RC}$
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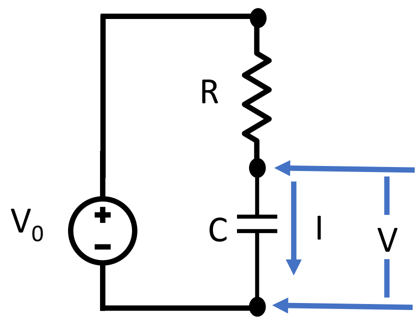
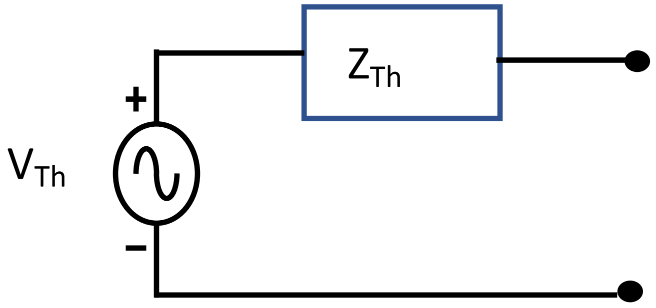
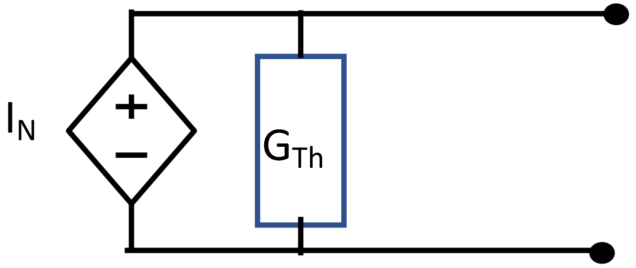


Thevenin and Norton

- Thevenin: Any combination of linear sources and passive elements terminating in two terminals is equivalent to a pure voltage source in series with an impedance



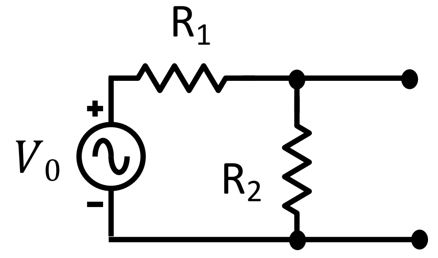
- Norton: Any combination of linear sources and passive elements terminating in two terminals is equivalent to a pure current source in parallel with a conductance



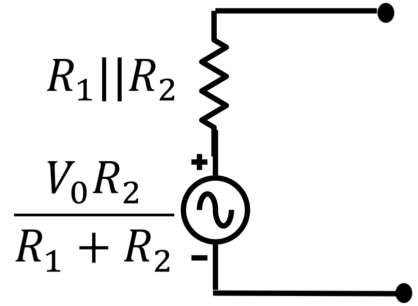
- Similar theorems for two terminal input and output devices (with transfer function)

Thevenin and Norton

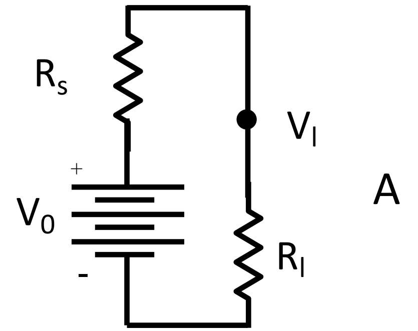
- We can use lookback resistance to calculate the Thevenin equivalent resistance and ideal source.
- To find the lookback resistance, short the source and apply the usual laws.
  - Here $R_s = R_1||R_2$
- To find the new ideal source, notice R_1 and R_2 form a voltage divider.
  - The new source voltage is $\frac{V_0 R_2}{R_1+R_2}$



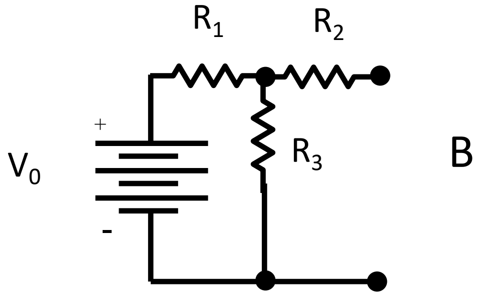
Is equivalent to



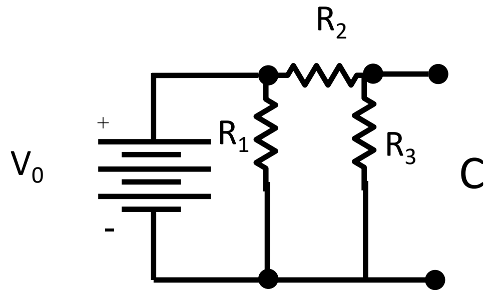
Exercise 1: Resistors



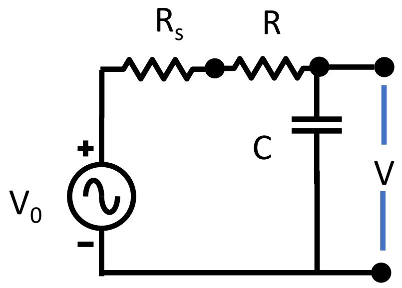
1. Consider (A). Find the formula for power in the load. Find the R_l that maximizes the power to the load.
   - $V_l = \frac{R_l}{R_s+R_l} V_0, I_l = \frac{V_0}{R_s+R_l}$.
   - $P_l = V_l I_l = \frac{R_l}{(R_s+R_l)^2} {V_0}^2$, which is maximum when $R_l = R_s$



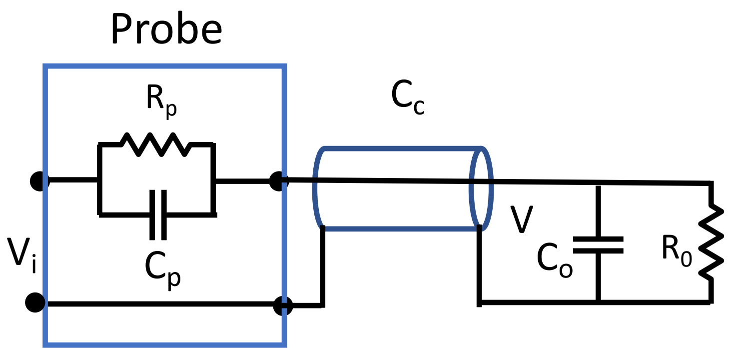
2. Find the Thevenin and Norton parameters fore (B).
   - $V_{Th} = \frac{R_3}{R_1+R_3} V_0$
   - $R_{Th} = R_2 + R_1||R_3$
3. Find the Thevenin and Norton parameters fore (C).
   - $V_{Th} = \frac{R_3}{R_2+R_3} V_0$
   - $R_{Th} = R_2||R_3$



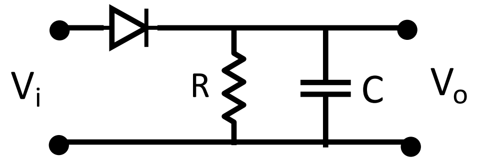
Exercise 3: Capacitors



1. In the circuit on the right, V_0 is a 2 volt pp ideal square wave source of frequency 20Hz, $R_S = 50\Omega$, $R = 300k\Omega$ and $C = 10\ nF$. Period is $50\ millisec$
2. What is the voltage, V, at the output? The scope has an input resistance of $1M\Omega$.
   - About a volt at peak
3. Let t_2, the time to discharge to 0V. Calculate τ and t_2.
   - $\tau = 3\times10^5\times10^{-8}\ sec = 3\ millisec$
   - $t_{12} \approx 1.5ms$
4. Capacitance on the scope prevents the delay from being 0. Measure the new t_2 with these changes.
5. Given C$_0$ and C$_p$ and R$_{p.}$
   - $C_0 = 100pf/m, C_o = 50pF, C_p = 10pF$
6. Now calculate the new t_{12}.
   - $\tau = 6\mu$-sec



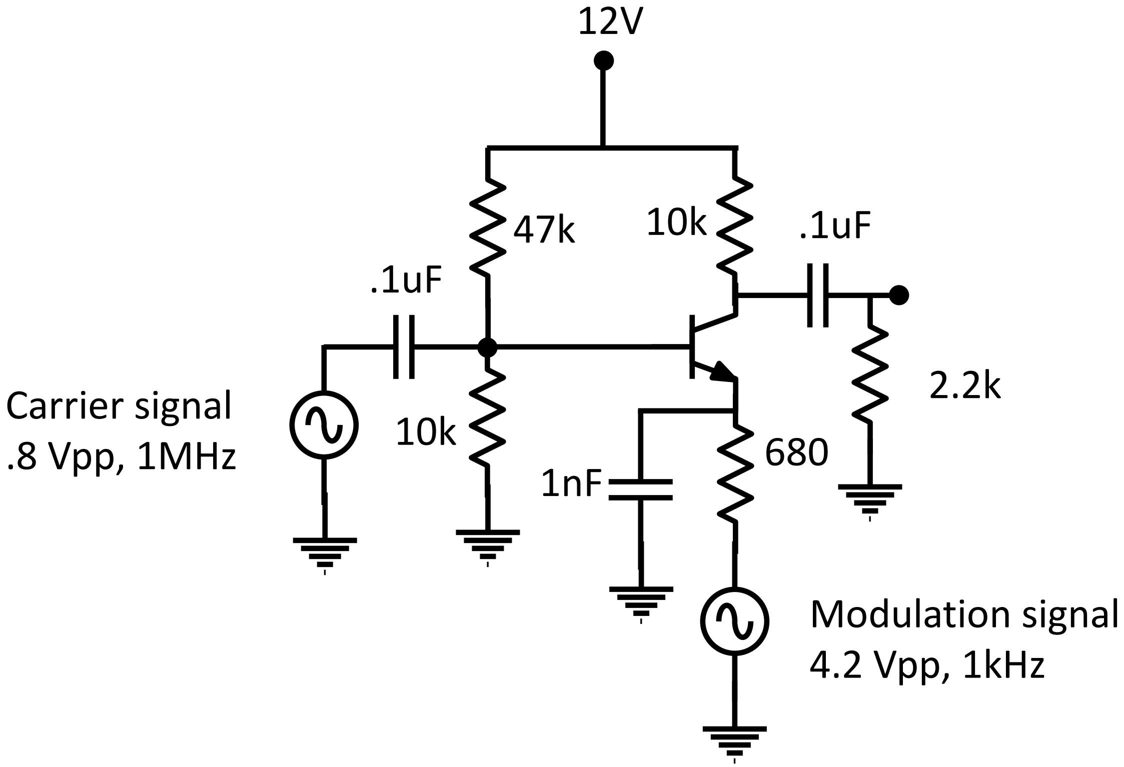
Exercise 4: Diode detectors



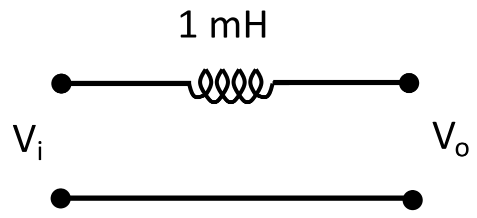
- For AM, $V(t) = V_c \cos(\omega_c t) + a(t)\cos(\omega_c t)$, Define the modulation depth $m = \frac{a_p}{V_c}$
- In circuit on the right, $R = 10k\Omega$, $C = 10\ nF$
- Set function generator for $f_c = 1MHz$, $V_{c,pp} = 5V$, $f_m = 1kHz$, $m = .7$
  1. Calculate τ for the RC circuit. $\tau = 10^4 \times 10^{-8}\ \text{sec} = .1ms$.
     - T_m is period of modulating signal. $T_m = 10^{-3}sec = 1ms$. So $\tau \ll T_m$
     - T_c is period of modulating signal. $T_c = 10^{-6}sec = 1\mu s$. $\tau \gg T_c$
     - As you change f_m does the frequency of V_o track it? (It better)
  2. Compare the max voltage of the AM signal to the max of V_0.
     - $V_0, p \approx .8V, V_{i,p} \approx 1.4V$
  3. What happens when we make $m = 1.0$

AM Modulator for previous exercise

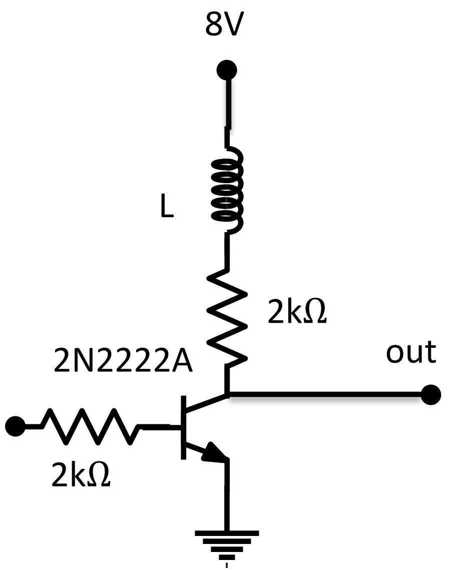
- I didn't have a signal generator that produced an AM signal, so I used the modulator on the right with the indicated inputs to produce the AM needed for the detector in the previous exercise.



Exercise 5: Inductors



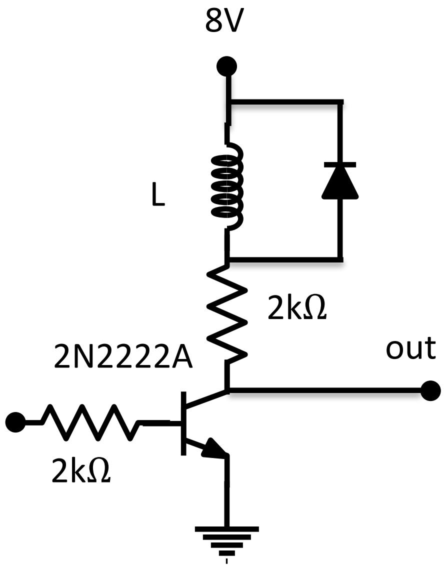
- Set function generator for 5V V_{pp}, $1kHz$. Connect a 50Ω load, L=1mH
  1. Observe square wave with rounded corners, measure the time, t_2 to decay to 0
  2. Calculate pp inductor current and the expected delay, t_2
  3. Use 2 scope channels: one at input, one at output



Exercise 6: Diodes and snubbers

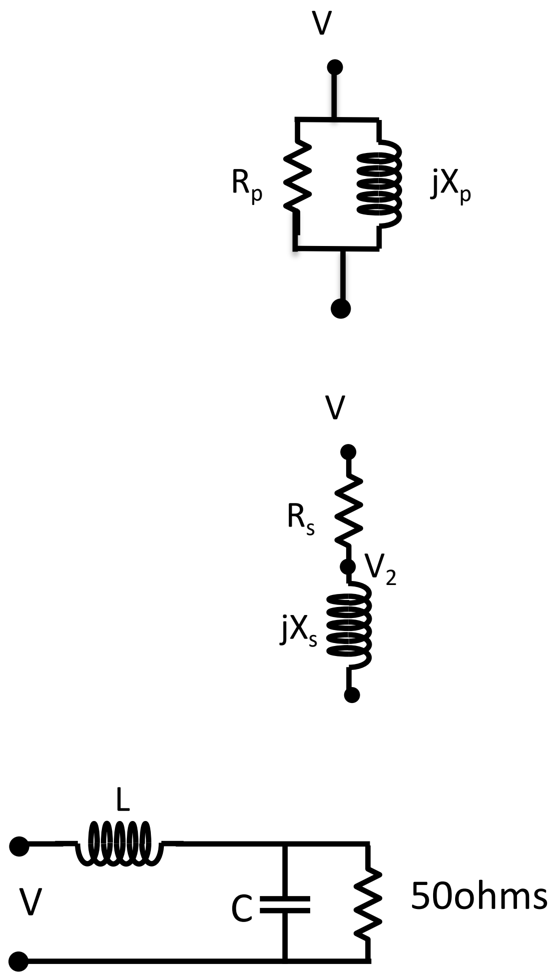
- Add indicated snubber diode.

1. What is its effect on ringing?
2. The ringing frequency comes from LC resonance in the circuit. Measure the frequency of the ringing and calculate C from $\omega = \frac{1}{\sqrt{LC}}$.
3. Diode should be on when transistor is off.

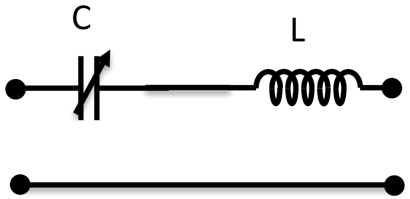


Exercise 7: Parallel to Series conversion

- For series: $Z_s = R_s + j\omega L$, $Q_s = \frac{\omega L}{R_s}$
- For parallel: $\frac{1}{Z_p} = \frac{1}{R_p} + \frac{1}{j\omega L}$, so $Z_p = \frac{j\omega L R_p}{R_p + j\omega L}$ and $Q_p = \frac{R_p}{\omega L}$



Exercise 8: Series resonance

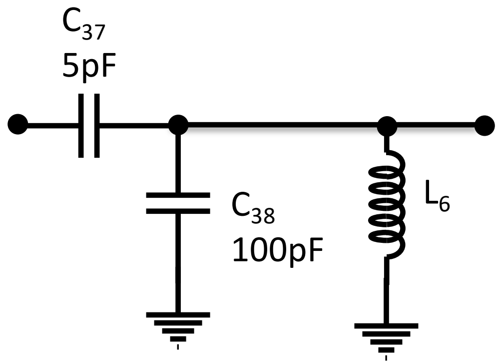


- For the circuit on the right, C$= 8 - 50pf$, $L = 15\mu H$forming a bandpass filter.
- If $C = 35pf$, the resonant frequency is $\omega = \frac{1}{\sqrt{35\times10^{-12}\times15\times10^{-6}}} = \frac{10^9}{\sqrt{525}} \approx 43.6$, so the resonant frequency is $\frac{43.6}{2\pi} \approx 6.9MHz$
- Tune the resonant frequency to $7MHz$ and find f_u, f_l and Δf and thus Q.
- Compute what these values should be

Exercise 9: Parallel resonance

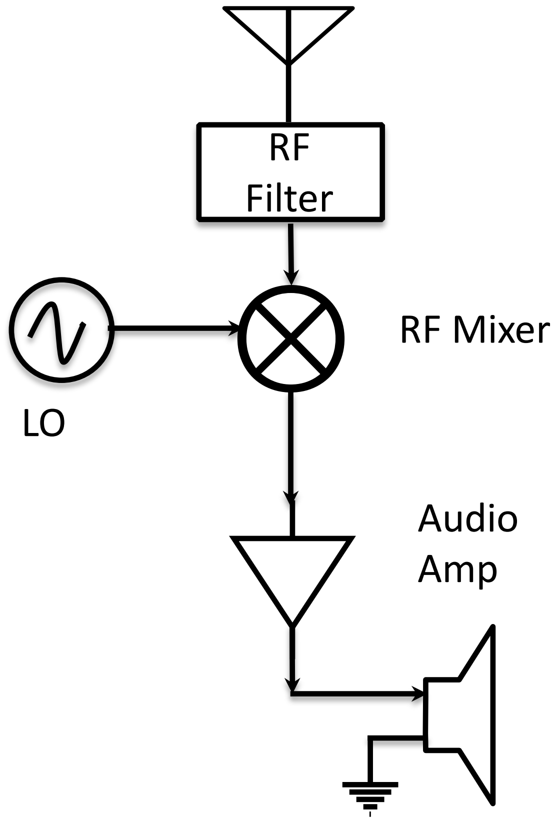
- $L = \ A_l \, N^2, A_l = 4 \frac{nH}{turn^2}$ for T37-2 core so for 28 turns, $L_6 = 3.1\mu H$

1. Again, find the resonant frequency, the frequencies corresponding to a 3db falloff, the bandwidth and the Q of this circuit. This circuit is in the transmit oscillator



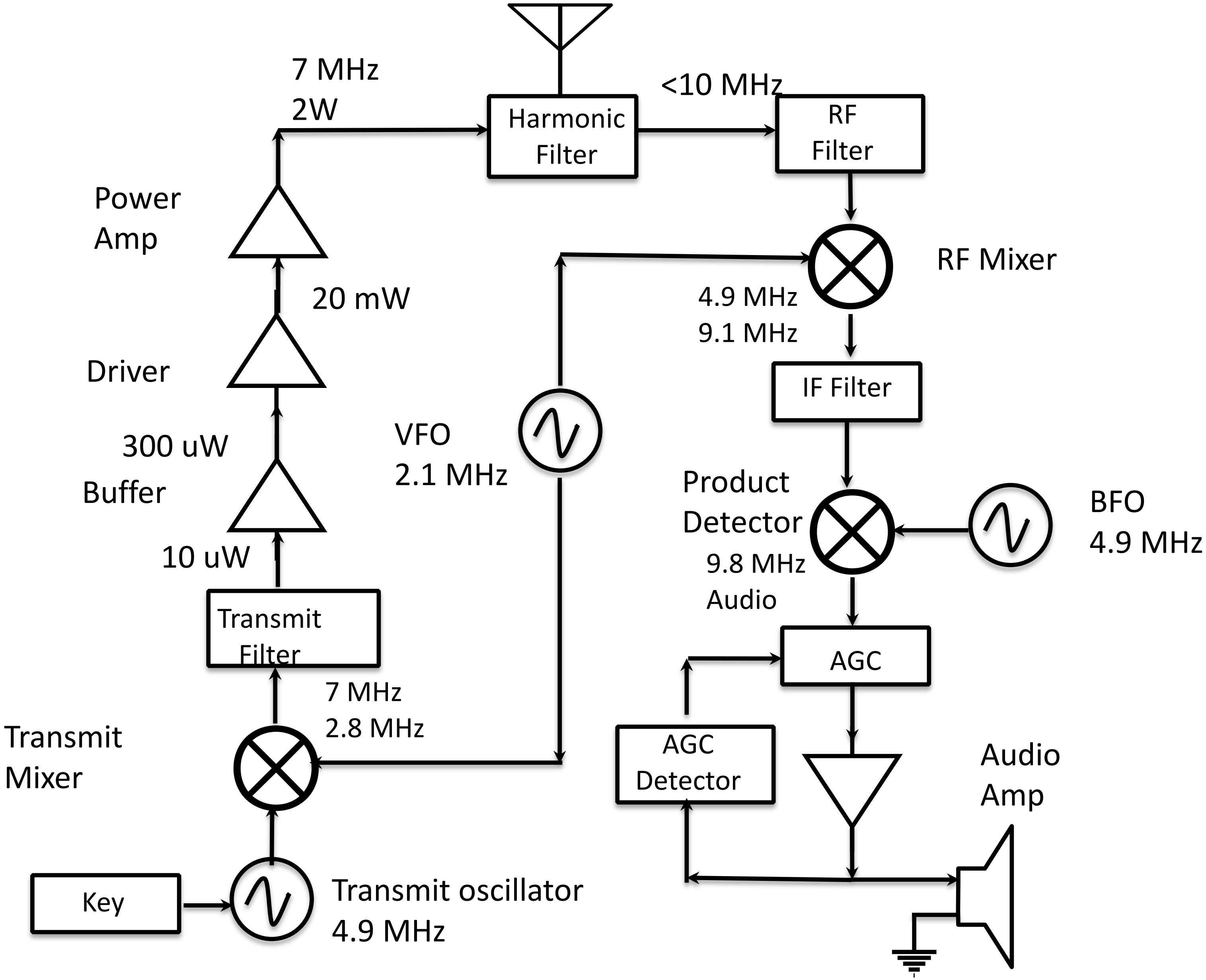
Direct conversion and superhet receivers

