Electronics of Radio, Part 2

Notes on David Rutledge’s book

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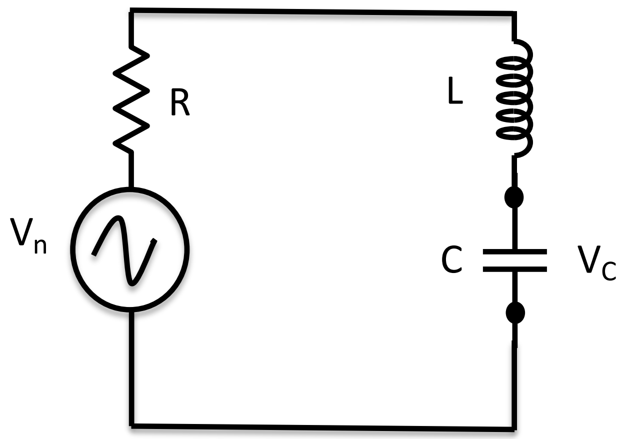
Noise

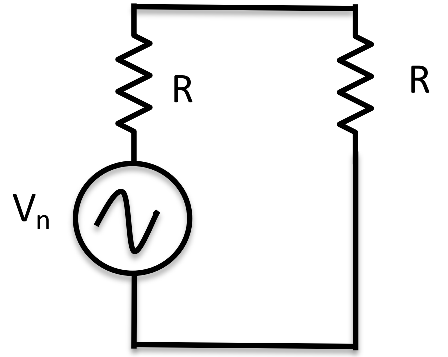
- $V_{n(rms)} = \sqrt{\frac{1}{\tau}\int_0^\tau V(t)^2}\ dt$ so the average noise power is $P_n = \frac{{V_{n(rms)}}^2}{R}$, where R isthe load resistance
- Define $SNR = \frac{P}{P_n}$ and $MDS = \frac{P_n}{G}$.
- $NEP = \frac{N}{G}$ (Noise equivalent power)
- Noise phasors
  - $\overline{|V_n|^2} = \int |V_n|^2 p\ dA$
  - $N = \frac{\overline{|V_n|^2}}{2R}$
  - $P_n = NB$, N is noise power density, B is bandwidth.
  - The units for B are ${}^{volts}/_{\sqrt{Hz}}$

Nyquist

- $V_C = \frac{1}{j\omega C}\frac{V_n}{R+j\omega L+\frac{1}{j\omega C}}$
- $\overline{|V_c|^2} = \frac{\overline{|V_n|^2}}{|1-\omega^2 LC+j\omega RC|^2}$
- Expected the energy stored in C, at resonance is $kT = \frac{C}{2}\int_0^\infty |V_c|^2 df$, by equipartition theorem
- $\int_0^\infty \frac{1}{|1-\omega^2 LC+j\omega RC|^2} df = \frac{1}{4RC}$
- So, $\overline{|V_n|^2} = 8\mathrm{kTR}$, $V_{n(rms)} = \sqrt{4kTR}$
- $N = kT = \frac{|\frac{V_n}{2}|^2}{2R} = \frac{\overline{|V_n|^2}}{8R}$ (Noise density)
- $T_e = \frac{N}{k}$
- $T_n = \frac{NEP}{k}$



Noise model for Nyquist



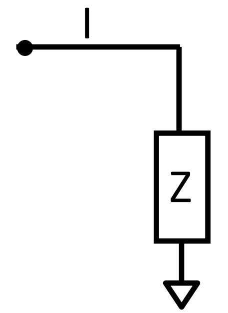
Matched load

Antennas

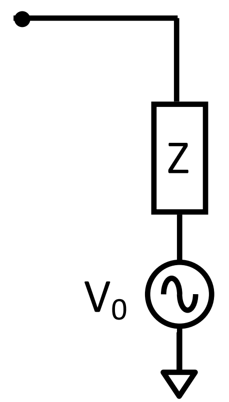
- From Maxwell, for a plane wave (E in x direction, H in y direction), wave is of form $\exp(j\omega t - j\beta z)$
- $\nabla \times E = -j\mu_0 \omega H, \nabla \times B = j\epsilon_0 \omega E$
- $\beta \hat{z} \times E = \mu_0 \omega H, \beta E_x \hat{y} = \mu_0 \omega H$
- Substituting and taking the cross product, we get:
  - $\beta E_x = \omega \mu_0 H_y$ and $\beta H_y = -\omega \epsilon_0 E_x$
  - $\eta_0 = \frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$
- Write $\beta = \omega\sqrt{\mu_0 \epsilon_0}$
- Power density
  - $S = Re\left(\frac{E_x \overline{H_y}}{2}\right) = \frac{(|E_x|)^2}{2\eta_0}$
- Impedance:
  - $P_t = \frac{R|I|^2}{2}$, R is real part of Z,
  - $R = R_r + R_l, \eta = \frac{R_r}{R}$

Transmitting Antennas

- Power density for isotropic antenna: $S_i = \frac{P_t}{4\pi r^2}$
- Define $G(\theta, \phi) = \frac{S(\theta,\phi)}{S_r}$. $S(\theta, \phi)$ is just the Poynting vector.
- $G(\theta, \phi) \equiv \frac{S(\theta,\phi)}{S_i} = \frac{4\pi r^2 S}{P_t}$.
- Since $S_\Omega((\theta, \phi) = r^2 S((\theta, \phi)$, $\int G(\theta, \phi)\, d\Omega = 4\pi$



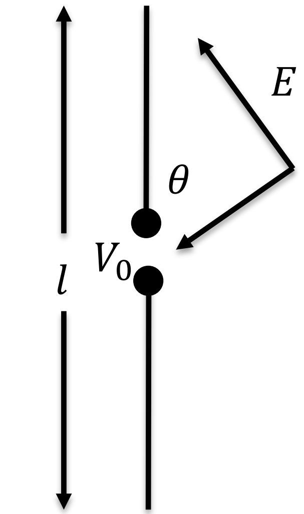
Transmitting Antenna



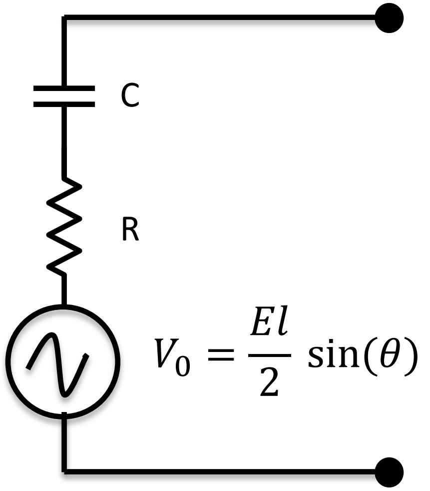
Receiving Antenna

Receiving Antenna

- $V_o = hE$, h is effective antenna length ($h = \frac{l}{2}$ for short antenna), V_o is open circuit Thevenin voltage.
- For dipole: $V_0 = \frac{l}{2}\, E\sin(\theta)$, so $h = \frac{l}{2}\sin(\theta)$.
- The effective area is $A(\theta,\phi) \equiv \frac{P_r}{S(\theta,\phi)}$ (This is proved later.)
- By reciprocity, $A(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)$
- $P_r = \frac{|V_0|^2}{8R_a} = \frac{|hE|^2}{8R_a}$, so $P_r = \frac{h^2 S(\theta,\phi)\eta_0}{4R}$, so $A = \frac{h^2\eta_0}{4R}$



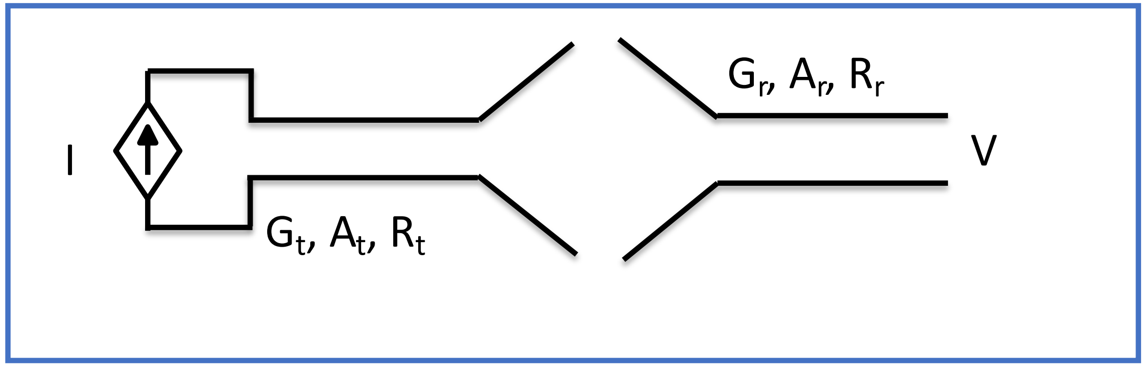
Receiving dipole



Dipole Thevenin equivalent circuit

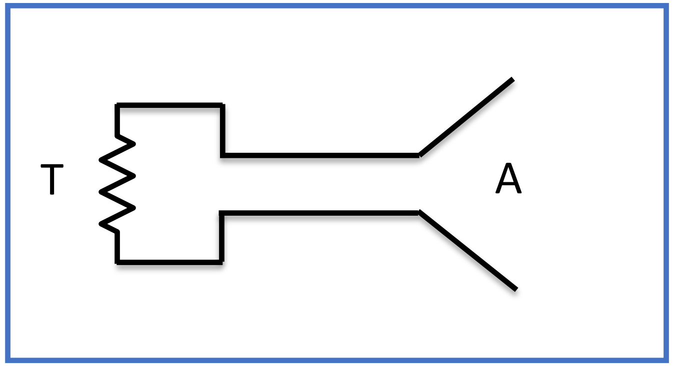
Friis formula

- For transmitting/receiving antenna pairs:
  - $G_1 A_2 = \frac{|V|^2 \pi r^2}{|I|^2 R_1 R_2} = G_2 A_1.$
- So $\frac{G_1}{A_1} = \frac{G_2}{A_2} = \frac{4\pi}{\lambda^2}$
- $S = \frac{P_t G}{4\pi r^2}$
- $P_r = SA = \frac{P_t G A}{4\pi r^2}$. --- Friis radiation formula
- For us, $G = 1, A = 150\ m^2, r = 2000km, P_t = 2W$
- $P_r = 6pW$



Antenna Theorem

- Antenna theorem: $\oint A(\theta,\phi)d\Omega = \lambda^2$
- For cavity on right, T is constant at thermodynamic equilibrium and the same power is emitted and absorbed, the Johnson noise is kT. The energy received is
  - $E = \frac{4\pi kT}{c\lambda^2}$.
  - Set $B = \frac{kT}{\lambda^2}$.
  - $kT = \oint BA\, d\Omega = \oint A \frac{kT}{\lambda^2}\, d\Omega$, giving the antenna theorem

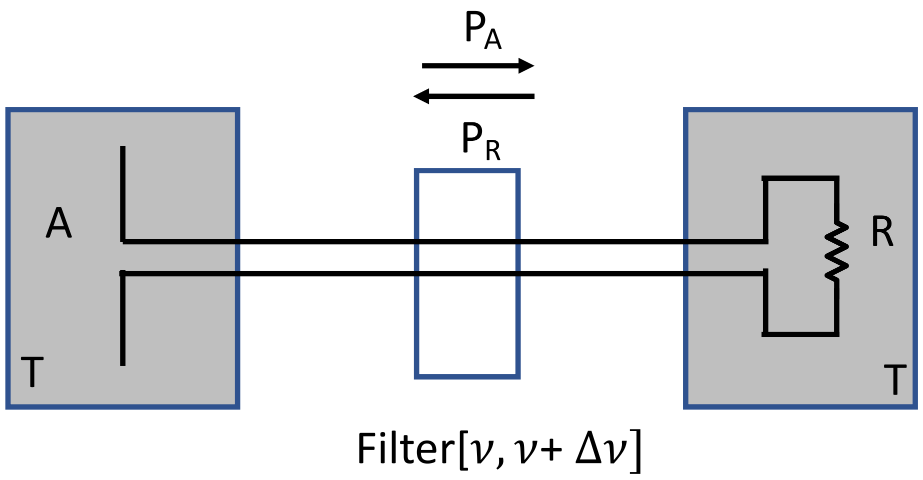


Insulated cavity

Antenna aperture

- Deriving antenna aperture uses thermodynamic argument: black body equilibrium

- $P_A = \frac{A_e}{2} B_\upsilon \Delta\nu \int_{[0,4\pi]} d\Omega = 2\pi\, A_e B_\upsilon \Delta\nu$
- $B_\upsilon = \frac{2\nu^2 kT}{c^2} = \frac{2kT}{\lambda^2}$ (Rayleigh-Jeans)
- $P_R = kT\Delta\nu$
- $P_A = P_R$
- $A_e = \frac{\lambda^2}{4\pi}$

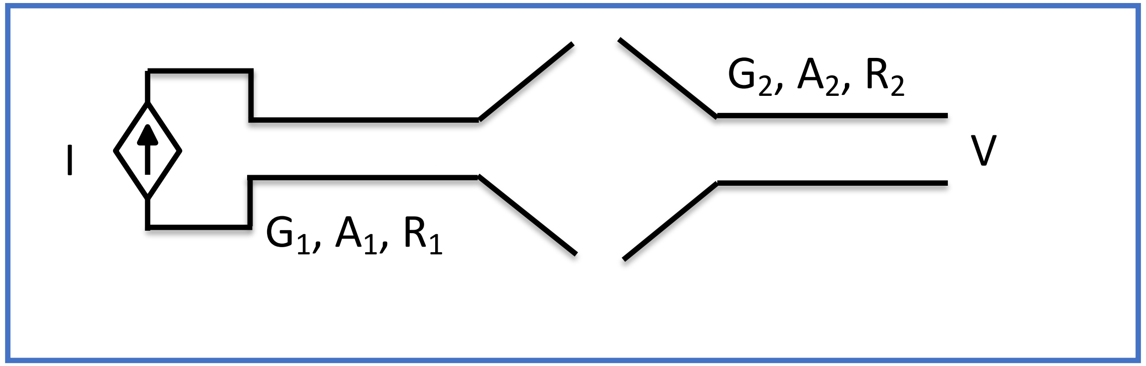


Radiation loss and antenna aperture

- Spreading loss: $L_s = 32 + \log(\mathrm{d}) + 20\log(\mathrm{f})$
  - d in kilometers
  - F in megahertz
- $W = A_e P_e, A_e = \frac{\lambda^2}{4\pi}$

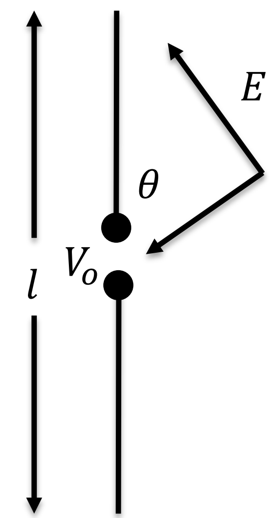
Reciprocity

- *Reciprocity:* The position of an ideal voltmeter and ideal current source can be interchanged without changing the voltmeter reading.
- $P_t = |I|^2 \frac{R_1}{2}$
- $P_r = \frac{P_t G_1 A_2}{4\pi r^2}$, *by Friis*
- $P_r = \frac{|V|^2}{8R_2}$
- $\frac{|V|^2}{8R_2} = \frac{|I|^2 R_1 G_1 A_2}{8\pi r^2}$
- $\frac{|V|^2 \pi r^2}{|I|^2 R_1 R_2} = G_1 A_2$, so
- $G_1 A_2 = G_2 A_1$ or $\frac{G_1}{A_1} = \frac{G_2}{A_2} = \frac{G}{A}$
- $\int G(\theta, \phi) d\Omega = 4\pi$ and by the antenna theorem, $\int A(\theta, \phi) d\Omega = \lambda^2$, and $\frac{G}{A} = \frac{4\pi}{\lambda^2}$

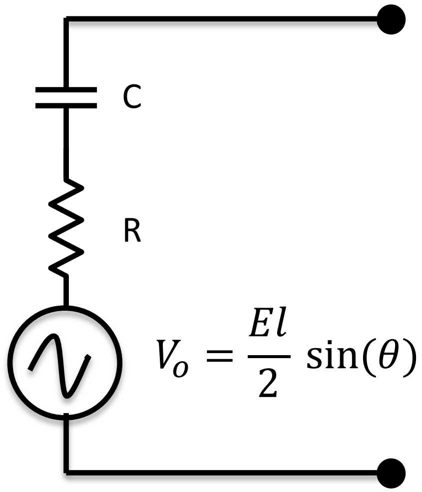


Reciprocity and dipoles

- For dipole, $h = \frac{l}{2}\sin(\theta)$. Remember $\eta = \frac{R_r}{R}$.
- $\lambda^2 = \int A(\theta,\phi)d\Omega = \int \frac{h^2\eta_0}{4R_r}\, d\Omega$, so
- $R_r = \frac{l^2\eta_0}{16\lambda^2}\int sin^2(\theta)d\Omega = \eta_0\frac{\pi}{6}\,(\frac{l}{\lambda})^2$
- Since $\eta_0 = 120\pi$ so for half-wave dipole, $R_r \approx 20\pi^2 \approx 49\Omega$ (It's closer to 73Ω
    - This explains the customary 73Ω or 73Ω coax impedance
- Since $A = \frac{h^2\eta_0}{4R}$, $A = \frac{3\lambda^2}{8\pi}\, sin^2(\theta)$
- Since $\frac{G}{A} = \frac{4\pi}{\lambda^2}$, $G = \frac{3}{2}\, sin^2(\theta)$.

