

Cryptanalysis

Block Ciphers 2

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Differential Cryptanalysis of DES

How input differentials affect output

- Expansion Matrix

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

P	1	2	3	4
1	16	7	20	21
2	29	12	28	17
3	1	15	23	26
4	5	18	31	10
5	2	8	24	14
6	32	27	3	9
7	19	13	30	6
8	22	11	4	25

Out	1	2	3	4
1	4,4	2,3	5,4	6,1
2	8,1	3,4	7,4	5,1
3	1,1	4,3	6,3	7,2
4	2,1	5,2	8,3	3,2
5	1,2	2,4	6,4	4,2
6	8,4	7,3	1,3	3,1
7	5,3	4,1	8,2	2,2
8	6,2	3,3	1,4	7,1

- After P
 - On average 1 bit difference affects 3 S boxes in next round after expansion.

How input differentials affect output

- Expansion Matrix

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

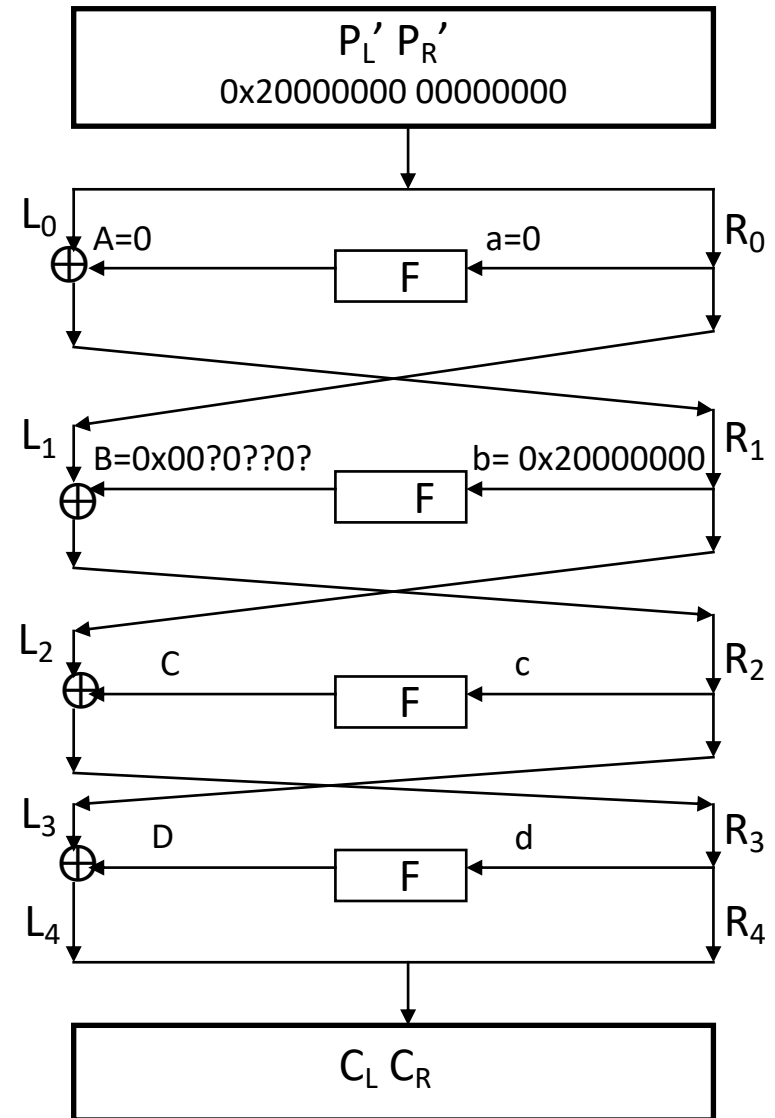
P	1	2	3	4
1	16	7	20	21
2	29	12	28	17
3	1	15	23	26
4	5	18	31	10
5	2	8	24	14
6	32	27	3	9
7	19	13	30	6
8	22	11	4	25

Out	1	2	3	4
1	4	2	5	6
2	8	3	7	5
3	1	4	6	7
4	2	5	8	3
5	1	2	6	4
6	8	7	1	3
7	5	4	8	2
8	6	3	1	7

- After P
 - Affected by box

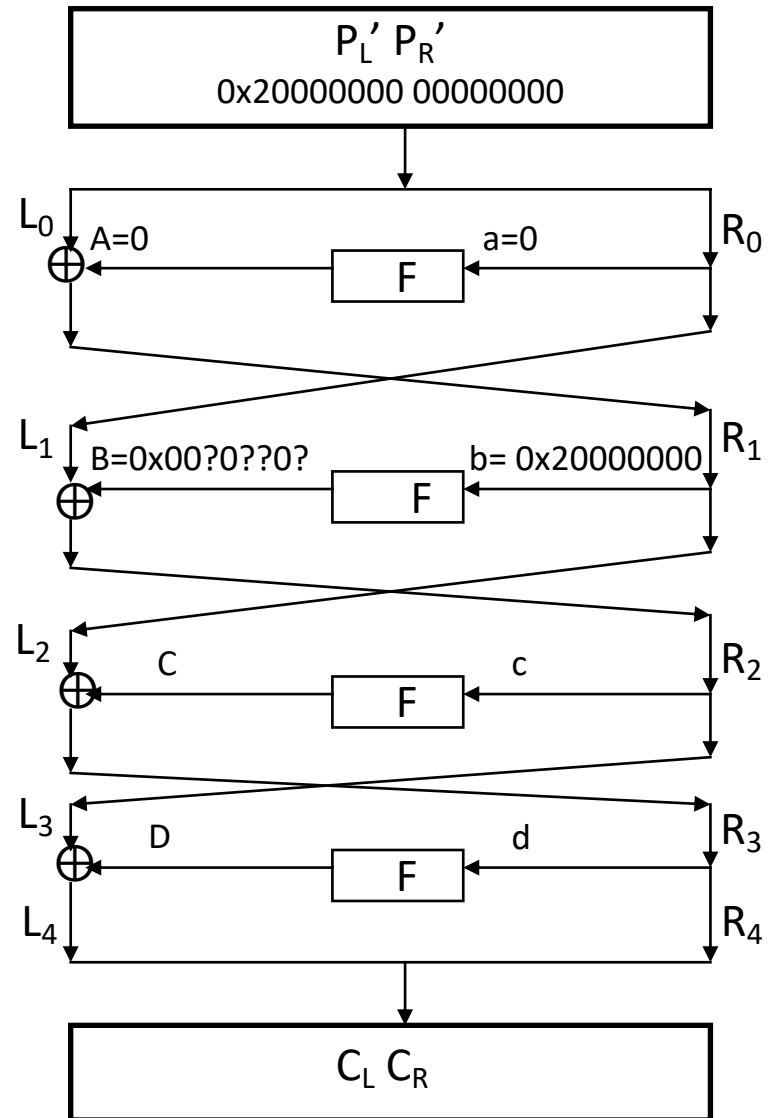
DC of DES, 4 rounds - 1

- Input differential: 0x20000000 00000000
- $A' = 0$, $a' = 0$; $b' = 0x20000000$, B' is affected (at most) as $\text{mask} = 0x00808202 = P(f0000000)$ since only the first S box is non-zero
- $d' = C_R'$ is known
- $D' = C_L' \oplus B'$ is known in 28 bits (all but the mask positions: 0x00808202)
- $S/N = pk/(\lambda\gamma)$, is the ratio of discarded pairs to all pairs, is the number of keys suggested by a pair. Remember only about .8 of xor output patterns are possible.
- Bits that leave all S-boxes but S_1 are valid.
- Weighted probabilities (next slide)
- For each S box, try all 2^6 keys and bump counts for each key which matches the differential, $d' \rightarrow D'$.

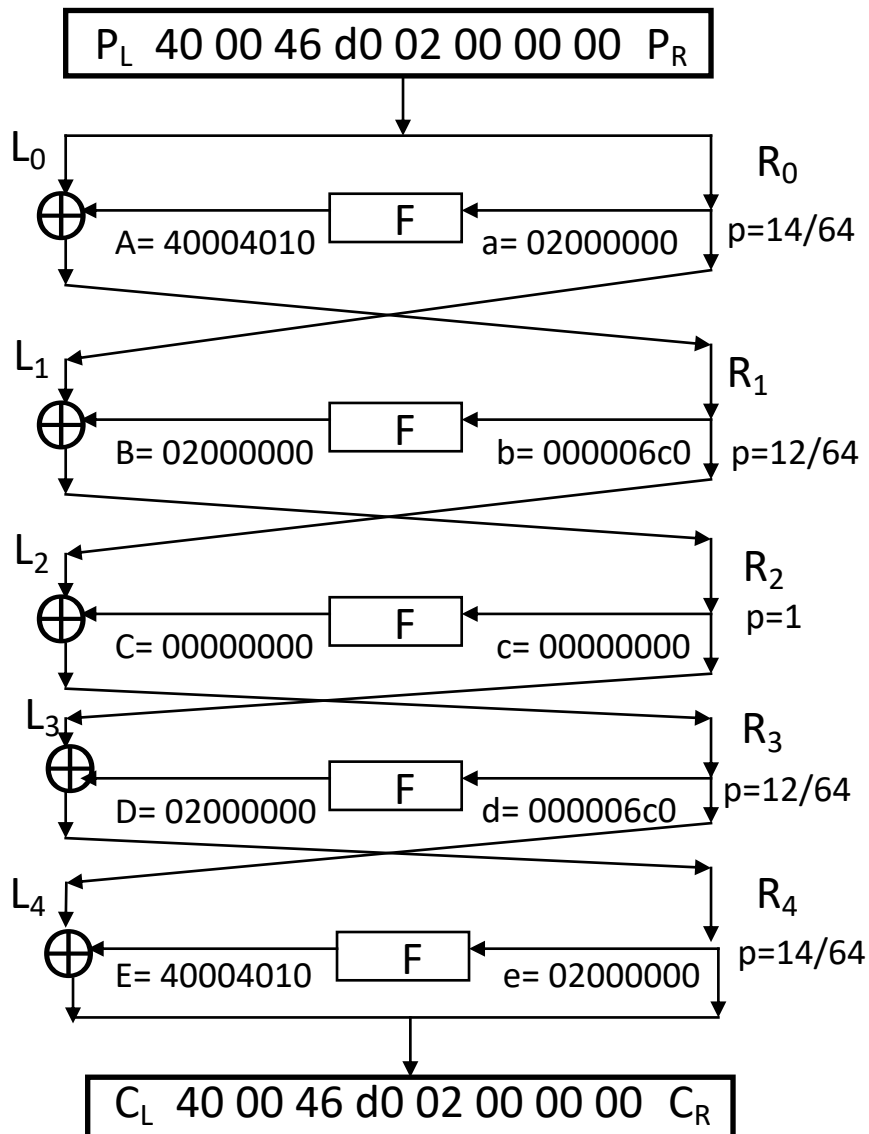


DC of DES, 4 rounds - 2

- For Sbox 1:
 - $0x04 \rightarrow 0x3$, $p = 6/64$ ($0x00000202$)
 - $0x04 \rightarrow 0x5$, $p = 10/64$ ($0x00800002$)
 - $0x04 \rightarrow 0x6$, $p = 10/64$ ($0x00800200$)
 - $0x04 \rightarrow 0x7$, $p = 6/64$ ($0x00800202$)
 - $0x04 \rightarrow 0x9$, $p = 4/64$ ($0x00008002$)
 - $0x04 \rightarrow 0xa$, $p = 6/64$ ($0x00008200$)
 - $0x04 \rightarrow 0xb$, $p = 4/64$ ($0x00008202$)
 - $0x04 \rightarrow 0xc$, $p = 2/64$ ($0x00808000$)
 - $0x04 \rightarrow 0xd$, $p = 8/64$ ($0x00808002$)
 - $0x04 \rightarrow 0xe$, $p = 6/64$ ($0x00808200$)
 - $0x04 \rightarrow 0xf$, $p = 2/64$ ($0x00808202$)