- Image frequency
  - $\omega_{rf} = \omega_{LO} - \omega_a$
  - $\omega_i = \omega_{LO} + \omega_a$
- Superheterodyne designs
  - $\omega_{rf} = \omega_{IF} + \omega_{VFO}$
  - $\omega_{vi} = \omega_{IF} - \omega_{VFO}$
  - $\omega_{IF} = \omega_{BFO} - \omega_a$
  - $\omega_{bi} = \omega_{BFO} + \omega_a$
  - $\omega_{usb} = \omega_{VFO} + \omega_{BFO} + \omega_a$
  - $\omega_{lsb} = \omega_{VFO} + \omega_{BFO} - \omega_a$



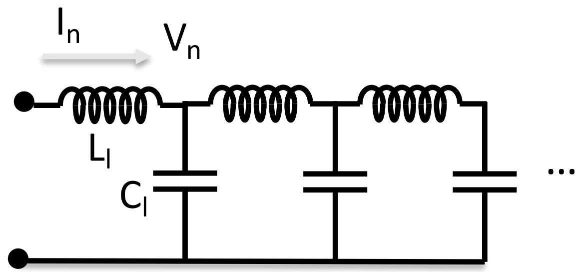
Direct conversion

Norcal 40A



7 MHz
2W
Harmonic Filter
<10 MHz
RF Filter
Power Amp
RF Mixer
20 mW
4.9 MHz
9.1 MHz
Driver
IF Filter
VFO
2.1 MHz
300 uW
Buffer
Product Detector
BFO
4.9 MHz
9.8 MHz
Audio
10 uW
Transmit Filter
AGC
7 MHz
2.8 MHz
AGC Detector
Transmit Mixer
Audio Amp
Key
Transmit oscillator
4.9 MHz

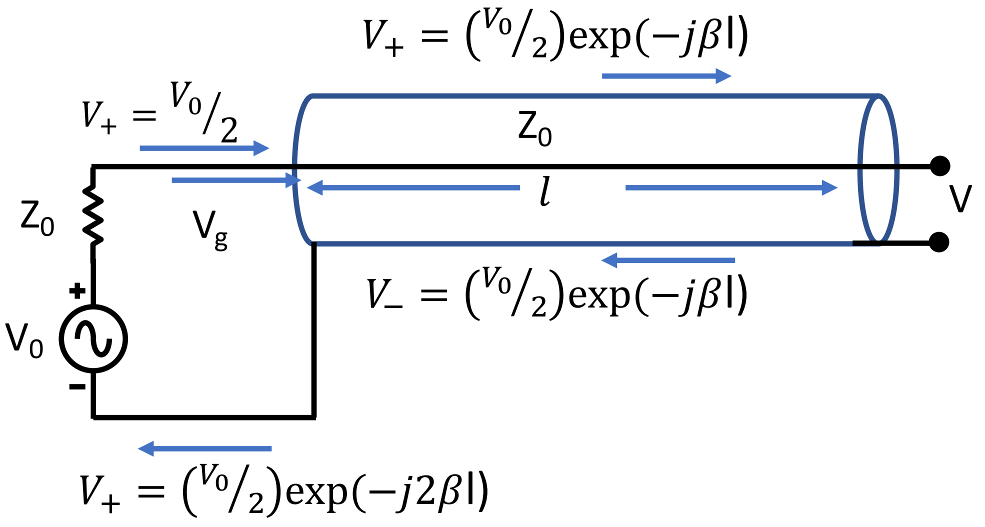
Transmission Lines



- $V_{n+1} - V_n = -L_l \frac{\partial I_{n+1}}{\partial t},\ L = \frac{L_l}{l}$
- $I_{n+1} - I_n = -C_l \frac{\partial V_n}{\partial t},\ C = \frac{C_l}{l}$
- $\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2}$ and $\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2}$
- Solution is $V(z - vt), v = \frac{1}{\sqrt{LC}}$, for forward wave
- $V' = vLI',\ \frac{V}{I} = \sqrt{\frac{L}{C}},\ Z_0 = \sqrt{\frac{L}{C}}$
- Another solution is $V(z + vt), v = \frac{1}{\sqrt{LC}}$, for reverse wave
- $Z_0 = \frac{V_+}{I_+}, -Z_0 = \frac{V_-}{I_-}, V = V_+ + V_-$
- $P_+(t) = \frac{{V_+}^2}{Z_0},\ P_-(t) = -\frac{{V_-}^2}{Z_0}$
- $\rho = \frac{V_-}{V_+},\ \mathrm{Z} = \frac{V}{I} = \frac{V_+ + V_-}{I_+ + I_-} = \frac{V_+}{I_+} \frac{1 + \frac{V_-}{V_+}}{1 + \frac{I_-}{I_+}} = Z_0 \frac{1+\rho}{1-\rho}$
- $\rho = \frac{Z - Z_0}{Z + Z_0}$
- $\rho_i = \frac{i_-}{i_+} = -\rho$

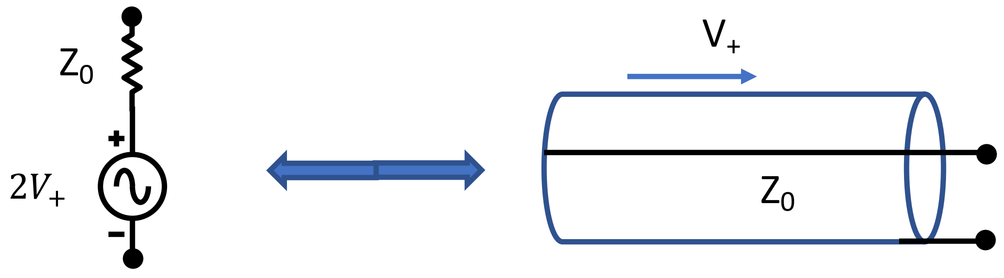
Transmission Lines - continued

- Phasor: $V(z-vt) = A\cos(\omega t - \beta z)$
- $\frac{dV}{dz} = -ZI, \frac{dI}{dt} = -YV'$
- $jk = \alpha + \beta j, jk = \sqrt{ZY}, Z_0 = \sqrt{\frac{Z}{Y}}$
- $jk = \sqrt{(j\omega L + R)(j\omega C + G)}, Z_o = \sqrt{\frac{(j\omega L+R)}{(j\omega C+G)}}$
- $\alpha = \sqrt{\frac{\omega RC}{2}}, v = \sqrt{\frac{2\omega}{R}}$



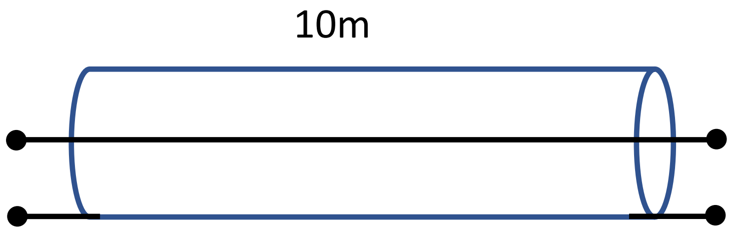
Power

- $\tau = \frac{V}{V_+} = 1 + \rho = \frac{2Z}{Z+Z_0}, V = 2V_+$
- Lookback resistance is $R_s = Z_0$
- $P_+ = \frac{V_+{}^2}{2Z_0} = \frac{V_0{}^2}{8Z_0}$, This is the total available power



Exercise 10: Coax

- Measure the velocity of. The waves in the coax by connecting one channel of the scope to the input and one to the output. Try different frequency.



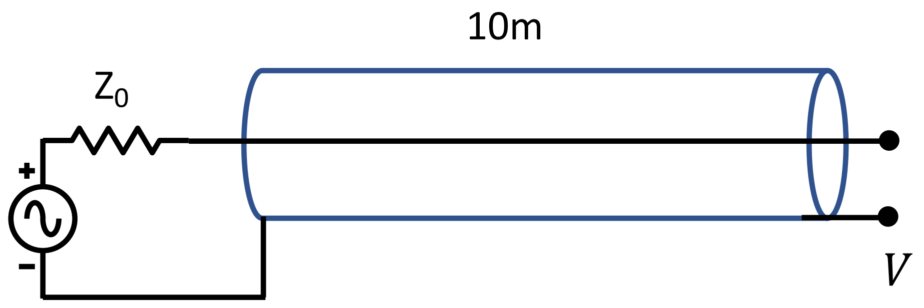
Exercise 11: Waves

- Suppose we want to send voice over 100km of coax.

1. Measure the SWR which is the ratio of the maximum to minimum output
2. If $L = 250\frac{nH}{m}, C = 100pf/m$ and the distributed resistance at voice is 50Ω, calculate total dB loss at 500, 1000 and 2000Hz using the high frequency approximation.
3. Add a 100mH inductor every 1km. Now what's the loss?

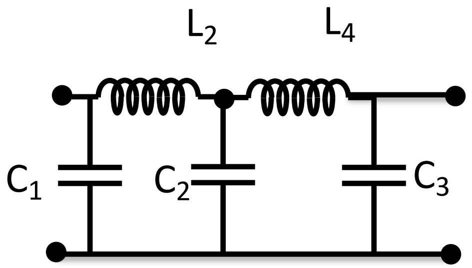
Exercise 12: Resonance

- $RG58U$ has a capacitance of about $100\ pF/m$
- Let α be the attenuation constant and β be the phase
- Derive an expression for $|\frac{V_g}{V}|$ and use it to calculate α
- Find the wave velocity by calculating the resonant frequency and noting the time delay with a scope on the input and output
- Find, as usual, f_u, f_u, and Q.
- Confirm $Q = \frac{\alpha}{2\beta}$

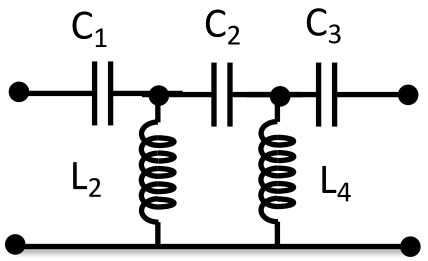


Filters

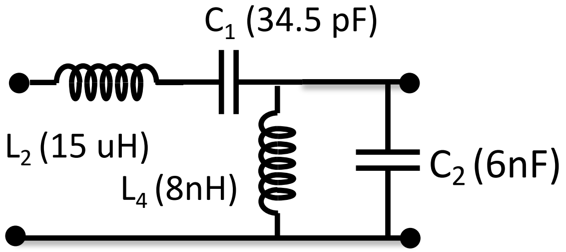
Low pass



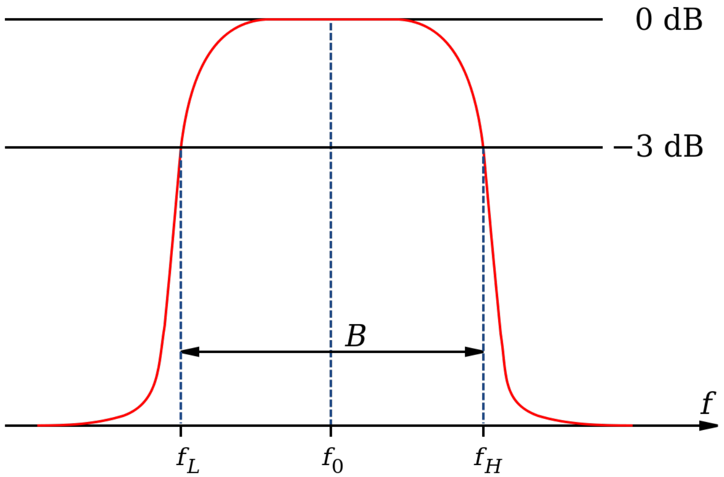
High pass



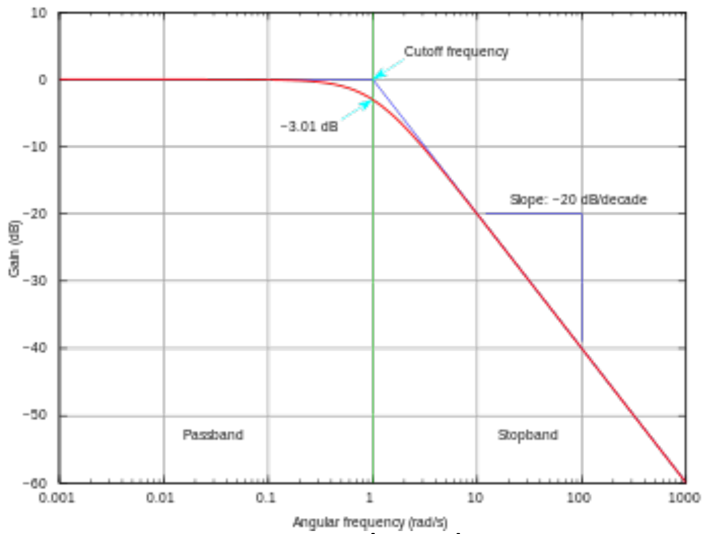
7 MHz bandpass



- Filters

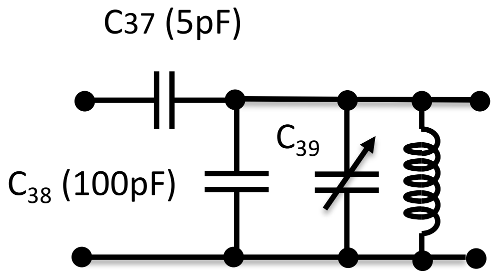


Bandpass - WIkipedia



Lowpass - WIkipedia

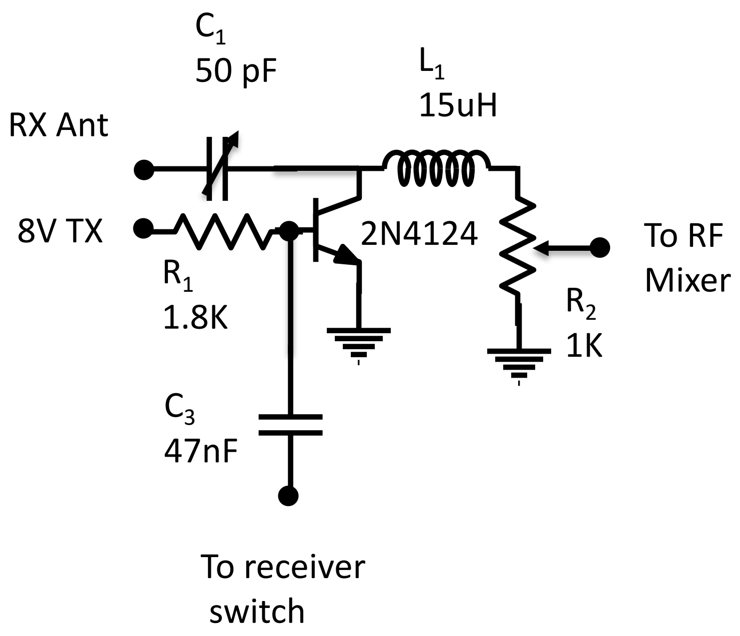
Norcal transmit bandpass filter



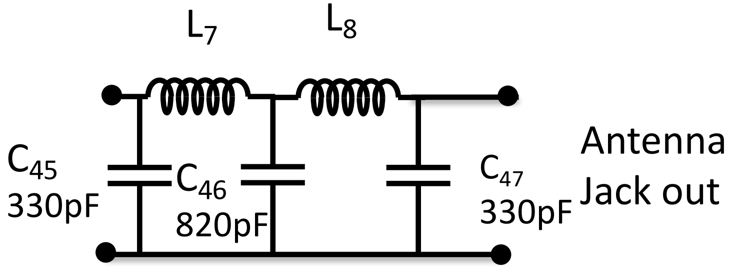
- $C_{39} = 50pF,$
- L_6 is 36 turns #28 on T37-2 which has $A_l = 4\,\frac{nH}{turn^2}$
- $L_6 = A_l \cdot 36^2 = 3.1\mu H$
- $Z_2 = -\frac{j}{(C_{38}+C_{39})\omega_o}$, $Z_3 = jL_6\omega_o, Z_1 = \frac{j}{C_{37}\omega_o}$

  L_6
- $Z_{2,3-eq} = \frac{jL_6\omega_0}{L_6(C_{38}+C_{39}){\omega_0}^2-1}$
- Resonance is when $Z_{2,3-eq} \rightarrow \infty, {\omega_o}^2 = \frac{1}{(C_{38}+C_{30})L_6} \approx \frac{10^{18}}{465}$, when almost all the voltage drop is across $Z_{2,3-eq}$ $\omega_o = \frac{10^9}{\sqrt{465}} \approx 50.8{\times}10^6, f_0 = \frac{\omega_o}{2\pi} \approx 7.1\ MHz$
- Q of filter is: $Q_s = \frac{X_s}{R_s}$. R_s comes from the other components and must be measured
- Note that $Z_{2,3-eq}$ is small for the other modulation product

Norcal RF Filter



Exercise 13: Norcal Harmonic Filter



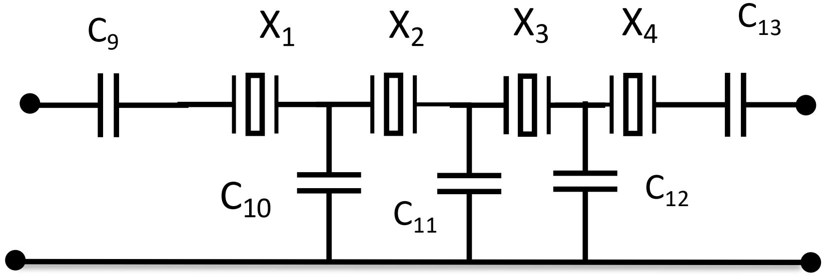
- L_7, L_8 use T37-2 core, 18 turns, 1.3uH. Use 50Ω termination and set function generator at 10Vpp.

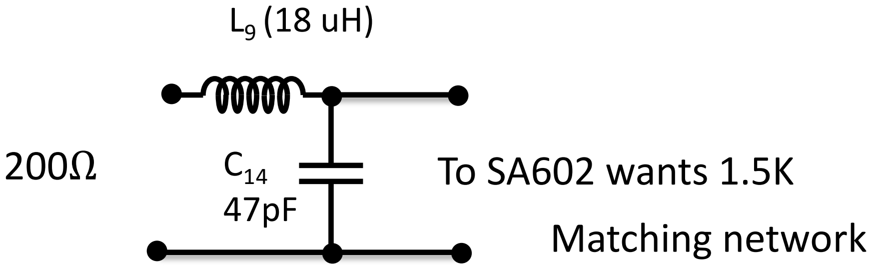
1. Compute and compare loss at 7MHz and 14MHz.
2. From $A_l = 5nH/turn^2$, calculate L_7 and L_8.
3. What is the spur strength at 7, 14 and 28MHz? Measure and calculate.