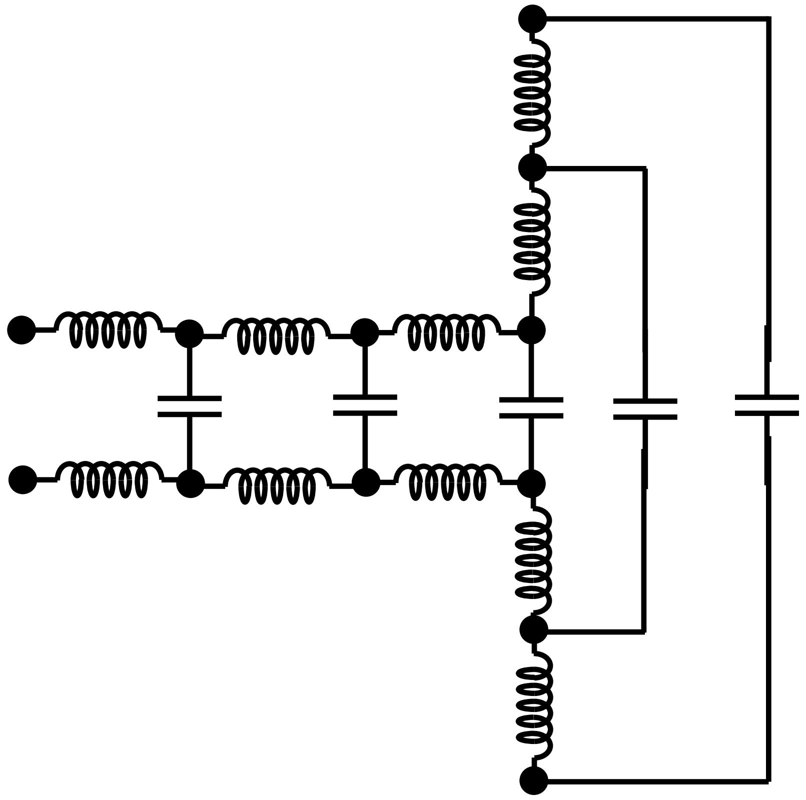


Receiving dipole



Dipole Thevenin equivalent circuit

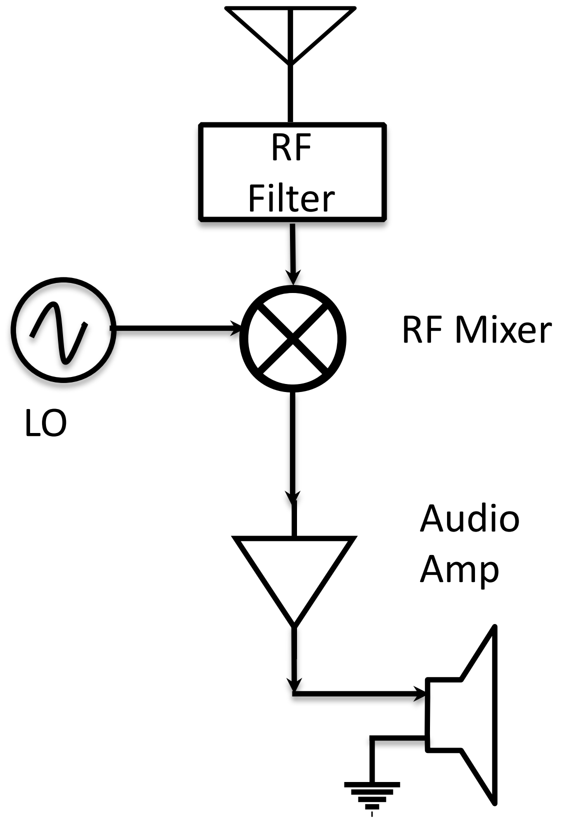
Another antenna model



- $E = \frac{j\omega\mu}{4\pi} I_m \frac{e^{j\beta r}}{r}$
- $P_R = P_B G_A G_B (\frac{\lambda}{4\pi r_{AB}})^2$

Direct conversion and superhet receivers

- Image frequency
  - $\omega_{rf} = \omega_{LO} - \omega_a$
  - $\omega_i = \omega_{LO} + \omega_a$
- Superheterodyne designs
  - $\omega_{rf} = \omega_{IF} + \omega_{VFO}$
  - $\omega_{vi} = \omega_{IF} - \omega_{VFO}$
  - $\omega_{IF} = \omega_{BFO} - \omega_a$
  - $\omega_{bi} = \omega_{BFO} + \omega_a$
  - $\omega_{usb} = \omega_{VFO} + \omega_{BFO} + \omega_a$
  - $\omega_{lsb} = \omega_{VFO} + \omega_{BFO} - \omega_a$



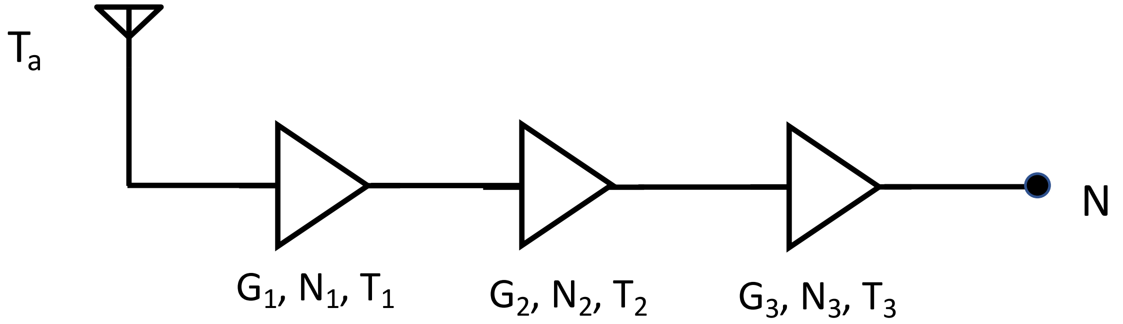
Direct conversion

System noise

- Attenuator (and filter) noise
  - $N_a = kT\left(1 - \frac{1}{L}\right), T_a = T(L-1)$

- $N = G_3 G_2 G_1 k T_a + G_2 G_3 N_1 + G_3 N_2 + N_3$
- $T_r = T_a + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}$
- $\frac{T_n}{T_0} = F - 1, T_0 = 290\ K$
- For NorCal, $L = 3.2$, each filter has $T_f = 290(L-1) = 630K$
- $T_r = T_f + LT_m + \frac{T_f L}{G} + T_m \frac{L^2}{G} = 2780K$

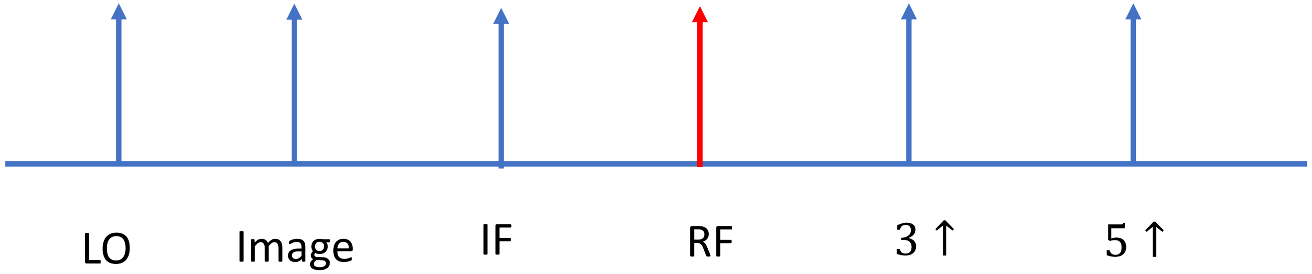
- Want receiver noise << Antenna noise
- For NorCal, antenna noise is at least 30 dB bigger than receiver noise
- To measure noise, inject a signal that doubles the Noise power



Process Cascade

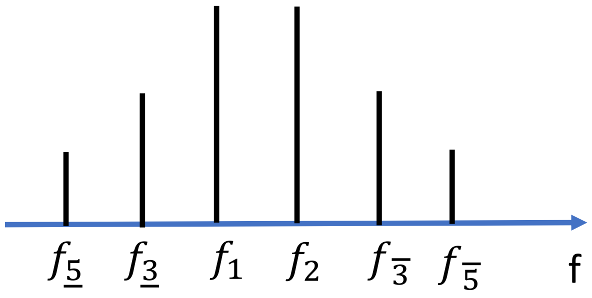
Mixers

- $V_{lo}(t)$ is a square wave with period ω_{lo}. Expanding this in a Fourier series, we get:
- $V_{lo}(t) = \frac{4}{\pi}(\cos(\omega_{lo}t) - \frac{\cos(3\omega_{lo}t)}{3} + \frac{\cos(5\omega_{lo}t)}{5} \ldots), V_{rf}(t) = V_{rf}\cos(\omega_{rf}t)$
- $V_{lo}(t)V_{rf}(t) = \frac{2V_{rf}}{\pi}(\cos(\omega_{-}t) - \frac{\cos(3\omega_{-}t)}{3} + \frac{\cos(5\omega_{-}t)}{5} \ldots) + \frac{2V_{rf}}{\pi}(\cos(\omega_{+}t) - \frac{\cos(3\omega_{+}t)}{3} + \frac{\cos(5\omega_{+}t)}{5} \ldots)$
- $\omega_{+} = \omega_{lo} + \omega_{rf}$ and $\omega_{-} = |\omega_{lo} - \omega_{rf}|$
- We define $\omega_{k+} = (k\omega_{lo}+\omega_{rf})$ and $\omega_{k-} = |k\omega_{lo} - \omega_{rf}|$ and $V_{k+}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k+}t)$ and $V_{k-}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k-}t)$
- $\omega_i = \omega_{if} - \omega_{lo}$ and $\omega_{if} = \omega_{if} + \omega_i$, ω_i is a spurious signal. ω_{k+} and ω_{k-} are the spurs from the kth harmonic



Intermodulation

- If frequency components are $k_1 f_1 + k_2 f_2 + \cdots + k_n\, f_n$, modulation order is $|k_1| + |k_2| + \cdots + |k_n\,|$
- Intermodulation response
  - $V = \ G_v V_i + G_2 {V_i}^2 + G_3 {V_i}^3 + \ldots$
  - $V_i = \ V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)$
- MDS is where input power (P_i) is $2P_n$. MDI is where intermodulation power is $2P_n$.
- Dynamic range = MDI-MDS.
- Good receivers have DR $\sim 100dB$



Intermodulation products

Exercise 35: Intermodulation

- Only $f_{3\uparrow} = 2f_1 - f_2, f_{3\downarrow} = 2f_2 - f_1, f_{5\uparrow} = 3f_1 - 2f_2\ and\ f_{5\downarrow} = 3f_2 + 2f_1$ are close enough to the rf frequency to matter for intermodulation

1. Find coefficients and frequencies for $[\cos(\omega_1 t) + [\cos(\omega_2 t)]^5$
2. Find $f_{3\uparrow}$, $f_{3\downarrow}$, $f_{5\uparrow}$ and f_1
3. Find the MDS and the antenna limited MDR

- $MDS = \frac{P_n}{G}$, MDI is input that gives output tone + $2P_n$
- $MDR = MDI - MDS$
- This needs measurements to measure the minimum detectable signal.

Exercise 37: Antennas

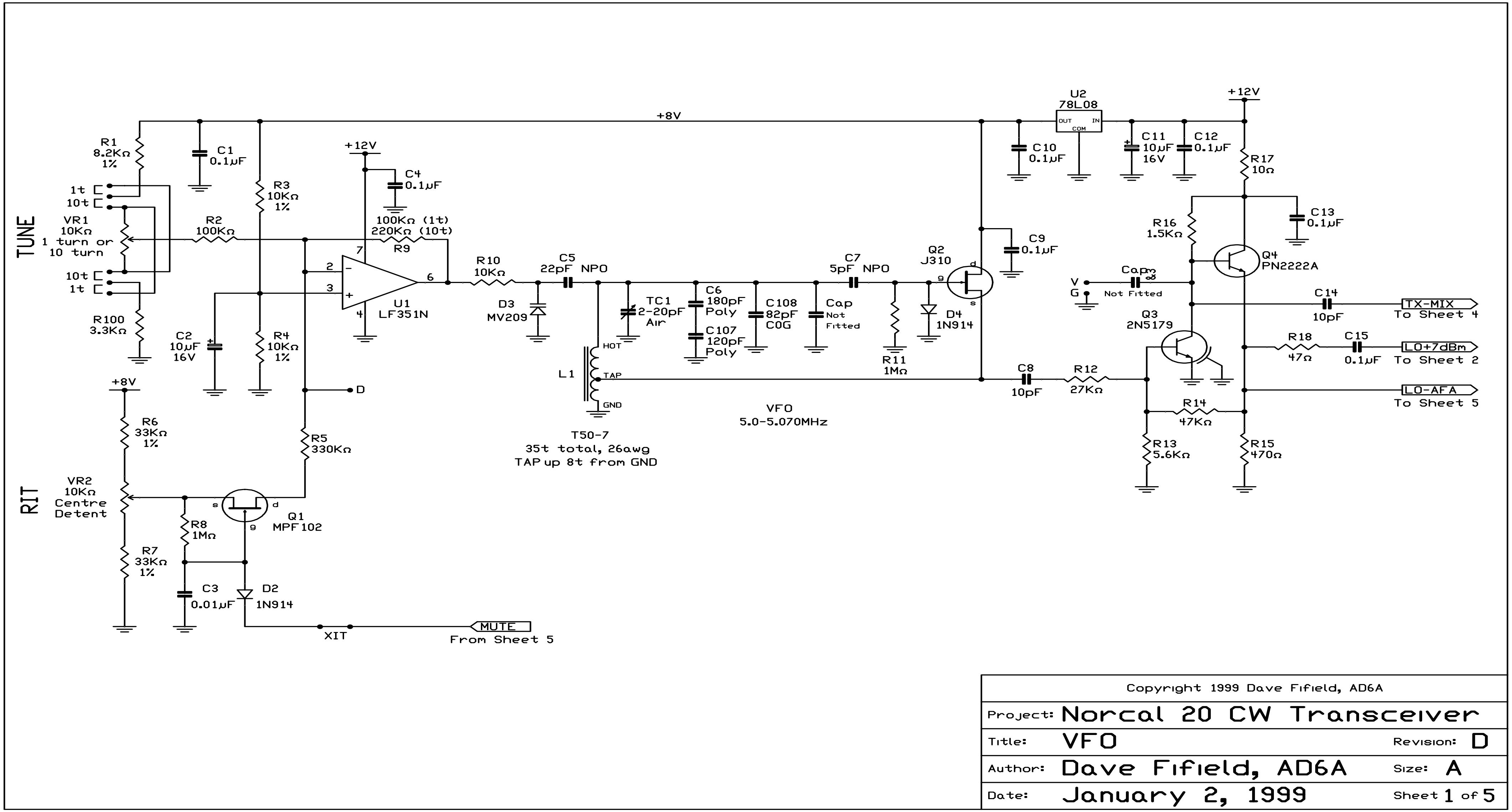
1. Use the relation between gain and effective area to rewrite the Friis transmission formula in terms of gain only. Consider UHF for airplanes. If the frequency makes the quarter length stub antenna have gain 2, find the maximum possible LOS at 10km height. Required receiver power is –90 dBm. Find the minimum transmission power.
   - Friis radiation formula is $P_r = \frac{P_t G A}{4\pi r^2}$, $\lambda = \frac{c}{f}$ and $\frac{G}{A} = \frac{4\pi}{\lambda^2}$, so $P_r = P_t(\frac{Gc}{4\pi r f})^2$
   - P_r requires $-90dBm$ power. $-90 = 10\log\left(\frac{P_r}{1mW}\right)$, so P_r must be at least $10^{-12}\ W$.
   - $P_t(f) = (\frac{4\pi r f}{Gc})^2 \times 10^{-12} W$. $P_t(10^8) = 4.4mW$, $P_t(2\times10^8) = 19.24mW$, $P_t(3\times10^8) = 40mW$ and $P_t(10^9) = 440mW$.
2. Find the inductance to resonate with a 3m whip. Assuming the Q of the coil is 200, find the turns ratio required to give a transceiver a 50 ohm load. What is the radiation efficiency?
   - For whip, $R_r = 160\pi^2(\frac{l}{\lambda})^2 \approx 7.8\Omega$. $Q = 200 = \frac{L}{7.8}$. $L = 1551\Omega$.
   - $Z_s = 7.8 + 1551j$. $L = 3.6\mu H$
   - $\frac{1551}{50} = \frac{Z_p}{Z_s} = (\frac{N_p}{N_s})^2$, $\frac{N_p}{N_s} = \sqrt{31} = 5.2$.
   - Radiation efficiency is $\frac{P_{radiated}}{P_{input}} = \frac{R_r}{R_r + R_L} = \frac{7.8}{57.8} \approx .14$