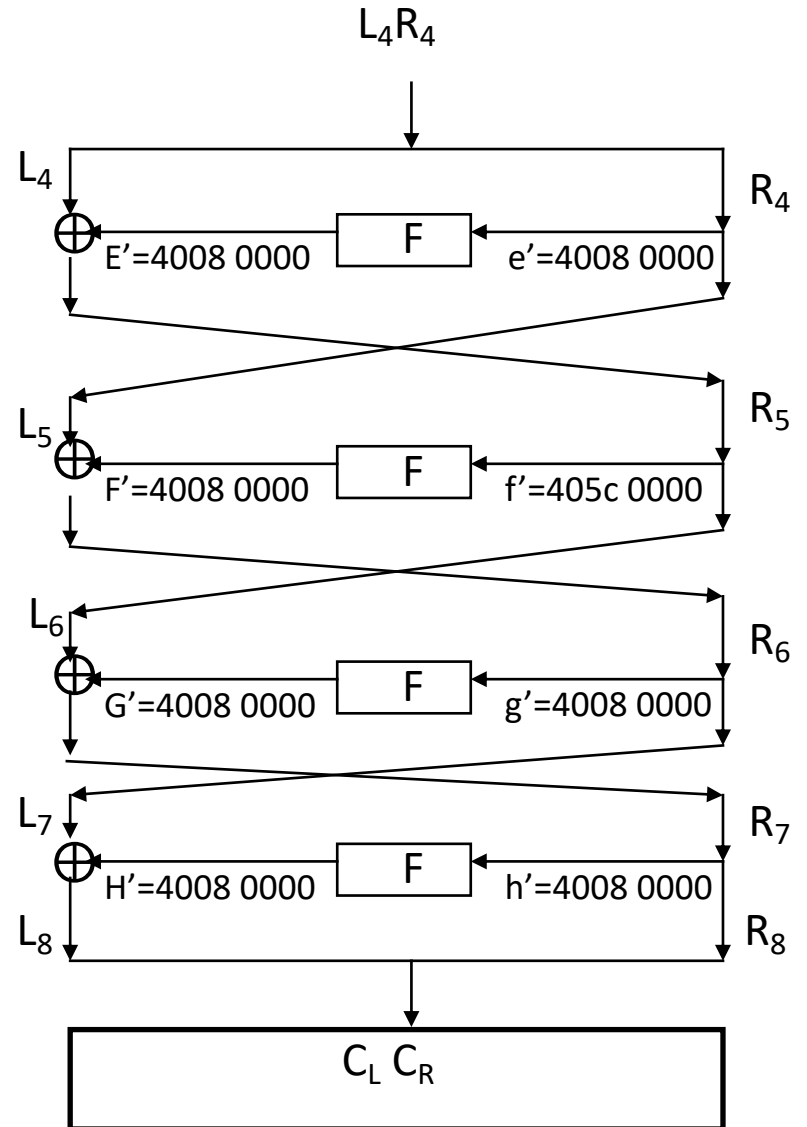
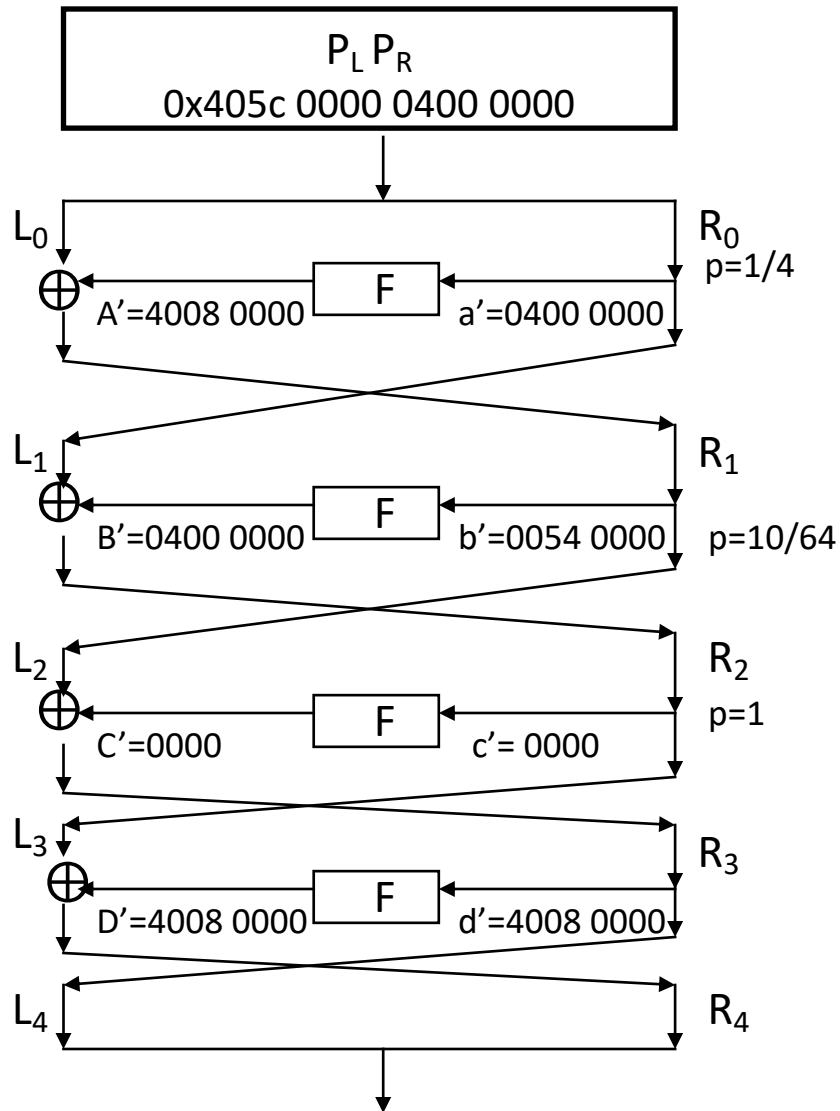


DC of DES, 5 rounds



- $(P_1, P_2) \rightarrow (C_1, C_2)$ gives information about K_5 in S_2 .
- $C_L \oplus L_4 = E$
- $L_2 \oplus L_4 = 0$
- $L_2 \oplus L_0 = A$
- So, $L_0 \oplus L_4 = E \oplus A$
- $02000000 \rightarrow 40004010, p=14/64$
- $000006c0 \rightarrow 02000000, p=12/64$
- Need 3-5 right pairs
- $\Pr[\text{wrong pair}] = 2^{-64}$
- Expected # of wrong pairs is $m2^{-64}$

DC of DES, 8 rounds - 1



DC of DES, 8 rounds - 2

- Requires 25,000 cipher texts. Finds 30 bits in K_8 .
- Uses 5 round differential 405c 0000 0400 0000 \rightarrow 405c 0000 0400 0000 for five rounds, $p = 1/10485.76$.
- $f' = d' \oplus E' = b' \oplus A' = L'$, $H' = l' \oplus g' = l' \oplus e' \oplus F'$
- $S/N = 2^{30}/(4^5 \cdot 10485.76) = 100$
- 4008 0000 = P(0a00 0000), 0400 0000 = P(0010 0000)
- $S/N = 2^{30}/(4^5 \cdot 10485.76) = 100$ for 30 bits --- too many counters.
- Reduce to 24 bit search with enhanced probability.
 - $e' \rightarrow E' = P(0W 00 00 00) = X0 0Y Z0 00 = f' = X0 5V Z0 00$.
 - $W \in \{1,2,3,8,9,a,b\}$, $X \in \{0,4\}$, $Y \in \{0,8\}$, $Z \in \{0,4\}$. $V = Y \oplus 4$.
 - $Z=0$, 0400 0000 \rightarrow 4008 0000, $p=1/4$, all others $Z=4$, $p=20/64$
 - $p_{e' \rightarrow E'} = 1/4 + .8(20/64) = 1/2$
 - $\text{Pr}(24 \text{ bit, differential}) = [(16 \cdot 10 \cdot 16)/64^3] \cdot [(16 \cdot 10 \cdot 32)/64^3] = 1/5243$

DC of DES, 8 rounds - 3

- For enhanced probability, 24 bits, find keys in S_2, S_6, S_7, S_8 .
- $e' = 0400\ 0000 \rightarrow E' = P(0w\ 00\ 00\ 00) = x0\ 0y\ z0\ 00 = f' = x0\ 5v\ z0\ 00$.
- $S/N = 2^{24} / (4^4 \cdot .8 \cdot 5243) = 15.6$
- Alternatively use 18 bit count (S_6, S_7, S_8), requiring 150,000 pairs with $S/N = 1.2$ followed by 12 bits.
- These keys allow us to calculate 20 bits of H, H^* .
- Can use this to complete K_8 (48 bits).
- Final 8 bits from exhaustive search.

DC of DES, 8 rounds - 4

- 18 bits of key, 150,000 pairs from S_2, S_6, S_7, S_8
 1. Set up 2^{18} counter
 2. Preprocess $S_i, S_i' \rightarrow S_o'$.
 3. For each cipher text pair
 - a. Calculate $S_{EH}' = S_{ih}', S_{Oh}'$ for S_2, S_5, S_6, S_7, S_8
 - b. For each of S_2, S_5, S_6, S_7, S_8 , check is $S_{ih}' \rightarrow S_{Oh}'$ is not satisfied for any S-box. If so, discard.
 - c. For S_6, S_7, S_8 , get all S_{ih} which are possible for $S_{ih} \rightarrow S_{ih}$. Calculate $S_{Kh} = S_{ih} \oplus S_{Eh}$
 4. Get entry of maximal count

Full Differential Attack on DES

- Use $0 \rightarrow 0$ and concatenated 2R characteristic with $p = \frac{1}{2^{34}}$ to get 13th round with $p=2^{-47.2}$.
- Want 1960 0000 0000 0000
- Candidate in round 16 has 20 ciphertexts with 0, use 2^{24}
- 2^{-20} of these
- Additional filter: 3 xors can only produce 15 outputs
- Survival rate: .0745, get 1.19 for $2^{35.2}$ structures
- Rate of values not discarded in round 16 is $2^{-32}/(4/5)^8$
- This gives $1.19 \times .84 = 1$ key

Summary of DES DC Attacks

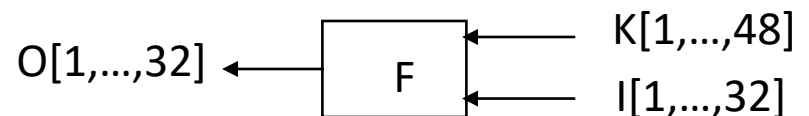
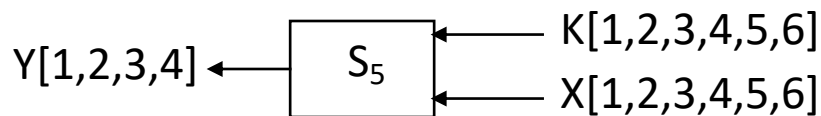
# Rounds	# Pairs needed	# Pairs used	# bits found	# chrtstcs	p	S/N	l	g
4	2^3	2^3	24	1	1	16		
6	2^7	2^7	30	3	1/16	2^{16}		
8	2^{15}	2^{13}	30	5	1/104656	15.6		
8	2^{17}	2^{13}	30	5	1/104656	1.2		
8	2^{20}	2^{19}	30	5	1/55000	1.5		
9	2^{25}	2^{24}	30	6	10^{-6}	1.0		
9	2^{26}	2^8	48	7	10^{-24}	2^{23}		

- For simple attacks

Linear Cryptanalysis of DES

One round linear constraint

- $S_5(x_1 \oplus k_1, x_2 \oplus k_2, x_3 \oplus k_3, x_4 \oplus k_4, x_5 \oplus k_5, x_6 \oplus k_6) \oplus x_2 = k_2 \oplus 1$, $p=52/64$
- Output of F from S_5 is permuted (by P) into positions 3,8,14,25 of round output, O.
- Input to S_5 for F comes from bits 16,17,18,19,20,21 of round input, I (after expansion).
- Key bits for S_5 are from bits 25,26,27,28,29,30 of the round key, K.
- After renaming input, output and key bits in this way, the constraint becomes $O[3,8,14,25] \oplus I[17] = K[26] \oplus 1$.



Matsui's Per Round Constraints

	SBx	Sbox Equation	w	ht(w)	Prob	Round Equation
A	5	$X[2] \oplus Y[1,2,3,4] = K[2] \oplus 1$	40_8	40	12/64	$X[17] \oplus Y[3,8,14,25] = K[26]$
B	1	$X[2,3,5,6] \oplus Y[2] = K[2,3,5,6] \oplus 1$	27_8	20	22/64	$X[1,2,4,5] \oplus Y[17] = K[2,3,5,6]$
C	1	$X[4] \oplus Y[2] = K[4] \oplus 1$	4_8	4	30/64	$X[3] \oplus Y[17] = K[4]$
D	5	$X[2] \oplus Y[1,2,3] = K[2]$	10_8	20	42/64	$X[17] \oplus Y[8,14,25] = K[26]$
E	5	$X[1, 5] \oplus Y[1,2,3] = K[1,5] \oplus 1$	22_8	32	16/64	$X[16,20] \oplus Y[8,14,25] = K[25,29]$

Ht(w) is (unnormalized) Hadamard weight. Note that $a-d=ht(w)$ and $a+d=2^n$ so $a = (2^n + ht(w))/2$ where $a = \#$ places linear appx agrees and $d = \#$ places linear appx disagrees.

Matsui: Linear Cryptanalysis Method for DES Cipher. Eurocrypt, 98. By the way, Matsui's bit numbering scheme differs from ours.

S-Box constraints

- S-1, Y[4]:

w :	000	001	002	003	004	005	006	007	008	009	010	011	012	013	014	015
ht:	000	000	004	004	-04	004	000	008	-08	000	004	-04	004	-12	000	000
w :	016	017	018	019	020	021	022	023	024	025	026	027	028	029	030	031
ht:	-04	-04	-08	-08	-08	000	-12	-04	-04	004	000	-08	008	-08	-04	-04
w :	032	033	034	035	036	037	038	039	040	041	042	043	044	045	046	047
ht:	000	000	-04	-04	-04	004	-08	000	-08	000	-04	020	-12	004	008	008
w :	048	049	050	051	052	053	054	055	056	057	058	059	060	061	062	063
ht:	004	004	008	008	-16	-08	-12	-04	020	-04	000	-08	000	-16	-04	028

- S5, Y[1 2 3 4]:

w :	000	001	002	003	004	005	006	007	008	009	010	011	012	013	014	015
ht:	000	000	008	008	000	-08	000	008	-08	008	000	000	008	000	008	000
w :	016	017	018	019	020	021	022	023	024	025	026	027	028	029	030	031
ht:	040	-08	000	000	000	-08	000	-08	008	008	000	000	000	-08	000	008
w :	032	033	034	035	036	037	038	039	040	041	042	043	044	045	046	047
ht:	000	000	024	008	000	008	000	008	000	000	-08	-08	000	-08	000	008
w :	048	049	050	051	052	053	054	055	056	057	058	059	060	061	062	063
ht:	-08	-08	000	000	000	-08	000	008	000	000	008	-08	-08	000	-08	000

S-Box constraints to round constraints

- S-Box output bit use

S[1]: 9 17 23 31

S[2]: 13 28 2 18

S[3]: 24 16 30 6

S[4]: 26 20 10 1

S[5]: 8 14 25 3

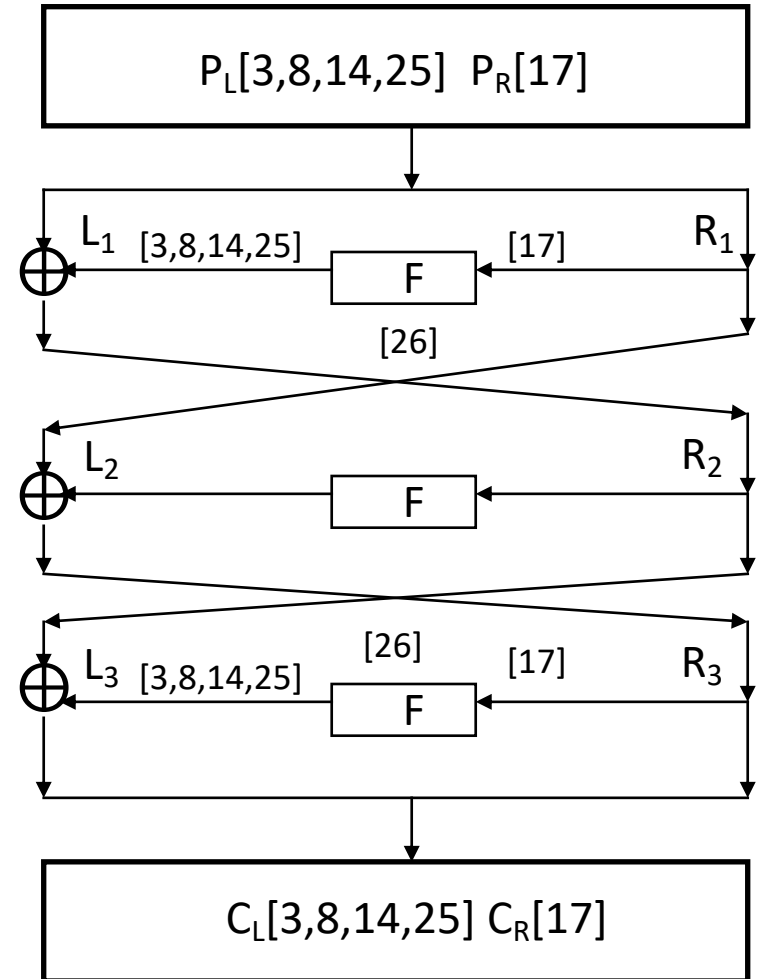
S[6]: 4 29 11 19

S[7]: 32 12 22 7

S[8]: 5 27 15 21

LC of DES, 3 rounds - 1

- Input at round 1 to activate S_5 constraint is
 - $P_R[17]$.
- Output at round 1 for constraint is
 - $O[3,8,14,25] = P_L[3,8,14,25] \oplus R_2[3,8,14,25]$ which holds with probability 52/64.
- Key bits are $K_1[26]$ and $K_3[26]$.
- First round thus yields
 - $P_L[3,8,14,25] \oplus R_2[3,8,14,25] \oplus P_R[17] = K_1[26] \oplus 1$
- Similarly using the same S_5 relation, round 3 is
 - $C_L[3,8,14,25] \oplus R_2[3,8,14,25] \oplus C_R[17] = K_3[26] \oplus 1$, which holds with probability 52/64.
- Adding we get
 - $P_L[3,8,14,25] \oplus C_L[3,8,14,25] \oplus P_R[17] \oplus C_R[17] = K_1[26] \oplus K_3[26]$.
- This holds with probability
- $p = (52/64)^2 + (12/64)^2 = .6953$