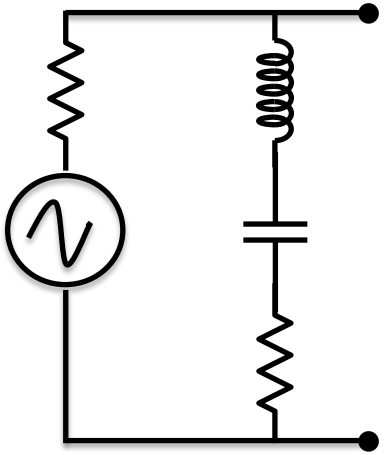
Exercise 14: Norcal IF Cohn Filter

- X_1 through X_4 are 4.91 MHz
- C_{10}, C_{11}, C_{12} are 270 pF
- Set function generator to $50mV_{pp}$ from function generator
- Calculate R and X for filter

1. Measure the resonant frequency of one of the crystals
2. Calculate the parameters of the crystal







Equivalent circuit for crystal and generator

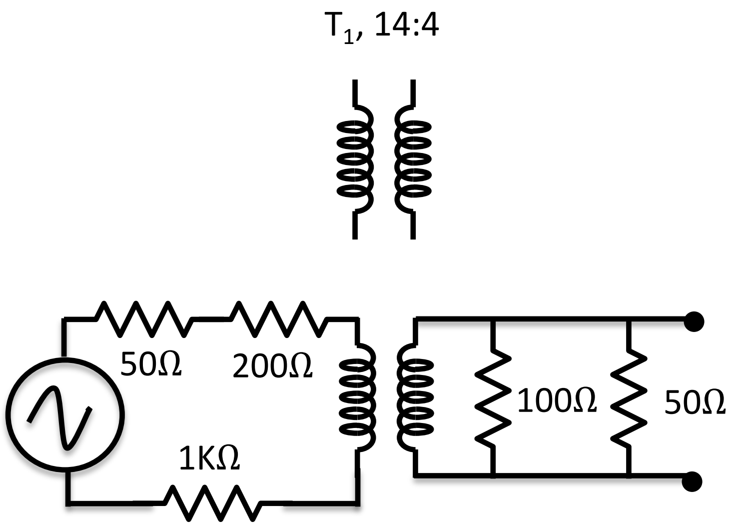
Transformers

- For solenoid, $\oint B \cdot ds = \mu_0 n I$ inside
- $LI = \Phi_B$. Since there are n turns in the solenoid, over the solenoid, $LI = \mu_0 n^2 I$, so $L = \mu_0 n^2$.
- This is the source of $L = A_l n^2$
- $V_s = \frac{N_s}{N_p} V_p$

Exercise 15: Norcal Driver Transformers

- T_1 is driver matcher uses FT 37−43

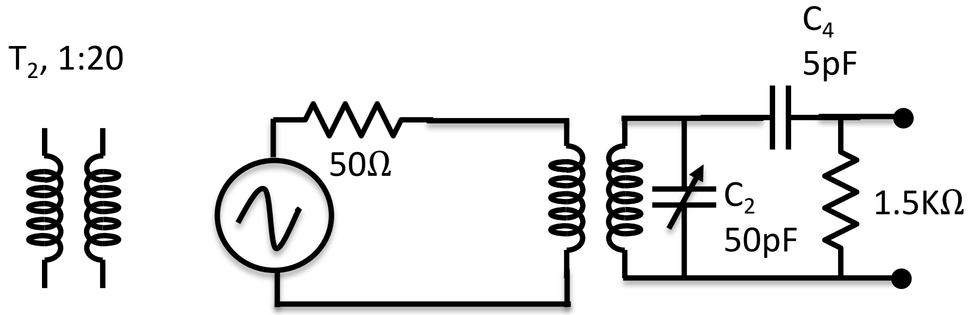
1. Measure the output V.
2. Calculate V
3. Measure the 3dB cutoff, f_c.
4. Use f_c to calculate A_l

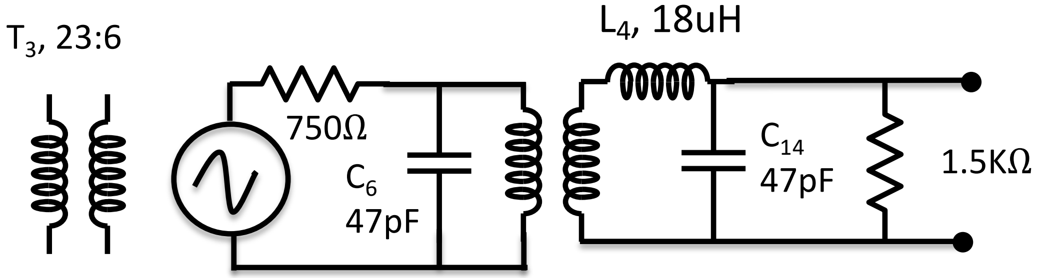


Exercise 16: Norcal Tuned Transformers

- T_2, T_3 are IF matchers using FT 37−61
- .5Vpp sine at 7MHz

1. Measure 3dB bandwidth
2. Find P/P_+





Acoustics

- Section of air of length l, U is average velocity, P is the pressure
- $\frac{\partial P}{\partial z} l = -\rho l \frac{\partial U}{\partial t}$
- $\frac{dl}{dt} = l \frac{\partial U}{\partial z}$
- $PV^{\gamma} = C$
- $\frac{\partial^2 P}{\partial t^2} = \frac{\gamma P}{\rho} \frac{\partial^2 P}{\partial x^2}$
- $v = \sqrt{\frac{\gamma P}{\rho}} = = 332 \frac{m}{s}$
- $SWR = \frac{\lambda^2}{2\pi A}$, A is the area of the tube

Sound	L_p	Power density
rustling leaves	10dB	1pW/m^2
broadcast studio	20dB	1pW/m^2
classroom	50dB	10nW/m^2
heavy truck	90dB	1nW/m^2
Shout at 1m	100dB	10mW/m^2
jackhammer	110db	100mW/m^2
jet takeoff at 50m	120dB	1W/m^2

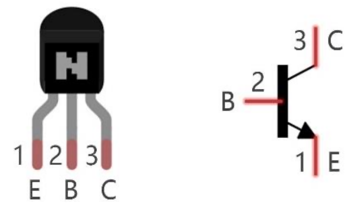
Exercise 17: Tuned Speaker

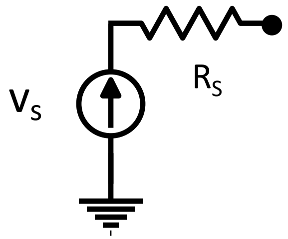
- Connect speaker to function generator 600Hz, 25mVrms.

1. Sound peaks at resonance. Find resonant frequency L_p.
2. Measure f_l, f_u by noting the 3dB loss. Calculate Q.
3. Use voltmeter to find resonance with speaker (nominally 8ohm) to calculate impedance
4. Calculate the resonant frequency from a transmission line equivalent circuit.

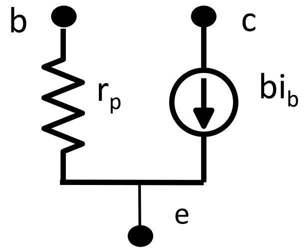
Bipolar Transistors

- NPN, PNP types
- Model
  - Conducts when $V_{be} > .7V$
  - $i_c = \beta i_b$
  - $i_c = \alpha i_e$
  - $\beta = \frac{\alpha}{1-\alpha}$
  - $\beta \sim 100$
- Switch
  - $G_S = \frac{i_b}{15mV}$
  - $R_S = 2\Omega$



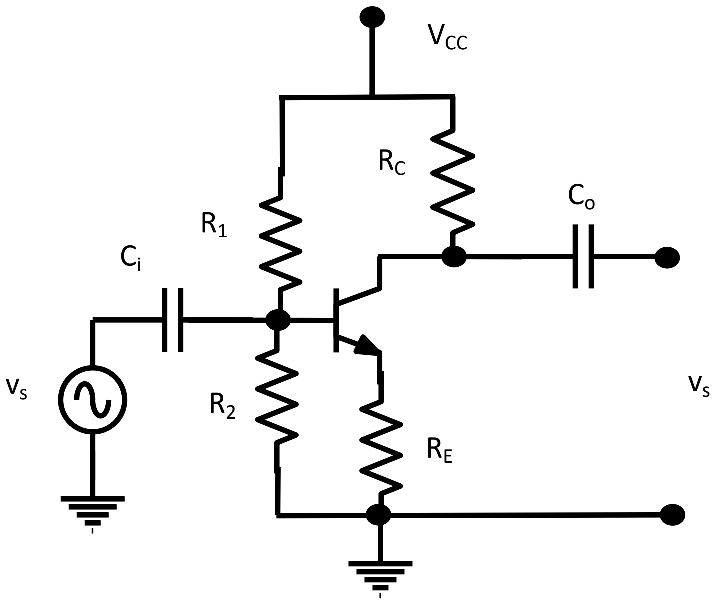


Bipolar source model



Bipolar equivalent circuit

BJT common emitter amplifier



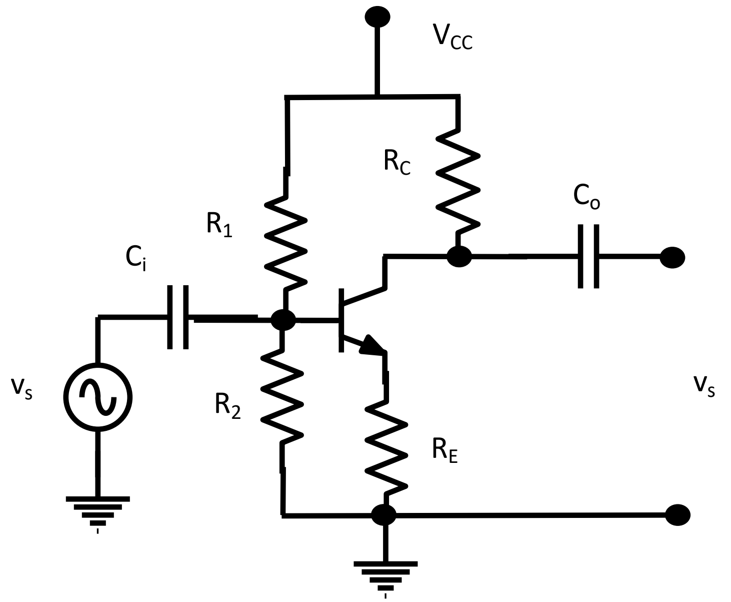
Common emitter amp

- Here's how to design a common emitter amplifier. We use a 2n3904 transistor with β=150. This circuit will work! Build it.

1. Pick the supply voltage V_{cc}=12V.
2. Choose a gain (amplification factor), A = 5.
3. Choose the "Q point" of the conducting transistor (4mA).
4. $V_{cc} = (i_c \cdot R_C) + V_{ce} + i_e R_E \sim i_e \cdot (R_C+R_E) + V_{ce}$ with i_c=4mA. We get $(R_C+R_E) = (V_{cc} - V_{ce})/(4mA)$ =1.75 k**Ω**.
5. Since A = 5 and $A=R_C/R_E$, R_C= 5 R_E so $R_E \sim 270$ **Ω** (this is a standard resistor value) and R_C= 1.5k**Ω**.

Credit: Ward, Hands on Radio.

BJT common emitter amplifier continued



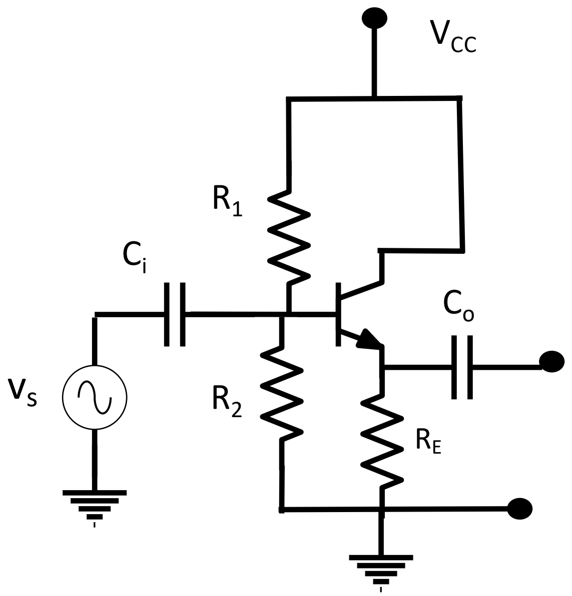
Common emitter amp

6. i_b = 4mA/β = 27 μA.
7. Since V_{be} must be greater than .7V throughout the input signal range, we want the voltage across R_2 to satisfy $V_{be} + i_c R_E$ = 1.8V.
8. We insert a voltage divider consisting of R_1 and R_2, so that R_1= (12-1.8)/270 μA ~ 39 kΩ.
9. C_o and C_i are picked to offer small resistance to the frequency range we're interested in and $C_o = C_i$ = 5 μF.

- I haven't explained why we want R_E but it provides thermal stability for the transistor over the range we care about. The fact that $A=R_C/R_E$ can be calculated using Kirchhoff's laws.

Credit: Ward, Hands on Radio.

BJT common collector amplifier

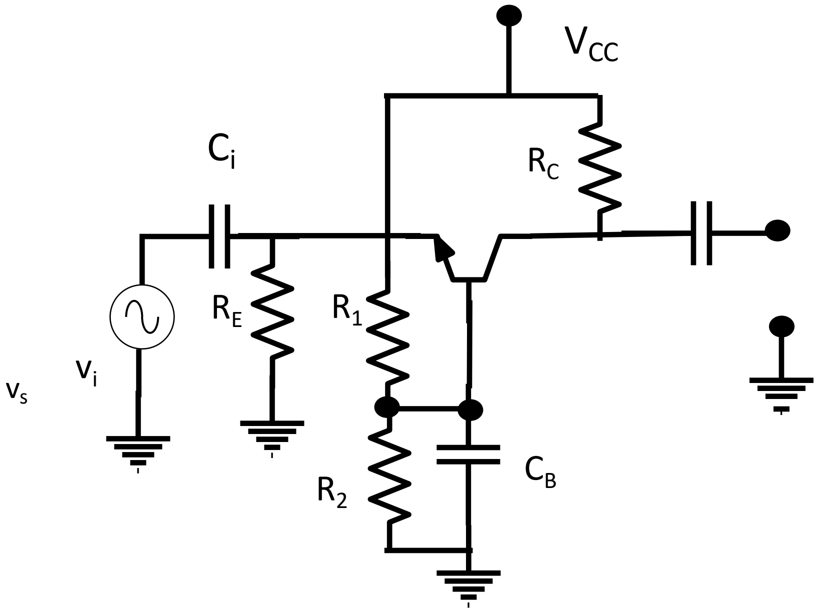


Common collector amp (Emitter Follower)

1. $\beta = 150,\ A_V = 1, V_{cc} = 12v$
2. Q-pt: $i_{ce} = 5mA$, $V_{ce,q} = 6v$ (rule of thumb), $v_{be} = .7V$.
3. $i_{R_1 \to R_2} = 10 i_b$ (ROT), $V_{ce} = v_{be} + i_{ce,q} R_E$, $R_E = 1.2k\Omega$, $i_b = \frac{V_{ce,q}}{\beta} = 33\mu A$
4. $V_{R_2} = V_{be} + i_C R_E = 6.7V$, $V_{R_1} = 5.3V$
5. $R_2 = \frac{6.7}{330\mu A} = 20k\Omega$, $R_1 = \frac{5.3}{330\mu A} = 16k\Omega$
6. $Z_{in} = R_1 || R_2 || (\beta + 1) R_E$, $R_{in} = 50\Omega$, $Z_{out} = 5\Omega$

Credit: Ward, Hands on Radio.

BJT common base amplifier



- $A_I = \frac{i_C}{i_E} = \frac{\beta}{\beta+1}$, $A_V = \frac{R_C||R_L}{r_e}$, $Z_{out} \approx R_C$

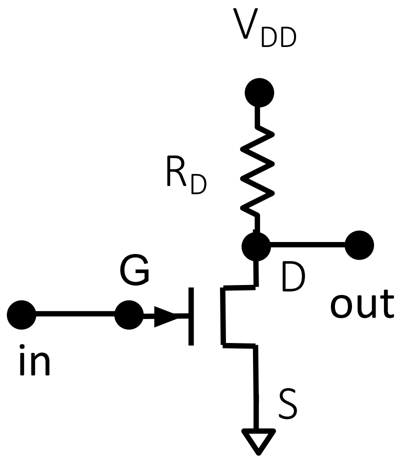
1. $V_{CC} = 12, V_{be} = .7V, R_E = 50\Omega, R_L = 1k\Omega, i_{ce,q} = 5mA, V_{ce,q} = 6V$
2. $i_b = \frac{i_{ce,q}}{\beta} = 33\mu A$, $i_{R_1 \to R_2} = 10\ i_b = 330\mu A$ (ROT)
3. $V_{R_2} = V_{be} + i_C R_E = 6.7V$, $V_{R_1} = 5.3V$
4. $R_1 = \frac{5.3}{330\mu A} = 16k\Omega$, $R_C = \frac{V_{cc} - i_{c,Q} R_E - V_{ce,Q}}{i_{c,Q}} = 1.35k\Omega$
5. $A_V = \frac{R_C||R_L}{26/i_e} = 115$

Common base amp

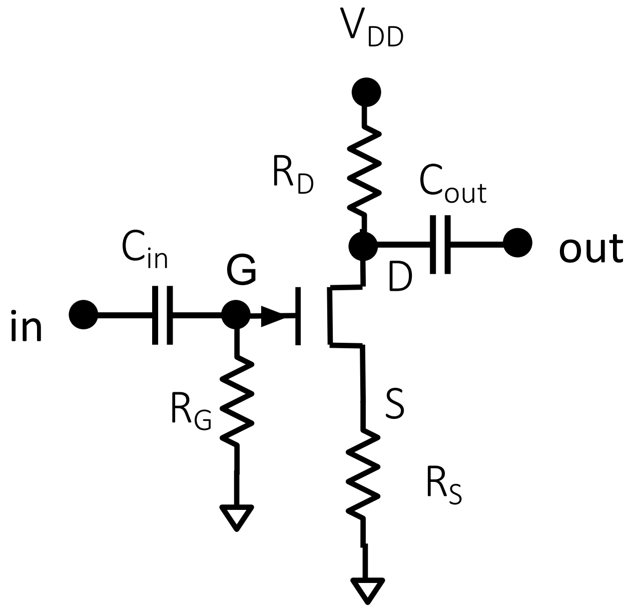
JFETs

- JFET circuit model: $I_{ds} = V_{ds}(\frac{2I_{dss}}{V_c{}^2})(V_{gs} - V_c - \frac{V_{ds}}{2})$
- $g_m = \frac{\Delta i_{ds}}{\Delta v_{gs}}$
- For circuit on right, $g_m \Delta v_{gs} = \Delta i_{ds}$ and $R_D \Delta i_{ds} = V_{out}$, so $-g_m R_D \Delta v_{gs} = V_{out}$
- Similar model for MOSFETs

- Op amp: $V_{out} = A_{OL}(V_+ - V_-)$, input resistance is very high



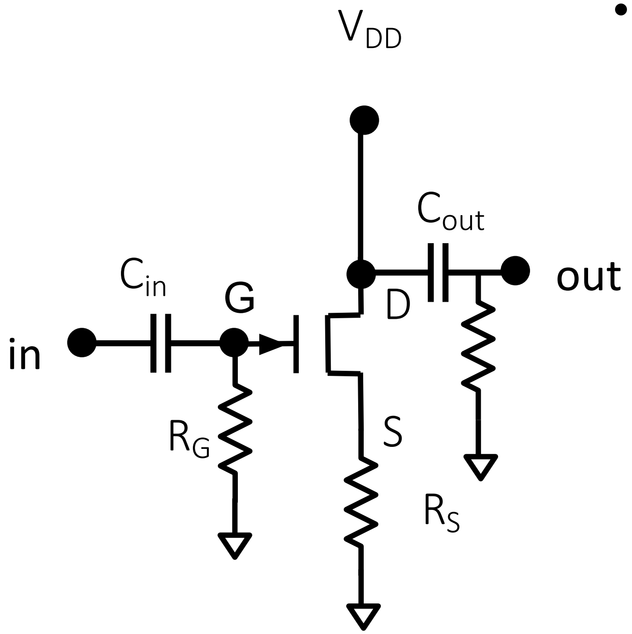
JFET common source amplifier



- $A_V = \frac{g_m R_D}{1+g_m R_S} = -\frac{R_D}{R_L}$, $R_S = \frac{-V_P}{i_{dd}}(1 - \sqrt{\frac{i_{dd}}{i_{dss}}})$. $g_m \approx 15mA/V$

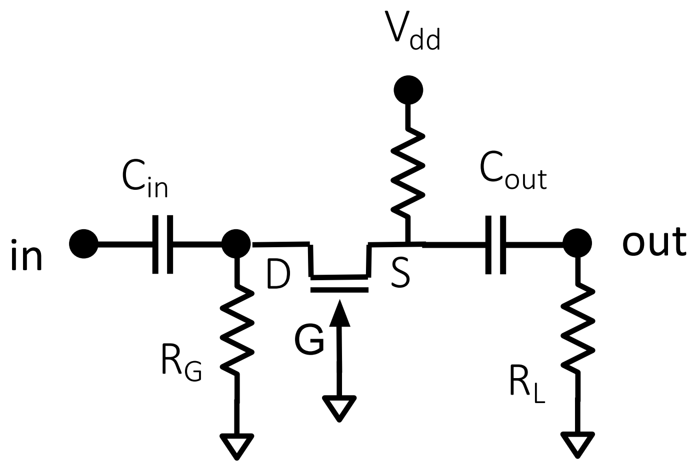
1. $V_{dd} = 12V, i_{dss} = 35mA, V_P = 3.0V$, $A_V = 10, i_{dd} = 10mA$
2. From equation above, $R_S = 139\Omega$, $R_D = 10R_S = 1390\Omega$
3. $A_V = -g_m(R_D||R_L)$

JFET common drain amplifier



- Similar to BJT emitter follower

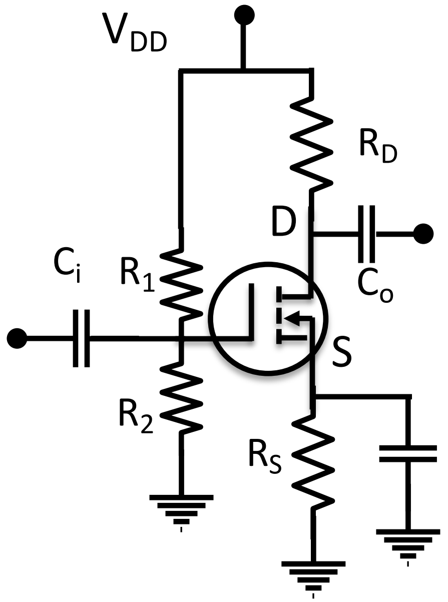
JFET common gate amplifier



- $V_{DD} = 12V, i_{dss} = 60mA, V_P = -6, A_V = 10, R_L = 1k\Omega, R_S = 50\Omega$
- $i_{d,q} = \frac{V_P}{2{R_S}^2 i_{dss}}\left(V_P + \sqrt{{V_P}^2 - 4R_S i_{dss} V_P}\right) - \frac{V_P}{R_S}$

1. Solve for R_D: $10 = g_m \times R_D || R_L$, $R_D = 2k\Omega$
2. Find $i_{d,q} = \frac{V_P}{2{R_S}^2 i_{dss}}\left(V_P + \sqrt{{V_P}^2 - 4R_S i_{dss} V_P}\right) - \frac{V_P}{R_S} = 10\text{mA}$

CMOS common emitter amplifier

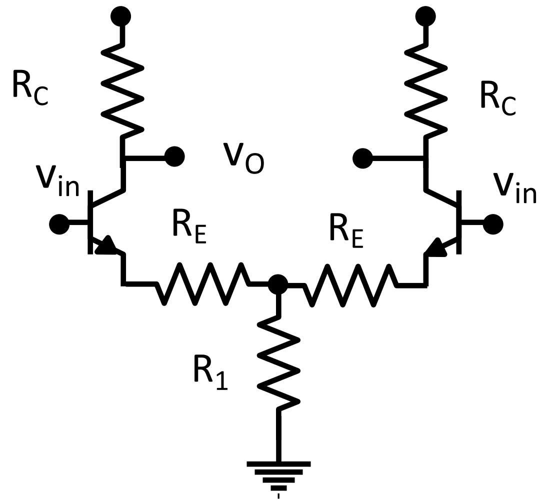


- Pick power
- $V_{DD} = i_D R_D + V_{DS} + i_D R_S$
- $V_{GS} = V_G - i_S R_S$
- $V_G = V_{DD} \frac{R_1}{R_1 + R_2}$
- $i_D = k(V_G - V_{TH})^2$
- Bias around $\frac{V_{DD}}{3}$
- Pick gain, $A = \frac{R_D}{R_S + \frac{1}{g_m}}$