Exercise 17: Tuned Speaker

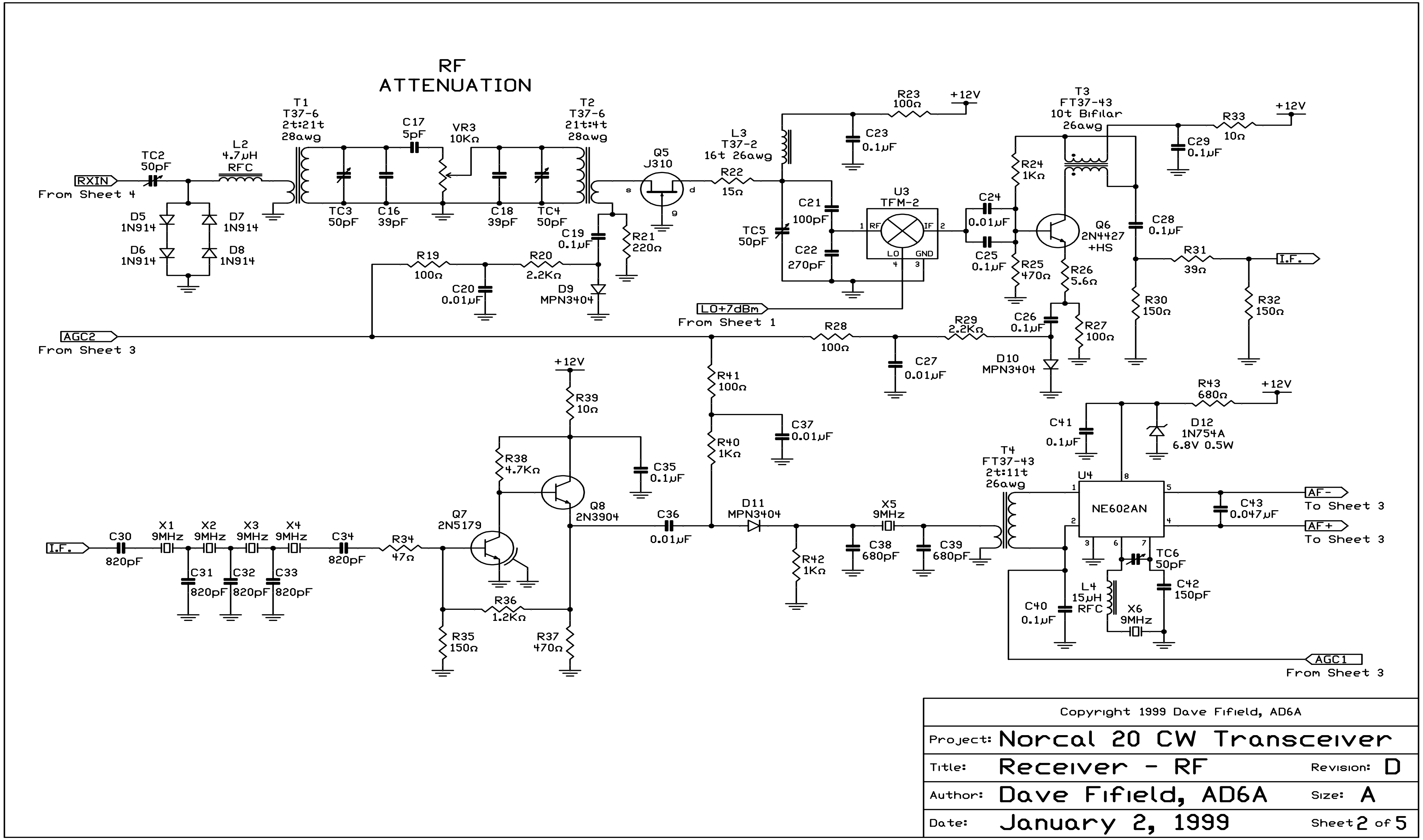
- Connect speaker to function generator 600Hz, 25mVrms.

1. Sound peaks at resonance. Find resonant frequency L_p.
2. Measure f_l, f_u by noting the 3dB loss. Calculate Q.
3. Use voltmeter to find resonance with speaker (nominally 8ohm) to calculate impedance
4. Calculate the resonant frequency from a transmission line equivalent circuit.

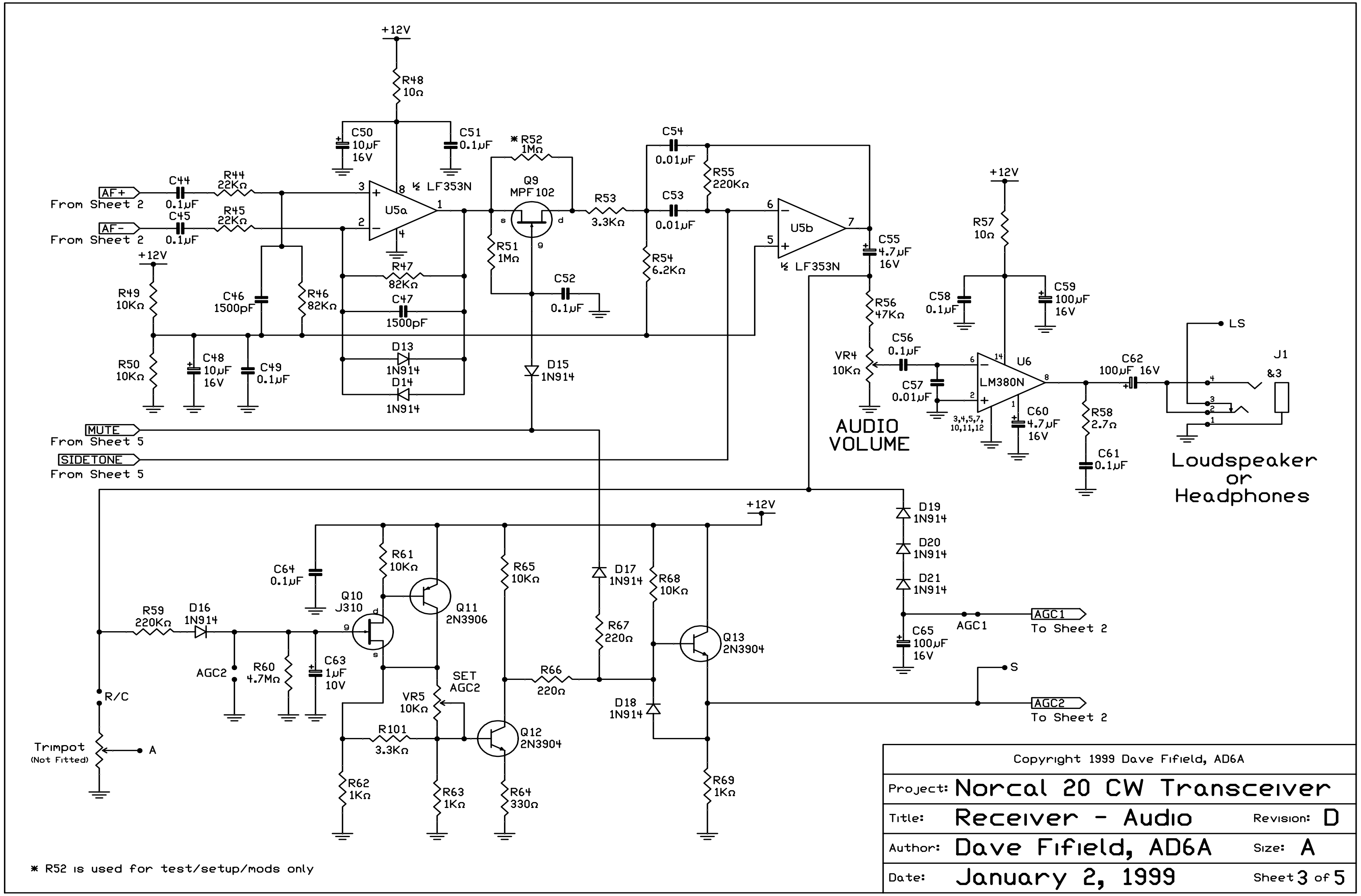
Norcal circuit diagram, 1



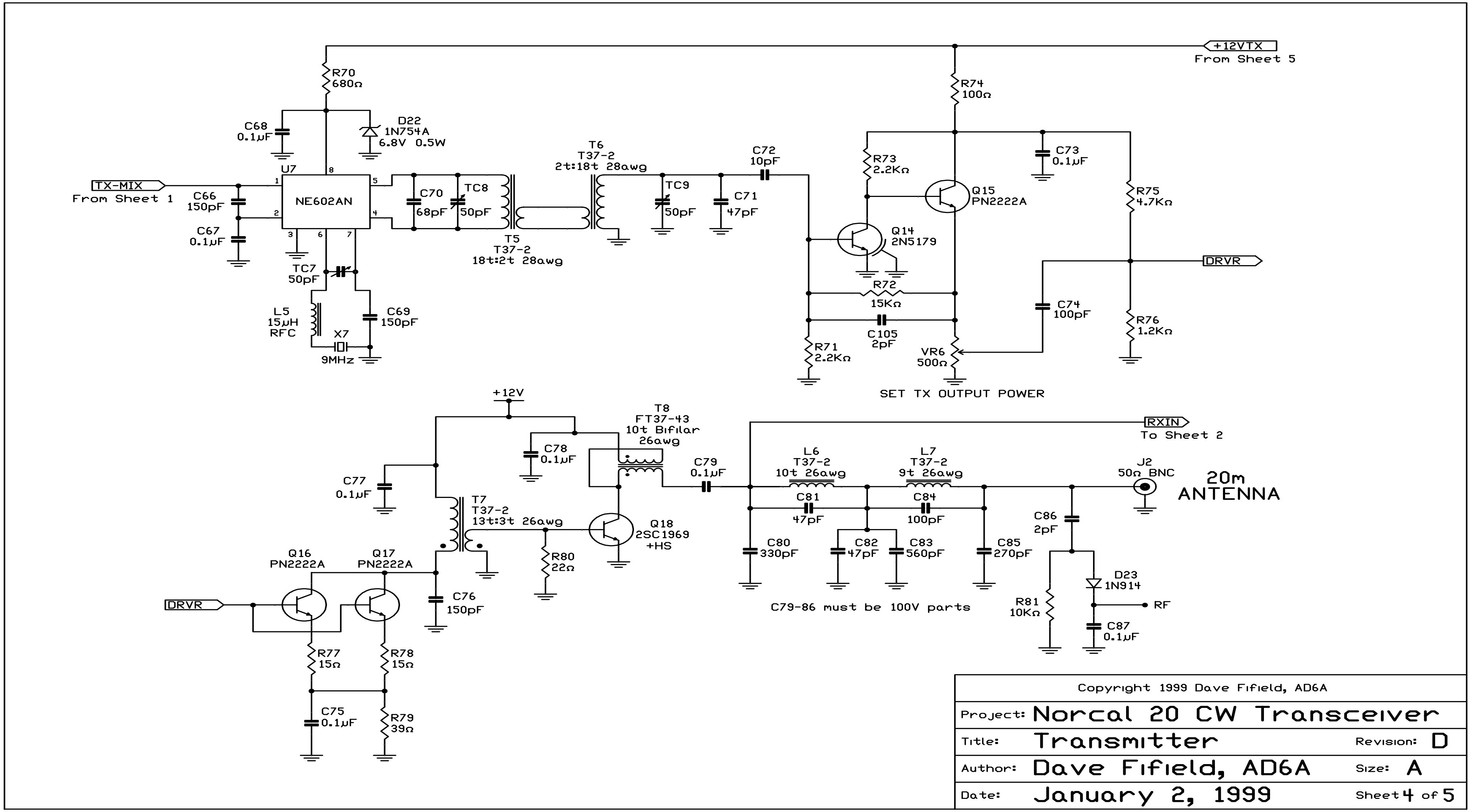
Norcal circuit diagram, 2



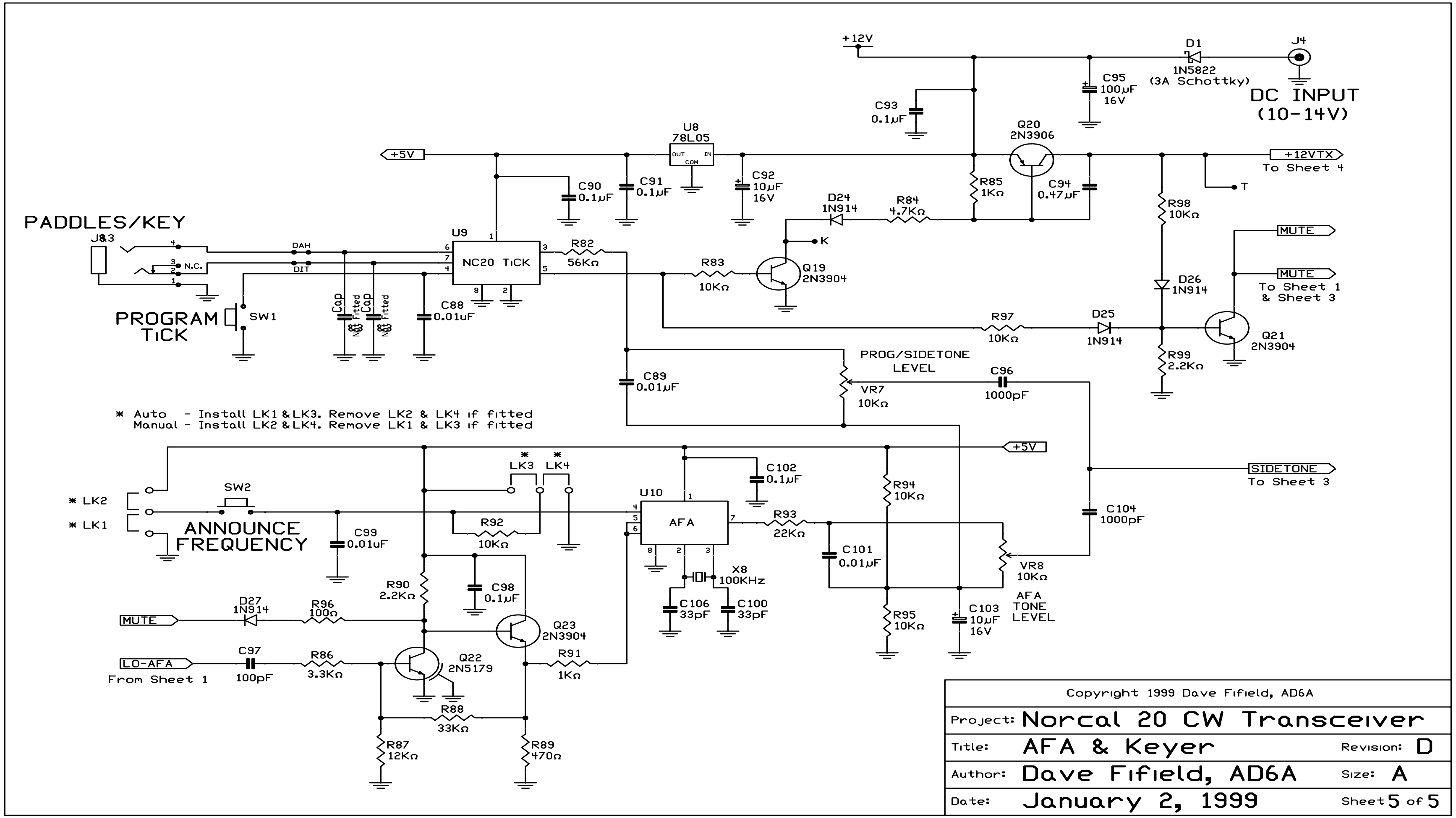
Norcal circuit diagram, 3



Norcal circuit diagram, 4



Norcal circuit diagram, 5



In the beginning…

- The laws of EM according to Clerk Maxwell are:
  1. $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$
  2. $\nabla \cdot \boldsymbol{B} = 0$
  3. $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
  4. $c^2\, \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$, $\epsilon_0 = 8.854 \times 10^{-12}\ \frac{C^2}{N{-}m^2}$, $\frac{1}{c^2} = \epsilon_0 \mu_0$
  5. $\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$
     - Here **E** is the electric field, **B** is the magnetic field, **j** is the current density through a closed surface, c is the speed of light and ρ is the charge density at a point.
     - In non-dispersive matter, $B = \mu H = \mu_0(H + M)$, $\mu = \mu_0(1 + \chi_m)$,
     - $D = \epsilon E = \epsilon_0 E + P, \epsilon = \kappa \epsilon_0$ and (1) becomes $\nabla \cdot \boldsymbol{D} = \rho_f$, (4) becomes $\nabla \times \boldsymbol{H} = \boldsymbol{j_f} + \frac{\partial \boldsymbol{D}}{\partial t}$

- The rest of classical physics, including special relativity, is:
  - Newton-Einstein: $\boldsymbol{p} = m\boldsymbol{v}$, $m = \frac{m_0}{\sqrt{1-(\frac{v}{c})^2}}$, $\boldsymbol{F} = m\frac{d\boldsymbol{p}}{dt}$.
  - Gravity: $\boldsymbol{F} = -\frac{Gm_1 m_2}{r^2}\ \boldsymbol{u}_r$ where $\mathbf{u_r}$ is the unit vector from m_1 to m_2 and **F** is the force on m_2.