Evaluating experimental outcome

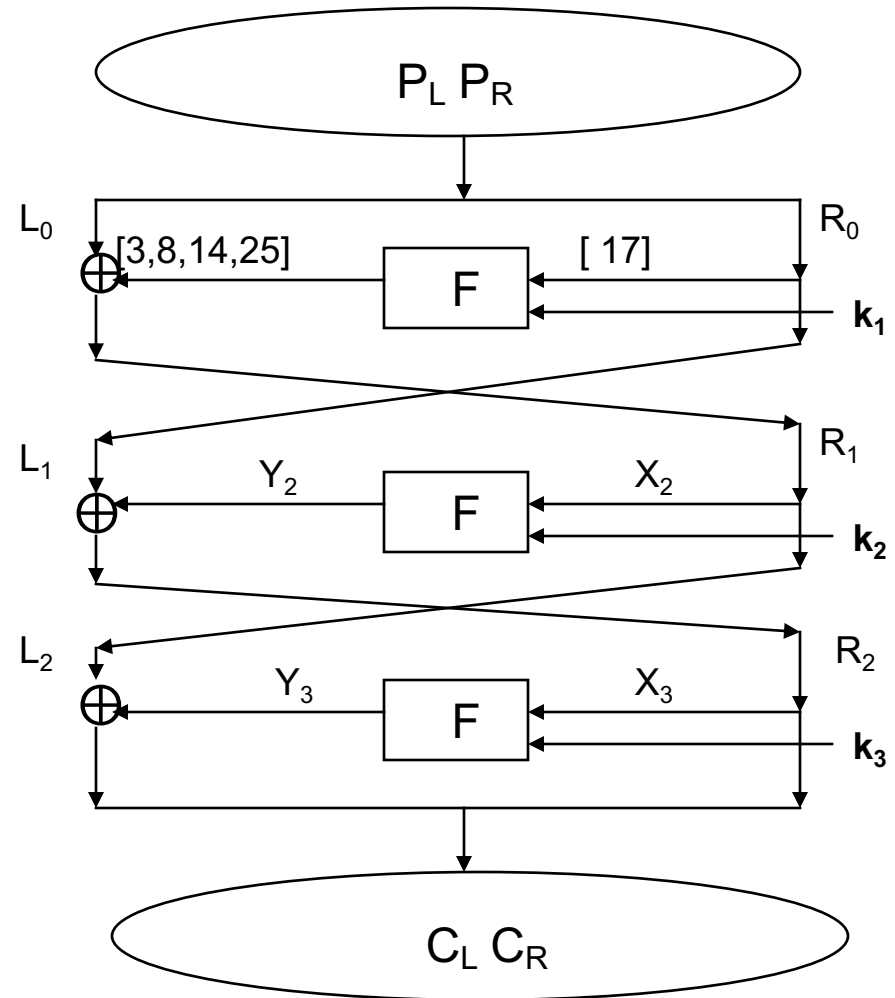
- Suppose an affine constraint $P[j_1, \dots, j_m] \oplus C[l_1, \dots, l_{m'}] = K[k_1, \dots, k_{m''}]$ holds with probability p . Put $\mathbf{x} = (x_1, \dots, x_n)$ where $x_i = P_i[j_1, \dots, j_m] \oplus C_i[l_1, \dots, l_{m'}]$ for the observed sequence (P_i, C_i) of corresponding plain and cipher text. \mathbf{x} is sampled from one of two populations: one with $K[k_1, \dots, k_{m''}] = 0$ and one with $K[k_1, \dots, k_{m''}] = 1$. We assume that the choice of population 1 or population 2 is made at random prior to observation of (P_i, C_i) .
- If \mathbf{x} is sampled from the first population ($q=0$), $\Pr(x_i | q=0) = p$ while if \mathbf{x} is sampled from the second population ($q=1$), $\Pr(x_i | q=1) = q = 1-p$.
- Denoting $p_0 = \Pr(q=0 | \mathbf{x})$ and $p_1 = \Pr(q=1 | \mathbf{x})$, from Bayes Theorem, we obtain $p_0 = \Pr(q=0 | \mathbf{x}) = \Pr(\mathbf{x} | q=0) \cdot \Pr(q=0) / \Pr(\mathbf{x})$ while $p_1 = \Pr(q=1 | \mathbf{x}) = \Pr(\mathbf{x} | q=1) \cdot \Pr(q=1) / \Pr(\mathbf{x})$.
- $\Pr(q=0) = \Pr(q=1) = 1/2$. Suppose we observe a 0's in \mathbf{x} and b 1's ($a+b=n$), then $\Pr(\mathbf{x} | q=0) = {}_n C_a p^a q^b$ and similarly, $\Pr(\mathbf{x} | q=1) = {}_n C_a q^a p^b$, while $\Pr(\mathbf{x}) = {}_n C_a (1/2)^a (1/2)^b = 2^{-n} {}_n C_a$.
- So $p_0 = 2^{n-1} p^a q^b$ and $p_1 = 2^{n-1} q^a p^b$.
- Thus, $p_0/p_1 = (p/q)^a (q/p)^b$.

LC of DES, 3 rounds - 2

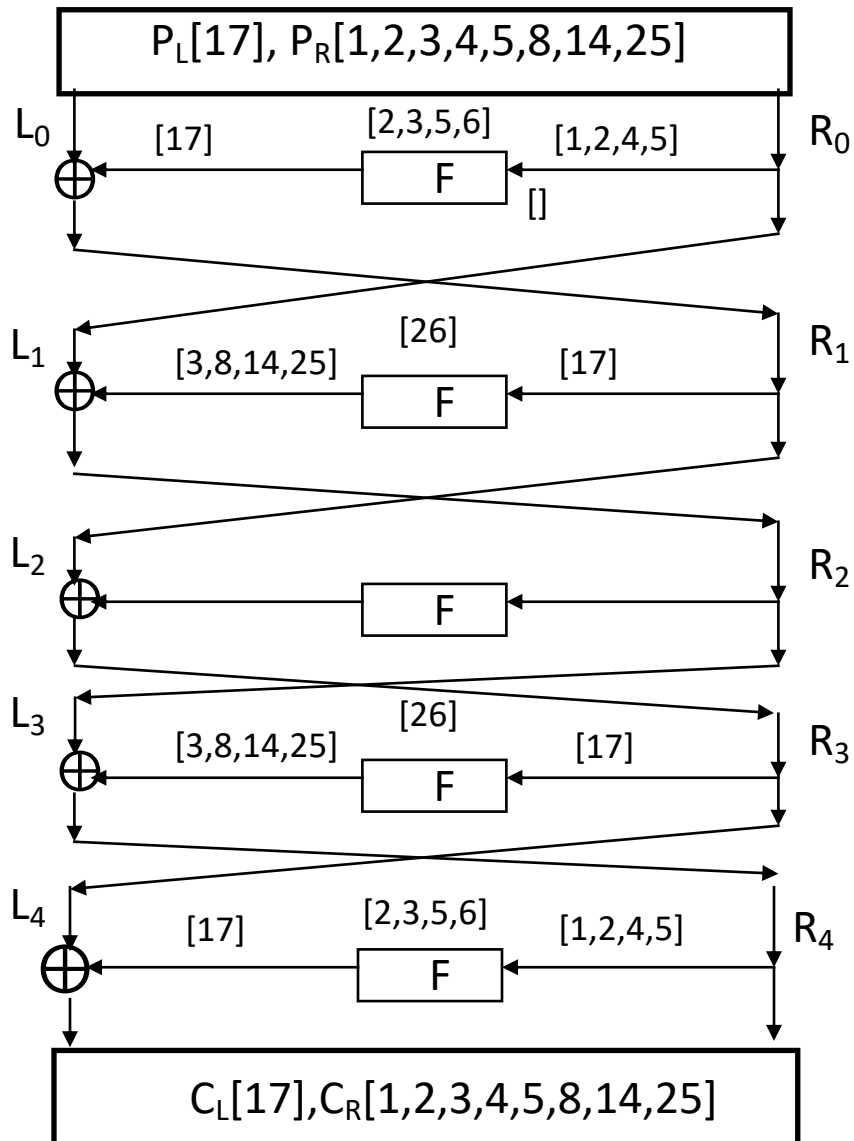
- $P_R[17] \oplus P_L[3,8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_1[26] \oplus K_3[26]$
- Recall $p = .6953$ so $q = .3047$.
- If we observe a 0's in x and b 1's, the previous result gives:

$$\Pr(q=0|\mathbf{x})/\Pr(q=1|\mathbf{x}) = (p/q)^a (q/p)^b.$$
- Equivalently, if $a > b$,

$$\Pr(q=0|\mathbf{x})/\Pr(q=1|\mathbf{x}) = (p/q)^{a-b} \cong (7/3)^{a-b}.$$
- So, if, for example, $a-b=5$, $p_0 \cong .99$.



LC of DES, 5 rounds - 1



LC of DES, 5 rounds - 2

1. $P_L[17] \oplus R_1[17] = K_1[2,3,5,6] \oplus P_R[1,2,4,5] \oplus 1 \quad \dots (\text{Eq B})$
2. $P_R[3,8,14,25] \oplus R_2[3,8,14,25] = K_2[26] \oplus R_1[17] \oplus 1 \quad \dots (\text{Eq A})$
3. $R_2[3,8,14,25] \oplus C_R[3,8,14,25] = K_4[26] \oplus C_R[17] \oplus 1 \quad \dots (\text{Eq A})$
4. $C_L[17] \oplus R_3[17] = K_5[2,3,5,6] \oplus C_R[1,2,4,5] \oplus 1 \quad \dots (\text{Eq B})$

- Adding yields:

$$P_L[17] \oplus P_R[1,2,3,4,5,8,14,25] \oplus C_L[17] \oplus C_R[1,2,3,4,5,8,14,25] = \\ K_1[2,3,5,6] \oplus K_2[26] \oplus K_4[26] \oplus K_5[2,3,5,6]$$

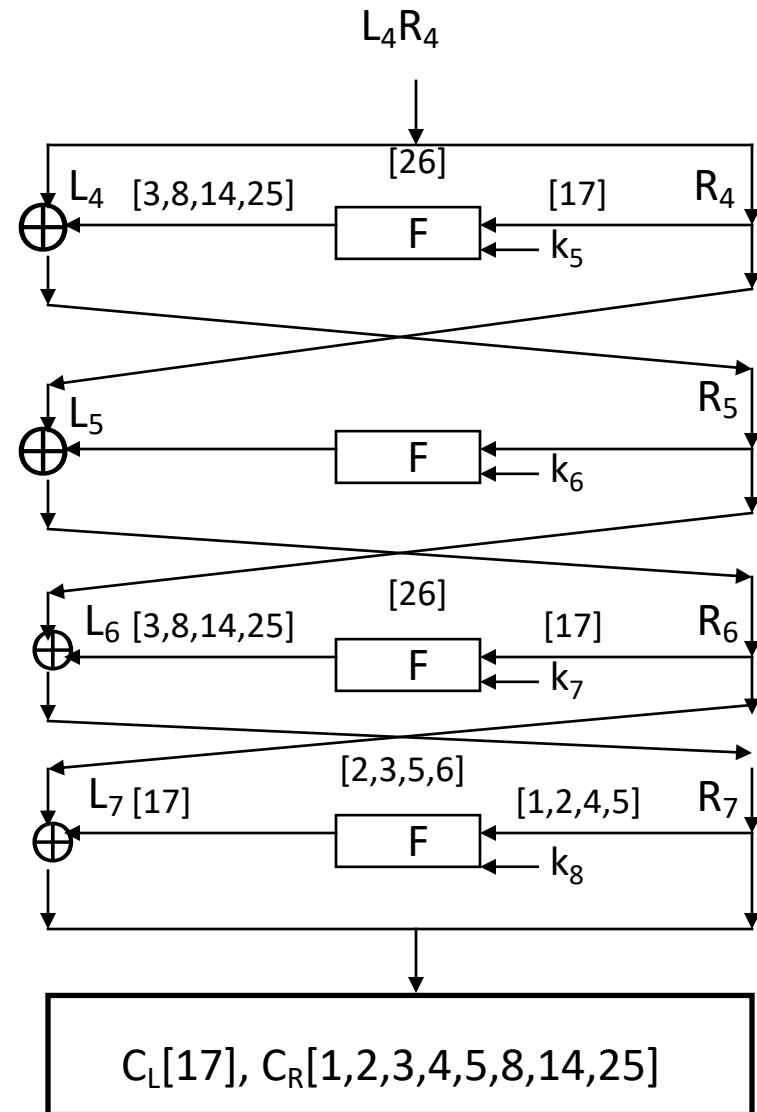
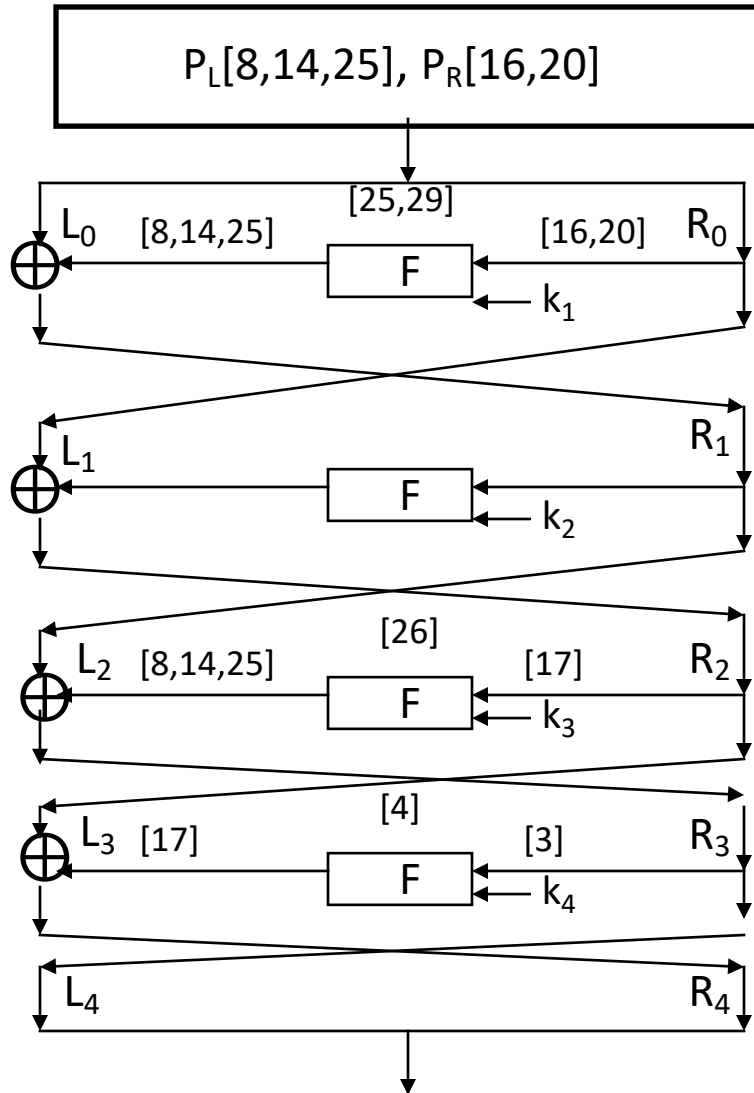
- This holds with probability:

$$p = p_B^2 p_A^2 + p_B^2 q_A^2 + p_A^2 q_B^2 + 4(q_A p_B q_B p_A) + q_B^2 q_A^2 \cong .519 = .5 + 1.22 \times 2^{-6},$$

where $q_i = 1 - p_i$. $p/q = 1.07927..$

- Suppose we decide, based on an excess (e), of LHS values. Odds of right answer is $r = (p/q)^e$. For example, if $e = 64$, $r \cong 131.92$.

LC of DES, 8 rounds - 1



LC of DES, 8 rounds - 2

1. $P_L[8,14,25] \oplus R_1[8,14,25] = K_1[25, 29] \oplus P_R[16,20] \oplus 1$ (Eq E)
 2. $R_1[8,14,25] \oplus R_3[8,14,25] = K_3[26] \oplus R_2[17]$ (Eq D)
 3. $R_3[3,8,14,25] \oplus R_5[3,8,14,25] = K_5[26] \oplus R_4[17]$ (Eq A)
 4. $R_2[17] \oplus R_4[17] = K_4[4] \oplus R_3[3] \oplus 1$ (Eq C)
 5. $R_5[3,8,14,25] \oplus R_7[3,8,14,25] = K_7[26] \oplus R_6[17]$ (Eq A)
 6. $C_L[17] \oplus R_6[17] = K_8[2,3,5,6] \oplus C_R[1,2,4,5] \oplus 1$ (Eq B)
- $P_L[8,14,25] \oplus P_R[16,20] \oplus C_R[1,2,3,4,5,8,14,25] \oplus C_L[17] =$
 $K_1[25,29] \oplus K_3[26] \oplus K_4[4] \oplus K_5[26] \oplus K_7[26] \oplus K_8[2,4,5,6] \oplus 1.$
 - This holds with probability: $p \cong 0.500596 = .50 + 1.22 \times 2^{-11}.$

15 Round Linear Approximation

Pattern: E-DCA-ACD-DCA-A. Note $L_i = R_{i-1}$, $L_i \oplus R_{i+1} = L_i \oplus L_{i+2}$.

$$\begin{array}{llll} 1 & P_L[8,14,25] \oplus R_2[8,14,25] \oplus P_R[16,20] & = & K_1[23,25] \\ 3 & L_3[8,14,25] \oplus R_4[8,14,25] \oplus R_3[17] & = & K_3[26] \\ 4 & L_4[17] \oplus R_5[17] \oplus R_4[3] & = & K_4[4] \\ 5 & L_5[3,8,14,25] \oplus R_6[3,8,14,25] \oplus R_5[17] & = & K_5[26] \\ 7 & L_7[3,8,14,25] \oplus R_8[3,8,14,25] \oplus R_7[17] & = & K_7[26] \\ 8 & L_8[17] \oplus R_9[17] \oplus R_8[3] & = & K_8[4] \\ 9 & L_9[8,14,25] \oplus R_{10}[8,14,25] \oplus R_9[17] & = & K_9[26] \\ 11 & L_{11}[8,14,25] \oplus R_{12}[8,14,25] \oplus R_{11}[17] & = & K_{11}[26] \\ 12 & L_{12}[17] \oplus R_{13}[17] \oplus R_{12}[3] & = & K_{12}[4] \\ 13 & L_{13}[3,8,14,25] \oplus R_{14}[3,8,14,25] \oplus R_{13}[17] & = & K_{13}[26] \\ 15 & L_{15}[3,8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] & = & K_{15}[26] \end{array}$$

15 Round Linear Approximation

Adding and canceling:

- $$\begin{aligned} &P_L[8,14,25] \oplus P_R[16,20] \oplus C_L[3,8,14,25] \oplus C_R[17] = \\ &\quad K_1[23,25] \oplus K_3[26] \oplus K_4[4] \oplus K_5[26] \oplus K_7[26] \oplus K_8[4] \\ &\quad \oplus K_9[26] \oplus K_{11}[26] \oplus K_{12}[4] \oplus K_{13}[26] \oplus K_{15}[26] \end{aligned}$$

which holds (Piling-up Lemma) with the indicated probability.

Full Linear Attack on DES

- Linear cryptanalysis can be accomplished with $\sim 2^{43}$ known plaintexts, using a more sophisticated estimation 14 round approximation
 - For each 48 bit last round sub-key, decrypt cipher-text backwards across last round for all sample cipher-texts
 - Increment count for all sub-keys whose linear expression holds true to the penultimate round
 - This is done for the first and last round yielding 13 key bits each (total: 26)

- Here they are:

$$P_R[8,14,25] \oplus C_L[3,8,14,25] \oplus C_R[17] = K_1[26] \oplus K_3[4] \oplus K_4[26] \oplus K_6[26] \oplus K_7[4] \oplus K_8[26] \oplus K_{10}[26] \oplus K_{11}[4] \oplus K_{12}[26] \oplus K_{14}[26]$$

with probability $\frac{1}{2} - 1.19 \times 2^{-21}$

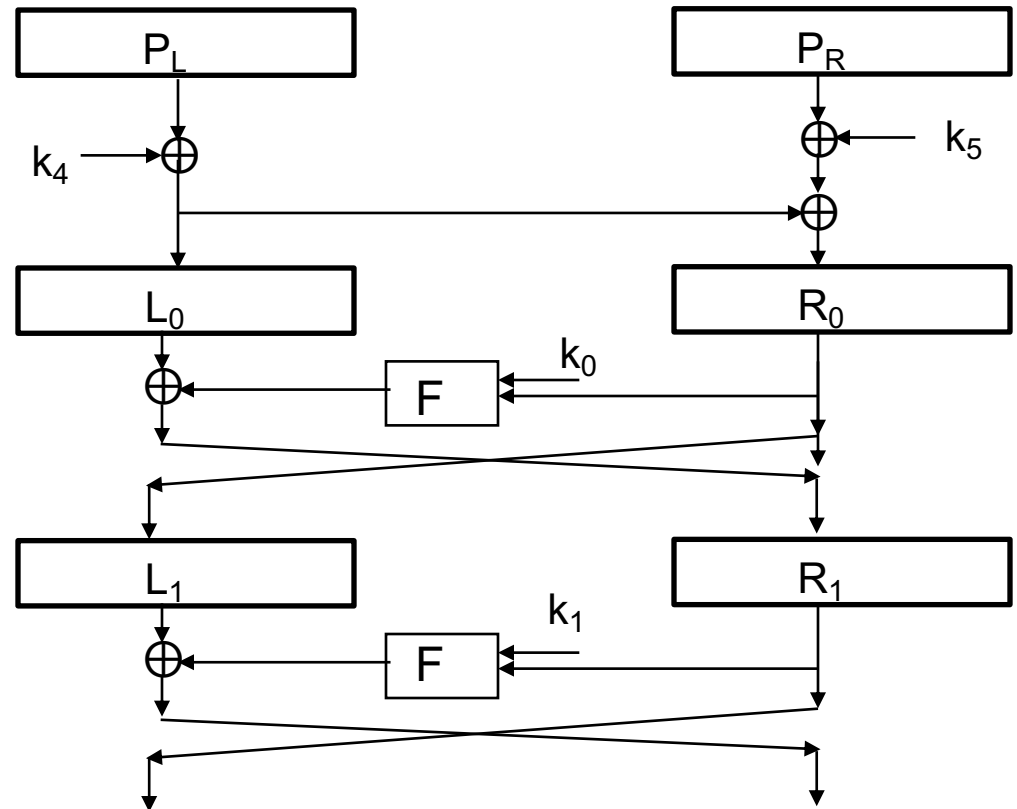
$$C_R[8,14,25] \oplus P_L[3,8,14,25] \oplus P_R[17] = K_{13}[26] \oplus K_{12}[24] \oplus K_{11}[26] \oplus K_9[26] \oplus K_8[24] \oplus K_7[26] \oplus K_5[26] \oplus K_4[4] \oplus K_3[26] \oplus K_1[26]$$

with probability $\frac{1}{2} - 1.19 \times 2^{-21}$

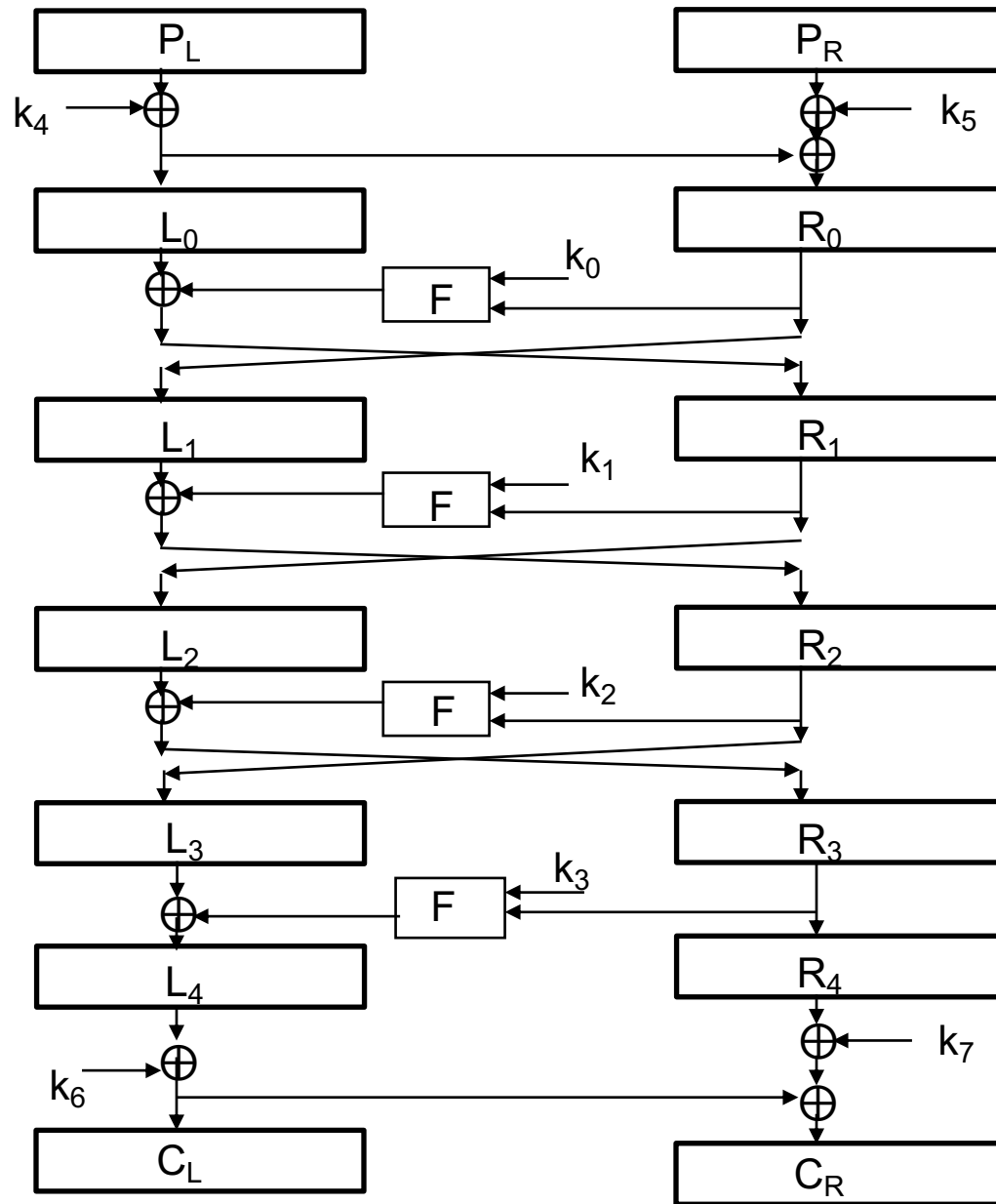
FEAL
(A fortunate mistake)

FEAL-4

- Four round Feistel cipher with a 64-bit block and 64-bit key
- Plaintext: P, Cipher-text: C
- Round function: F
- 32-bit sub-keys: K_0, K_1, \dots, K_7
- Most important failed cipher: showed the power of differential cryptanalysis and linear cryptanalysis



Original FEAL-4



FEAL-4 Round Function

- $G_0(a,b) = (a+b \pmod{256}) \lll 2$
- $G_1(a,b) = (a+b+1 \pmod{256}) \lll 2$
where “ \lll ” is left cyclic shift (rotation)
- $F(x_0, x_1, x_2, x_3) = (y_0, y_1, y_2, y_3)$ where
 1. $y_1 = G_1(x_0 \oplus x_1, x_2 \oplus x_3)$
 2. $y_0 = G_0(x_0, y_1)$
 3. $y_2 = G_0(y_1, x_2 \oplus x_3)$
 4. $y_3 = G_1(y_2, x_3)$