Differential Amplifier

- Two port model
- $\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$

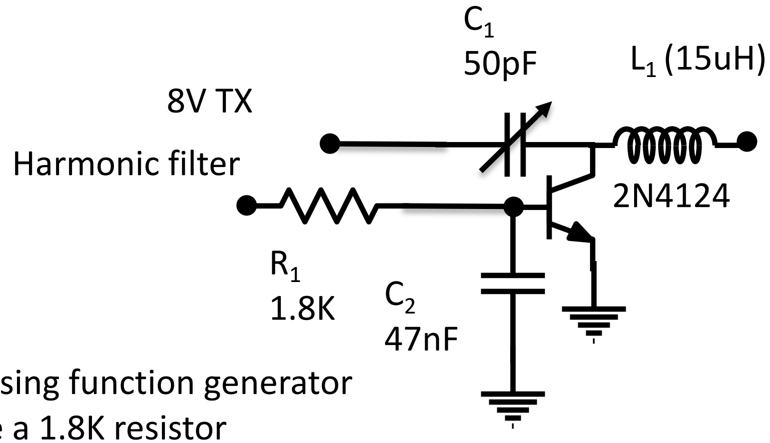
Differential amplifier



- Pick power ∓ 12
- Choose collector current ($2mA$) by picking R_1
- Pick gain, $A = \frac{R_C}{2R_E}$

Exercise 19: Norcal receiver switch

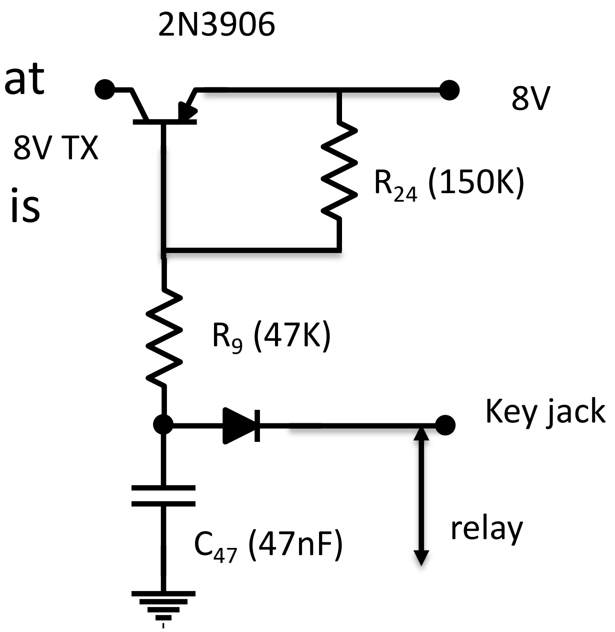
1. Consider the rising part of the base voltage waveform. Calculate slope.
2. Do the same for the falling part for voltage below .6V. Calculate t_2.
3. Measure switch attenuation
4. Measure the voltage with the switch on. Measure output voltage and calculate on-off rejection ratio $R=20\log(V_{off}/V_{on})$
5. Find the saturation resistance R_s.
6. Calculate the expected attenuation



If using function generator
use a 1.8K resistor

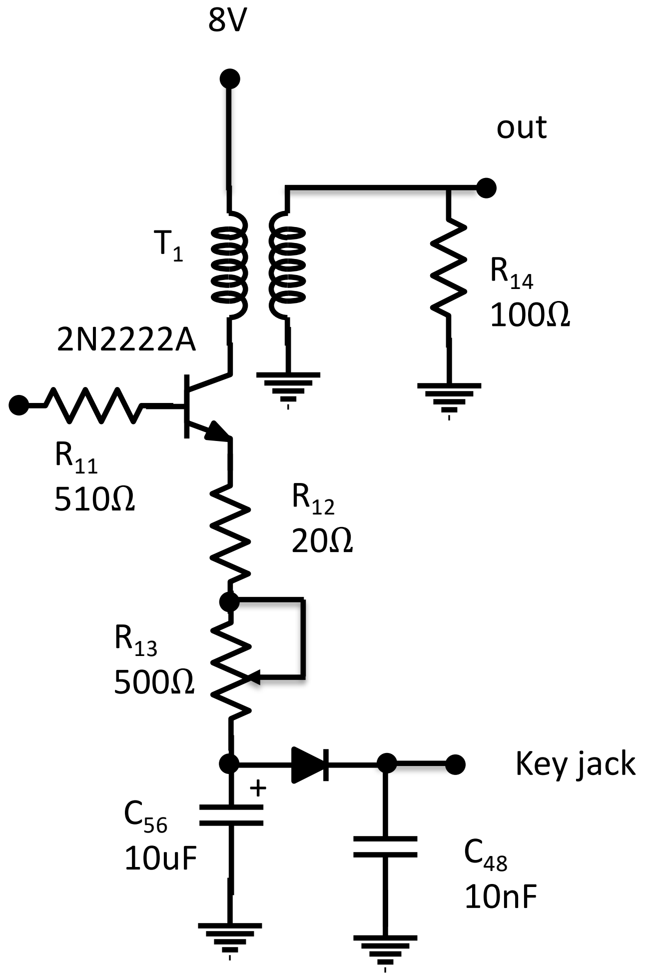
Exercise 20: NorCal transmitter switch

1. Calculate voltage on C_{57}.   Measure time for capacitor to charge half-way. Calculate what the time should be.
2. Calculate the approximate current ic when Q4 is on. Assume base voltage on Q1 is700 mV. Neglect saturation voltage on Q4. Calculate base current i_b required to produce this collector current assuming $\beta = 100$.
3. Calculate i_b at key down assuming a 700 mV dropin base-emitter of Q4 and at 600mV at D11
4. Sketch collector voltage at Q4 showing where transistor is saturated. What is the delay in going active?
5. Use the delay to measure β.



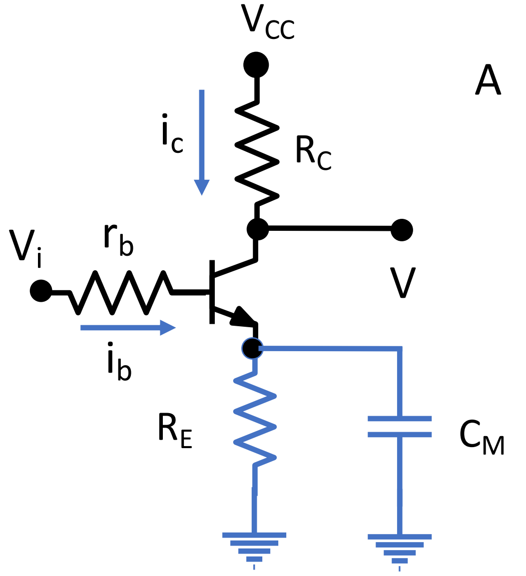
Exercise 21: Norcal Driver

1. Measure the voltage gain $G_v = \frac{v}{v_i}$ with R13 at minimum and maximum gain.
2. Calculate expected voltage gain at each setting.
3. 560ohm source resistance $V_o = 2V$

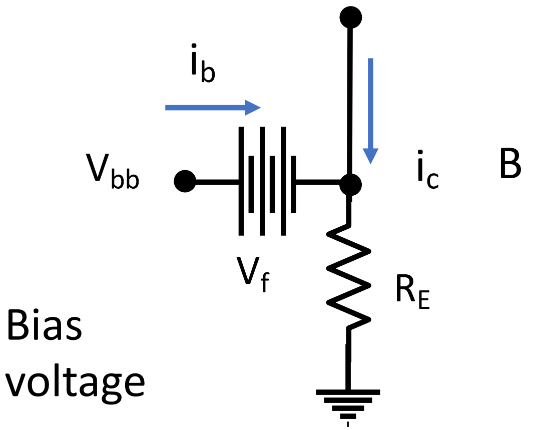


Emitter degeneration

- To the usual transistor circuit (A), on the right, we add R_E. (B) is an equivalent circuit.
- $V_{bb} \approx V_f + i_c R_E$. Let V be the output AC and V_i be the input AC, then the gain is $G = \frac{V}{V_i}$.
- $V_i = i_b r_b + i_E R_E \approx i_C R_E$, $Z_i = \frac{V_i}{i_{b,}}$,
- $V = -i_c R_C$. So $G_v = -\frac{R_C}{R_E}$ (Doesn't depend on β).
- $V_i \approx \beta i_b R_E$ and $Z_i = \frac{V_i}{i_{b,}}$, so $Z_i = \beta R_E$.
- C_M is called a Miller capacitor, $i_m = j\omega(V_i - V) = j\omega C_M(1 + |G_v|)V_i$
- So with the Miller capacitor, $Z_i = \beta R_E || (1 + |G_v|)C_M$
- $r_c \approx \frac{V_{early}}{i_c}$, r_c is the collector resistance
- $R_s{}' = R_S + r_b$, r_b is the base resistance
- $z_c = r_c || C_C$, C_c is specified in data sheet (8pF), z_C is the collector impedance
- $Z_o = \frac{V}{i_C}$, $i = i_c - \beta i_b$, $i_b = -\frac{i_C R_S}{R_S{}' + R_E}$, $i = i_c(1 + \frac{\beta R_E}{R_S{}' + R_E})$
- $V = i z_C + i_C\,(R_s{}' || R_E)$
- $Z_o = \frac{V}{i_C} = z_C\left(1 + \frac{\beta R_E}{R_S{}' + R_E}\right) + R_s{}' || R_E$.
- $|z_c| \gg R_E$, so $Z_o = z_C\left(1 + \frac{\beta R_E}{R_S{}' + R_E}\right)$



Adding R_E (and (C_M)



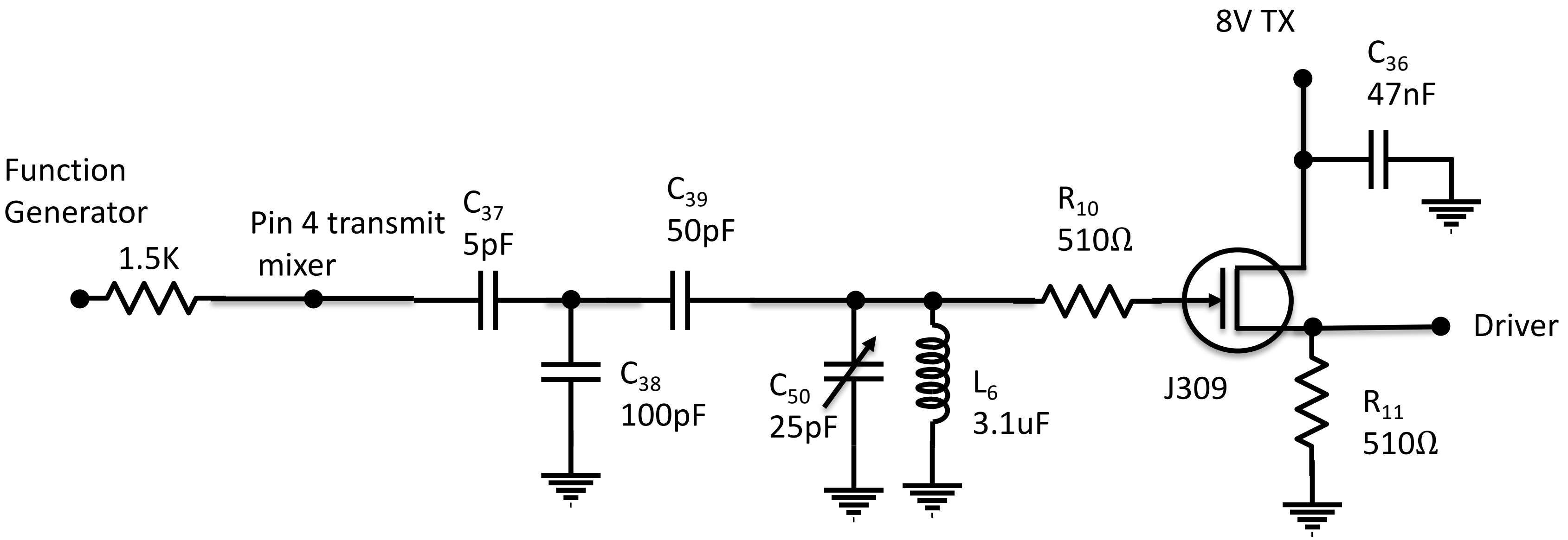
Exercise 22: Emitter degeneration

- In Driver amplifier, add probe to R_{11}, this allows us to measure the AC voltage, V_i

1. Measure $G_v = \frac{V}{V_i}$ with R_{13} turned fully counterclockwise
2. Calculate the expected voltage gain for each setting
3. Measure V_i at the maximal gain setting
4. The open circuit voltage is $V_0 = 2V$, calculate V_i in terms of C_M

Exercise 23: Norcal Buffer amplifier

1. Measure the DC voltage at source of the JFET
2. Calculate the source and drain voltages you should expect
3. Measure the voltage gain
4. Find the transconductance you should expect
5. Calculate the available power P_+ from the function generator through a 1.5Kohm load. Calculate gain in in dB

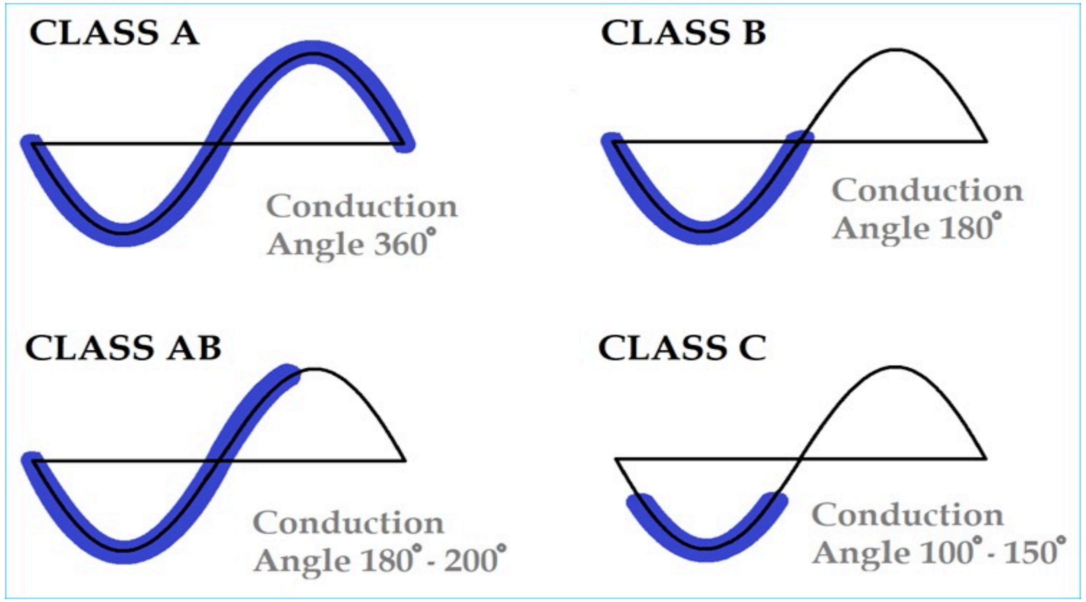


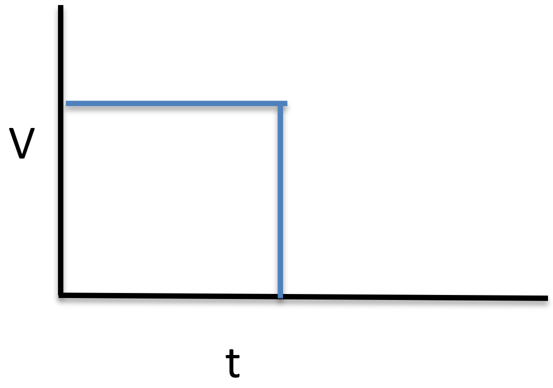
Amplifier classes

Class	Efficiency	Characteristics
A	35%	Full bias
B	60%	Low bias
C	75%	Saturating
D	75%	Switch in pass-band
E	90%	Voltage switch
F	80%	Harmonic resonators

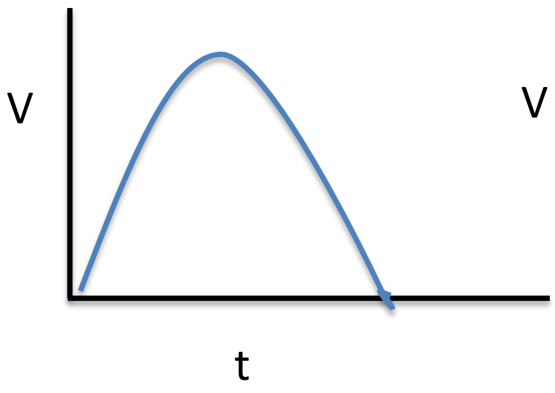
$$\eta = \frac{P}{P_0}, P_d = P_0 - P_i$$

$$P_d = P_a + P_{on}$$

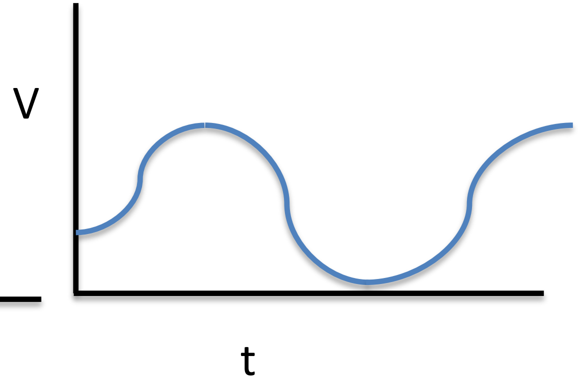




Class D



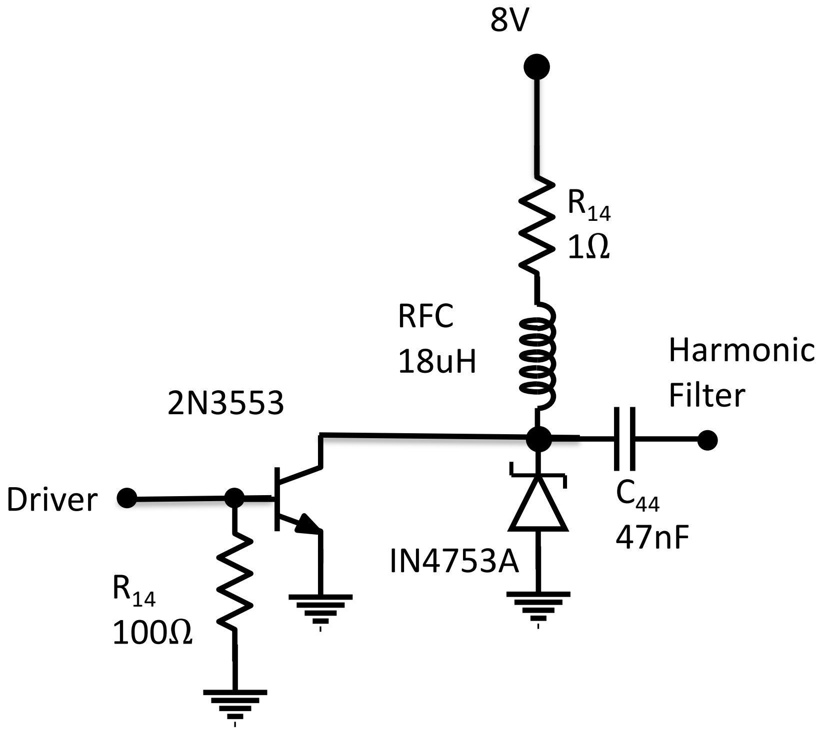
Class E



Class F

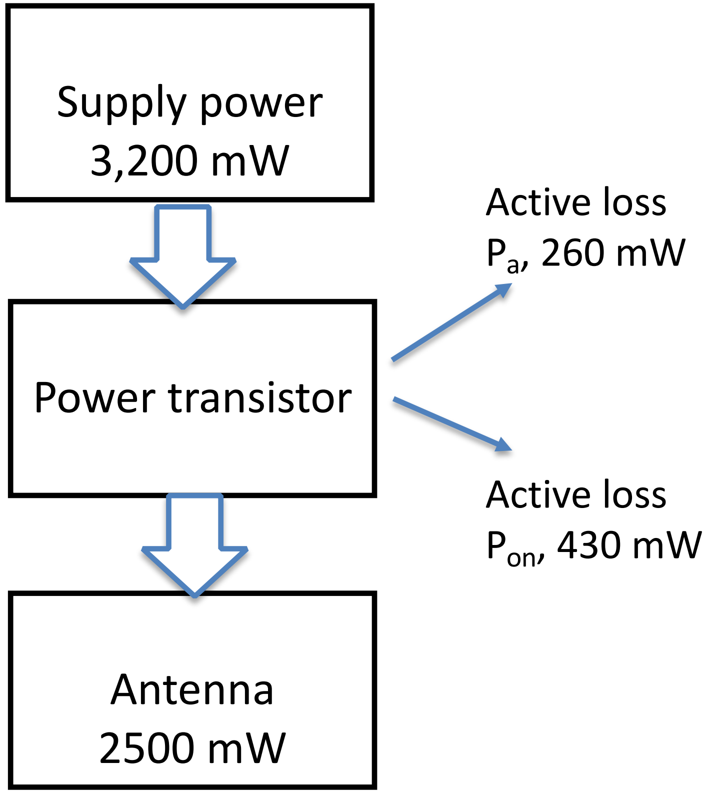
Exercise 24: Norcal Power Amp

Norcal-40 Power amp is class C



- $R_t = \frac{T - T_0}{P_d}$
- T_0 is ambient temperature, T is heat sink temperature

1. Calculate pp across 50ohm load required for output of 2W
2. Find pp output voltages or 5, 10, 15, 20, 25 and 30V. Calculate power supply current subtracting 2mA for regulator
3. Plot efficiency $\eta = \frac{P}{P_0}$. Plot dissipated power $P_d = P_o - P_i$