Solutions to the wave equation

- The solution of $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = -s$ is $\psi(x, y, z, t) = \frac{S(t-\frac{r}{c})}{4\pi r}$ where $S = \int_V s\, dV$
- Later, we will use this to find the ”general” solution to Maxwell’s equations
  - $\phi(r_1, t) = \int_{V_2} \frac{\rho(r_2, t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0|r_1-r_2|} dV_2$ and $\mathbf{A}(r_1, t) = \int_{V_2} \frac{\mathbf{j}(r_2,\ t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0 c^2|r_1-r_2|} dV_2$, where
  - $B = \nabla\times A$, $E = -\nabla\phi - \frac{\partial A}{\partial t}$, and $c^2\nabla\cdot A = -\frac{\partial\phi}{\partial t}$
- You are not expected to have guessed this answer
- To do this, we’ll need the “BAC-CAB” identity: $A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$
- When we apply this to $\nabla\times(\nabla\times\mathbf{A})$, we get $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot A) - \nabla^2 A$

General solution to Maxwell's equations

- Returning to the general Maxwell equations, from $\nabla \cdot B = 0$, we get $B = \nabla\times A$
- Substituting into $c^2\ \nabla\times\mathbf{B} = \ \frac{\boldsymbol{j}}{\epsilon_0} + \ \frac{\partial\mathbf{E}}{\partial t}$, we get $c^2\ \nabla\times(\nabla\times A) = \frac{j}{\epsilon_0} + \frac{\partial E}{\partial t}$
- Applying "BAC-CAB", we get $\nabla\ (\nabla \cdot A) - \ \nabla^2 A = \ \frac{j}{c^2\epsilon_0} + \ \frac{1}{c^2}\frac{\partial E}{\partial t}$ (Equation 1)
- Now, $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, so substituting for B, we get $\nabla\times\left(\mathrm{E} + \frac{\partial A}{\partial t}\right) = 0$ and so $E = -\nabla\phi - \frac{\partial A}{\partial t}$
- Substituting into equation 1, $\nabla\ (\nabla \cdot A) - \ \nabla^2 A = \ \frac{j}{c^2\epsilon_0} + \ \frac{1}{c^2}\frac{\partial}{\partial t}(-\nabla\phi - \frac{\partial A}{\partial t})$, or
- $\nabla\ (\nabla \cdot A) - \ \nabla^2 A = \ \frac{j}{c^2\epsilon_0}$ - $\frac{1}{c^2}\ \nabla\frac{\partial\phi}{\partial t}\ - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$.
- $\nabla^2 A$ - $\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\ \frac{j}{c^2\epsilon_0} + \nabla[\frac{1}{c^2}\ \frac{\partial\phi}{\partial t} + (\nabla \cdot A)]$
- Now if A and ϕ give $B = \ \nabla\times A$ and $E = -\nabla\phi - \frac{\partial A}{\partial t}$, then $A' = A + \ \nabla\varphi$ and $\phi' = \phi - \frac{\partial\varphi}{\partial t}$ give $B = \ \nabla\times A'$ and $E = -\nabla\phi' - \frac{\partial A'}{\partial t}$, for any function φ

General solution to Maxwell's equations

- Thus, we can pick a solution $(A, \phi)$with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$. Then we get
- $\nabla^2 A$ - $\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2 \epsilon_0}$
- Substituting for $E = -\nabla\phi - \frac{\partial A}{\partial t}$ into $\nabla \cdot \mathrm{E} = \frac{\rho}{\epsilon_0}$, we get
- $\nabla \cdot (\nabla\phi) + \frac{\partial\, \nabla \cdot A}{\partial t} = -\frac{\rho}{\epsilon_0}$, or $\nabla \cdot (\nabla\phi) - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$, or $\nabla^2\phi$ - $\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$
- The solutions are $\phi(r_1, t) = \int_{V_2} \frac{\rho(r_2, t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 |r_1 - r_2|} dV_2$ and
- $\mathbf{A}(r_1, t) = \int_{V_2} \frac{\mathbf{j}(r_2,\ t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 c^2 |r_1 - r_2|} dV_2$ with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$, with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$.

Solution to Maxwell's equations in free space

- Free space is defined by $\rho = 0$ and $j = 0$, so our potentials satisfy
- $\nabla^2 A$ - $\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$ =0 and $\nabla^2\phi$ - $\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0$
- These have the usual wave equation solutions $\phi(x, y, z, t) = \ f(k \cdot r \ - \ \omega t)$, etc
- Thus, in free space ϕ and **A** and hence E and B propagate as waves.

Solution to Maxwell's equations in conductors

- In conductors, $\mathbf{j} = \sigma\mathbf{E}$
  - $c^2\ \nabla\times\mathbf{B} = \frac{\boldsymbol{j}}{\epsilon} + \frac{\partial\mathbf{E}}{\partial t} = \frac{\sigma}{\epsilon}\mathbf{E} + \frac{\partial\mathbf{E}}{\partial t}$
  - This becomes $c^2\frac{\partial(\nabla\times\mathbf{B})}{\partial t} = \frac{\sigma}{\epsilon}\frac{\partial\mathbf{E}}{\partial t} + \frac{\partial^2\mathbf{E}}{\partial t^2}$
  - $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, so we get $c^2\nabla\times\frac{\partial B}{\partial t} = -c^2\nabla\times(\nabla\times E) = \frac{\sigma}{\epsilon}\frac{\partial\mathbf{E}}{\partial t} + \frac{\partial^2\mathbf{E}}{\partial t^2} = -c^2[\nabla(\boldsymbol{\nabla}\cdot\mathbf{E}) - \nabla^2\mathbf{E}] = \boldsymbol{c^2\nabla^2}\mathbf{E}$ (since $\rho = 0$ in a conductor)
  - Applying the trial solution E= E_0 exp(ωt-kr), we get –k^2-i$\omega\mu\sigma$ + $\omega^2\mu\epsilon$ = 0.
  - Putting k = α-βi, $\alpha = \frac{\omega}{2}\sqrt{\mu\epsilon}\ (1 + \sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}})$ and $\beta = \frac{\omega\mu\sigma}{2\alpha}$.
  - For copper, σ = 5.78 × 107 $\boldsymbol{\Omega}$-m. This explains the "skin effect" in conductors.

Radiation, antennas

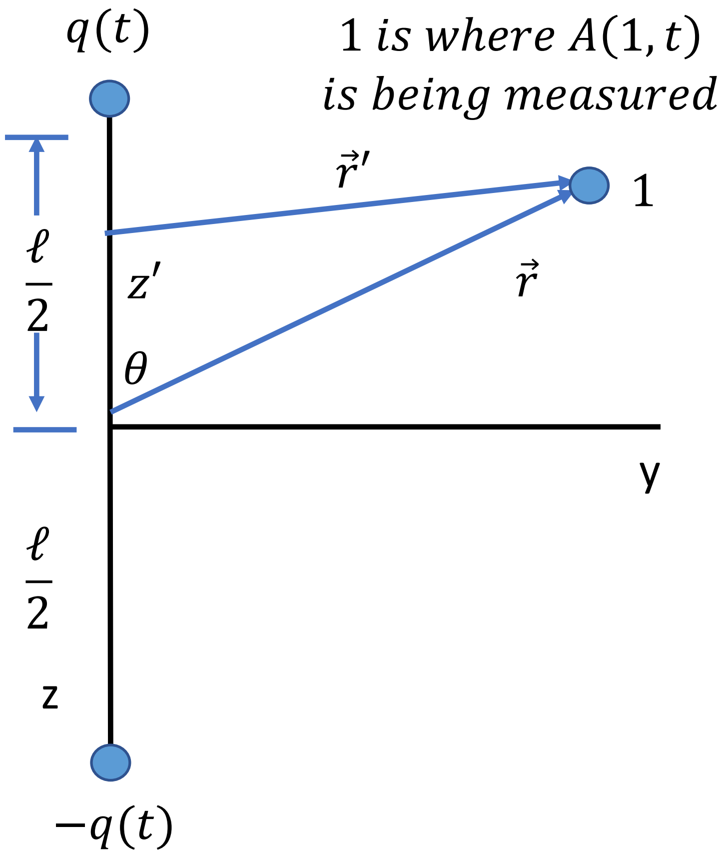
- Accelerating charges radiate energy in the form of electromagnetic waves (companion E and B fields).
- The radiation from accelerating charge q is $\mathbf{E}_{rad} = -\frac{1}{4\pi\epsilon_0 c^2} \frac{q}{r} \boldsymbol{a}_{\perp}(t - \frac{r_{12}}{c})$.
- Here, $\mathbf{a}_{\perp}$ is the acceleration $\perp$ to the line from r_1 to r_2.
- For example, applying a time varying potential $V_0\sin(\omega t)$ to an antenna will cause the antenna to radiate power since the voltage and hence charges affected accelerate within the antenna, that is, their positions have a non-zero second derivative. That's how a transmitter "couples" to the antenna of a receiver. In the receiver, the radiated wave accelerates charges in the antenna replicating the original wave (at much reduced power).
- These simple radio waves are carrier waves of frequency $\frac{\omega}{2\pi}$. To transfer information (voice, images, binary data), we modulate carrier waves combining them with an "information source" signal. Receivers demodulate the incoming wave and recreate the original "information source" signal.

Maxwell's equations in a non-dispersive media

- $\mathrm{B} = \mathrm{\mu H}, \mathrm{D} = \epsilon \mathrm{E}$
- $\nabla \cdot D = \rho$
- $\nabla \cdot B = 0$
- $\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}}$
- $\nabla \times \mathrm{H} = \mathrm{j} + \frac{\partial \mathrm{E}}{\partial \mathrm{t}}$
- $\nabla \cdot j = -\frac{\partial \rho}{\partial t}$

Radiation from a small dipole

- $A(1,t) = \int \frac{\vec{j}\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}}\, dV_2$. From figure on right, $j_z\, dV_2 = I\, dz'$.
- Note $\vec{p}(t) = q(t)l\vec{k}$, $k = \hat{z}$ and $\dot{q}(t) = I$. ϕ is the angle from the x axis to the projection of r on the x-y plane.
- $\vec{r}' + z'\vec{k} = \vec{r}$ and $|\vec{r}\text{-}\, z'\vec{k}| = \mathrm{r} - z'\cos(\theta)$
- If $l \ll cT = \lambda$, $I\left(\vec{z}', t - \frac{r'}{c}\right) \approx I\left(0, t - \frac{r'}{c}\right)$ and we get:
- $A_z(r,t) = \frac{\mu_0}{4\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{I\left(\vec{z}', t-\frac{r'}{c}\right)}{4\pi\epsilon_0 c^2 r'}\, dz' = \frac{\mu_0}{4\pi}\frac{l}{r} I(0, t - \frac{r'}{c})$
- Choosing gauge, $\nabla \cdot \mathrm{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$
- $\frac{\partial\varphi}{\partial t} = -\frac{l}{4\pi\epsilon_0}\frac{\partial}{\partial z}(\frac{1}{r}\, I\left(t - \frac{r}{c}\right)) = \frac{z}{r^2}(\frac{q(t-\frac{r}{c})}{r} - \frac{I((t-\frac{r}{c})}{c})$
- Pick and oscillating dipole: $q\left(t - \frac{r}{c}\right) = q_0 \cos\left(\omega\left[t - \frac{r}{c}\right]\right)$
- $I\left(t - \frac{r}{c}\right) = I_0 \sin\left(\omega\left[t - \frac{r}{c}\right]\right) = -\omega q_0 \sin\left(\omega\left[t - \frac{r}{c}\right]\right)$



Radiation from a small dipole

- $\nabla^2 H - \epsilon\mu \frac{\partial^2 H}{\partial t^2} - \sigma\mu \frac{\partial H}{\partial t} = 0$
- $\nabla^2 E - \epsilon\mu \frac{\partial^2 E}{\partial t^2} - \sigma\mu \frac{\partial E}{\partial t} = 0$
- $A_r = \frac{\mu_0}{4\pi} \frac{I_0 l}{r} \cos(\theta) \sin(\omega \left[t - \frac{r}{c}\right])$
- $A_\phi = 0,\ A_\theta = -\frac{\mu_0}{4\pi} \frac{I_0 l}{r} \cos(\theta) \sin(\omega \left[t - \frac{r}{c}\right])$
- $B_\phi = \frac{1}{r} \frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r} \frac{\partial A_r}{\partial \theta} = \frac{\mu_0}{4\pi} \frac{I_0 l}{r} \sin(\theta)[\frac{\omega}{r} \cos\left(\omega \left[t - \frac{r}{c}\right]\right) + \frac{1}{r} \sin(\omega[t - \frac{r}{c}])]$
- $E_r = -\frac{\partial \phi}{\partial t} - \frac{\partial A_r}{\partial t} = \frac{2lI_0 \cos(\theta)}{4\pi\epsilon_0} [\frac{\sin(\omega\left[t - \frac{r}{c}\right])}{r^2 c} - \frac{\cos(\omega\left[t - \frac{r}{c}\right])}{\omega r^3}]$
- $E_\theta = \frac{-I_0 \,\mathrm{l}\sin(\theta)}{4\pi\epsilon_0} (\left[\frac{1}{r^3\omega} - \frac{\omega}{rc^2}\right] \cos\left(\omega \left[t - \frac{r}{c}\right]\right) - \frac{1}{cr^2} \sin\left(\omega \left[t - \frac{r}{c}\right]\right))$
- $E_\phi = -\frac{1}{r sin(\theta)} \frac{\partial \varphi}{\partial \phi} - \frac{\partial A_\phi}{\partial t} = 0$

Radiation from a small dipole

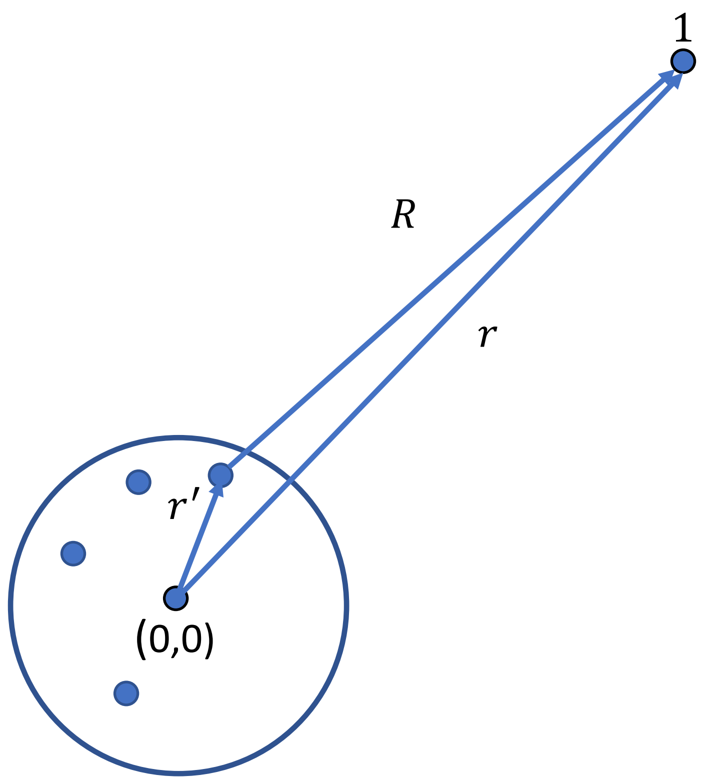
- $\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\mathbf{H}\frac{\partial \mathbf{B}}{\partial t} - \mathbf{E}\frac{\partial \mathbf{D}}{\partial t} - \mathbf{E} \cdot \mathbf{j}$
- $\mathbf{S} = \mathbf{E} \times \boldsymbol{H}, \nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} = -\mathbf{E} \cdot \mathbf{j}$ (u is energy density)
- $\int S \cdot dA = \frac{(l\, I_0\, \omega)^2}{6\pi\epsilon_0 c^3} \cos(\omega \left[t - \frac{r}{c}\right])^2$
- $P_{av} = \frac{(l\omega)^2}{6\pi\epsilon_0 c^3} \frac{{I_0}^2}{2} = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{l}{\lambda}\right)^2 \frac{{I_0}^2}{2}$
- $R_r = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{l}{\lambda}\right)^2$