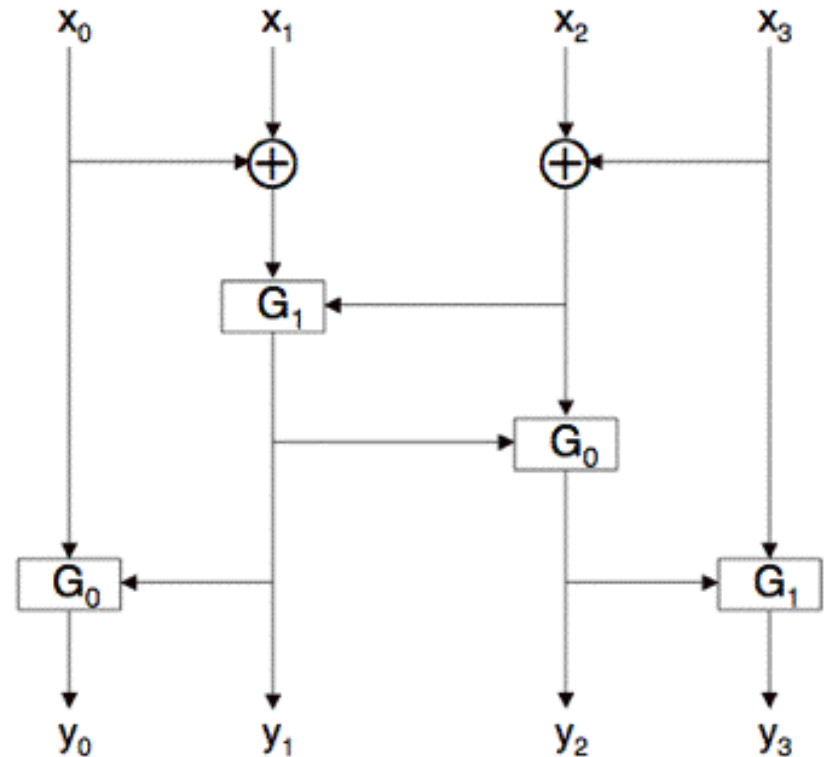
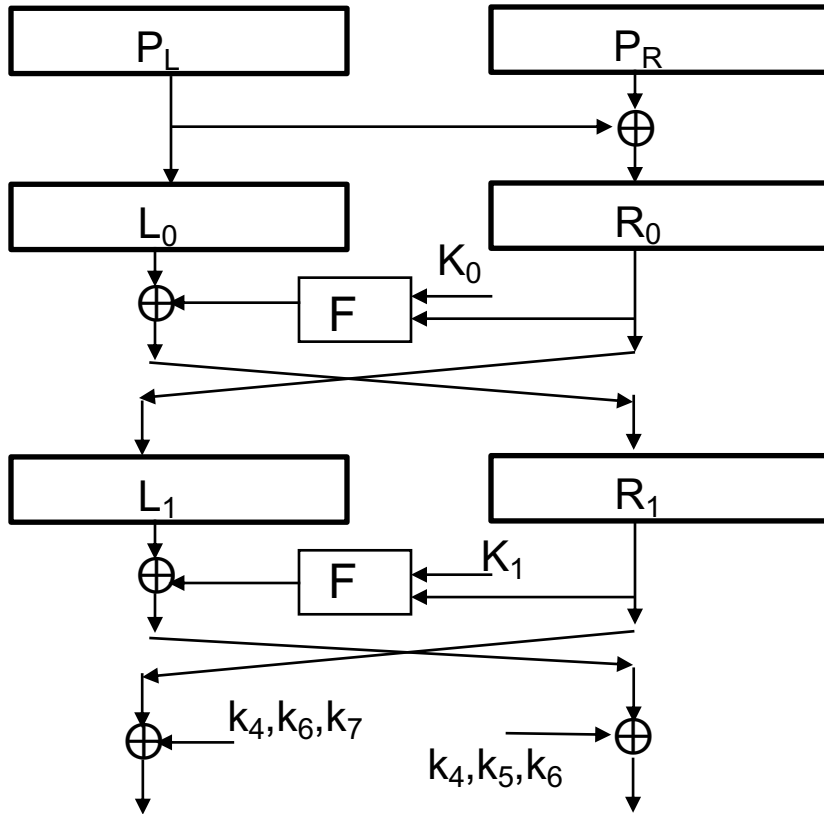


Diagram from Mark Stamp

FEAL-4 Key Schedule

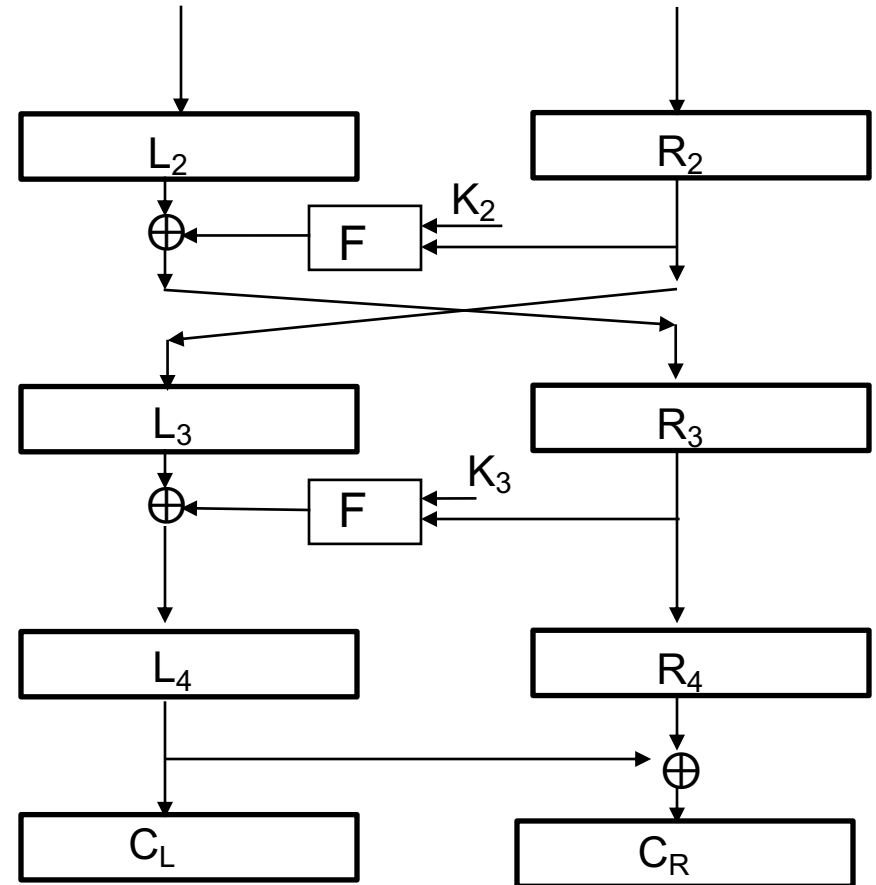
- $F_K(a_0 || a_1 || a_2 || a_3, b_0 || b_1 || b_2 || b_3) = c_0 || c_1 || c_2 || c_3$ by
 - $d_1 = a_0 \oplus a_1$
 - $d_2 = a_2 \oplus a_3$
 - $c_1 = G_1(d_1, a_2 \oplus b_0)$
 - $c_2 = G_0(d_2, c_1 \oplus b_1)$
 - $c_0 = G_0(a_0, c_1 \oplus b_2)$
 - $c_3 = G_1(a_3, c_2 \oplus b_3)$
- $k_{-2} = 0$
- $k_{-1} = k_L$
- $k_0 = k_R$
- $k_i = f_K(k_{i-2}, k_{i-1} \oplus k_{i-3})$

Refactored FEAL-4



$$K_0 = k_0 + k_4 + k_5$$

$$K_1 = k_1 + k_4$$



$$K_3 = k_3 + k_6 + k_7$$

$$K_2 = k_2 + k_6$$

Encryption Equations

- $L_0 = P_L + k_4, \quad R_0 = P_L + P_R + k_4 + k_5$
- $L_4 = C_L + k_6, \quad R_4 = C_L + C_R + k_6 + k_7$

- $L_1 = R_0$
- $L_2 = R_1 = L_0 + F(k_0 + R_0)$
- $L_3 = R_2 = R_0 + F(k_1 + L_0 + F(k_0 + R_0))$
- $L_4 = L_3 + F(k_3 + R_4) = R_0 + F(k_1 + L_0 + F(k_0 + R_0)) + F(k_3 + R_4)$

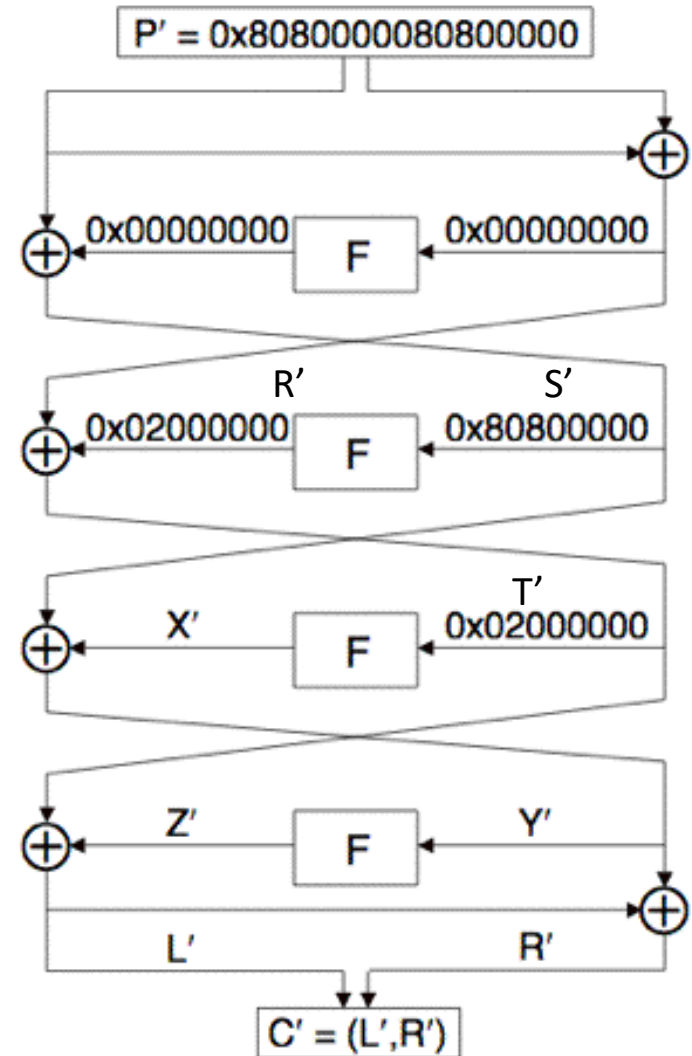
- $R_1 = L_0 + F(k_0 + R_0)$
- $R_2 = L_1 + F(k_1 + R_1) = R_0 + F(k_1 + L_0 + F(k_0 + R_0))$
- $R_3 = L_2 + F(k_2 + R_2) = L_0 + F(k_0 + R_0) + F(k_2 + R_0 + F(k_1 + L_0 + F(k_0 + R_0)))$
- $R_3 = R_4$

Refactored FEAL-4 Equations

- $K_0 = k_0 + k_4 + k_5$, $K_1 = k_1 + k_4$, $K_2 = k_2 + k_6$, $K_3 = k_3 + k_6 + k_7$
- $K_4 = k_4 + k_5 + k_6$, $K_5 = k_4 + k_6 + k_7$
- $L_1 = P_L + P_R$, $R_1 = P_L + f(P_L + P_R + K_0)$
- $L_2 = R_1 + K_5$, $R_2 = L_1 + K_4 + f(R_1 + K_1)$
- $L_3 = R_2$, $R_3 = L_2 + f(R_2 + K_2)$
- $C_L = L_3 + f(R_3 + K_3)$, $C_R = C_L + R_3$
- Substituting,
 - $C_L = P_L + P_R + k_4 + k_5 + k_6 + f(P_L + k_4 + k_1 + f(P_L + P_R + k_4 + k_5 + k_0))$
 - $C_R = C_L +$
 $(P_L + k_4) + k_6 + k_7 + f(P_L + P_R + k_4 + k_5 + k_0) +$
 $f(P_L + P_R + k_4 + k_5 + k_2 + f(P_L + k_4 + k_1 + f(P_L + P_R + k_4 + k_5 + k_0)))$

FEAL-4 Basic Differential Attack

- If $A_0 \oplus A_1 = 0$ then $F(A_0) = F(A_1)$, $p=1$.
- If $A_0 \oplus A_1 = 0x80800000$ then $F(A_0) \oplus F(A_1) = 0x02000000$, $p=1$
- Choose (P_0, P_1) :
- $P_0 \oplus P_1 = 0x8080000080800000$
- $P' = P_0 \oplus P_1$, $C' = C_0 \oplus C_1$
- $L' = 0x02000000 \oplus Z'$, $Y' = 0x80800000 \oplus X'$
- For $C = (L, R)$ we have $Y = L \oplus R$
- Solve for sub-key K_3 : $Z' = 0x02000000 \oplus L'$
- Compute $Y_0 = L_0 \oplus R_0$, $Y_1 = L_1 \oplus R_1$
- Guess K_3 and compute guessed Z_0, Z_1
 - Note: $Z_i = F(Y_i \oplus K_3)$
- Compare true Z' to guessed Z'



FEAL-4 Improved Differential Attack

- Using 4 chosen plaintext pairs
 - Work is of order 2^{32}
 - Expect one K_3 to survive
- Can reduce work to about 2^{17}
 - For 32-bit word $A=(a_0,a_1,a_2,a_3)$, define $M(A) = (z, a_0 \oplus a_1, a_2 \oplus a_3, z)$, where z is all-zero byte
 - For all possible $A=(z, a_0, a_1, z)$, compute $Q_0 = F(M(Y_0) \oplus A)$ and $Q_1 = F(M(Y_1) \oplus A)$
 - Can be used to find 16 bits of K_3
- When $A = M(K_3)$, we have $\langle Q_0 \oplus Q_1 \rangle_{8...23} = \langle Z' \rangle_{8...23}$ where $\langle X \rangle_{i...j}$ is bits i thru j of X . Can recover K_3 with about 2^{17} work
- Once K_3 is known, can successively recover K_2, K_1, K_0 and finally K_4, K_5
- Second characteristic: 0xa200 8000 0x2280 8000