Thermal modelling

- T is heat sink temperature, T_0 is ambient temperature, P_d is power dissipated.
- $R_t = \frac{T - T_0}{P_d}$, R_t is the thermal resistance
- $C_t\dot{T} = P_d$, C_t is the thermal capacitance
- $R_j = \frac{T_j - T}{P_d}$, T_j is the junction temperature
- $f(t) + \tau \dot{f(t)} = f_\infty, f(t) = f_0 e^{-\frac{t}{\tau}}$
- $P_d = \frac{T(t) - T_0}{R_t} + C_t \dot{T(t)}, \tau = C_t R_t, T_\infty = P_d R_t + T_0$
- $T(t) + \tau \dot{T(t)} = T_\infty, \tau = C_t R_t.$
- $T_\infty = P_d R_t + T_0$
- $T(t) = T_\infty - P_d R_t e^{-\frac{t}{\tau}}$
- $T_j(t) = T(t) + R_j P_d$

Exercise 25: Thermal modelling

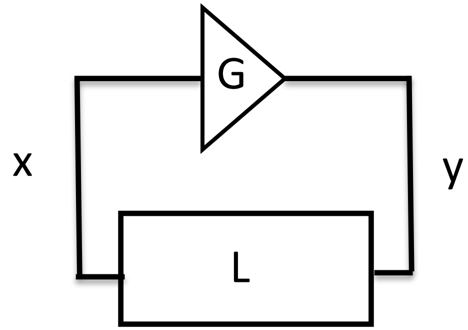
- For Motorola 2N3553, $T_j = 25\ {}^{C}/_{W}$

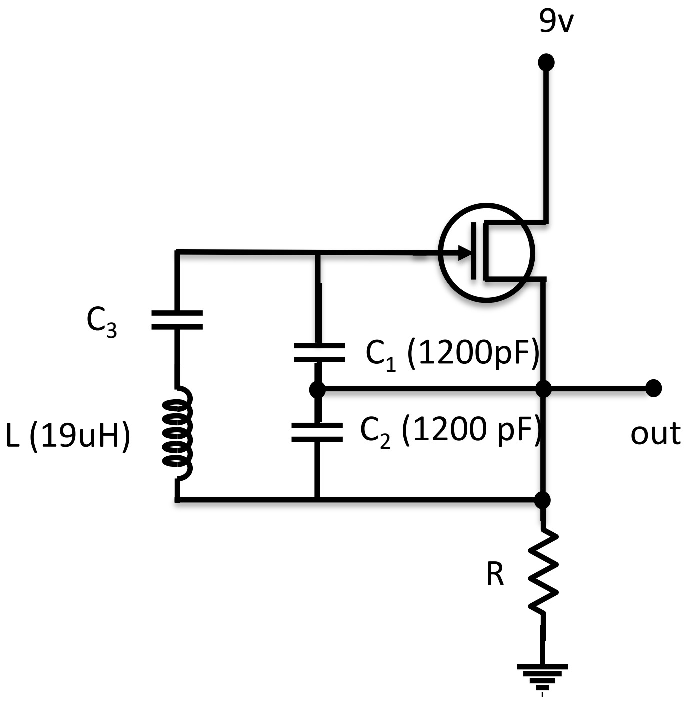
1. Measure ambient temperature
2. Turn function generator until output is 30Vpp
3. After 20 minutes, measure T_∞. Use this to calculate R_t and T_j
4. Plot heat sink temperature vs time. Measure t_2 and calculate C_t

Clapp oscillator

- Oscillation condition
  - $Gx = y$
  - Ly=x
  - $|G| = |L|$ and $\angle G = \angle L$

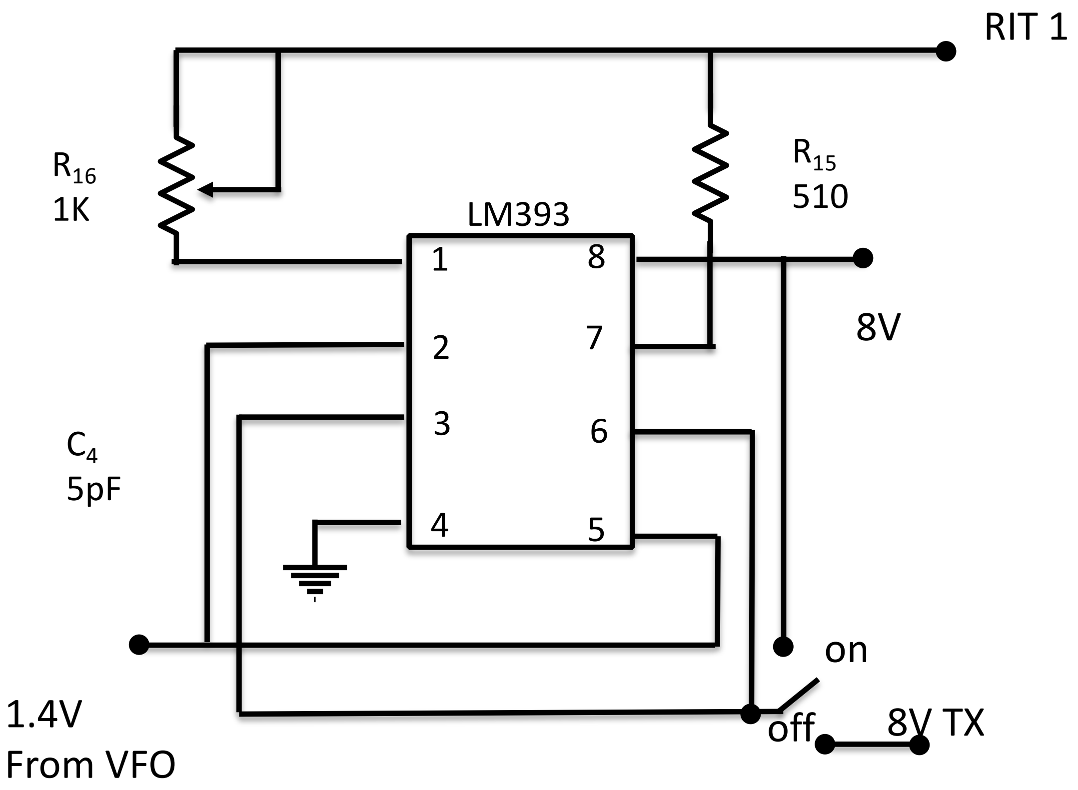
- Clapp (circuit on right)
  - $i_d = g_m v_{gs}$
  - Resonance: $-\frac{1}{j\omega_0 C_2} = j\omega_0 L + \frac{1}{j\omega_0 C_3} + \frac{1}{j\omega_0 C_1}$
  - $\omega_0 = \frac{1}{\sqrt{LC}}$, $C = C_1||C_2||C_3$
  - At resonance, $v_{gs} = Ri_d \frac{C_1}{C_2}$, $L = \frac{C_1}{RC_2}$
  - Oscillation continues if $g_m > \frac{C_1}{RC_2}$
  - $v_{gs} = 2v_s$





Norcal Receiver Incremental Tuning (RIT)

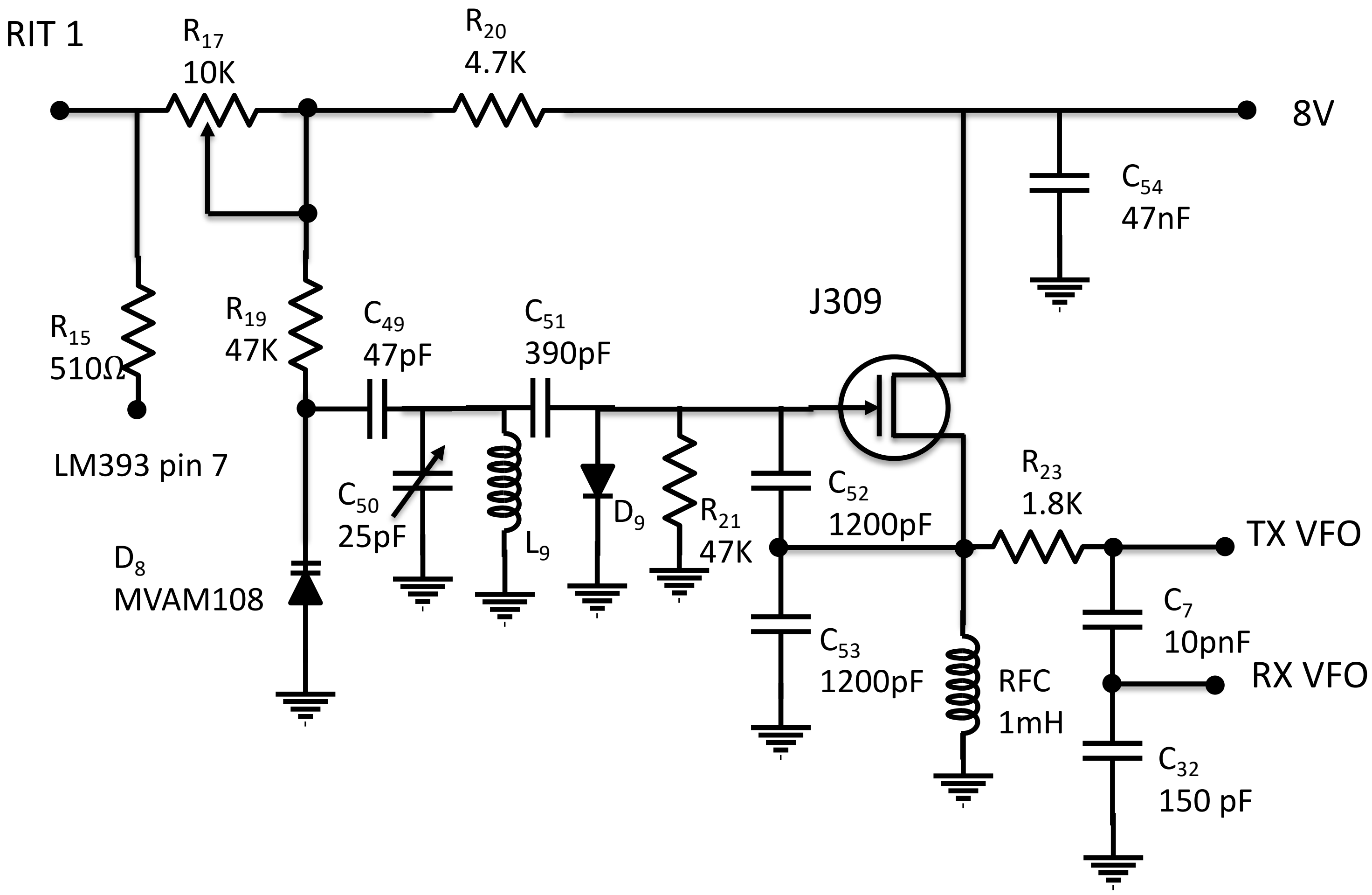
- LM393 is a comparator
- For function generator connect through 1.5K



Exercise 26: Norcal VFO

- L9: T68-7 62 turns
- Check MVAM108 capacitor when R_{17} is high and low
- Start resistor (R_{21}) pulls gat to ground at start
- When gain limiting diode (D9) conducts, it pulls gate negative
- Oscillator keeps growing as long as g_m>1/R

1. Measure DC voltage across wiper in R17
2. Calculate expected V for large signal oscillation
3. How does this change if we consider the inductor and source-drain resistance
4. How does the frequency change as R17 changes?
5. Calculate the oscillation frequency and the loss ratio $|V/V_l|$

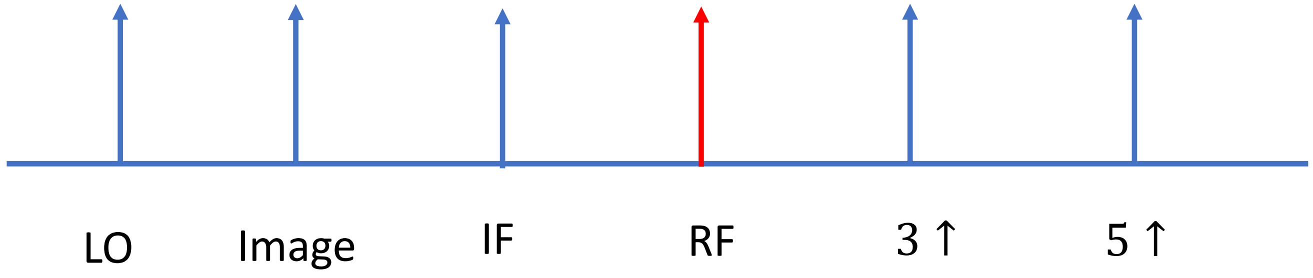


Exercise 27: Gain limiting

1. Measure the voltage, V, on R23
2. In deriving the oscillation condition, we neglected the inductor resistance and drain source resistance, r_d. How does this affect the conditions. L9 has a Q of 250 and $r_d = 5k\Omega$, now what is the predicted V.
3. Find the loss ratio $|\frac{V}{V_i}|$ and calculate what it should be.
4. Measure the temperature dependence of the VFO
5. How much does the temperature have to change to cause a 100Hz shift?
6. What is the oscillation change if we remove one turn of the inductor
7. What is the RIT tuning range?

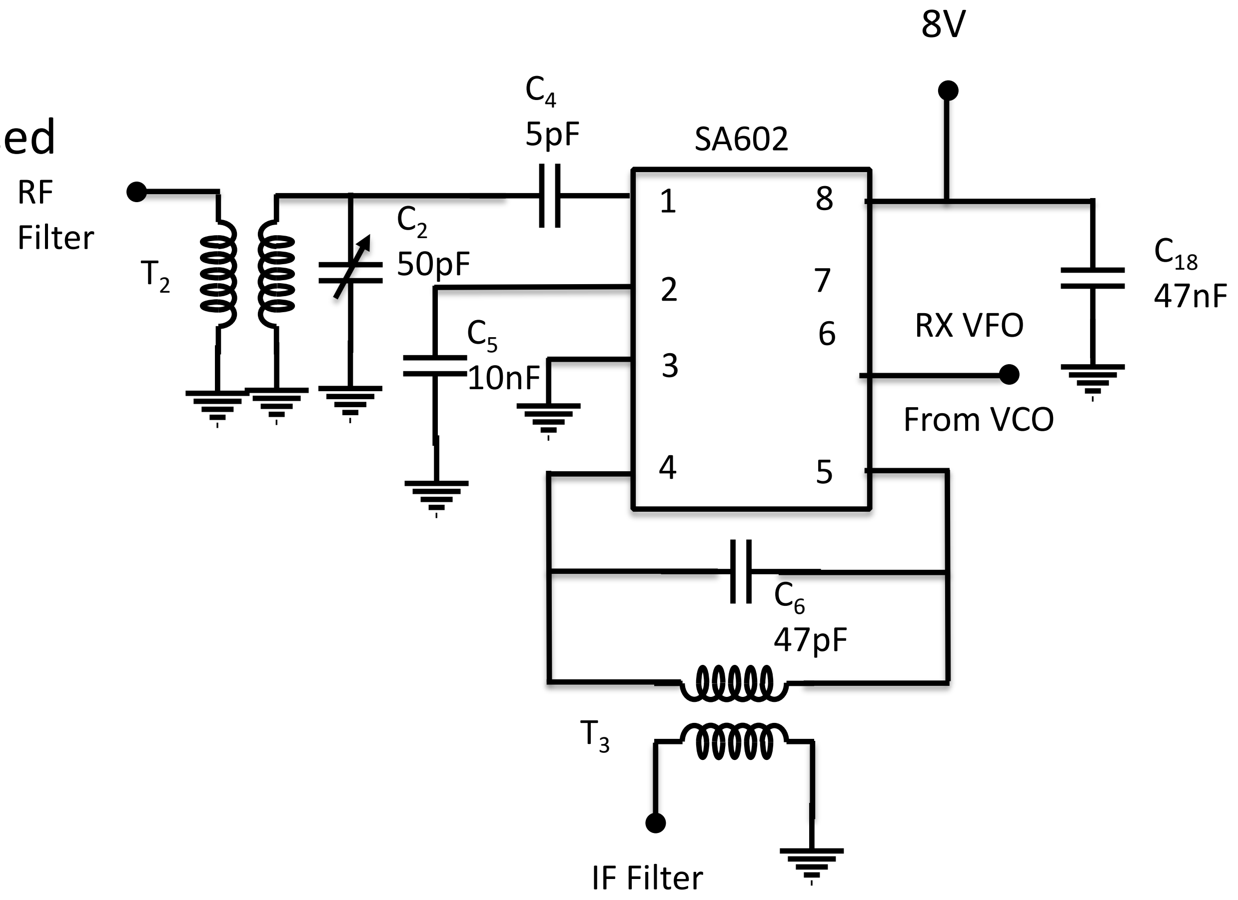
Mixers

- $V_{lo}(t)$ is a square wave with period ω_{lo}. Expanding this in a Fourier series, we get:
- $V_{lo}(t) = \frac{4}{\pi}(\cos(\omega_{lo}t) - \frac{\cos(3\omega_{lo}t)}{3} + \frac{\cos(5\omega_{lo}t)}{5} \ldots), V_{rf}(t) = V_{rf}\cos(\omega_{rf}t)$
- $V_{lo}(t)V_{rf}(t) = \frac{2V_{rf}}{\pi}(\cos(\omega_{-}t) - \frac{\cos(3\omega_{-}t)}{3} + \frac{\cos(5\omega_{-}t)}{5} \ldots) + \frac{2V_{rf}}{\pi}(\cos(\omega_{+}t) - \frac{\cos(3\omega_{+}t)}{3} + \frac{\cos(5\omega_{+}t)}{5} \ldots)$
- $\omega_{+} = \omega_{lo} + \omega_{rf}$ and $\omega_{-} = |\omega_{lo} - \omega_{rf}|$
- We define $\omega_{k+} = (k\omega_{lo} + \omega_{rf})$ and $\omega_{k-} = |k\omega_{lo} - \omega_{rf}|$ and $V_{k+}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k+}t)$ and $V_{k-}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k-}t)$
- $\omega_i = \omega_{if} - \omega_{lo}$ and $\omega_{if} = \omega_{if} + \omega_i$, ω_i is a spurious signal. ω_{k+} and ω_{k-} are the spurs from the kth harmonic



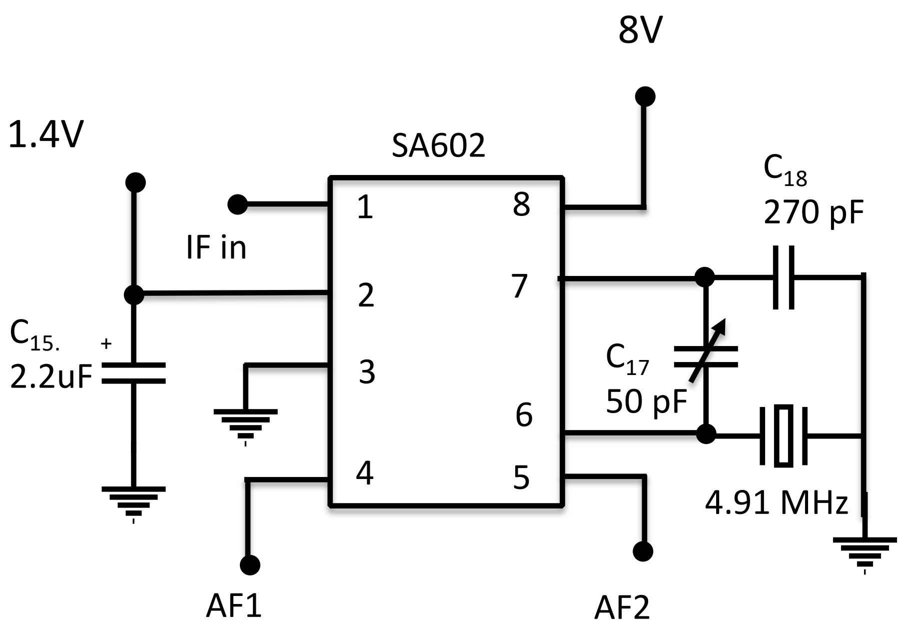
Exercise 28: Norcal RF Mixer

1. Measure conversion gain of the Mixer.
2. How much attenuation is provided by pot?
3. By how many dB is the image response suppressed



Exercise 29: Norcal Product Detector

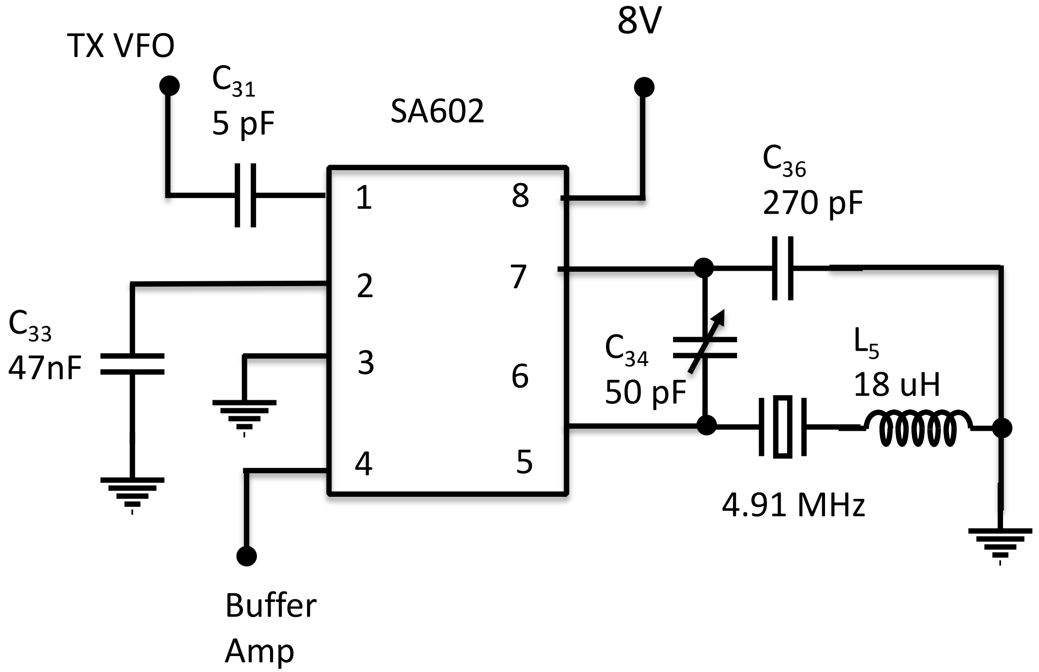
1. Adjust C17 for minimum oscillation frequency and record it
2. Calculate the minimum oscillation frequency you'd expect
3. Measure the temperature coefficient for the BFO
4. Measure the gain through the receiver from the antenna through the product detector
5. Find the f5 spur calculate the expected f3
6. By how much is the if spur suppressed