Large half wave dipole

- For large half wave, add small dipoles to produce half wave antenna.
- $dE_\theta = I_0 \, \frac{\sin(\theta)}{4\pi\epsilon_0 R c^2} \, \omega \cos(\omega) \cos\left(\frac{2\pi z\prime}{\lambda}\right) \, dz'$
- $dB_\phi = I_0 \, \frac{\mu_0 \omega}{4\pi R c} \, \omega \cos\left(\omega \left[\left[t - \frac{r}{c}\right]\right]\right) \cos\left(\frac{2\pi z\prime}{\lambda}\right) \, dz'$
- Suffices to find: $K = \int_{[-\frac{\pi}{2},\frac{\pi}{2}]} \frac{1}{R} \cos\left(t - \frac{R}{c}\right) \cos(u) \, du = \frac{1}{2\pi\epsilon_0 r c} \cos\left(\omega \left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin^2(\theta)}, u = \frac{2\pi z\prime}{\lambda}$
- $E_\theta = I_0 \, \frac{1}{2\pi\epsilon_0 r c} \, \cos\left(\omega \left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin\,(\theta)}$
- $B_\phi = I_0 \, \frac{\mu_0}{2\pi r} \, \omega \cos\left(\omega \left[\left[t - \frac{r}{c}\right]\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{sin\,(\theta)}$
- $P_{av} = \frac{1}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \, {I_0}^2 \int_{[0,\pi]} \frac{cos^2(\frac{\pi}{2}\cos(\theta))}{sin^2(\theta)} \, \sin(\theta) d\theta = 73.1\Omega \, \frac{{I_0}^2}{2}$

Radiation from charge distribution

- $\mathrm{r}' + \mathrm{R} = \mathrm{r},\ R = |r - r'|$
- $\varphi(r,t) = \frac{1}{4\pi\epsilon_0}\int_{V_1}\frac{\rho(r',t-\frac{R}{c})}{|r-r'|}dv' = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{r\cdot p(t-\frac{r}{c})}{r^3} + \frac{r\cdot\frac{dp}{dt}(t-\frac{r}{c})}{cr^2}\right]$
- $\boldsymbol{A}(r,t) = \frac{\mu_0}{4\pi}\int_{V_1}\frac{\boldsymbol{j}(r',t-\frac{R}{c})}{|r-r'|}dv' = \frac{\mu_0}{4\pi r}\frac{d}{dt}\boldsymbol{p}(t-\frac{r}{c})$
- $\boldsymbol{E} = -\frac{\partial\boldsymbol{A}}{\partial t} - \nabla\varphi$
- $\boldsymbol{B}(r,t) = \frac{-\mu_0}{4\pi c r^2}\boldsymbol{r}\times\ddot{\boldsymbol{p}}(t-\frac{r}{c})$
- $\boldsymbol{E}(r,t) = -\frac{c}{r}\boldsymbol{r}\times\boldsymbol{B}(r,t)$
- $\frac{d\boldsymbol{p}}{dt} = q\frac{dr'}{dt} = qv,\ \frac{d^2}{dt^2}\boldsymbol{p}(t-\frac{r}{c}) = q\frac{dv}{dt}$
- $S = \frac{1}{\mu_0}E\times B = \frac{\ddot{p}^2\sin(\theta)^2}{16\pi^2\epsilon_0 c^3 r^r}\frac{\vec{r}}{r}$
- $P_R = -\frac{dW}{dt} = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0 c^3}(\frac{dv}{dt})^2$
- Compare to Feynman later



Radiation from a single accelerating charge

- Near zone
  - $\varphi(r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R\prime(1+\frac{v\cdot n'}{c})}\right]$
  - $R^* = R' - \frac{v}{c}(x_0 - x'_1)$
  - $E(r,t) = \frac{1}{4\pi\epsilon_0}\frac{1}{R^{*3}}[\left(R' - \frac{R'\,v'}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \frac{R\prime}{c^2}\left(R' - \frac{R'v'}{c}\right)\times \frac{dv'}{dt}]$
  - $B = \frac{R'\times E}{R'c}$
  - $S = \frac{q^2}{16\pi^2 c^3 \epsilon_0}\,\frac{R'(R'\times v')^2}{(R')^5}$

Feynman's Treatment - 1

- Want to derive the Heaviside-Feynman equation:
  - $E = \frac{q}{4\pi\epsilon_0}[\frac{e_{r\prime}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\frac{e_{r\prime}}{r'^2} + \frac{1}{c^2}\frac{d^2 e_{r\prime}}{dt^2}]$
  - $cB = e_{r'} \times E$
- Solution to Maxwell:
  - $E = -\nabla\phi - \frac{\partial A}{\partial t}, B = \nabla \times A$
  - $\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}, \nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2\epsilon_0}$
  - With gauge $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$
- Consider solution to $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = -s$
- First solve $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$ =0

- The solution of $\frac{\partial^2\varphi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial x^2} = 0$ is
- $\varphi(x,t) = Af(x - ct) + Bg(x + ct)$
- $\frac{\partial\psi(r)}{\partial x} = \psi'\frac{\partial r}{\partial x}$ and $\frac{\partial r}{\partial x} = \frac{x}{r}$
- $\frac{\partial^2\psi}{\partial x^2} = \psi''(\frac{\partial r}{\partial x})^2 + \psi'\frac{\partial^2 r}{\partial x^2}$ and $\frac{\partial^2 r}{\partial x^2} = \frac{1}{r}(1 - \frac{x^2}{r^2})$
- So, $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \psi'' + \frac{2}{r}\psi'$
- $\nabla^2(r\psi) - \frac{1}{c^2}\frac{\partial^2(r\psi)}{\partial t^2} = 0$
- So, $\psi(x,t,z,t) = \frac{f(t - \frac{r}{c})}{r}$

Feynman's Treatment - 2

- As $r \to 0, \psi(x,t,z,t) = \frac{f(t)}{r}$.
- Consider the electrostatic case where $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$ where $\phi = \frac{q}{4\pi\epsilon_0 r}$.
- By analogy:
  - $f\left(t - \frac{r}{c}\right) = \frac{S(t-\frac{r}{c})}{4\pi}$
  - $S = \int s\, dV$
- So, solution is: $\psi(x,t,z,t) = \frac{1}{4\pi}\frac{S(t-\frac{r}{c})}{r}$. Thus,
  - $A(1,t) = \int \frac{\vec{j}\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}}\, dV_2$
  - $\phi(1,t) = \int \frac{\rho\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 r_{12}}\, dV_2$
  - $E = -\nabla\phi - \frac{\partial A}{\partial t}, B = \nabla\times A$
  - With gauge $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$

Feynman's Treatment - 3

- For small blob of charge moving with velocity v, $j = v\rho.$
- $A(1,t) = \int \frac{\vec{j}\left(2, t - \frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}} \, dV_2 = \frac{1}{r} \int v\rho dV_2 = \frac{qv}{r}$
- So, for a dipole with $\vec{p}(r,t) = qd\vec{k}$, $\frac{qv}{r} = \frac{\partial \vec{p}}{\partial t}$
- $A(1,t) = \frac{\dot{p}(t - \frac{r}{c})}{4\pi\epsilon_0 c^2 r}, B = \nabla \times A$ and $B_z = 0.$ A is due to dipole current. If the dipole charges oscillate $p(t) =$

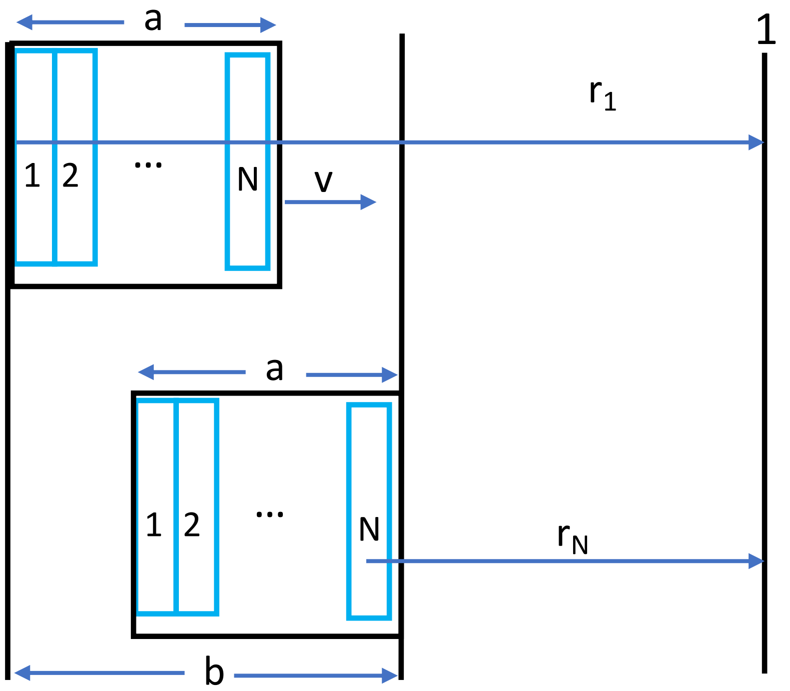
  $p_0 \sin(\omega t)$, $A_z(1,t) = \frac{1}{4\pi\epsilon_0 c^2 r} \, p_0 \omega \cos(\omega[t - \frac{r}{c}])$
- $B_x = \frac{\partial A_z}{\partial y} = \frac{1}{4\pi\epsilon_0 c^2 r} \frac{\partial \dot{p}(t - \frac{r}{c})}{\partial y}.$
- $B_y = -\frac{\partial A_z}{\partial x} = -\frac{1}{4\pi\epsilon_0 c^2 r} \frac{\partial \dot{p}(t - \frac{r}{c})}{\partial x}$
- $B_x = \frac{1}{4\pi\epsilon_0 c^2 r} \left[-\frac{y\left(t - \frac{r}{c}\right)}{r^3} - \frac{y\ddot{p}\left(t - \frac{r}{c}\right)}{cr^2} \right]$, $B_y = \frac{1}{4\pi\epsilon_0 c^2 r} [\frac{x\dot{p}\left(t - \frac{r}{c}\right)}{r^3} + \frac{x\ddot{p}\left(t - \frac{r}{c}\right)}{cr^2}]$

Feynman's Treatment - 4

- Rewrite previous equation as
  - $\vec{B} = \frac{1}{4\pi\epsilon_0 c^2 r^3}\left[\dot{p}\left(t - \frac{r}{c}\right) + \frac{r}{c}\ddot{p}\left(t - \frac{r}{c}\right)\right] \times r$
  - Compare this to Biot Savart: $dB = \frac{1}{4\pi\epsilon_0 c^2}\frac{j \times r}{r^3} dV$
- Now suppose the dipole charges oscillate $q(t) = q_0 \sin(\omega t)$
- When $\frac{r}{c}$ is small, $\dot{p}\left(t - \frac{r}{c}\right) = \dot{p}(t) - \frac{r}{c}\ddot{p}(t) + \ \ldots$
- $E \perp B$, $E = cB$ and from $\frac{\partial \phi}{\partial t} = -\nabla \cdot A$, we get
- $\phi = \frac{1}{4\pi\epsilon_0 r^3}\left[p\left(t - \frac{r}{c}\right) + \frac{r}{c}\dot{p}\left(t - \frac{r}{c}\right)\right] \cdot r$, and
- $E = -\nabla\phi - \frac{\partial A}{\partial t} = \frac{1}{4\pi\epsilon_0 c^2 r^3}\left[\frac{3(p^* \cdot r)\vec{r}}{r^2} + \frac{1}{c^2}(\ddot{p}\left(t - \frac{r}{c}\right) \times r \times r)\right]$ where
- $p^* = p\left(t - \frac{r}{c}\right) + \frac{r}{c}\dot{p}\left(t - \frac{r}{c}\right)$

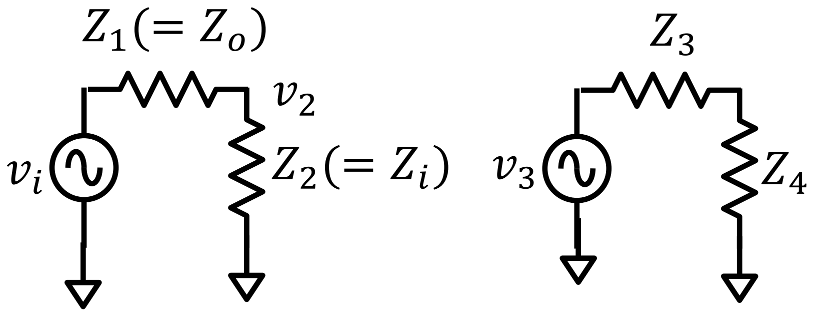
Lienard-Wierchert

- $\phi(1,t) = \int \frac{\rho\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 r_{12}}\, dV_2$
- $\phi(1,t) = \sum \frac{\rho_i \Delta V_i}{r_i{}'} = \sum \frac{\rho w a^2}{r'} = \frac{\rho a^3}{r'} \frac{Nw}{a} = \frac{q}{r'} \frac{b}{a}$
- $b = a + \frac{v}{c} b$
- So, we get
  - $\phi(1,t) = \frac{q}{4\pi\epsilon_0 [r - \frac{v \cdot r}{c}]_{ret}}$ and
  - $A(1,t) = \frac{q v_{ret}}{4\pi\epsilon_0 c^2 [r - \frac{v \cdot r}{c}]_{ret}}$
- Now we can compute E as above and verify the Heaviside formula

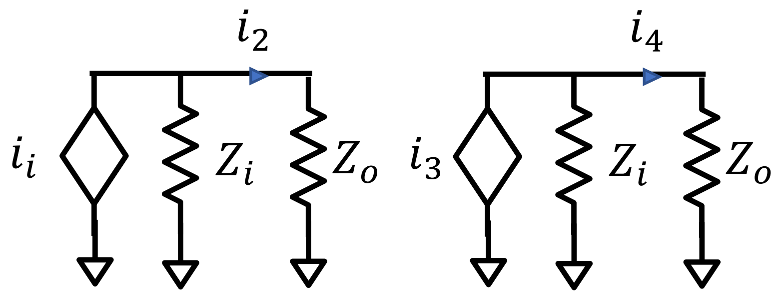


Coupling

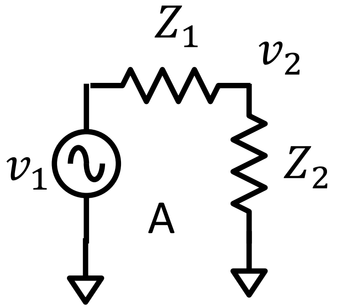
- Voltage gain, A
  - $v_o = v_3 = A\frac{Z_o}{Z_i+Z_o}v_i$
  - For two stages gain is $(A\frac{Z_o}{Z_i+Z_o})^2$



- Current gain, A
  - $i_o = i_3 = A\frac{Z_i}{Z_i+Z_o}i_i$
  - For two stages, gain is $(A\frac{Z_i}{Z_i+Z_o})^2$



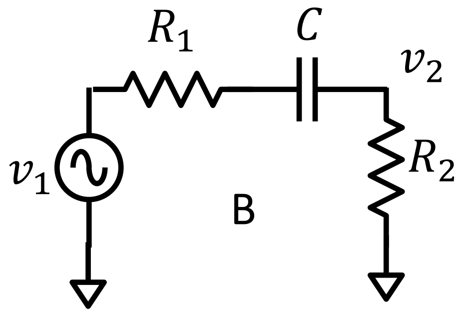
Power transfer

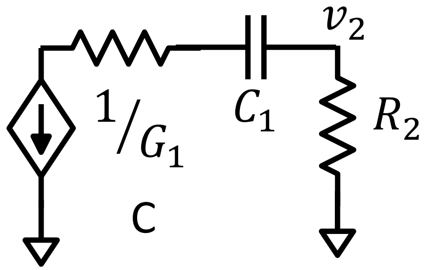


- Power transfer (A)
  - $Z_1 = R_1 + jX_1, Z_2 = R_2 + jX_2$
  - $P = \left| \frac{v_1}{R_1+R_2+j(X_1+X_2)} \right|^2 R_2$
- Power transfer (B)
  - $v_2 = \frac{v_1\left(\frac{R_1}{R_1+R_2}\right)}{1+\frac{1}{j\omega C(R_1+R_2)}}$
  - $P = i_2 v_2 \cos(\phi)$
  - Max power: $R_1 = R_2$
  - $v_2 = v_1 \frac{R_2/(R_1+R_2)}{1+1/[j\omega C(R_1+R_2)]}$