FEAL-4 Differential Attack

- Primary for K_3

```
// Characteristic is 0x8080000080800000
P0 = random 64-bit value
P1 = P0 ⊕ 0x8080000080800000
// Given corresponding ciphertexts
// C0 = (L0, R0) and C1 = (L1, R1)
Y0 = L0 ⊕ R0
Y1 = L1 ⊕ R1
L' = L0 ⊕ L1
Z' = L' ⊕ 0x02000000
for (a0, a1) = (0x00, 0x00) to (0xff, 0xff)
    Q0 = F(M(Y0) ⊕ (0x00, a0, a1, 0x00))
    Q1 = F(M(Y1) ⊕ (0x00, a0, a1, 0x00))
    if ⟨Q0 ⊕ Q1⟩8...23 == ⟨Z'⟩8...23 then
        Save (a0, a1)
    end if
next (a0, a1)
```

- Secondary for K_3

```
// P0, P1, C0, C1, Y0, Y1, Z' as in primary
// Given list of saved (a0, a1) from primary
for each primary survivor (a0, a1)
    for (c0, c1) = (0x00, 0x00) to (0xff, 0xff)
        D = (c0, a0 ⊕ c0, a1 ⊕ c1, c1)
        Z̃0 = F(Y0 ⊕ D)
        Z̃1 = F(Y1 ⊕ D)
        if Z̃0 ⊕ Z̃1 == Z' then
            Save D // candidate subkey K3
        end if
    next (c0, c1)
next (a0, a1)
```

Slide adapted from Mark Stamp

- Assuming only one chosen plaintext pair

FEAL-4 Linear Attack

- Now we'll use linear cryptanalysis to break Feal-4.
- Notation: let $Y=F(X)$. We use $X[i,j]$ to denote $X[i] \oplus X[j]$
- Using the definition of F , we will see (next slide) that the following linear constraints hold with probability 1.
 1. $Y[13] = X[7, 15, 23, 31] + 1$ (or $F(X)[13] = X[7,15,23,31] + 1$)
 2. $Y[5, 15] = X[7]$ (or $F(X)[5,15] = X[7]$)
 3. $Y[15, 21] = X[23, 31]$ (or $F(X)[15,21] = X[23,31]$)
 4. $Y[23, 29] = X[31] + 1$ (or $F(X)[23,29] = X[31] + 1$)

FEAL-4 Constraint Derivation

- $(a \oplus b)[7] = (a + b \pmod{256})[7]$, so
- $G_0(a, b)[5] = (a \oplus b)[7]$, similarly, $G_1(a, b)[5] = (a \oplus b \oplus 1)[7]$
- $y_1 = G_1(x_0 \oplus x_1, x_2 \oplus x_3) \rightarrow Y[13] = y_1[5] = x_0[7] \oplus x_1[7] \oplus x_2[7] \oplus x_3[7] \oplus 1 = X[7, 15, 23, 31] \oplus 1$
- $y_0 = G_0(x_0, y_1) \rightarrow Y[5] = y_0[5] = y_1[7] \oplus x_0[7] = Y[15] \oplus X[7]$
- $y_2 = G_0(y_1, x_2 \oplus x_3) \rightarrow$
 $Y[21] = y_2[5] = y_1[7] \oplus x_2[7] \oplus x_3[7] = Y[15] \oplus X[23, 31]$
- $y_3 = G_1(y_2, x_3) \rightarrow$
 $Y[29] = y_3[5] = y_2[7] \oplus x_3[7] \oplus 1 = Y[23] \oplus X[31] \oplus 1$

$$Y = F(X)$$

- $Y = (y_0, y_1, y_2, y_3)$
- $X = (x_0, x_1, x_2, x_3)$

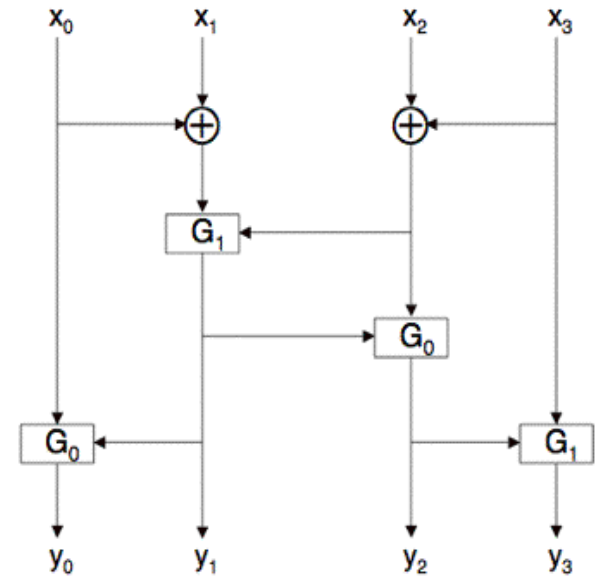


Diagram from Mark Stamp

FEAL-4 Linear Attack Equations

- Adapting the F constraint equations for each round, we get:
 - $Y_0 = F(R_0 \oplus k_0)$, $R_1 = L_0 \oplus Y_0$, $L_1 = R_0$
 - $Y_1 = F(R_1 \oplus k_1)$, $R_2 = L_1 \oplus Y_1$, $L_2 = R_1$
 - $Y_2 = F(R_2 \oplus k_2)$, $R_3 = L_2 \oplus Y_2$, $L_3 = R_2$
 - $Y_3 = F(R_3 \oplus k_3)$
- Looking at the *original* FEAL-4 diagram (using “+” instead of “ \oplus ”), we get
 - $L_4 = R_2 + Y_3$ and $R_2 = R_0 + Y_1$, “adding” these gives
 - $L_4 + R_0 = Y_1 + Y_3$, or
 - $L_4 + R_0 = F(R_1 + k_1) + F(R_4 + k_3)$
- Since $R_1 = L_0 + F(R_0 + k_0)$, we have finally
 - $L_4 + R_0 = F(R_4 + k_3) + F(L_0 + F(R_0 + k_0) + k_1)$
- Note that $L_0 = P_L + k_4$, $R_0 = P_L + P_R + k_4 + k_5$, $L_4 = C_L + k_6$ and $R_4 = C_L + C_R + k_6 + k_7$, so we get
 - $C_L + P_L + P_R + k_4 + k_5 + k_6 = F(P_L + k_4 + F(P_L + P_R + k_4 + k_5 + k_0) + k_1) + F(C_L + C_R + k_6 + k_7 + k_3)$

FEAL-4 Linear Attack using refactored FEAL-4

- Now we can explain why we refactored FEAL-4.
- If we knew L_0, R_0, L_4, R_4 , we could mount a standard linear attack on FEAL-4. Because of the “whitening” keys, $k_4, k_5, k_6, k_7, L_0, R_0, L_4, R_4$ are unknown.
- However, if we use the round key $K_0 = k_0 + k_4 + k_5$ for the first-round key and $K_3 = k_3 + k_6 + k_7$ for the last round key, we can express the inputs to F in the first and last rounds in terms of P_L, P_R, C_L, C_R, K_0 , and K_3 . This allows us to find K_0 , and K_1 .
- We can then use K_0 and K_3 to find K_2 and K_3
- Knowing K_0, K_3, K_2 , and K_3 allows us to compute the intermediate keys K_4, K_5 for refactored FEAL4 and then we can compute the original $k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7$.

FEAL-4 Linear Attack

- Let $K_4 = k_4 + k_5 + k_6, K_3 = k_3 + k_6 + k_7, K_1 = k_1 + k_4, K_0 = k_0 + k_6 + k_7$
- Then $(C_L + P_L + P_R) + K_4 = F(K_1 + P_L + F(K_0 + P_L + P_R)) + F(C_L + C_R + K_3)$
- From F-constraint 4,
 - $F(C_L + C_R + K_3)[23,29] = (C_L + C_R + K_3)[31] + 1$
 - $F(P_L + F(R_0 + K_0) + K_1)[23,29] = (P_L + F(P_L + P_R + K_0) + K_1)[31] + 1$
- Rearranging, we get “Equation A:”
$$K_3[31] + K_1[31] + K_4[23,29] = (C_L + P_L + P_R)[23,29] + P_L[31] + (C_L + C_R)[31] + F(P_L + P_R + K_0)[31]$$
- The attack consists of guessing K_0 and computing
$$h_A(P, C) = (C_L + P_L + P_R)[23,29] + P_L[31] + (C_L + C_R)[31] + F(P_L + P_R + K_0)[31]$$
for a number of corresponding $(P_L, P_R), (C_L, C_R)$.
- If the guessed K_0 is right, $h_A(P, C)$ will have the same value for each corresponding pair of plain-text and cipher-text.

Computing the Final Equations - A

- Remember, Equation A gave us
$$h_A(P,C) = (C_L + P_L + P_R)[23,29] + P_L[31] + (C_L + C_R)[31] + F(P_L + P_R + K_0)[31]$$
- It was derived from
 - $(L_4 + R_0)[23,29] = Y_1[23,29] + Y_3[23,29]$.
 - $Y_1[23,29] = F(R_1 + k_1)[31] + 1$, and $R_1[31] = L_0[31] + F(R_0 + K_0)[31]$, giving
 - $Y_1[23,29] = (L_0[31] + F(R_0 + K_0) + k_1)[31] + 1$
 - $Y_3[23,29] = (R_4 + K_3)[31] + 1$
- Combining, we got
 - $h_A(P,C) = f(K_i) = (L_4 + R_0)[23,29] + (R_4 + L_0 + F(R_0 + K_0))[31]$

Computing the Final Equations - B

- Analogously,
 - $(L_4+R_0)[13] = Y_1[13] + Y_3[13]$
 - $Y_1[13] = F(R_1+K_2)[13]+1 = (R_1+K_2)[7, 15, 23, 31]+1$
 - $R_1[7, 15, 23, 31] = (L_0[7, 15, 23, 31] + F(R_0+K_0))[7, 15, 23, 31]$, so
 - $Y_1[13] = (L_0[7, 15, 23, 31] + F(R_0+K_0))[7, 15, 23, 31] + K_2[7, 15, 23, 31]+1$
 - $Y_3[13] = F(R_4+K_3)[13]+1 = (R_4+K_3) [7, 15, 23, 31]+1$
 - $(L_4+R_0)[13] = (L_0[7, 15, 23, 31] + F(R_0+K_0))[7, 15, 23, 31]+$
 $K_2[7, 15, 23, 31]+(R_4+K_3) [7, 15, 23, 31]$
- This yields
 - $h_B(P,C) = (C_L+P_L+P_R)[13]+(P_L+(C_L+C_R)+F(P_L+P_R+K_0))[7, 15, 23, 31]$

Computing the Final Equations - C

- Similarly
 - $(L_4+R_0)[5, 15] = Y_1[5, 15] + Y_3[5, 15]$
 - $Y_1[5, 15] = F(R_1+K_2)[5, 15] + 1 = (R_1+K_2)[7]$
 - $R_1[7] = (L_0[7] + F(R_0+K_0))[7]$, so
 - $Y_1[5, 15] = (L_0[7]+F(R_0+K_0))[7] + K_2[7]$
 - $Y_3[5, 15] = F(R_4+K_3)[5, 15] = (R_4+K_3) [7]$
 - $(L_4+R_0)[5, 15] = (L_0[7]+F(R_0+K_0))[7]+K_2[7]+(R_4+K_3) [7]$
- This gives
 - $h_C(P,C) = (C_L+P_L+P_R)[5, 15] + (P_L+(C_L+C_R)+F(P_L+P_R+K_0))[7]$

Computing the Final Equations - D

- From $Y[15, 21] = X[23, 31]$
 - $(L_4 + R_0)[15, 21] = Y_1[15, 21] + Y_3[15, 21]$
 - $Y_1[15, 21] = F(R_1 + K_2)[15, 21] + 1 = (R_1 + K_2)[23, 31]$
 - $R_1[23, 31] = (L_0 + F(R_0 + K_0))[23, 31]$, so
 - $Y_1[15, 21] = (L_0 + F(R_0 + K_0))[23, 31] + K_2[23, 31]$
 - $Y_3[15, 21] = F(R_4 + K_3)[15, 21] = (R_4 + K_3)[23, 31]$
 - This gives
 - $(L_4 + R_0)[15, 21] = (L_0 + F(R_0 + K_0))[23, 31] + K_2[23, 31] + (R_4 + K_3)[23, 31]$
- This gives
 - $h_D(P, C) = (C_L + P_L + P_R)[15, 21] + (P_L + (C_L + C_R) + F(P_L + P_R + K_0))[23, 31]$

Computing the Final Equations - E

- We will use one more constraint. Adding all four round constraints, we get
 - $(L_4+R_0)[5,13,21] = Y_1[5,13,21]+Y_3[5,13,21] = F(R_1+K_1) [5,13,21] + F(R_4+K_3) [5,13,21]$
 - $F(R_4+K_3) [5,13,21] = (R_4+K_3) [15]+1$ and since $R_1 = L_0+F(L_0+Y_0+K_0)$,
 - $F(R_1+K_1) [5,13,21] = F(L_0+F(L_0+Y_0+K_0)+K_1) = (L_0+F(L_0+Y_0+K_0)+K_1)[15]+1$
- This gives
 - $h_E(P,C) = (C_L+P_L+P_R)[5,13,21]+P_L[15]+(C_L+C_R)[15]+F(P_L+P_R+K_0)[15]$
- Putting $P_L+P_R+K_0 = (x_0, x_1, x_2, x_3)$, we note that $F(P_L+P_R+K_0)[15]$ is only dependent on $(x_0 \oplus x_1, x_2 \oplus x_3)$
- Similar relations hold looking at FEAL-4 as a decryption algorithm. These constraints are summarized in the next two slides.