- 620 Hz output through AF1 and AF2

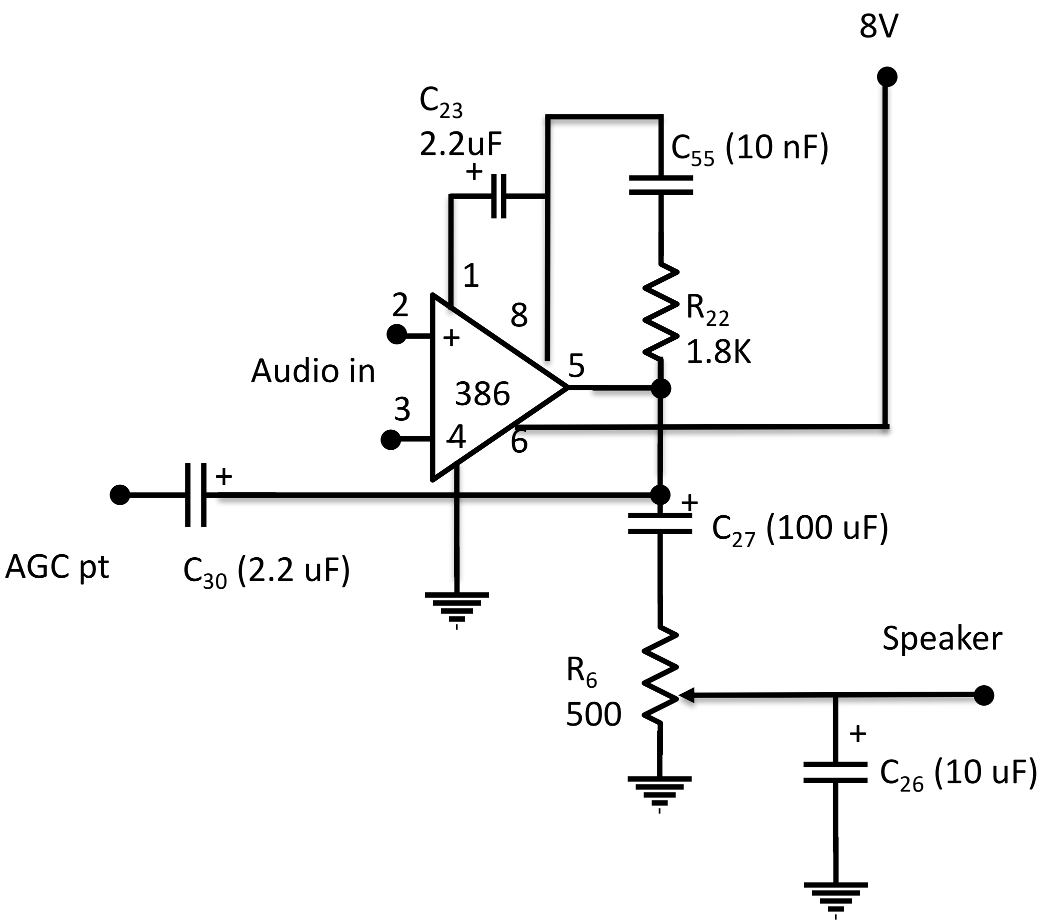
Exercise 30: Norcal transmit mixer and oscillator

1. How much would you expect the inductor to lower the oscillation frequency
2. Use the TX VFO and the voltage attenuation to calculate the input power from the transmit mixer. Calculate the gain through the entire chain
3. Measure the rise and fall time of keying response
4. There is a spurious $f_{mn}=mf_{vfo}+nf_{to}$.



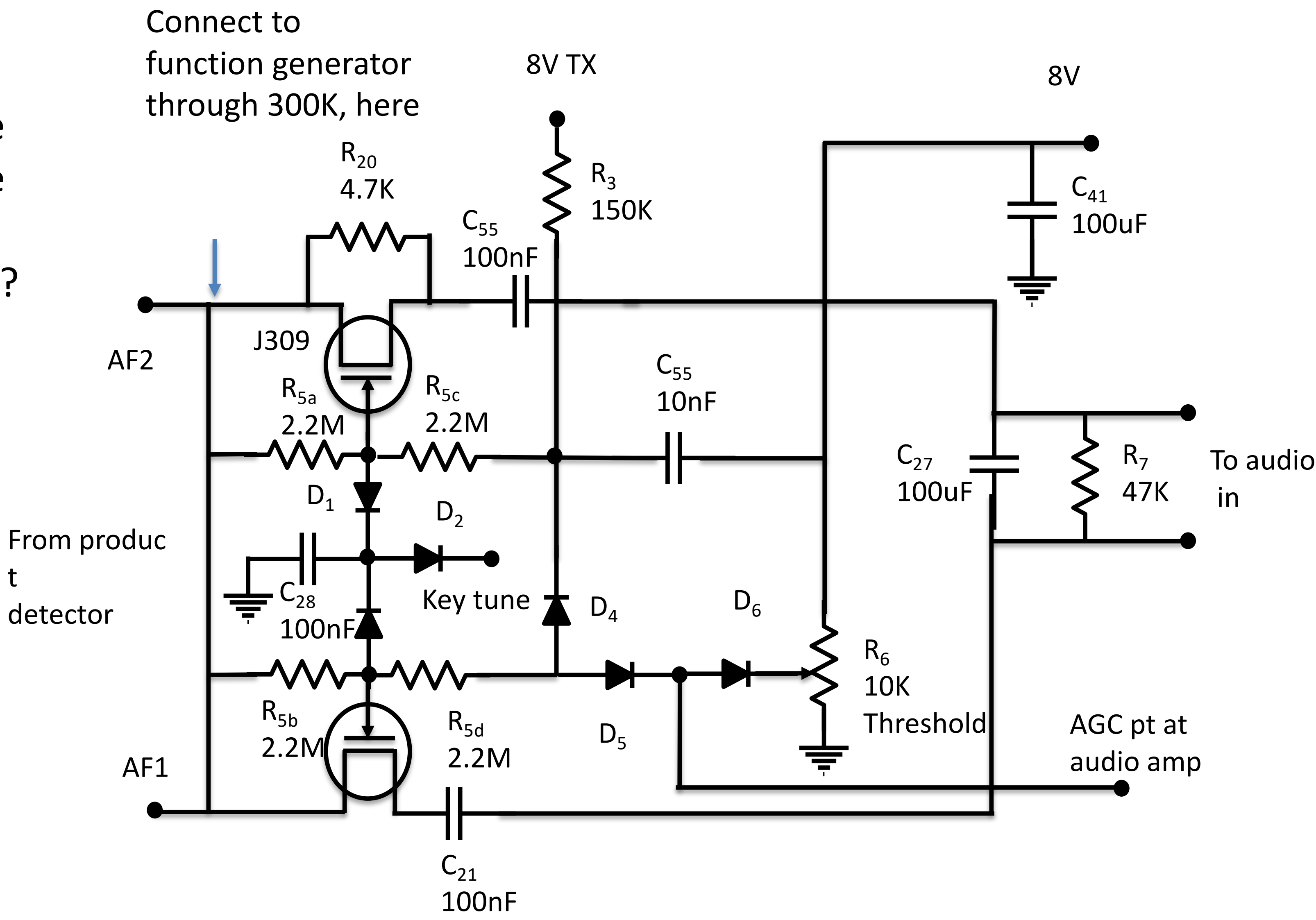
Exercise 31: Norcal Audio Amp

1. Calculate input V_i assuming very high input impedance
2. Measure the voltage gain Gv at high frequency and 3dB rolloff

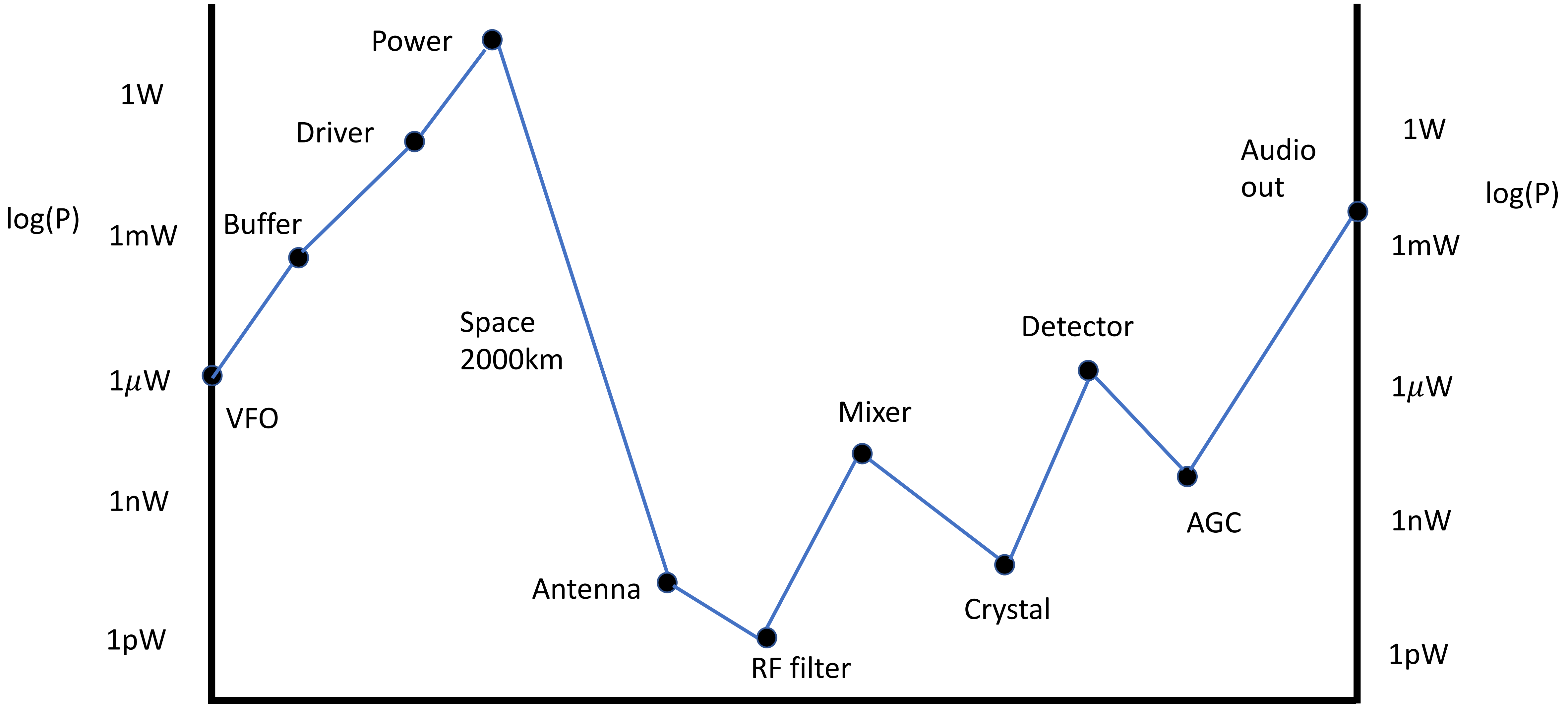


Exercise 32: Norcal AGC

1. Plot audio output v dc control
2. What is the maximum control voltage we can measure? Infer cutoff voltage V_c
3. What is the minimum control voltage?

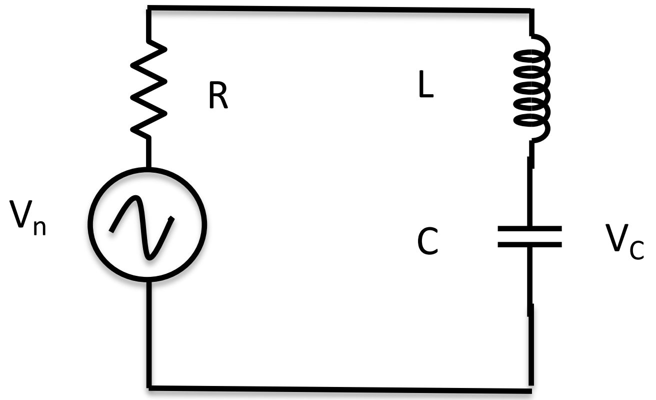


NorCal power levels



Noise

- $V_{n(rms)} = \sqrt{\frac{1}{\tau}\int_0^{\tau} V(t)^2\ dt}$
- $P_n = \frac{{V_{n(rms)}}^2}{R}$, R is load resistance
- $SNR = \frac{P}{P_n}$
- $MDS = \frac{P_n}{G}$
- $P_n = NB$, N is noise power density, B is bandwidth
- $NEP = \frac{N}{G}$

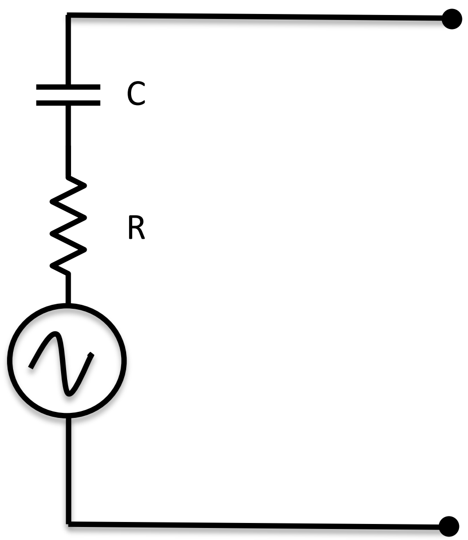


- Nyquist
  - $V_C = \frac{1}{j\omega C}\frac{V_n}{R+j\omega L+\frac{1}{j\omega C}}$
  - $\overline{|V_c|^2} = \frac{\overline{|V_n|^2}}{|1-\omega^2 LC+j\omega RC|^2}$
  - Expected energy at resonance is $kT = \frac{C}{2}\int_0^{\infty}|V_c|^2 df$, by equipartition theorem
  - $\int_0^{\infty}\frac{1}{|1-\omega^2 LC+j\omega RC|^2}df = \frac{1}{4RC}$
  - So, $\overline{|V_n|^2} = 8\mathrm{kTR}$
  - $N = kT = \frac{|\frac{V_n}{2R}|^2}{2R}$
  - $T_c = \frac{N}{k}$, $T_n = \frac{NEP}{k}$, $V_{rms} = \sqrt{4kTR}$

Antennas

- From Maxwell, for a plane wave (E in x direction, H in y direction), wave is of form $\exp(j\omega t - j\beta z)$
- $\nabla \times E = -j\mu_0 \omega H$
- $\nabla \times B = j\epsilon_0 \omega E$
- $\beta \hat{z} \times E = \mu_0 \omega H, \beta E_x \hat{y} = \mu_0 \omega H$
- Substituting and taking the restricted cross products, we get: $\beta E_x = \omega \mu_0 \frac{\omega \epsilon_0}{\beta}$, so $\beta = \omega \sqrt{\mu_0 \epsilon_0}$
- Power density: $S = Re\left(\frac{E_x \overline{H_y}}{2}\right) = \frac{(|E_x|)^2}{2\eta_0}$
- $\eta_0 = \frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$
- Impedance: $P_t = \frac{R|I|^2}{2}$, R is real part of Z, $R = R_r + R_l, \eta = \frac{R_r}{R}$
- Power density for isotropic antenna: $S_i = \frac{P_t}{4\pi r^2}$
- Define $G(\theta, \phi) = \frac{S(\theta,\phi)}{S_r}$. $S(\theta, \phi)$ is just the Poynting vector

Receiving antenna Thevenin



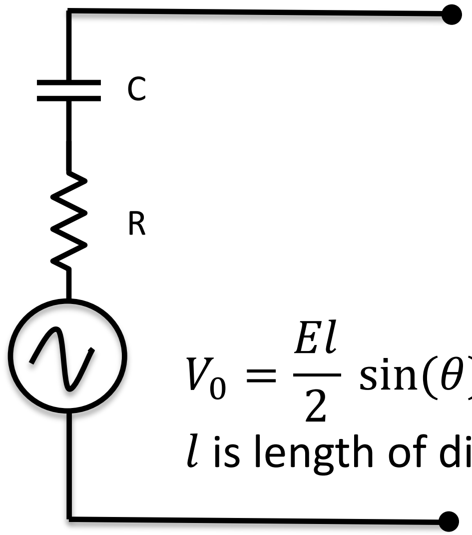
Transmitting Antenna

- Define $G(\theta, \phi) = \frac{S(\theta,\phi)}{S_r}$. $S(\theta, \phi)$ is just the Poynting vector
- For isotropic reference: $S_i = \frac{P_t}{4\pi r^2}$, $G = \frac{4\pi r^2 S}{P_t}$
- $\int G \, d\Omega = 4\pi$

Receiving Antenna

- $V_0 = hE$, h is effective antenna length ($h = \frac{l}{2}$ for short antenna)
- For dipole: $V_0 = \frac{l}{2}\, E \sin(\theta)$
- $A(\theta,\phi) = \frac{P_r}{S(\theta,\phi)}$. This is the definition of the effective area, A.
- By reciprocity, $A(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)$
- $P_r = \frac{|V_0|^2}{8R_a} = \frac{|hE|^2}{8R_a}$, so
- $P_r = \frac{h^2 S \eta_0}{4R}$
- $A = \frac{h^2 \eta_0}{4R}$

Dipole Thevenin equivalent circuit



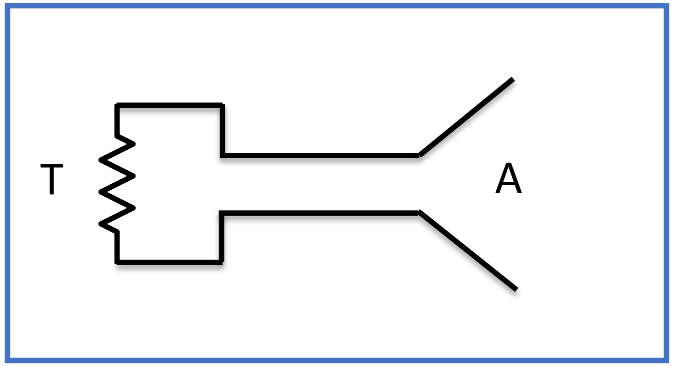
$V_0 = \frac{El}{2} \sin(\theta)$

l is length of dipole

Friis, blackbody and Antenna Theorem

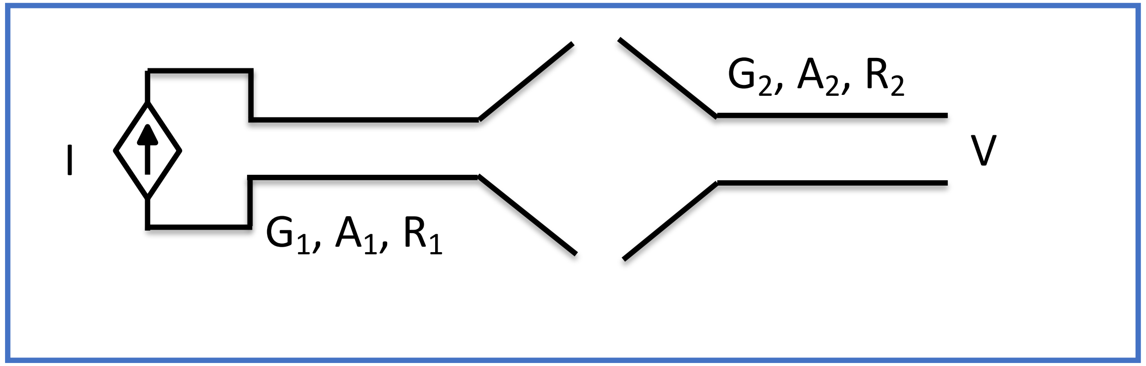
- For transmitting/receiving antenna pairs: $G_1 A_2 = \frac{|V|^2 \pi r^2}{|I|^2 R_1 R_2} = G_2 A_1$. So $\frac{G_1}{A_1} = \frac{G_2}{A_2} = \frac{4\pi}{\lambda^2}$
- $S = \frac{P_t G}{4\pi r^2}$
- $P_r = SA = \frac{P_t G A}{4\pi r^2}$. --- Friis radiation formula
- For us, $G = 1, A = 150\, m^2, r = 2000km, P_t = 2W$
- $P_r = 6pW$
- Antenna theorem: $\oint A \, d\Omega = \lambda^2$
- For cavity on right, T is constant at thermodynamic equilibrium and the same power is transmitted and emitted, the Johnson noise is kT. The energy received is
  - $E = \frac{4\pi kT}{c\lambda^2}$.
  - Set $B = \frac{kT}{\lambda^2}$.
  - $kT = \oint BA \, d\Omega = \oint A \frac{kT}{\lambda^2} \, d\Omega$, which gives the antenna theorem

Insulated cavity



Reciprocity

- *Reciprocity:* The position of an ideal voltmeter and ideal current source can be interchanged without changing the voltmeter reading.
- $\frac{G}{A} = \frac{4\pi}{\lambda^2}$
- $\frac{G_1}{A_1} = \frac{G_2}{A_2}$



Reciprocity and dipoles

- For dipole (Length: $l = \frac{\lambda}{2}$)
- $\lambda^2 = \int A \, d\Omega = \int \frac{h^2 \eta_0}{4R_r} \, d\Omega$, so
- $R_r = \frac{l^2 \eta_0}{16\lambda^2} \int sin^2(\theta) d\Omega = \eta_0 \frac{\pi}{6} (\frac{l}{\lambda})^2$
- $A = \frac{3\lambda^2}{8\pi} sin^2(\theta)$ and $G = 1.5 \, sin^2(\theta)$. . Note we used $h = \frac{l}{2} \sin(\theta)$
- $\frac{|V|^2}{8R_2} = \frac{|I|^2 R_1 G_1 A_2}{8\pi r^2}, G_1 A_2 = G_2 A_1$
- $P_t = \frac{|I|^2 R_1}{4\pi r^2}, P_t = \frac{|V|^2}{8R_2}$
- $P_r = \frac{P_t G_1 A_2}{4\pi r^2}$