1. ω, large: $G = \frac{R_2}{R_1+R_2} = B$
2. $\omega = \frac{1}{C(R_1+R_2)}$, large: $G = B\,\frac{1+j}{2} = \sqrt{2}B\angle 45$
3. ω, small: Let $\omega_1 = C(R_1 + R_2), G = B\,\frac{\omega}{\omega_1}$

- For C
  - $v_2 = \frac{i_1}{G_1} \frac{R_2}{R_2+1/G_1}$
- For voltage amplifiers, we want $R_2 \gg R_1$
- For current amplifiers, we want $R_2 \ll R_1$

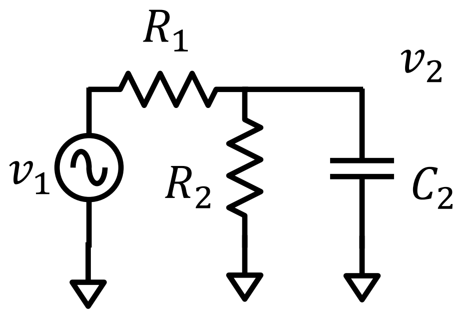




High frequency coupling

- Equivalent circuit on right
- $\frac{v_1 - v_2}{R_1} = \frac{v_2}{R_2} + j v_2 \omega C_2$
- $G = \frac{v_2}{v_1} = \frac{R_2/(R_1+R_2)}{1 + j\omega C_2 \, R_1 R_2/(R_1+R_2)}$

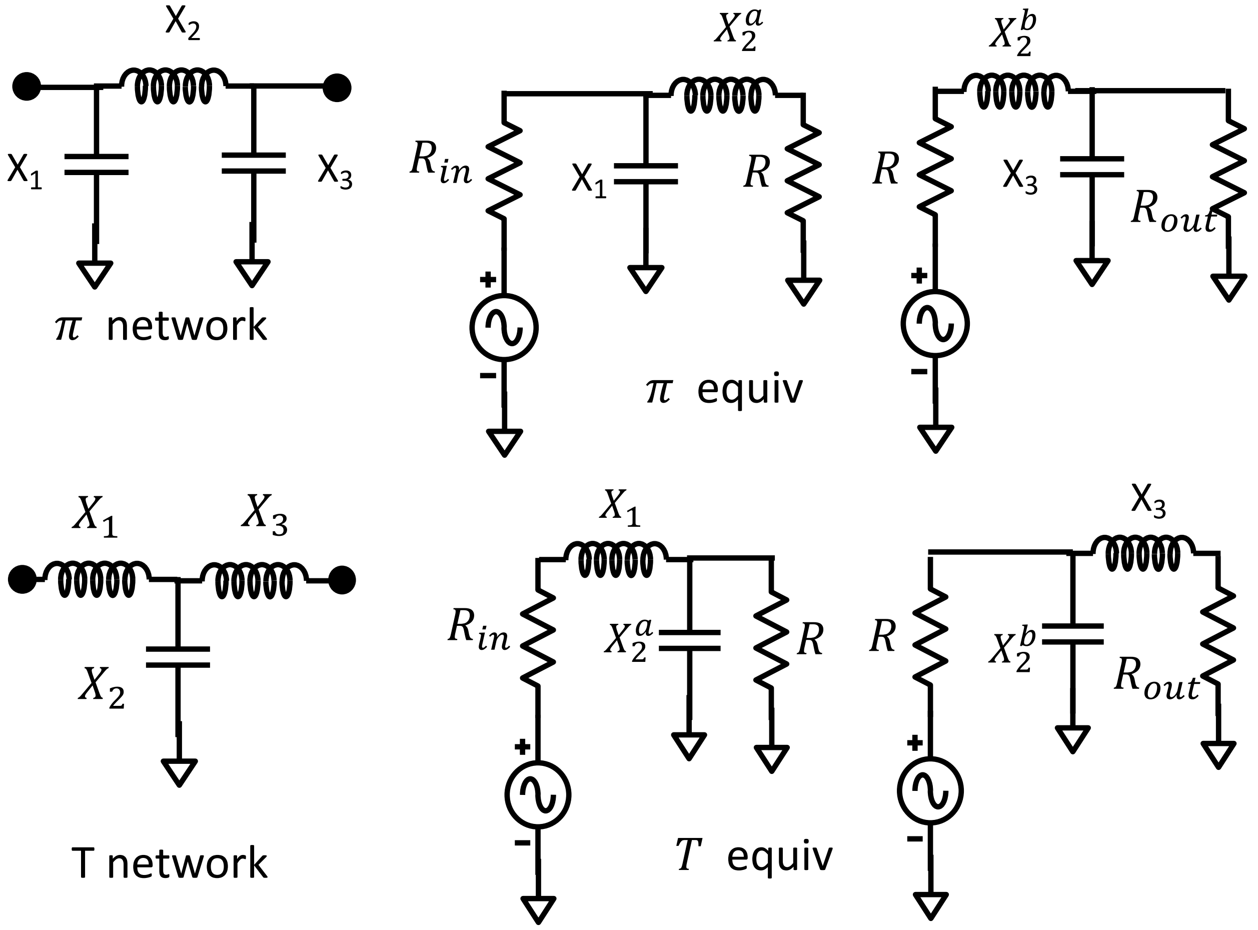
1. ω low: $G = \frac{v_2}{v_1} = R_2/(R_1+R_2) = \mathrm{B}$
2. For $\omega_2 = \frac{R_1+R_2}{C_2 R_1 R_2}$, $G = \frac{v_2}{v_1} = \frac{B}{1+j} = \frac{\sqrt{2}}{2} B \angle -45$
3. ω high: $G = \frac{v_2}{v_1} = \frac{R_2/(R_1+R_2)}{j\omega/\omega_2} = B \frac{\omega_2}{\omega} \angle -90$



Impedance matching

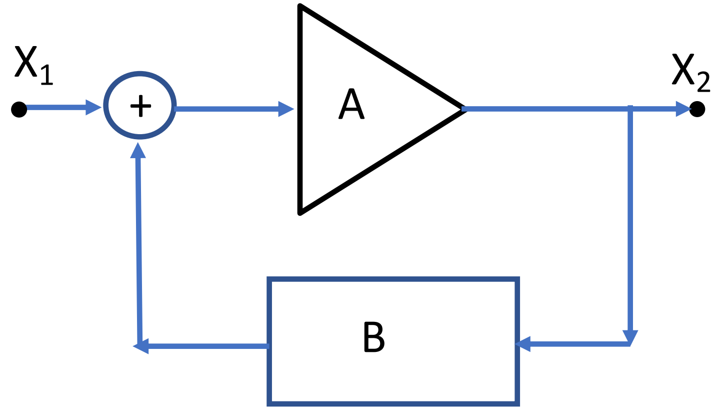
- π network
  - $Q = \sqrt{\max(R_{in}, R_{out})/_{R} - 1}$, *choose R for BW*
  - $X_2^a + X_2^b = X_2$
  - $R < \min(R_{in}, R_{out})$

- T networks
  - $Q = \sqrt{R/_{\min(R_{in}, R_{out})} - 1}$, *choose R for BW*
  - $X_2^a || X_2^b = X_2$
  - $R > \max(R_{in}, R_{out})$



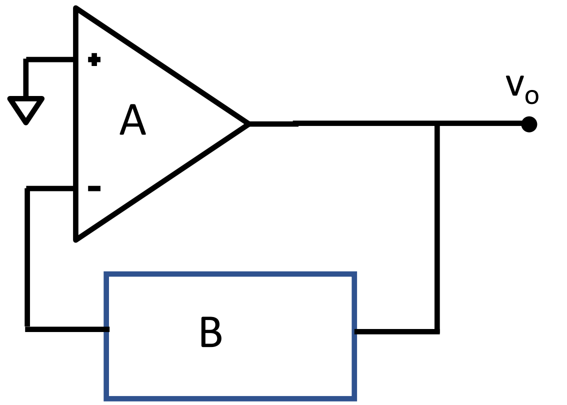
Negative feedback

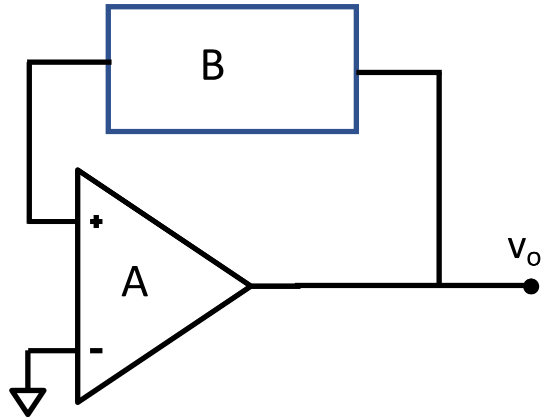
- $\frac{X_2}{A} = X_1 - BX_2$, $\frac{X_2}{X_1} = \frac{A}{1+AB} \equiv K$
- No feedback ($B = 0$):
  - $K = A$
- $AB < 0, 1 + AB < 1$
  - $K \gg A$
- $AB = -1$
  - $K = \infty$
- $AB \gg 0$
  - $K \approx \frac{1}{B}$
- $\frac{dK}{dA} = \frac{1}{(1+AB)^2}$ measures stability
- If forward gain is $\frac{A}{1+j\omega\tau_1}$, $K_{feebback} = \frac{(1+AB)K_{no-feedback}}{1+j\omega\tau_1}$
  - This increases frequency response
- Effect on impedance
  - $Z_1 \rightarrow Z_1(1 + AB)$, $Z_2 \rightarrow \frac{Z_2}{1+AB}$



Oscillators and feedback

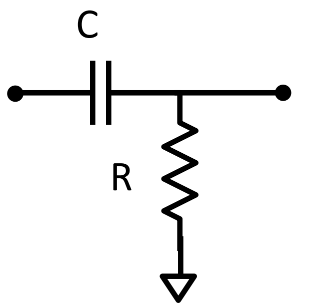
- $K = \frac{A}{1+AB}$
- On upper right, oscillation conditions is $AB = 1\angle 180$
- For lower right, $AB = 1\angle 0$





Some filter transfer functions

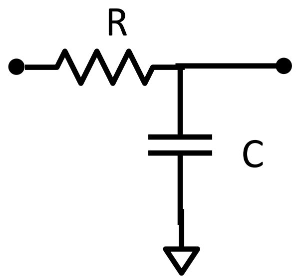
High pass



$$G_{HP} = \frac{1}{1 + \frac{\omega_0}{j\omega}}$$

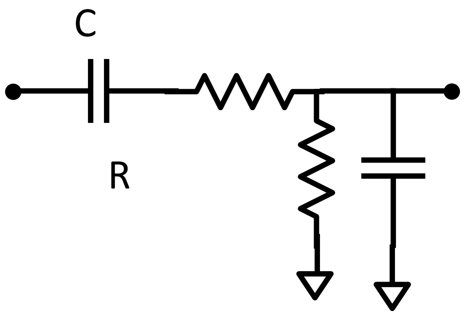
- For CR ladder filter of length 3
- $\frac{v_2}{v_1} = \frac{1}{1-5\omega^2C^2R^2+j((6\omega CR-\omega^3C^3R^3)}$
- Phase shift is 0 when $6 = \omega^2C^2R^2$

Low pass



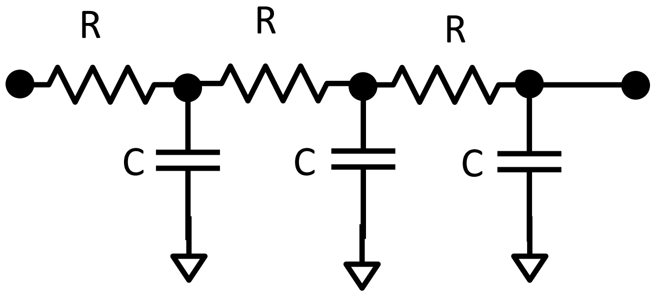
$$G_{LP} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

Wein



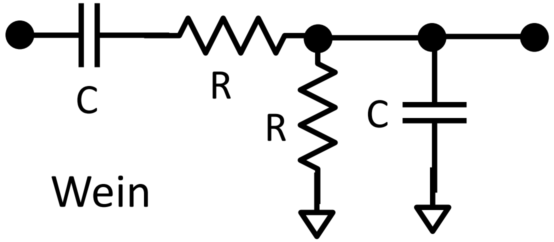
- $\frac{v_2}{v_1} = \frac{1}{3+j(\omega CR-\frac{1}{\omega CR})}$
- Oscillation when $\omega CR = 1$
- $\tan(\phi) = \frac{-R_1R_2(\omega CR-\frac{1}{\omega CR})}{R_1+R_2}$
- $\frac{d\phi}{d\omega} = -\frac{2Q}{\omega}$ so high Q improved stability

Feedback networks for oscillators



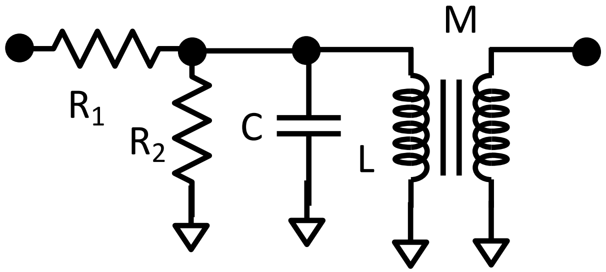
Phase Shift Ladder

- $\frac{v_2}{v_1} = {}^1\!/_{(1-5\omega^2C^2R^2)+j(6\omega CR-\omega^3C^3R^3)}$



Wein

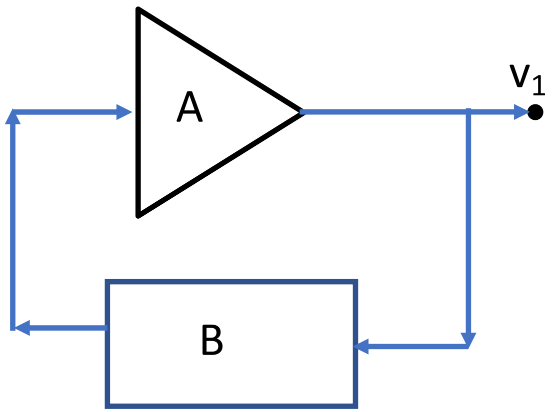
- $\frac{v_2}{v_1} = \frac{1}{3}$



Tuned circuit

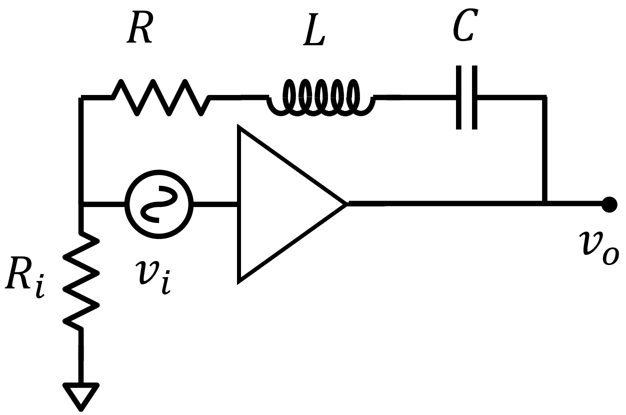
- $\frac{v_2}{v_1} = (\frac{M}{L})({}^1\!/_{(R_1/R_2+1)+j\left(R_1\omega C-{}^{R_1}\!/_{(\omega L)}\right)})$

More on oscillation

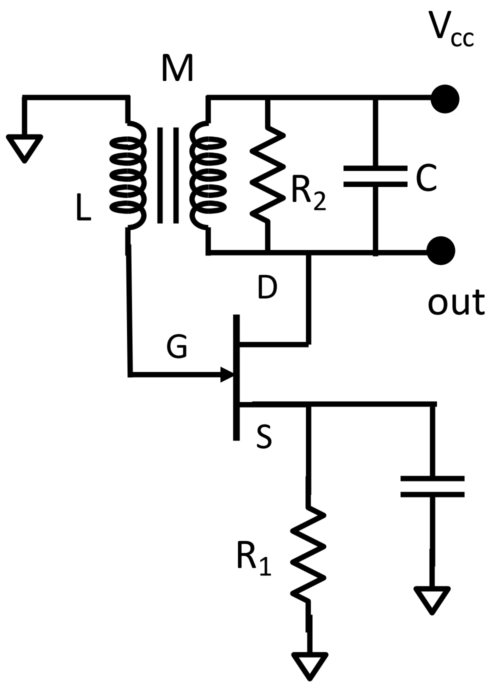


- For sine waves, $|v_1| = 1, AB > 1$
- $v_1 = ABv_1$
- You can use the feedback networks on the previous slide for B