FEAL-4 Summary of invariants

Name	First Round Equation	Key bits affecting outcome
A	$h_A(P,C) = (C_L + P_L + P_R)[23, 29] + P_L[31] + (C_L + C_R)[31] + F(P_L + P_R + K_0)[31]$	
B	$h_B(P,C) = (C_L + P_L + P_R)[13] + (P_L + (C_L + C_R) + F(P_L + P_R + K_0))[7, 15, 23, 31]$	
C	$h_C(P,C) = (C_L + P_L + P_R)[5, 15] + (P_L + (C_L + C_R) + F(P_L + P_R + K_0))[7]$	
D	$h_D(P,C) = (C_L + P_L + P_R)[15, 21] + (P_L + (C_L + C_R) + F(P_L + P_R + K_0))[23, 31]$	
E	$h_E(P,C) = (C_L + P_L + P_R)[5, 13, 21] + P_L[15] + (C_L + C_R)[15] + F(P_L + P_R + K_0)[15]$	9,...,15; 17,...,23

FEAL-4 Summary of invariants

Name	Fourth Round Equation	Key bits affecting outcome
A	$h_A'(P,C) = (P_L + C_L + C_R)[23, 29] + (C_L + (P_L + P_R))[31] + F(C_L + C_R + K_3)[31]$	
B	$h_B'(P,C) = (P_L + C_L + C_R)[13] + (C_L + (P_L + P_R))[7, 15, 23, 31] + F(C_L + C_R + K_3)[7, 15, 23, 31]$	
C	$h_C'(P,C) = (P_L + C_L + C_R)[5, 15] + (C_L + (P_L + P_R))[7] + F(C_L + C_R + K_3)[7]$	
D	$h_D'(P,C) = (P_L + C_L + C_R)[15, 21] + (C_L + (P_L + P_R)) + F(C_L + C_R + K_3)[23, 31]$	
E	$h_E'(P,C) = (P_L + C_L + C_R)[5, 13, 21] + (C_L + (P_L + P_R))[15] + F(C_L + C_R + K_3)[15]$	9,...,15; 17,...,23

Strategy for FEAL-4 Linear Attack

- We use $h_E(P,C)$ to estimate the xor of the first two and last two bytes of K_0 and R_0 to estimate the xor of the two halves of K_0 (see slide 47) then we use h_A, \dots, h_D to find K_0 .
- Next, we use $h'_E(P,C)$ to estimate the xor of the first two and last two bytes of K_3 and R_4 then we use h'_A, \dots, h'_D to find K_3 .
- Next compute candidate K_1 's; for successful candidates, compute
 - $k_4+k_5+k_6 = F(P_L+F(P_L+P_R+K_0)+K_1) + F(C_L+C_R+K_3) + (P_L+P_R+C_L)$
- Analogously, for round 3, compute candidate K_2 's; for successful, candidates compute
 - $k_4+k_6+k_7 = F(C_L+F(C_L+C_R+K_3)+K_2) + F(P_L+P_R+K_0) + (C_L+C_R+P_L)$
- The “vanilla” attack of guessing K_0 , also works but our modified attack is much faster --- on the order of 2^{16} , which is peanuts.

FEAL-4 Linear Attack in gory detail

- Remember $k_4+k_5+k_6 = F(P_L+F(P_L+P_R+K_0)+K_1)+F(C_L+C_R+K_3)+(P_L+P_R+C_L)$
 - If $X = P_L+F(P_L+P_R+K_0)$, $Y = F(C_L+C_R+K_3)$ and $Z = P_L+P_R+C_L$. Note that X , Y and Z are known once we know K_0 and K_3 .
 - $k_4+k_5+k_6 = Z+Y+F(X+K_1)$.
 - Guess $K_1[0,1]$, $K_1[2,3]$ and compute $X[0,1]$, $X[2,3]$, we can test the guess by checking that $(Z+Y+F(X+K_1))[8,9,...15]$ remains constant over a set of plain/cipher pairs. This requires 2^{16} time.
 - Next, guess $K_1[0]$, $K_1[3]$ and again confirm the guess by checking that $(Z+Y+F(X+K_1))$ is constant.
 - Now that we know K_1 , can compute $k_4+k_5+k_6 = Z+Y+F(X+K_1)$.
- By looking at the corresponding FEAL-4 decryption, we get K_2 in exactly the same way as well as the other invariants r intermediate key, $k_4+k_6+k_7$.
- Finally, we check the complete set of guesses to confirm all the sub-keys are right.
- The entire automated attack runs in about 1 second on my MAC using 128 pairs of corresponding plain and cipher text.

Automated attack

```
./new_feal4.exe -preparecorrespondingtext 1234567890abcdef  
23234545abababcdcd 2048 feal.in1 feal.in2
```

Key schedule

k0	:	90abcdef
k1	:	32b729f8
k2	:	ada42552
k3	:	d26ad875
k4	:	ed3f65e8
k5	:	5f452e24
k6	:	14ee3941
k7	:	dbcb9075
k0+k4+k5	:	22d18623
k1+k4	:	df884c10
k2+k6	:	b94a1c13
k3+k6+k7	:	1d4f7141
k4+k5+k6	:	a694728d
k4+k6+k7	:	221accdc

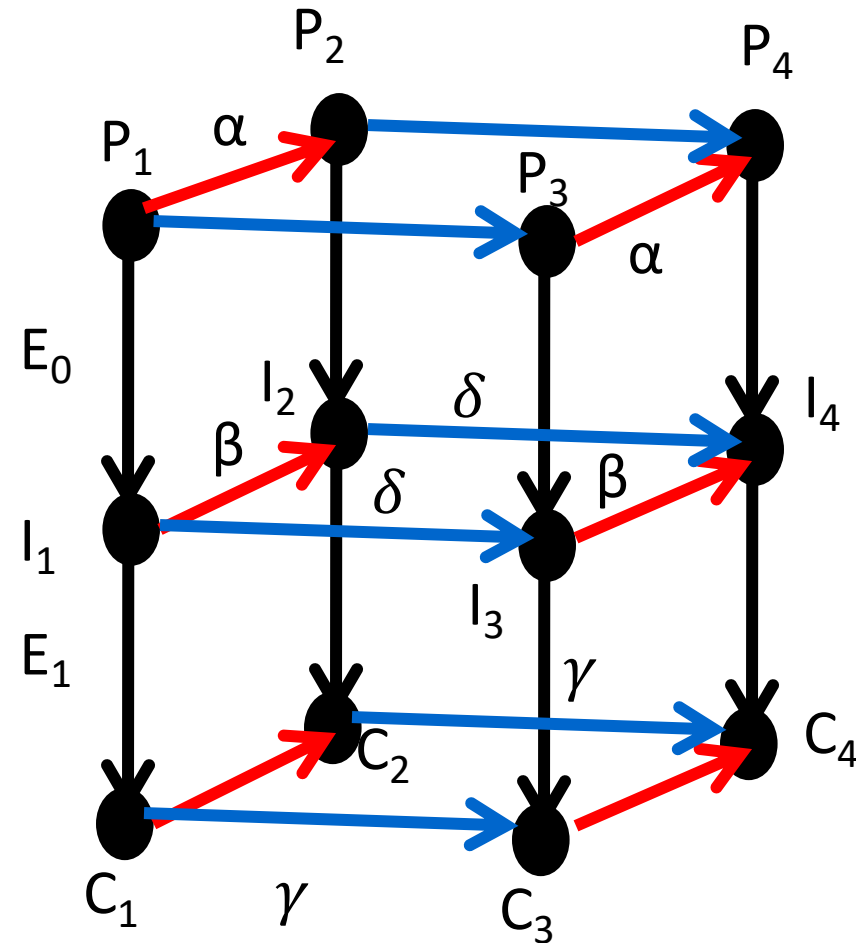
Automated attack

```
./new_feal4.exe -linearattack feal.in1 feal.in2
256 pairs examined
Plain: a1b24026 54a3e397, Cipher: c259fa58 99a44084
Plain: 44392b89 3e28d016, Cipher: b01696d4 59d70a09
...
...
```

Final check

```
Round 1 trial key: 22d18623
Round 2 trial key: df884c10
Round 3 trial key: b94a1c13
Round 4 trial key: 1d4f7141
k4k5k6 trial key: a694728d
k4k6k7 trial key: 221accdc
succeeded
```

Boomerang Attack



- $E_0: \alpha \rightarrow \beta$ with probability, p .
- $E_1: \delta \rightarrow \gamma$ with probability, q .
- For each pair (P_1, P_2) with $E_0: \alpha \rightarrow \beta$, obtain (C_1, C_2) and compute $C_3 = C_1 \oplus \gamma$ and $C_4 = C_2 \oplus \gamma$. Request the decryption of (C_3, C_4) as (P_3, P_4) .
- Probability that $P_3 \oplus P_4 = \alpha$, is $p^2 q^2$.
- For random permutation, the probability that $P_3 \oplus P_4 = \alpha$, is 2^{-n} .
- Can also be mounted for all possible β 's and γ 's as long as $\beta \neq \gamma$, with $p^2 = [\sum_{\beta, \alpha \rightarrow \beta} \text{Pr}^2(\alpha \rightarrow \beta)]^{1/2}$, $q^2 = [\sum_{\gamma, \gamma \rightarrow \delta} \text{Pr}^2(\gamma \rightarrow \delta)]^{1/2}$

End

DES Data

S Boxes as Polynomials over GF(2)

1,1: 56+4+35+2+26+25+246+245+236+2356+16+15+156+14+146+145+13+135+134+1346+1345+13456+125+1256+1245+123+12356+1234+12346

1,2: C+6+5+4+45+456+36+35+34+346+26+25+24+246+2456+23+236+235+234+2346+1+15+156+134+13456+12+126+1256+124+1246+1245+12456+123+1236+1235+12356+1234+12346

1,3: C+6+56+46+45+3+35+356+346+3456+2+26+24+246+245+236+16+15+145+13+1356+134+13456+12+126+125+12456+123+1236+1235+12356+1234+12346

1,4: C+6+5+456+3+34+346+345+2+23+234+1+15+14+146+135+134+1346+1345+1256+124+1246+1245+123+12356+1234+12346

2,1: C+4+456+3+36+35+26+245+2456+235+2356+1+16+156+1456+13+136+135+1356+12+125+1256+1246+1236+12356

2,2: C+5+4+35+34+346+345+2+256+246+2456+236+1+156+145+13+135+134+1346+1345+12+126+125+124+1246+12456+123+1235+12356+1234

2,3: C+6+5+4+456+36+3456+2+24+246+23+1+1245+12456+1235+12356

2,4: C+6+5+45+3+26+24+245+23+236+1+156+145+1456+1356+126+1256+1245+12456+123+1236

Legend: C+6+56+46 means $1 \oplus x_6 \oplus x_5 x_6 \oplus x_4 x_6$

S boxes as polynomials

3,1: 6+4+45+35+2+1+16+15+146+145+13+135+12+126+125+1256+123+1236+1235+12346

3,2: C+6+5+4+46+456+36+35+356+34+346+345+3456+2+25+256+24+245+23+236+
234+2346+1+16+14+146+145+1456+135+1356+1346+13456+126+125+
1256+124+1246+12456+1234+12346

3,3: 6+46+45+456+3+35+26+25+256+24+246+23+236+235+2356+234+1+1456+
13456+12+126+125+1256+124+123+1236+1235+12356+1234

3,4: C+5+46+45+456+3+35+34+3456+2+24+245+2456+235+2356+234+16+14+146+
145+1456+13+1356+134+13456+12+124+1245+12456+123+1234

4,1: C+56+4+46+45+3+3456+26+25+256+245+2456+23+236+2346+1+16+156+
146+1456+13+136+135+13456+12+125+124+1245+123+1236+12356+1234

4,2: C+6+5+56+46+45+3+345+3456+2+26+256+2456+236+234+2346+16+15+
156+14+146+145+1456+136+135+1345+13456+12+125+124+1245+1236+1235+
12356+1234

4,3: C+56+46+45+456+3+36+35+2+26+256+2456+23+2356+234+2346+1+15+156+
146+135+1356+1346+13456+1256+124+1245+12356+1234

4,4: 6+5+56+4+46+456+36+35+26+25+256+245+2456+23+235+2356+2346+1+
156+14+146+1356+134+1346+1345+13456+125+1256+124+1245+1235+12356+1234

S boxes as polynomials

5,1: $56+45+3+36+35+356+346+345+3456+26+25+256+24+246+2456+235+16+14+145+13+136+1346+1345+13456+12+126+125+1256+124+1245+123+1236+1235+12356+1234$

5,2: $C+5+56+4+46+45+36+35+34+346+345+3456+2+25+256+246+245+235+2356+234+2346+1+16+156+14+145+13+136+135+134+1346+1345+13456+126+125+124+12456+123+12356+1234+12346$

5,3: $6+5+4+3+36+356+346+3456+24+236+2346+1+156+145+1456+1345+126+1246+123+1236+1234+12346$

5,4: $6+5+56+46+45+36+34+346+345+3456+2+24+246+245+236+2356+15+156+146+13+136+1356+1345+1256+124+1246+1245+12456+1236+1234$

6,1: $5+456+3+34+346+345+3456+24+2456+23+234+2346+1+16+145+1456+135+134+1346+1345+13456+126+1246+12456+1236$

6,2: $6+4+456+35+256+245+23+235+16+15+1456+13+136+135+1356+12+1245+12456+123+12356$

6,3: $C+6+5+4+3+35+345+2+24+2456+1+145+1456+13+136+1356+1345+1245+123+1236+1235+12356+12346$

6,4: $C+5+56+46+45+456+36+356+34+346+345+3456+2+23+2346+16+15+156+146+1456+13+136+135+1356+1246+12456+1236+12356+12346$

S boxes as polynomials

7,1: 6+5+45+3+34+345+2+246+2456+23+1+146+1456+1346+13456+1256+1246+1236

7,2: 5+56+4+45+456+3+36+346+3456+2+245+2456+2346+16+15+156+13+135+1356+
1346+13456+124+1245+123+1236+1235+12356+12346

7,3: C+5+4+3456+2+26+24+2456+23+1+16+14+13+1345+12+1246+12456+1236+1234

7,4: 6+5+3+345+3456+24+23+236+234+2346+16+15+156+14+1456+136+135+1345+
13456+12+124+1245+123+1236+1235+1234+12346

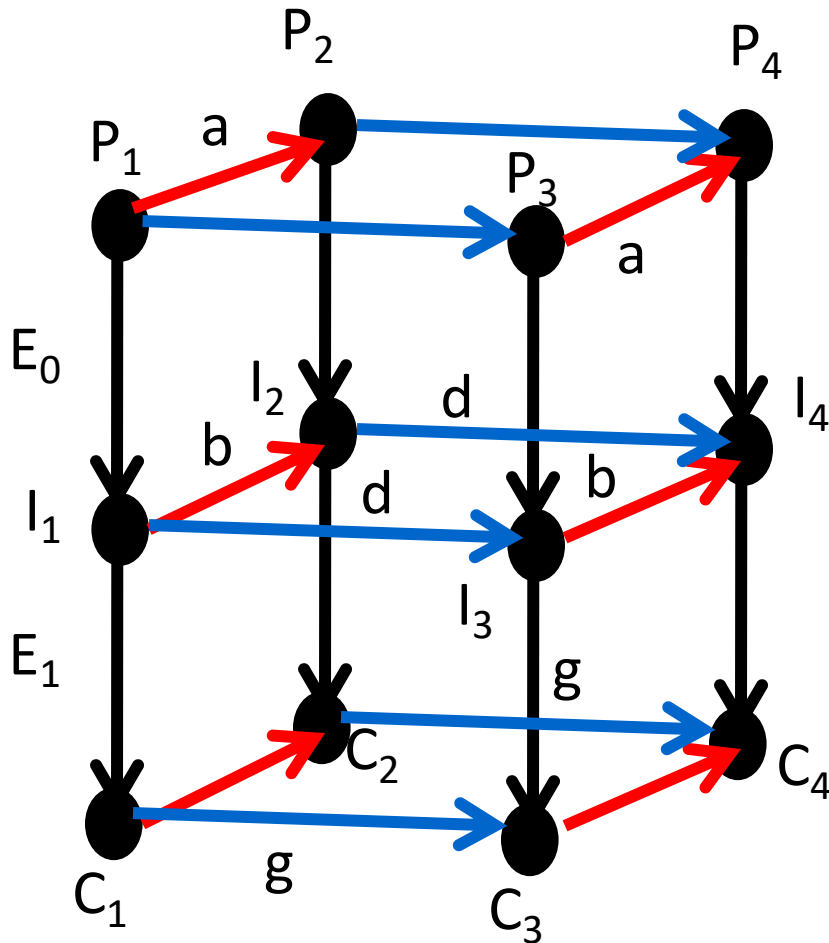
8,1: C+5+56+4+46+45+3+356+346+3456+2+256+245+236+16+15+1456+13+135+1356+
1346+1256+124+1246+1245+123+1235+12356+12346