Exercise 35: Intermodulation

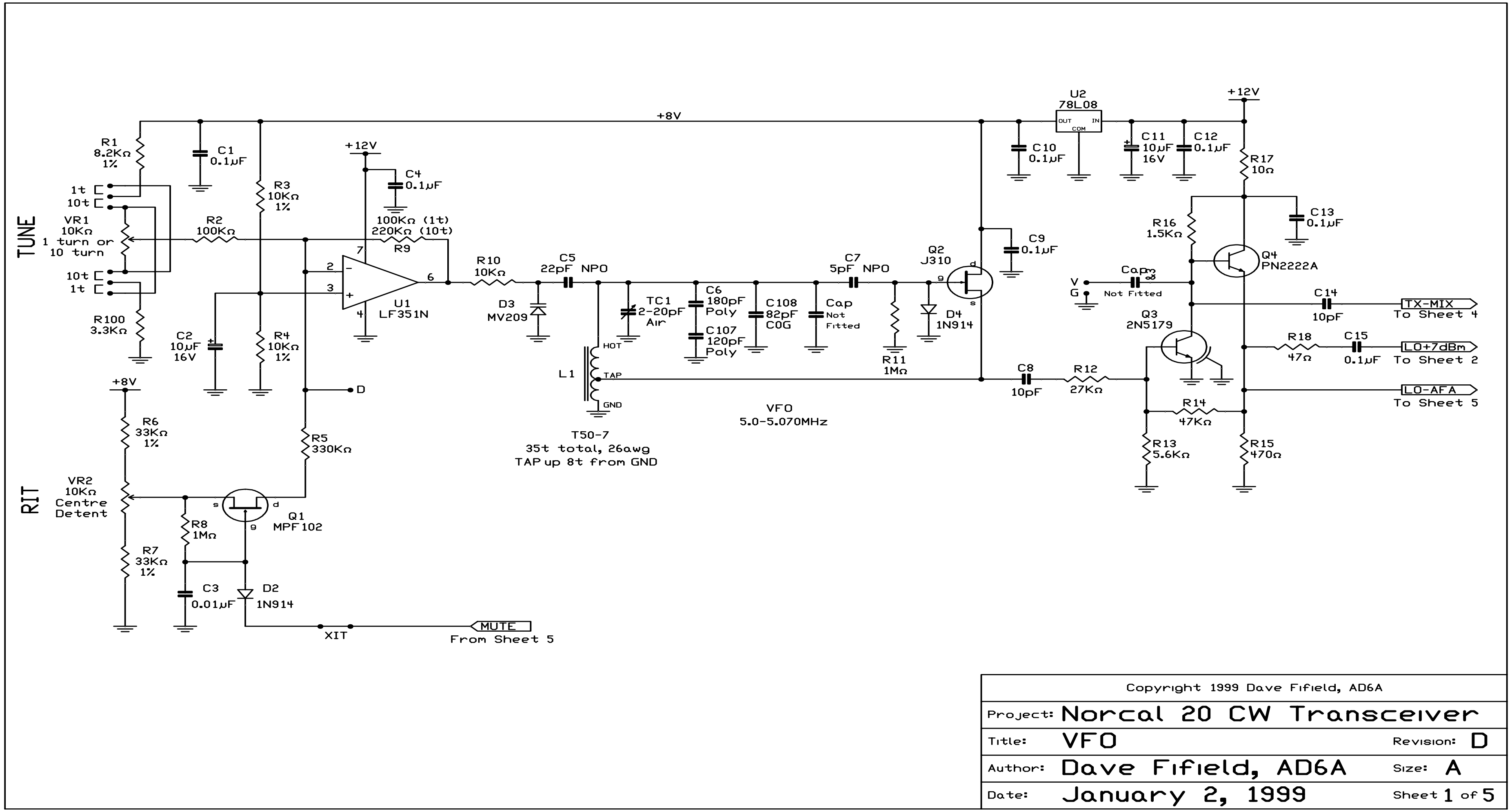
- Only $f_{3\uparrow} = 2f_1 - f_2, f_{3\downarrow} = 2f_2 - f_1, f_{5\uparrow} = 3f_1 - 2f_2\ and\ f_{5\downarrow} = 3f_2 + 2f_1$ are close enough to the rf frequency to matter for intermodulation

1. Find coefficients and frequencies for $[\cos(\omega_1 t) + [\cos(\omega_2 t)]^5$
2. Find $f_{3\uparrow}$, $f_{3\downarrow}$, $f_{5\uparrow}$ and f_1
3. Find the MDS and the antenna limited MDR

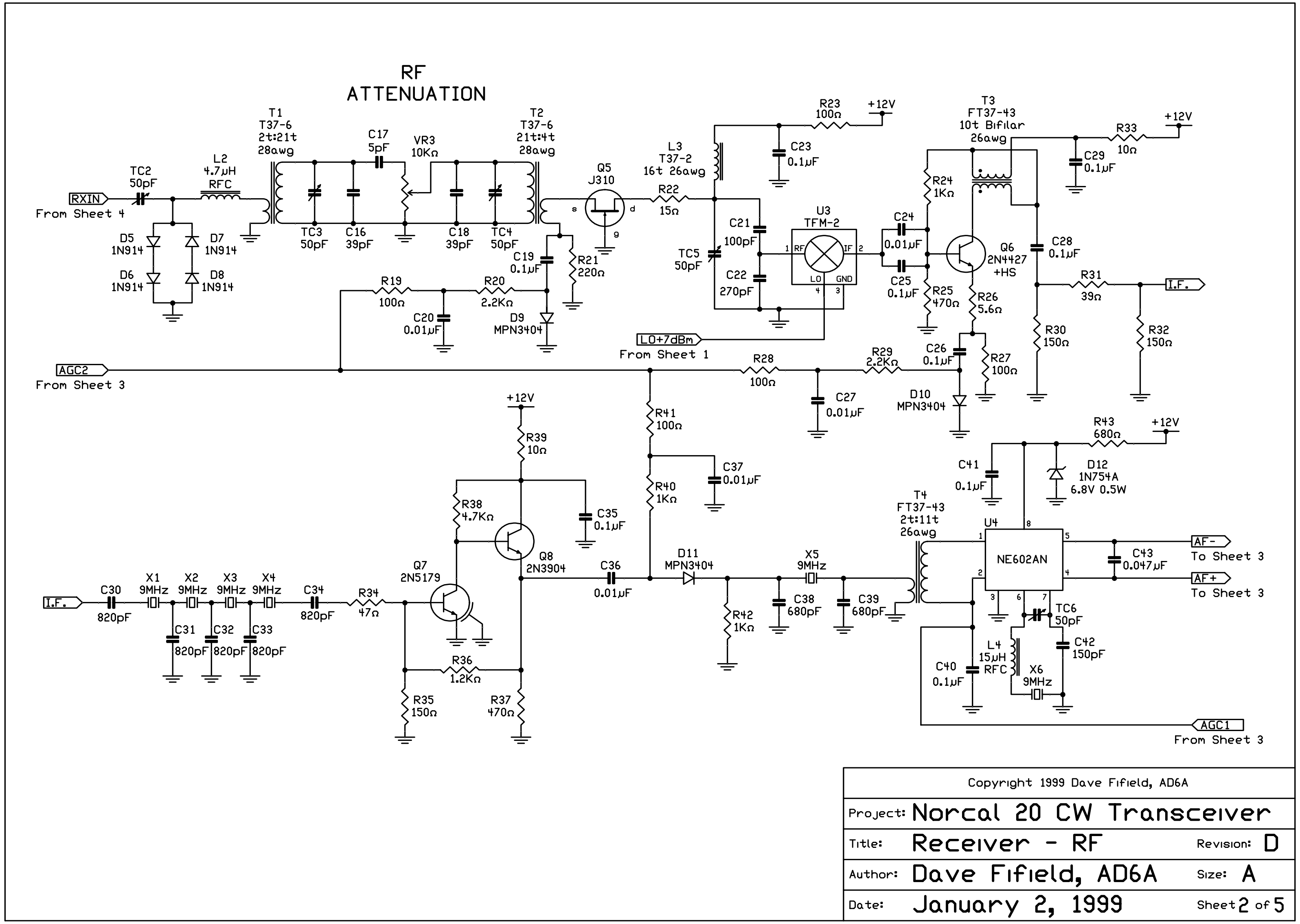
Exercise 37: Antennas

1. Use the relation between gain and effective area to rewrite the Friis transmission formula in terms of gain only. Consider UHF for airplanes. If the frequency makes the quarter length stub antenna have gain 2, find the maximum possible LOS at 10km height. Required receiver power is –90 dBm. Find the minimum transmission power.
2. Find the inductance to resonate with a 3m whip. Assuming the Q of the coil is 200, find the turns ratio required to give a transceiver a 50 ohm load. What is the radiation efficiency?
3. Repeat 2 with a capacitive end loading, assuming the capacitance doubles.

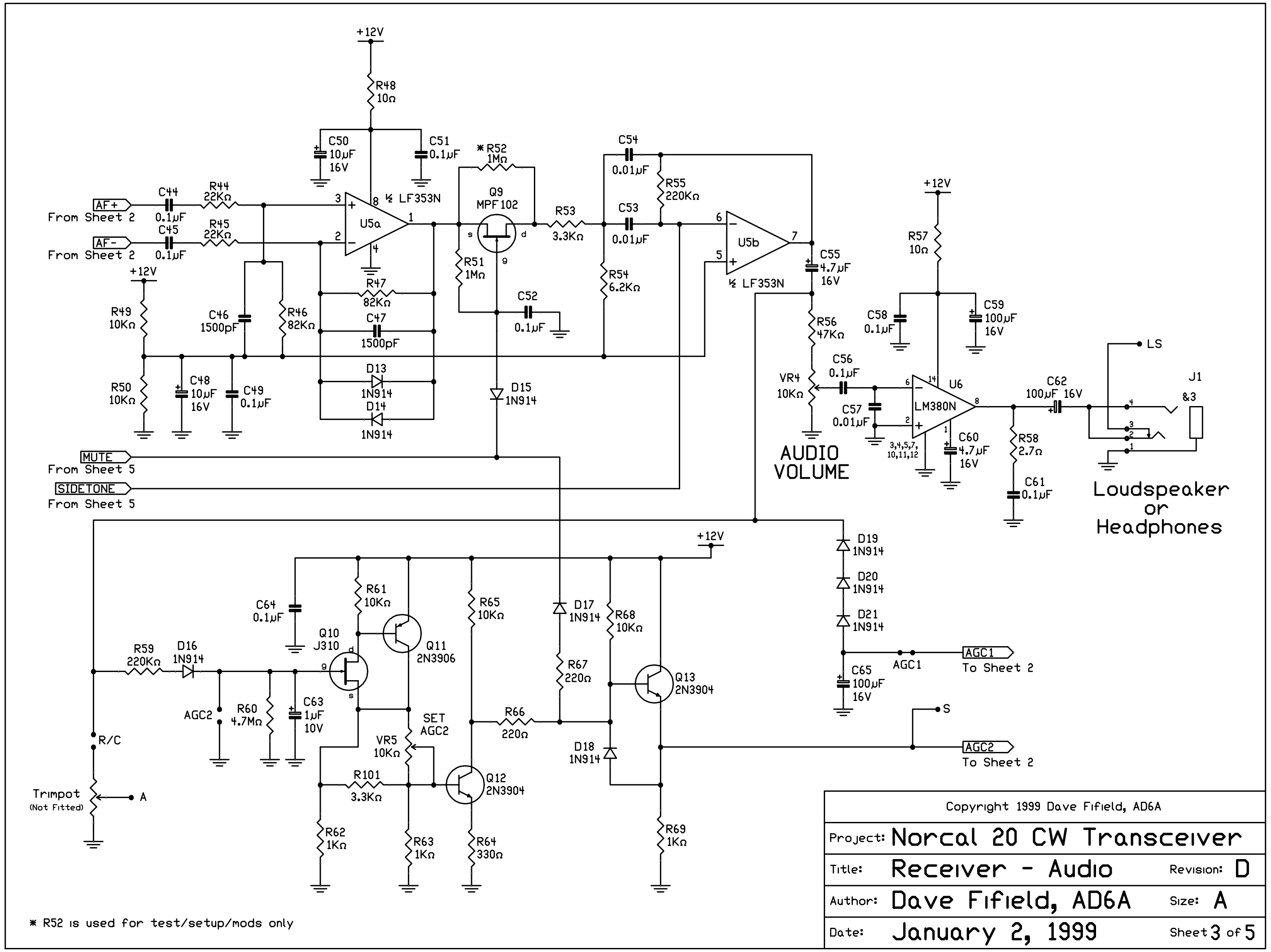
Norcal circuit diagram, 1



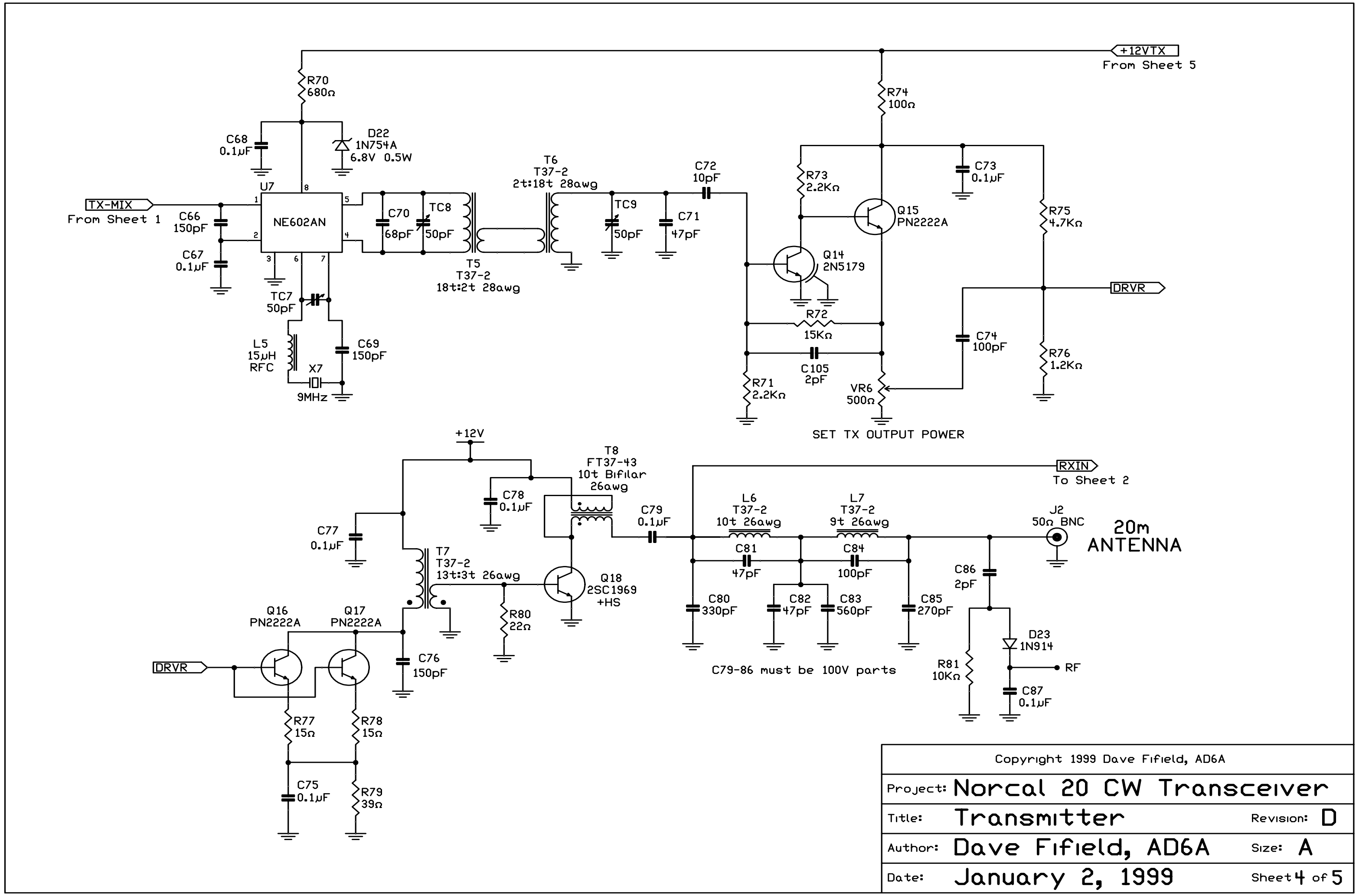
Norcal circuit diagram, 2



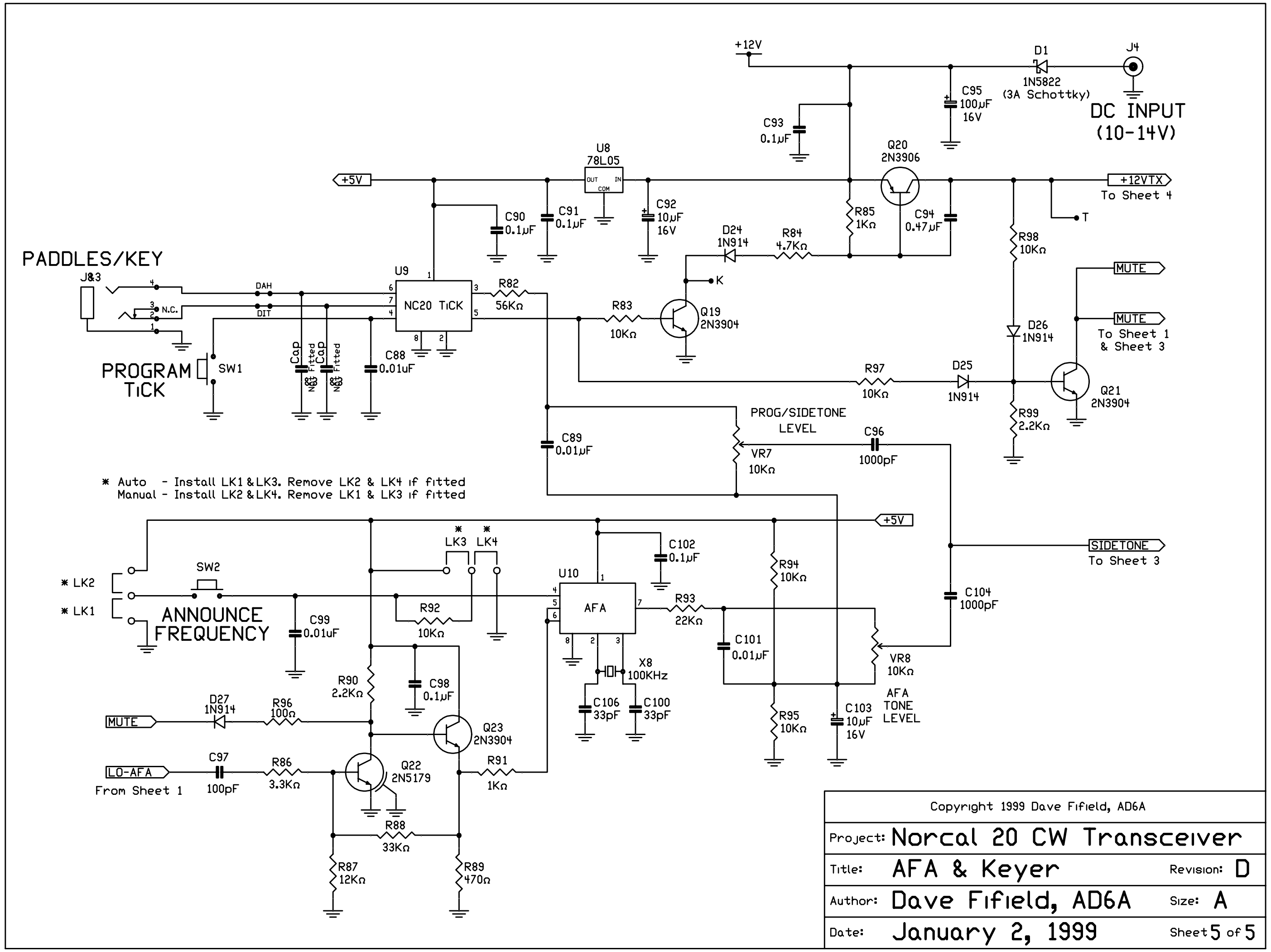
Norcal circuit diagram, 3



Norcal circuit diagram, 4



Norcal circuit diagram, 5

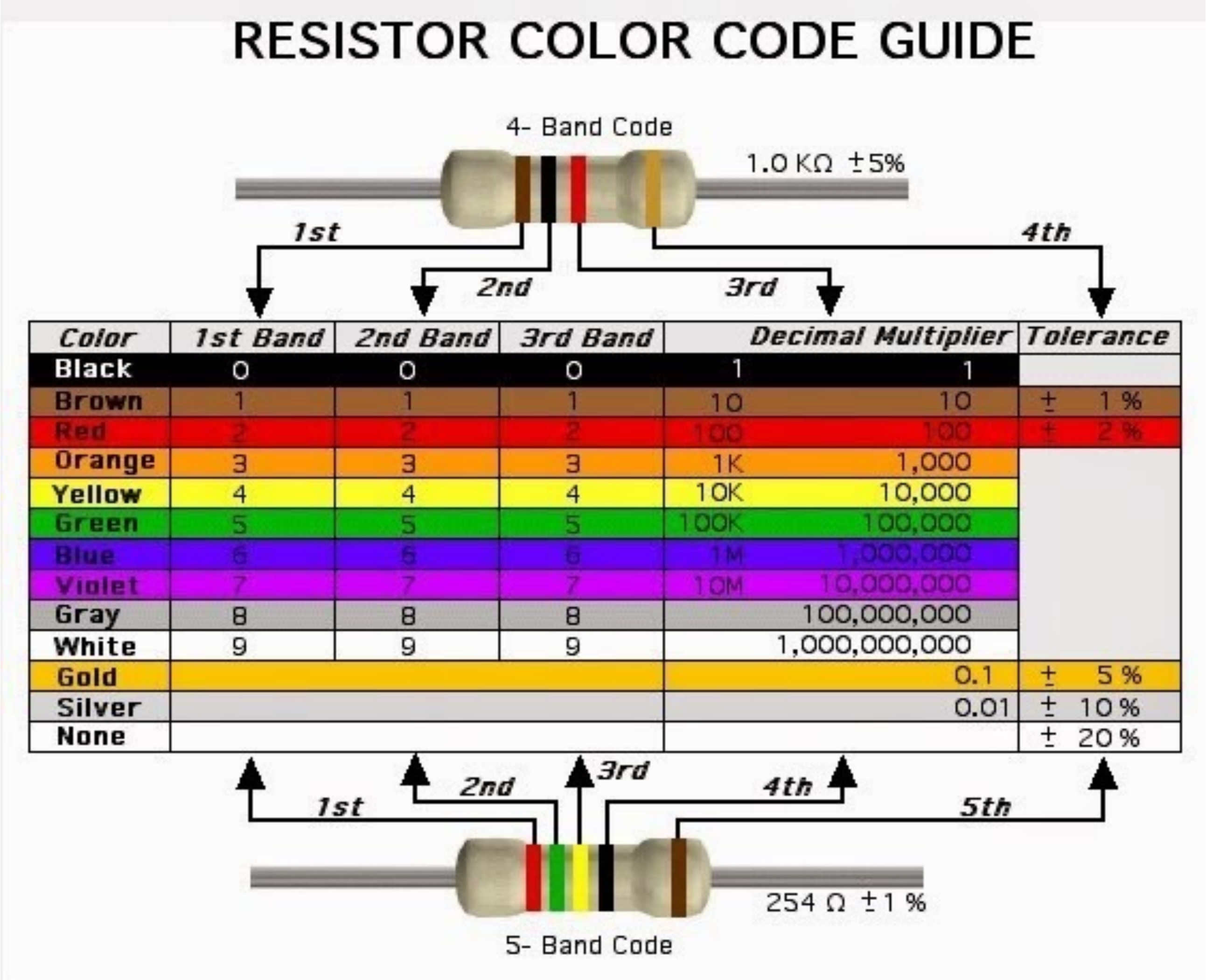


Appendix

Morse

Symbol	Code	Symbol	Code	Symbol	Code
a	._	m	__	y	_.__
b	_...	n	_.	z	__..
c	_._.	o	___	0	_____
d	_..	p	.__.	1	.____
e	.	q	__._	2	..___
f	.._.	r	-_.	3	...__
g	__.	s	...	4	...._
h	....	t	_	5	.....
i	..	u	.._	6	_....
j	.___	v	..._	7	__...
k	_._	w	.__	8	___..
l	._..	x	_.._	9	____.