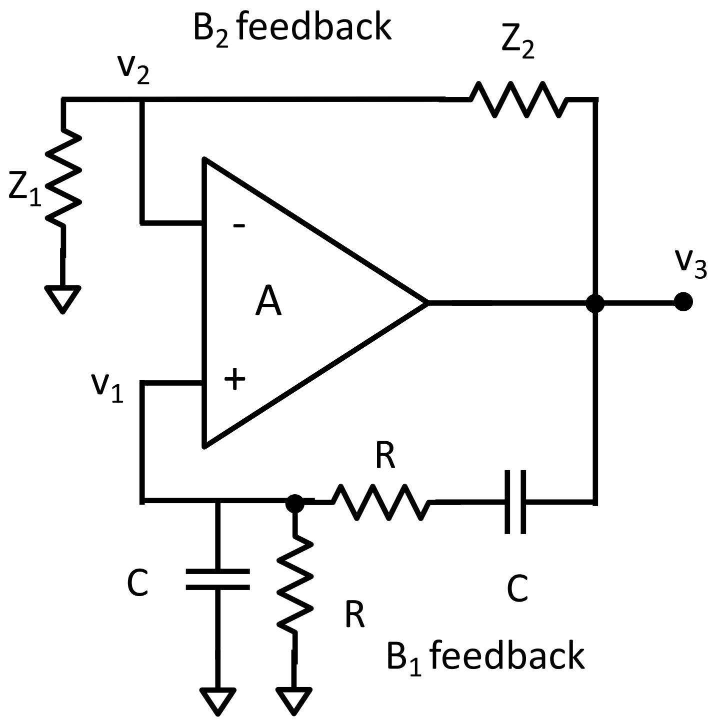
- Want hi Q for stability
- $v_o = \frac{A}{1 - \frac{R_i A}{R_i + R + j(\omega L - {}^1/_{\omega C})}} v_i$
- $Q = \frac{\omega_0 L}{R_i + R}$



Some oscillators

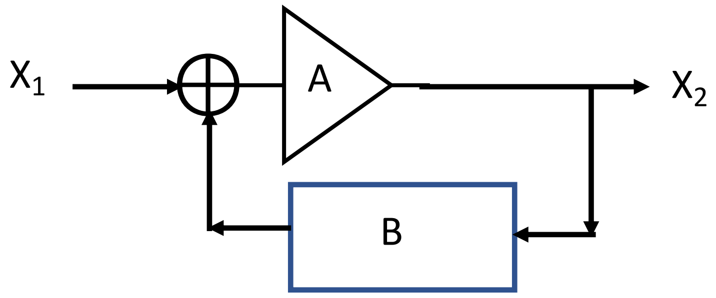


- $B = \frac{M}{L}\left(\frac{R_2}{R_1+R_2}\right)$
- $\omega = \frac{1}{\sqrt{LC}}$



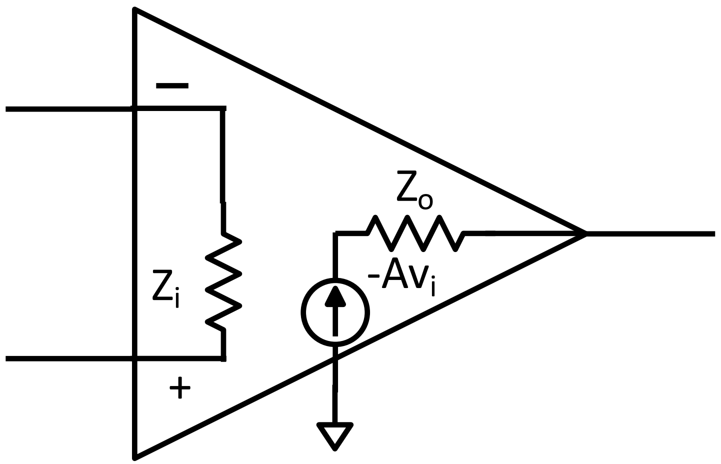
- $v_3 = A(v_1 - v_2)$
- $K = \frac{1}{B_1}$

Op Amp models



Feedback

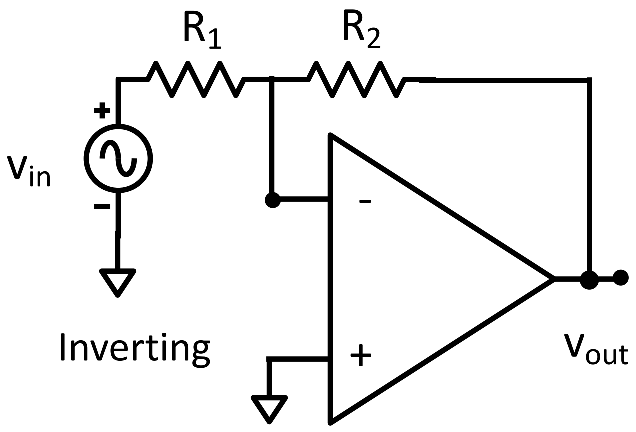
- $A(f) = \frac{A_0}{1+j(^{f}/_{f_0})}$
- $A(X_1 - BX_2) = X_2$ so $\frac{X_2}{X_1} = K = \frac{A}{1+AB}$
- $\frac{dK}{dA} = \frac{1}{(1+AB)^2}$
- $AB < 0$, positive feedback
- $AB > 0$, negative feedback

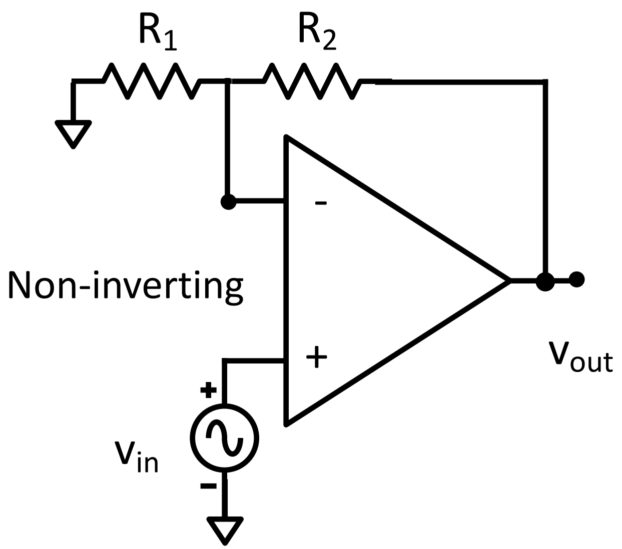


- For 741, $Z_i \approx 2\times10^6\Omega$, $Z_o \approx 150\Omega$

Op Amps

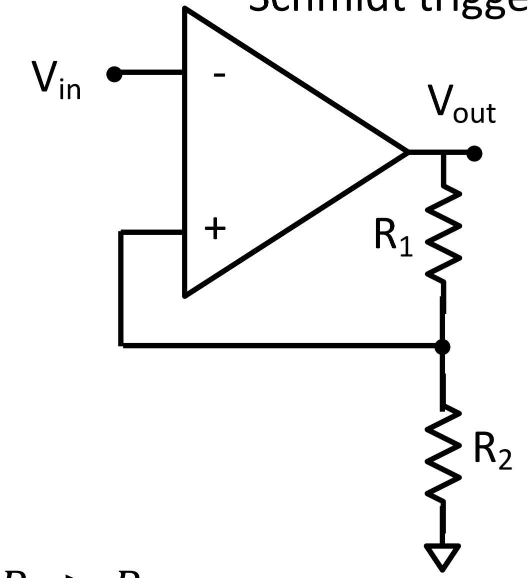
- Ideal op amp
  - $Z_{in} = \infty$
  - $A_V = \frac{V_o}{V_{i,+} - V_{i,-}}$, A_V is an op amp parameter between 10^4 and 10^6.
  - $V_- = V_+$ for negative feedback
  - Output voltage increases when $v_+ > v_-$, decreases when $v_+ < v_-$.
- Example 1: Inverting amp, we'll show the gain is $\frac{R_2}{R_1}$
  - $i_1 = \frac{V_{in}}{R_1}$, $i_2 = \frac{V_{out}}{R_2}$ since the op amp has infinite input impedance
  - By Kirchhoff, $i_1 = -i_2$, so $V_{out} = \frac{R_2}{R_1}\ V_{in}$
  - $Z_{in} = R_1$
  - Z_{out} is same as non-inverting.
- Example 2: Non-inverting amp
  - $V_- = V_+$, so $V_{out} = (1 + \frac{R_2}{R_1})\ V_{in}$
  - $Z_{in} > 10^6 \Omega$
  - $Z_{out} = \frac{R_{o,Th}}{1 + A_V \beta}$, $\beta = \frac{R_2}{R_1 + R_2}$, $R_{o,Th}$ is the Thevenin resistance of the op amp





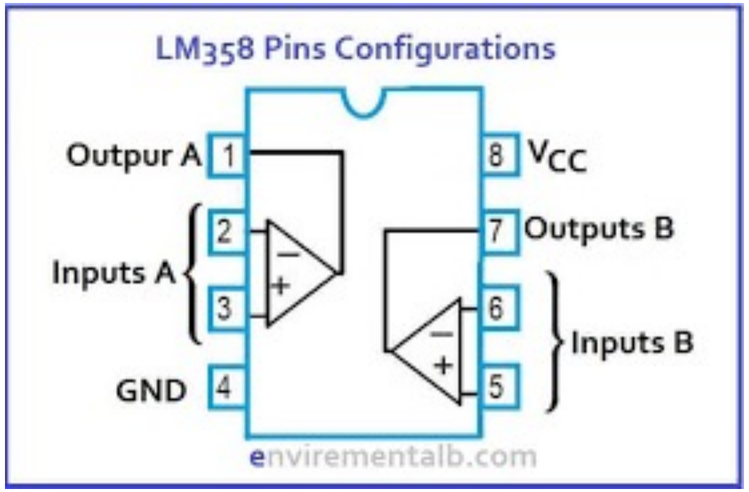
Miscellaneous op amps

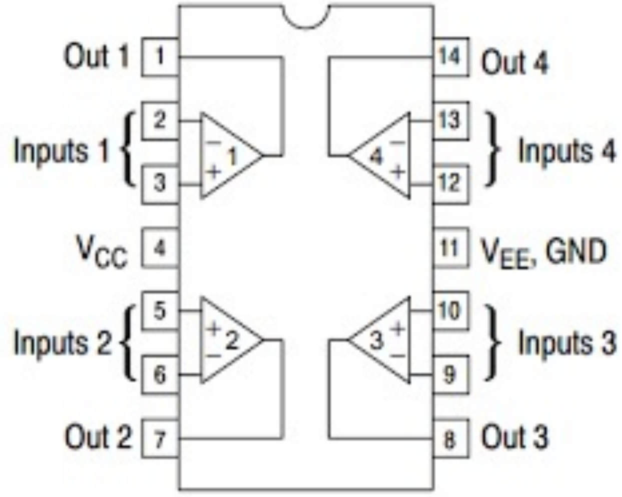
Schmidt trigger



- $R_1 > R_2$
- $\left(V_{Th,+} - V_{Th,-}\right) = \frac{R_2}{R_1+R_2}\left(V_{sat,+} - V_{sat,-}\right)$

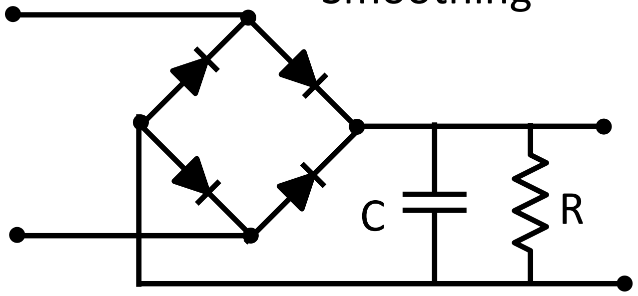
- Simple op amp model
- $\frac{1}{g(f)} = \frac{1}{g_{DC}} + j\frac{f}{f_{BW}}$