8,2: 5+45+3+35+2+26+256+246+2456+236+2346+1+15+156+14+146+145+1456+135+
125+12456+1235+12356

8,3: C+6+5+4+35+2+25+24+245+23+156+14+146+13+135+1356+134+1346+125+124+
1245+123+1234+12346

8,4: C+6+5+46+456+3+34+346+26+25+256+24+246+245+234+2346+1+16+156+145+
1456+136+135+134+1346+1246+12456+1236+12356+1234+12346

Amplified Boomerang Attack



- Given plaintext pair $(P_1, P_2)(P_3, P_4)$
- For random permutations, the probability that $P_1 \oplus P_2 = P_3 \oplus P_4 = a$,
- E_0 : $a \rightarrow b$ with probability, p .
- When both pairs satisfy $E_0(P_1) \oplus E_0(P_2) = E_0(P_3) \oplus E_0(P_4) = b$, $E_0(P_1) \oplus E_0(P_3) = (E_0(P_1) \oplus b) \oplus (E_0(P_3) \oplus b) = E_0(P_2) \oplus E_0(P_4)$.
- If $E_0(P_1) \oplus E_0(P_3) = E_0(P_2) \oplus E_0(P_4) = g$, each has a probability, q , to be a right pair wrt $g \rightarrow d$. $C_1 \oplus C_3 = C_2 \oplus C_4 = d$
- $\Pr(\text{quartet becomes right quartet with difference } a) = (Np)^2/2$ quartets
- Expected number of right quartets is $Np C_2 2^{-n} q^2$

Truncated Differentials

- A *truncated differential* predicts that the differences are restricted to some set. For example, in the description of the 2R-attack on 7-round DES for a right pair with respect to the 5-round characteristic, there are some cipher text bits with a zero difference for sure. This can be described as a 7-round truncated differential of DES with probability $p=1/9511$ that predicts the difference of 12 output bits.
- Truncated differentials can be used in the differential 1R- and 2R-attacks, to discard wrong pairs. Another application of truncated differentials is to define a distinguisher for the cipher (resulting in a key recovery attack at the end). For example, there is a 12-round truncated differential (in rounds 5–16) of Skipjack with probability 1 that predicts 16 bits of difference.

Rectangle Attack

- Given N pairs with difference a , pN pairs satisfy $a \rightarrow b$.
- pN pairs satisfy $a \rightarrow b$.
- $\sim (Np)^2/2$ quartets that satisfy differentials.
- Given Np pairs $(P_1, P_2), (P_3, P_4)$, expected number of right quartets is $_{Np}C_2 2^{-n}$
 $q^2 = N^2 2^{-n+1} (pq)^2$
- $E' = E_f \cdot E_1 \cdot E_0 \cdot E_b, Z_i = E_0(P_i)$
- Instead of just looking for $g \rightarrow d$, look for any $g' \rightarrow d$.

Rectangle Distinguisher

- $P_1 \oplus P_2 = P_3 \oplus P_4 = a, C_1 \oplus C_3 = C_2 \oplus C_4 = b$
- $\Pr[(P_1, P_2), (P_3, P_4) \text{ is a right quartet}] = 2^{-n} \sum_{a,b} ([\Pr(a \rightarrow a) \Pr(b \rightarrow b)] \sum_g ([\Pr(g \rightarrow d) \Pr(g \oplus a \oplus b \rightarrow d)])$
- $E' = E_f \cdot E_1 \cdot E_0 \cdot E_b, Z_i = E_0(P_i)$
- Steps
 1. Data collection
 2. Initialize
 3. Insert
 4. Generate Quartet
 5. Find and analyze quartets
 6. Count sub-keys

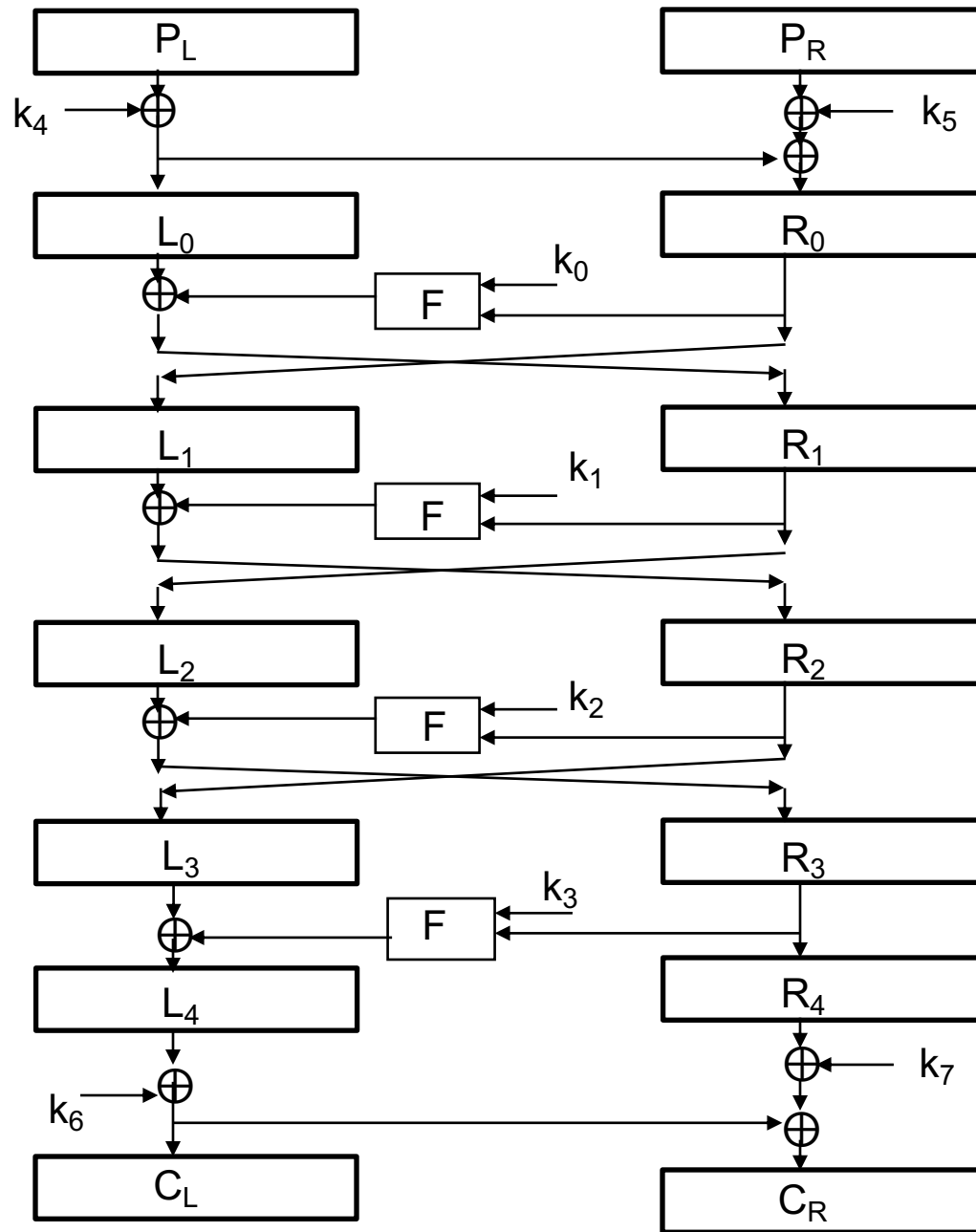
Bilinear Attack

- Let $L_r[0, 1, 2, \dots, n-1]$, $R_r[0, 1, 2, \dots, n-1]$ are the input to round r and $L_r[0, 1, 2, \dots, n-1]$, $O_r[0, 1, 2, \dots, n-1]$ are the input (without key) and output to the round functions.
- If $\alpha \subseteq \{0, 1, 2, \dots, n-1\}$, define $L_r[\alpha] = \bigoplus_{s \in \alpha} L_r[s]$.
- Consider the bilinear $L_{r+1}[\beta] \cdot R_{r+1}[\alpha] \oplus R_r[\beta] \cdot L_r[\alpha] = L_r[\beta] \cdot O_r[\alpha]$.

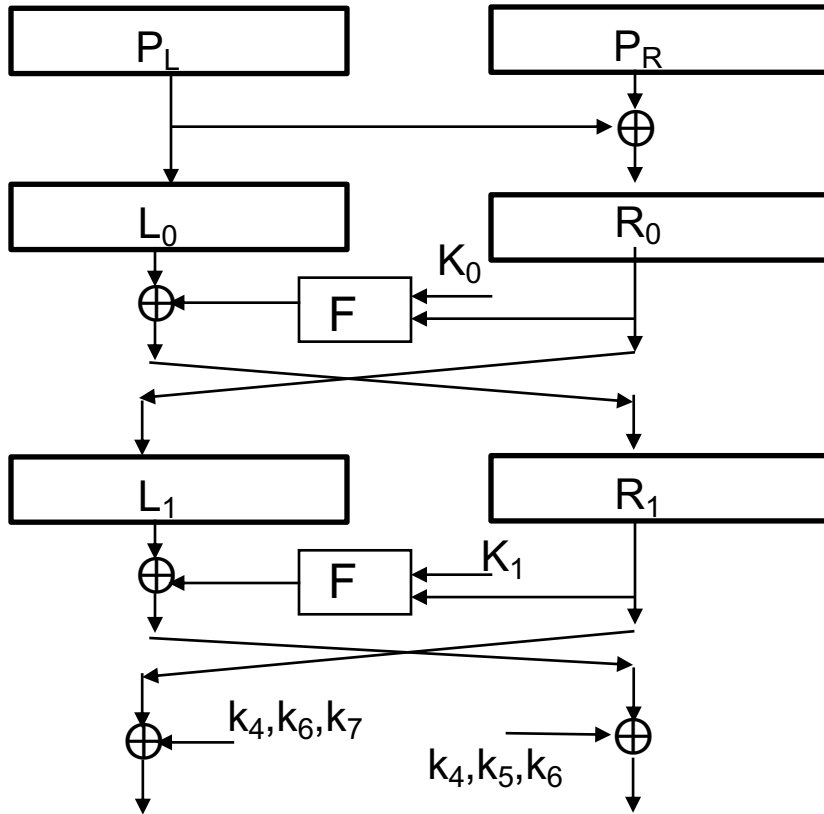
Slide Attack

- Let F be a per-round function.
- If $C = E_K(P) = F_K^m(P)$, $P, C \in GF(2)^n$ and $P' = F(P)$
- $C' = E(P') = F(C)$. To find slide pairs, let $a_F(P, C) = K$ which is easy to calculate. Store $2^{n/2}$ (and possibly less as in DES) pairs (P, C) if $a_F(P, C) = a_F(P', C')$, $P' = F_K(P)$ and $C' = F(C)$. By birthday collision, this will happen.
- Effective against rounds which implement weak permutations.

Original FEAL-4

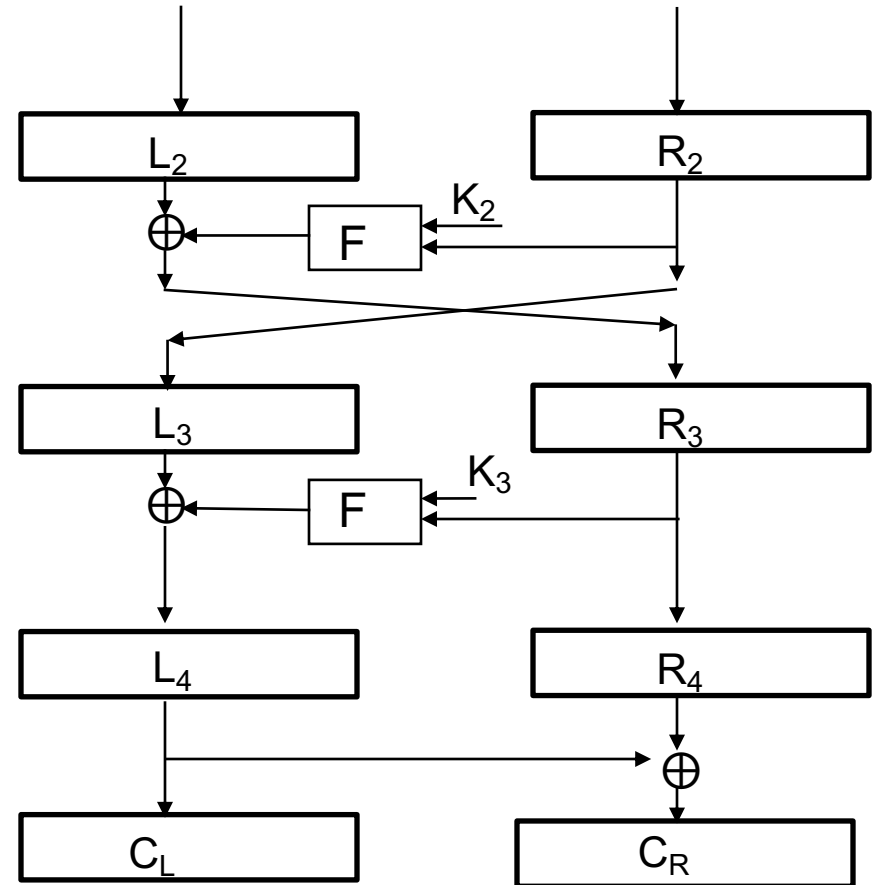


Refactored FEAL-4



$$K_0 = k_0 + k_5 + k_6$$

$$K_1 = k_1 + k_4$$



$$K_3 = k_3 + k_6 + k_7$$

$$K_2 = k_2 + k_6$$