Color codes



Color	1st Band	2nd Band	3rd Band	Decimal Multiplier	Tolerance
Black	0	0	0	1 1	
Brown	1	1	1	10 10	± 1 %
Red	2	2	2	100 100	± 2 %
Orange	3	3	3	1K 1,000	
Yellow	4	4	4	10K 10,000	
Green	5	5	5	100K 100,000	
Blue	6	6	6	1M 1,000,000	
Violet	7	7	7	10M 10,000,000	
Gray	8	8	8	100,000,000	
White	9	9	9	1,000,000,000	
Gold				0.1	± 5 %
Silver				0.01	± 10 %
None					± 20 %

- Resistors: ohms
- Capacitors: picoFards
- Inductors: milliHenries

In the beginning)

- The laws of EM according to Clerk Maxwell are:
  1. $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$
  2. $\nabla \cdot \boldsymbol{B} = 0$
  3. $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
  4. $c^2\, \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$, $\epsilon_0 = 8.854 \times 10^{-12}\ \frac{C^2}{N{-}m^2}$, $\frac{1}{c^2} = \epsilon_0 \mu_0$
  5. $\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$
- Here **E** is the electric field, **B** is the magnetic field, **j** is the current density through a closed surface, c is the speed of light and ρ is the charge density at a point. The rest of classical physics, including special relativity, is:
  - Newton-Einstein: $\boldsymbol{p} = m\boldsymbol{v}$, $m = \frac{m_0}{\sqrt{1-(\frac{v}{c})^2}}$, $\boldsymbol{F} = m\frac{d\boldsymbol{p}}{dt}$.
  - Gravity: $\boldsymbol{F} = -\frac{Gm_1 m_2}{r^2}\ \boldsymbol{u}_r$ where $\mathbf{u_r}$ is the unit vector from m_1 to m_2 and **F** is the force on m_2.

Solutions to the wave equation

- The solution of $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = -s$ is $\psi(x,y,z,t) = \frac{S(t-\frac{r}{c})}{4\pi r}$ where $S = \int_V s\, dV$
- Later, we will use this to find the ”general” solution to Maxwell’s equations
  - $\phi(r_1,t) = \int_{V_2} \frac{\rho(r_2, t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0|r_1-r_2|} dV_2$ and $\mathbf{A}(r_1,t) = \int_{V_2} \frac{\mathbf{j}(r_2,\ t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0 c^2|r_1-r_2|} dV_2$, where
  - $B = \nabla\times A$, $E = -\nabla\phi - \frac{\partial A}{\partial t}$, and, $c^2\nabla\cdot A = -\frac{\partial\phi}{\partial t}$
- You are not expected to have guessed this answer
- To do this, we’ll need the “BAC-CAB” identity: $A\times(B\times C) = B(A\cdot C) - \mathrm{C}(A\cdot B)$
- When we apply this to $\nabla\times(\nabla\times\mathbf{A})$, we get $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot A) - \nabla^2 A$

General solution to Maxwell's equations

- Returning to the general Maxwell equations, from $\nabla \cdot B = 0$, we get $B = \nabla \times A$
- Substituting into $c^2\, \nabla \times \mathbf{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \mathbf{E}}{\partial t}$, we get $c^2\, \nabla \times (\nabla \times A) = \frac{j}{\epsilon_0} + \frac{\partial E}{\partial t}$
- Applying "BAC-CAB), we get $\nabla\, (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0} + \frac{1}{c^2} \frac{\partial E}{\partial t}$ (Equation 1)
- Now, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, so substituting for B, we get $\nabla \times \left(\mathrm{E} + \frac{\partial A}{\partial t} \right) = 0$ and so $E = -\nabla \phi - \frac{\partial A}{\partial t}$
- Substituting into equation 1, $\nabla\, (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0} + \frac{1}{c^2} \frac{\partial}{\partial t} (-\nabla \phi - \frac{\partial A}{\partial t})$, or
- $\nabla\, (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0}$ - $\frac{1}{c^2}\, \nabla \frac{\partial \phi}{\partial t} - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2}$.
- $\nabla^2 A$ - $\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2 \epsilon_0} + \nabla [\frac{1}{c^2} \frac{\partial \phi}{\partial t} + (\nabla \cdot A)]$
- Now if A and ϕ give $B = \nabla \times A$ and $E = -\nabla \phi - \frac{\partial A}{\partial t}$, then $A' = A + \nabla \varphi$ and $\phi' = \phi - \frac{\partial \varphi}{\partial t}$ give $B = \nabla \times A'$ and $E = -\nabla \phi' - \frac{\partial A'}{\partial t}$, for any function φ

General solution to Maxwell's equations

- Thus, we can pick a solution $(A, \phi)$with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$. Then we get
- $\nabla^2 A$ - $\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2 \epsilon_0}$
- Substituting for $E = -\nabla\phi - \frac{\partial A}{\partial t}$ into $\nabla \cdot \mathrm{E} = \frac{\rho}{\epsilon_0}$, we get
- $\nabla \cdot (\nabla\phi) + \frac{\partial\, \nabla\cdot A}{\partial t} = -\frac{\rho}{\epsilon_0}$, or $\nabla \cdot (\nabla\phi) - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$, or $\nabla^2\phi$ - $\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$
- The solutions are $\phi(r_1, t) = \int_{V_2} \frac{\rho(r_2, t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 |r_1 - r_2|} dV_2$ and
- $\mathbf{A}(r_1, t) = \int_{V_2} \frac{\mathbf{j}(r_2,\ t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 c^2 |r_1 - r_2|} dV_2$ with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$, with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$.

Solution to Maxwell's equations in free space

- Free space is defined by $\rho = 0$ and $j = 0,$ so our potentials satisfy
- $\nabla^2 A$ - $\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$ =0 and $\nabla^2\phi$ - $\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0$
- These have the usual wave equation solutions $\phi(x, y, z, t) = \ f(k \cdot r\ -\ \omega t)$, etc
- Thus, in free space ϕ and **A** and hence E and B propagate as waves.

Solution to Maxwell's equations in conductors

- In conductors, $\mathbf{j} = \sigma\mathbf{E}$
  - $c^2\,\nabla\times\mathbf{B} = \frac{\boldsymbol{j}}{\epsilon} + \frac{\partial\mathbf{E}}{\partial t} = \frac{\sigma}{\epsilon}\mathbf{E} + \frac{\partial\mathbf{E}}{\partial t}$
  - This becomes $c^2\frac{\partial(\nabla\times\mathbf{B})}{\partial t} = \frac{\sigma}{\epsilon}\frac{\partial\mathbf{E}}{\partial t} + \frac{\partial^2\mathbf{E}}{\partial t^2}$
  - $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, so we get $c^2\nabla\times\frac{\partial B}{\partial t} = -c^2\nabla\times(\nabla\times E) = \frac{\sigma}{\epsilon}\frac{\partial\mathbf{E}}{\partial t} + \frac{\partial^2\mathbf{E}}{\partial t^2} = -c^2[\nabla(\boldsymbol{\nabla}\cdot\mathbf{E}) - \nabla^2\mathbf{E}] = \boldsymbol{c^2\nabla^2}\mathbf{E}$ (since $\rho = 0$ in a conductor)
  - Applying the trial solution E= E_0 exp(ωt-kr), we get –k^2-i$\omega\mu\sigma$ + $\omega^2\mu\epsilon$ = 0.
  - Putting k = α-βi, $\alpha = \frac{\omega}{2}\sqrt{\mu\epsilon}\,(1 + \sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}})$ and $\beta = \frac{\omega\mu\sigma}{2\alpha}$.
  - For copper, σ = 5.78 × 107 $\boldsymbol{\Omega}$-m. This explains the “skin effect” in conductors.

Radiation, antennas

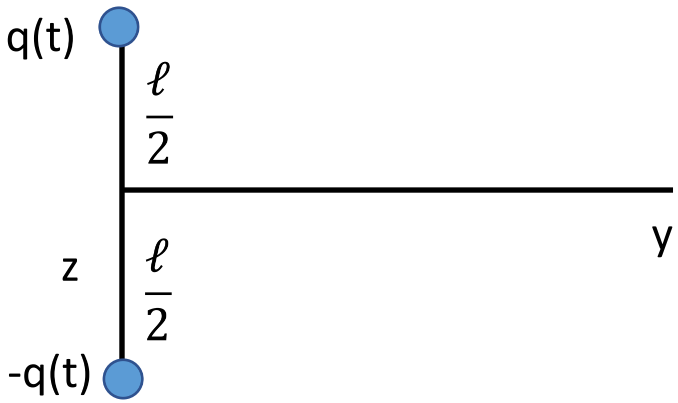
- Accelerating charges radiate energy in the form of electromagnetic waves (companion E and B fields).
- The radiation from accelerating charge q is $\mathbf{E}_{rad} = -\frac{1}{4\pi\epsilon_0 c^2}\frac{q}{r}\,\boldsymbol{a}_{\perp}(t - \frac{r_{12}}{c})$.
- Here, $\mathbf{a}_{\perp}$ is the acceleration $\perp$ to the line from r_1 to r_2.
- For example, applying a time varying potential $V_0\sin(\omega t)$ to an antenna will cause the antenna to radiate power since the voltage and hence charges affected accelerate within the antenna, that is, their positions have a non-zero second derivative. That’s how a transmitter “couples” to the antenna of a receiver. In the receiver, the radiated wave accelerates charges in the antenna replicating the original wave (at much reduced power).
- These simple radio waves are carrier waves of frequency $\frac{\omega}{2\pi}$. To transfer information (voice, images, binary data), we modulate carrier waves combining them with an “information source” signal. Receivers demodulate the incoming wave and recreate the original “information source” signal.
- We’ll talk about modulation in the section on software defined radios.

Maxwell’s equations in a non-dispersive media

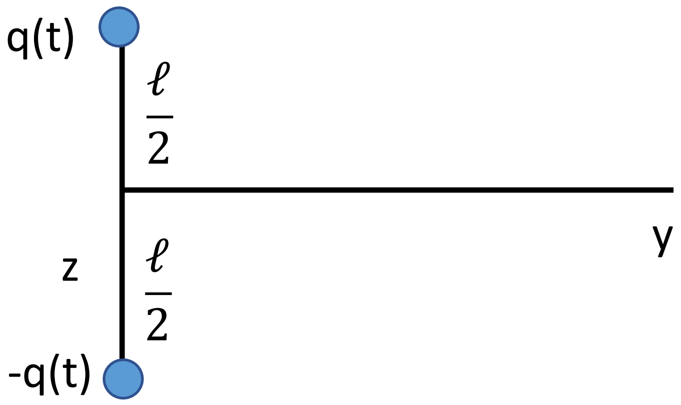
- $\mathrm{B} = \mu\mathrm{H},\ \mathrm{D} = \epsilon\mathrm{E}$
- $\nabla \cdot D = \rho$
- $\nabla \cdot B = 0$
- $\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}}$
- $\nabla \times \mathrm{H} = \mathrm{j} + \frac{\partial \mathrm{E}}{\partial \mathrm{t}}$
- $\nabla \cdot j = -\frac{\partial \rho}{\partial t}$

Radiation from a small dipole

- $A_z(r,t) = \frac{\mu_0}{4\pi} \int_{[\frac{-l}{2},\frac{l}{2}]} \frac{I(z',t-\frac{z'}{c}k)}{|r-z'k|}\, dz'$
- If $l \ll cT = \lambda$
- $A_z(r,t) = \frac{\mu_0}{4\pi} \frac{l}{r} I(z', t - \frac{r}{c})$
- Choosing gauge, $\nabla \cdot \mathrm{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$
- $\frac{\partial\varphi}{\partial t} = -\frac{l}{4\pi\epsilon_0}\frac{\partial}{\partial z}(\frac{1}{r}\, I\left(t-\frac{r}{c}\right)) = \frac{z}{r^2}(\frac{q(t-\frac{r}{c})}{r} - \frac{I((t-\frac{r}{c})}{c})$
- $q\left(t-\frac{r}{c}\right) = q_0 \cos\left(\omega\left[t-\frac{r}{c}\right]\right),\ I\left(t-\frac{r}{c}\right) = I_0 \sin(\omega\left[t-\frac{r}{c}\right])$



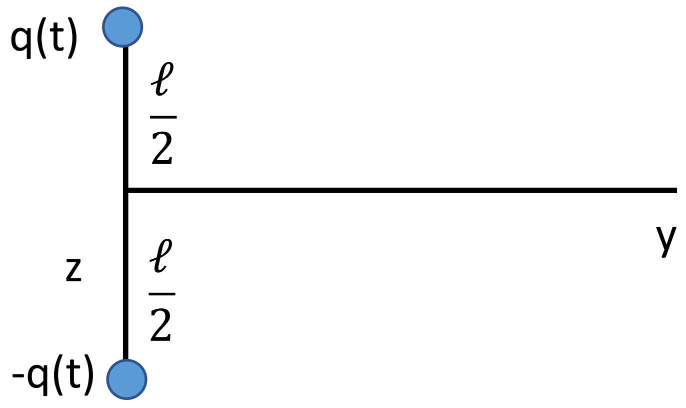
Radiation from a small dipole



- $\nabla^2 H - \epsilon\mu\frac{\partial^2 H}{\partial t^2} - \sigma\mu\frac{\partial H}{\partial t} = 0$
- $\nabla^2 E - \epsilon\mu\frac{\partial^2 E}{\partial t^2} - \sigma\mu\frac{\partial E}{\partial t} = 0$
- $A_r = \frac{\mu_0}{4\pi}\frac{I_0 l}{r}\cos(\theta)\sin(\omega\left[t - \frac{r}{c}\right])$
- $A_\phi = 0,\ A_\theta = -\frac{\mu_0}{4\pi}\frac{I_0 l}{r}\cos(\theta)\sin(\omega\left[t - \frac{r}{c}\right])$
- $B_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial\theta} = \frac{\mu_0}{4\pi}\frac{I_0 l}{r}\sin(\theta)[\frac{\omega}{r}\cos\left(\omega\left[t - \frac{r}{c}\right]\right) + \frac{1}{r}\sin(\omega[t - \frac{r}{c}])]$
- $E_r = -\frac{\partial\phi}{\partial t} - \frac{\partial A_r}{\partial t} = \frac{2lI_0\cos(\theta)}{4\pi\epsilon_0}[\frac{\sin(\omega\left[t - \frac{r}{c}\right])}{r^2 c} - \frac{\cos(\omega\left[t - \frac{r}{c}\right])}{\omega r^3}]$
- $E_\theta = \frac{-I_0\ l\sin(\theta)}{4\pi\epsilon_0}(\left[\frac{1}{r^3\omega} - \frac{\omega}{rc^2}\right]\cos\left(\omega\left[t - \frac{r}{c}\right]\right) - \frac{1}{cr^2}\sin\left(\omega\left[t - \frac{r}{c}\right]\right))$
- $E_\phi = -\frac{1}{r\sin(\theta)}\frac{\partial\varphi}{\partial\phi} - \frac{\partial A_\phi}{\partial t}=0$

Radiation from a small dipole

- $\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\mathbf{H}\frac{\partial \mathbf{B}}{\partial t} - \mathbf{E}\frac{\partial \mathbf{D}}{\partial t} - \mathbf{E} \cdot \mathbf{j}$, $\mathbf{S} = \mathbf{E} \times \boldsymbol{H}$, $\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} = -\mathbf{E} \cdot \mathbf{j}$ (u is energy density)
- $\int S \cdot dA = \frac{(l\, I_0\, \omega)^2}{6\pi\epsilon_0 c^3} \cos(\omega \left[t - \frac{r}{c}\right])^2$
- $P_{av} = \frac{(l\omega)^2}{6\pi\epsilon_0 c^3} \frac{{I_0}^2}{2} = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{l}{\lambda}\right)^2 \frac{{I_0}^2}{2}$
- $R_r = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{l}{\lambda}\right)^2$



Large half wave dipole

- For large half wave, add small dipoles to produce half wave antenna.

- $dE_\theta = I_0 \frac{\sin(\theta)}{4\pi\epsilon_0 Rc^2}\ \omega \cos(\omega) \cos\left(\frac{2\pi z\prime}{\lambda}\right)\ dz'$
- $dB_\phi = I_0 \frac{\mu_0 \omega}{4\pi Rc}\ \omega \cos\left(\omega\left[\left[t - \frac{r}{c}\right]\right]\right) \cos\left(\frac{2\pi z\prime}{\lambda}\right)\ dz'$
- $K = \int_{[-\frac{\pi}{2},\frac{\pi}{2}]} \frac{1}{R} \cos\left(t\ -\ \frac{R}{c}\right) \cos(u)\, du = \frac{1}{2\pi\epsilon_0 rc} \cos\left(\omega\left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin^2(\theta)}\ \ ,u = \frac{2\pi z\prime}{\lambda}$
- $E_\theta = I_0 \frac{1}{2\pi\epsilon_0 rc}\ \cos\left(\omega\left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin\ (\theta)}$
- $B_\phi = I_0 \frac{\mu_0}{2\pi r}\ \omega \cos\left(\omega\left[\left[t - \frac{r}{c}\right]\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin\ (\theta)}$
- $P_{av} = \frac{1}{4\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\ {I_0}^2 \int_{[0,\pi]} \frac{cos^2(\frac{\pi}{2}\cos(\theta))}{sin^2(\theta)}\ \sin(\theta) d\theta = 73.1\Omega\ \frac{{I_0}^2}{2}$

Radiation from an accelerating charge

- $r' + R = r$, R= |r-r'|
- $\varphi(r,t) = \frac{1}{4\pi\epsilon_0}\int_{V_1} \frac{\rho(r',t-\frac{R}{c})}{|r-r'|} dv' = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{r\cdot p(t-\frac{r}{c})}{r^3} + \frac{r\cdot\frac{dp}{dt}(t-\frac{r}{c})}{cr^2}\right]$
- $\boldsymbol{A}(r,t) = \frac{\mu_0}{4\pi}\int_{V_1} \frac{\boldsymbol{j}(r',t-\frac{R}{c})}{|r-r'|} dv' = \frac{\mu_0}{4\pi r}\frac{d}{dt}\boldsymbol{p}(t-\frac{r}{c})$
- $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\varphi$
- $\boldsymbol{B}(r,t) = \frac{-\mu_0}{4\pi c r^2}\boldsymbol{r} \times \frac{d^2}{dt^2}\boldsymbol{p}(t-\frac{r}{c})$
- $\boldsymbol{E}(r,t) = -\frac{c}{r}\boldsymbol{r}\times \boldsymbol{B}(r,t)$
- $\frac{d\boldsymbol{p}}{dt} = q\frac{dr\prime}{dt} = qv,\ \frac{d^2}{dt^2}\boldsymbol{p}(t-\frac{r}{c}) = q\frac{dv}{dt}$
- $P_R = -\frac{dW}{dt} = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0 c^3}(\frac{dv}{dt})^2$

Radiation from a single accelerating charge

- Near zone
  - $\varphi(r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R\prime(1+\frac{v\cdot n\prime}{c})}\right]$
  - $R^* = R' - \frac{v}{c}(x_0 - x'_1)$
  - $E(r,t) = \frac{1}{4\pi\epsilon_0}\frac{1}{R^{*3}}[\left(R' - \frac{R'\,v\prime}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \frac{R\prime}{c^2}\left(R' - \frac{R'v\prime}{c}\right) \times \frac{dv\prime}{dt}]$
  - $B = \frac{R' \times E}{R'c}$
  - $S = \frac{q^2}{16\pi^2 c^3 \epsilon_0}\frac{R\prime(R' \times v')^2}{(R')^5}$

Radiation loss and antenna aperture

- Spreading loss: $L_s = 32 + \log(\mathrm{d}) + 20\log(\mathrm{f})$
  - d in kilometers
  - F in megahertz
- $W = A_e P_e, A_e = \frac{\lambda^2}{4\pi}$

Antenna aperture

- Deriving antenna aperture uses thermodynamic argument: black body equilibrium

- $P_A = \frac{A_e}{2} B_\upsilon \Delta\nu \int_{[0,4\pi]} d\Omega = 2\pi\, A_e B_\upsilon \Delta\nu$
- $B_\upsilon = \frac{2\nu^2 kT}{c^2} = \frac{2kT}{\lambda^2}$ (Rayleigh-Jeans)
- $P_R = kT\Delta\nu$
- $P_A = P_R$
- $A_e = \frac{\lambda^2}{4\pi}$