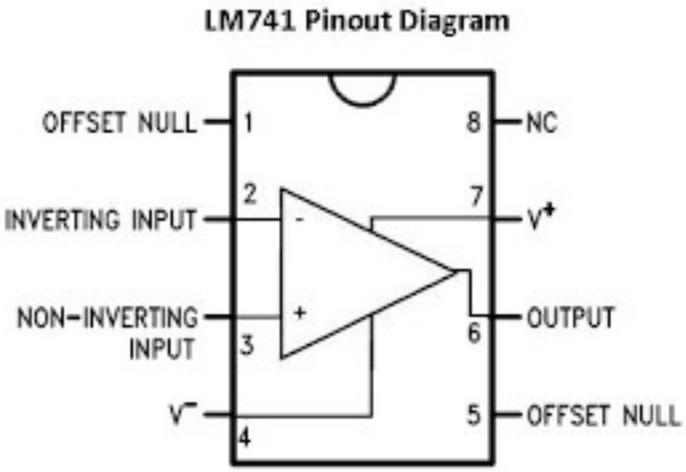


LM324

Smoothing

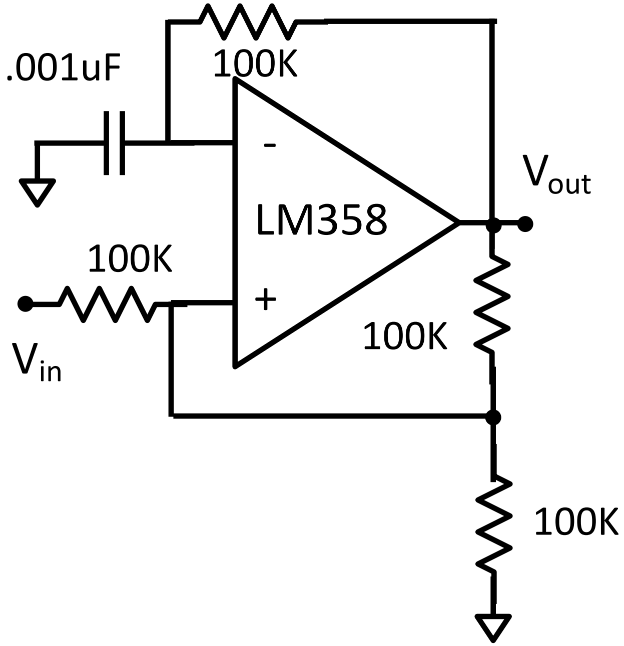


- $RC = \frac{10}{f}$

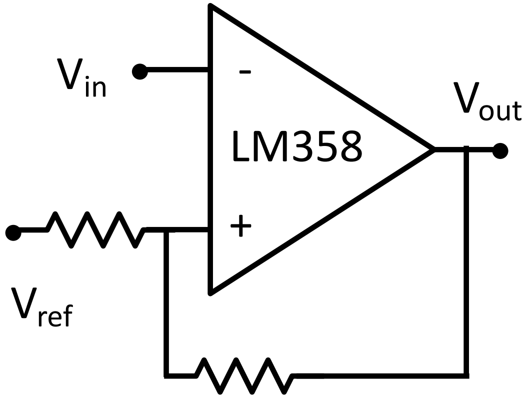


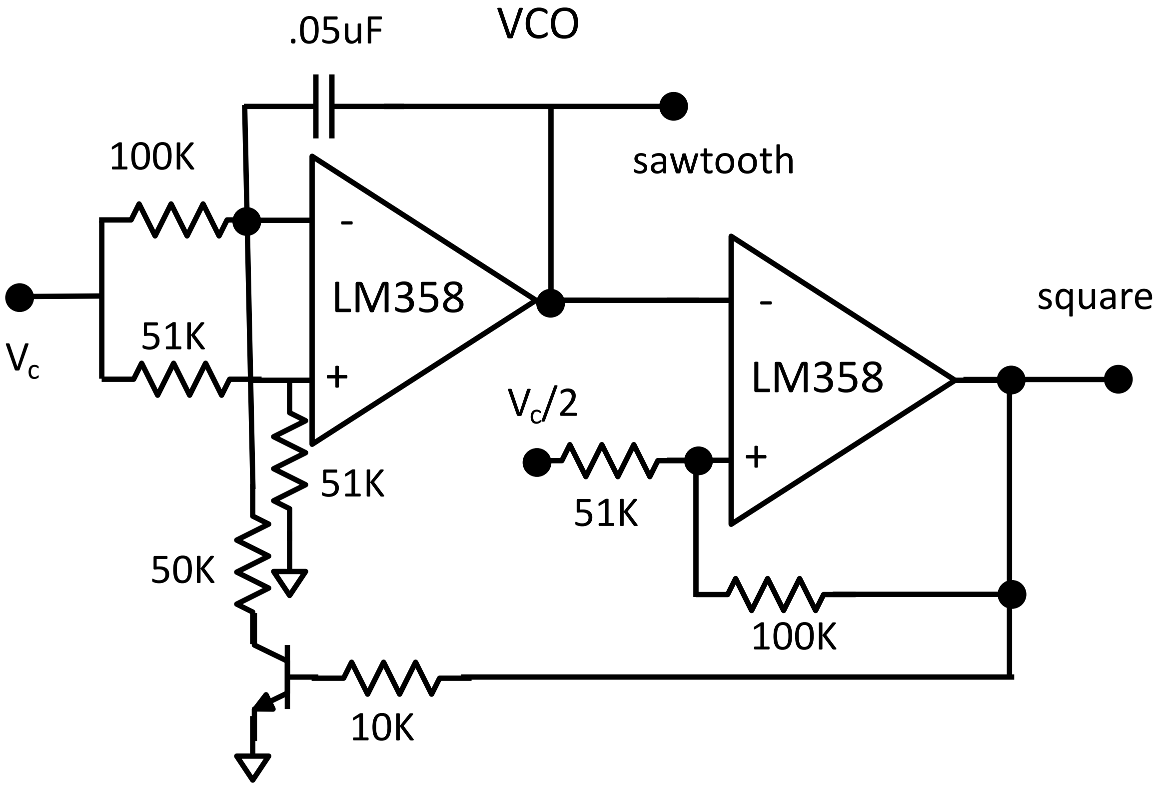
358 op amp

Square wave



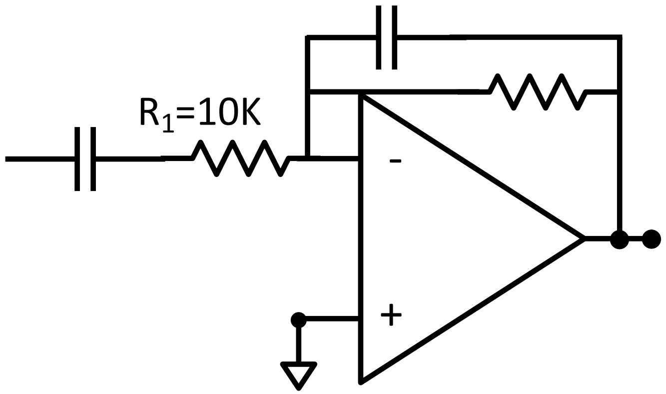
Peak



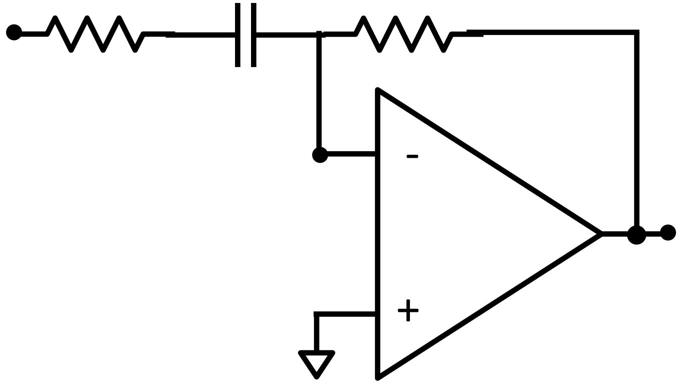


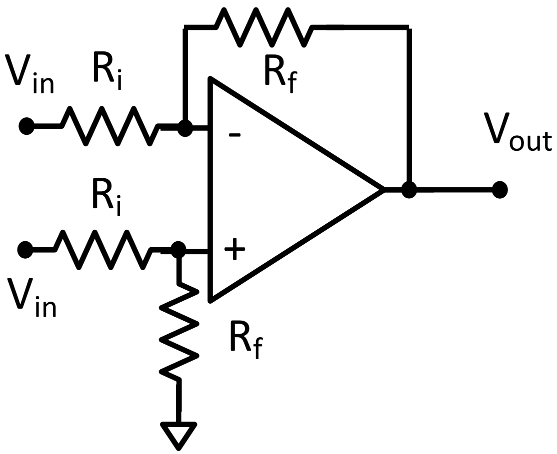
Op integrators and differentiators and differential amp

Integrator



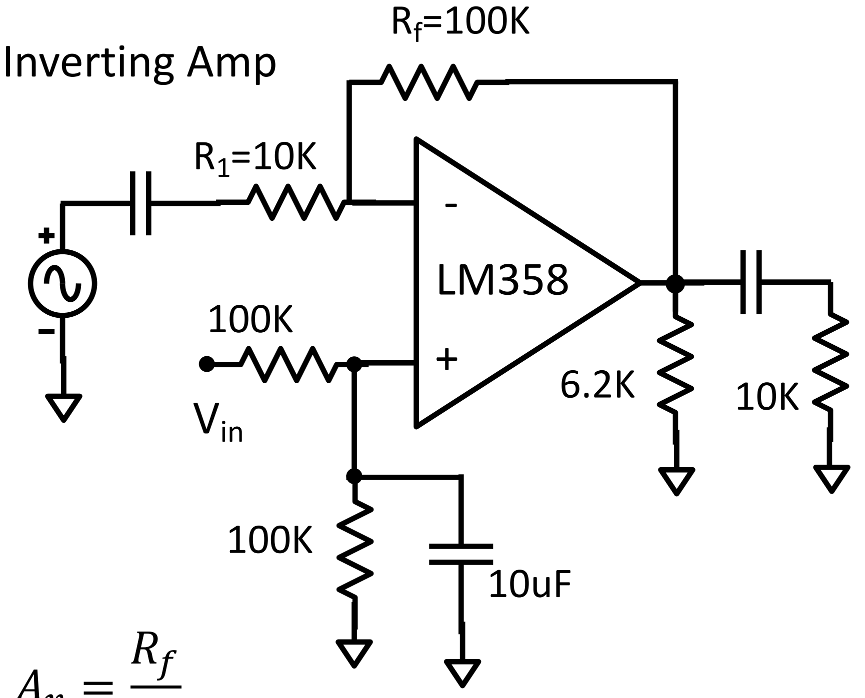
Differentiator



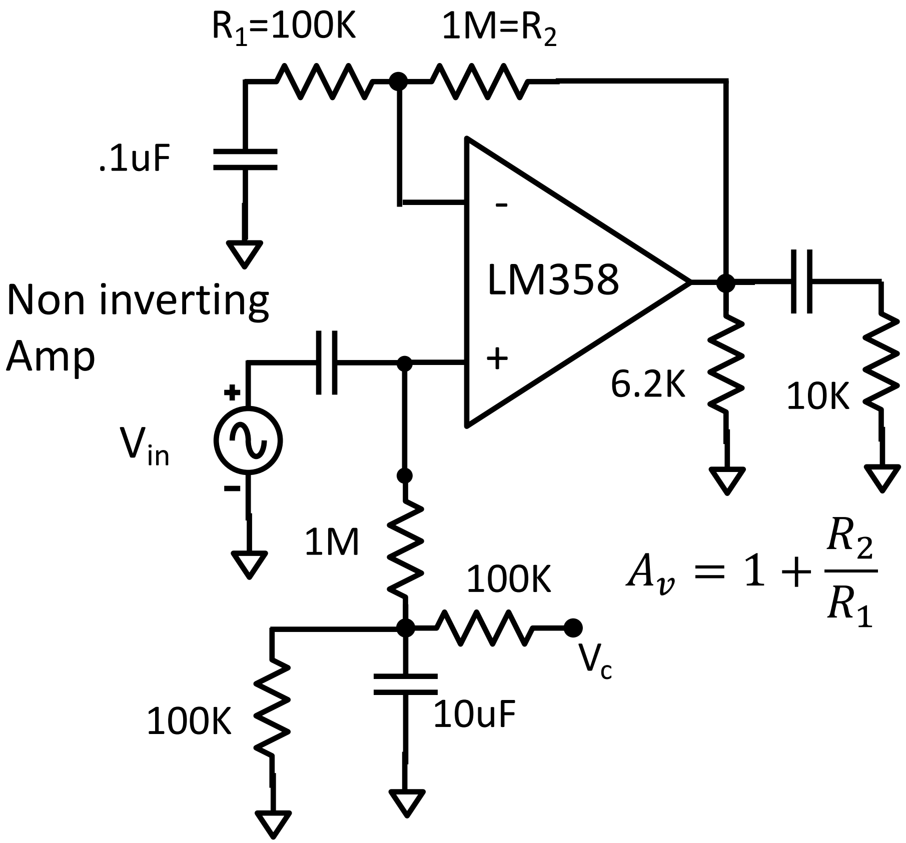


Differential amp

Real 358 based ops

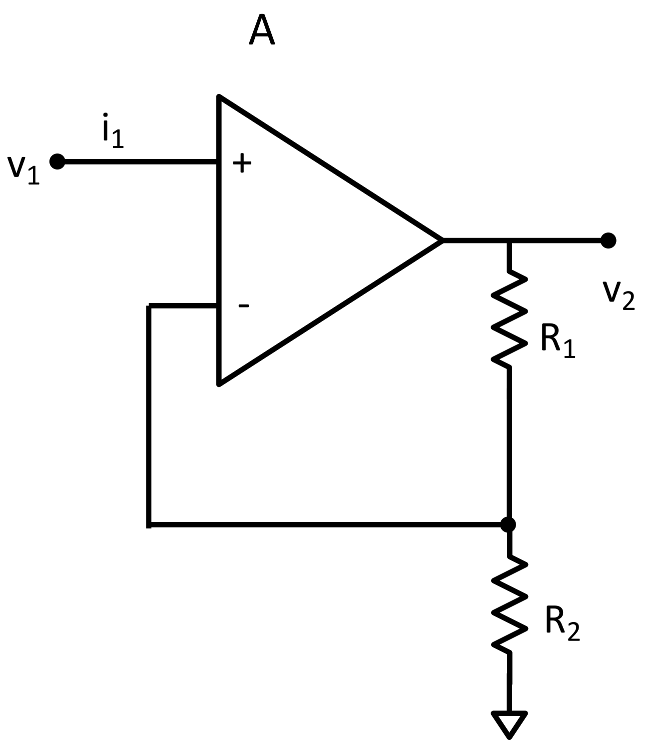


$$A_v = \frac{R_f}{R_1}$$



$$A_v = 1 + \frac{R_2}{R_1}$$

Op amp feedback

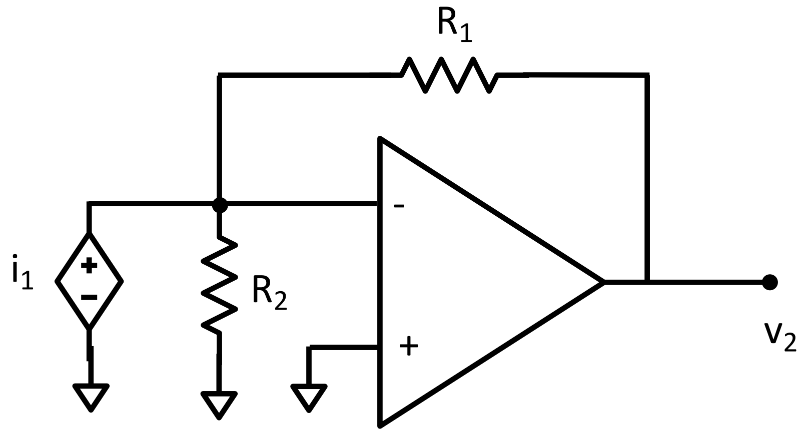


Nyquist

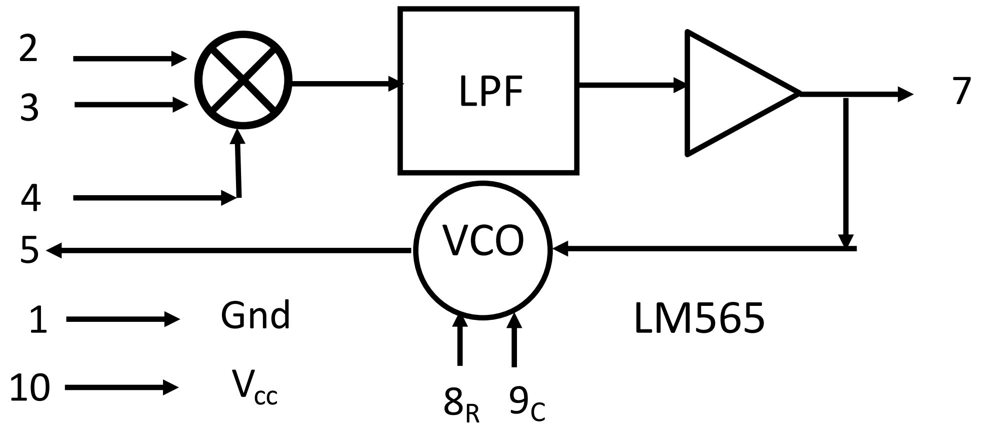
- Polar plot of AB

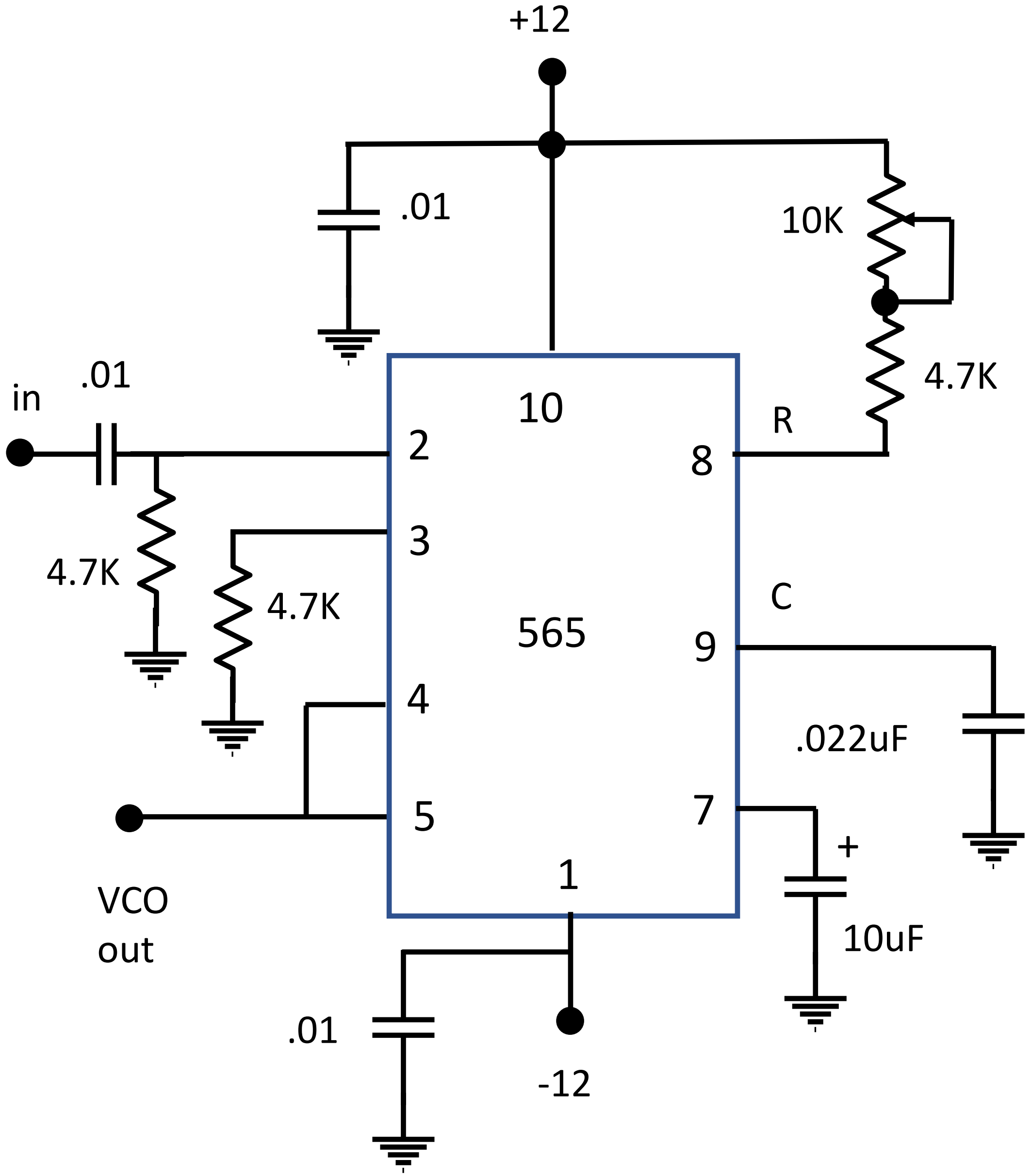
- $v_{-} = v_2 \frac{R_2}{R_1+R_2}$
- $v_2 = A\left(v_1 - v_2 \frac{R_2}{R_1+R_2}\right)$
- $\frac{v_2}{v_1} = \frac{A}{1+A\frac{R_2}{R_1+R_2}} = \frac{A}{1+AB}$
- $K = \frac{A}{1+AB}$
- $\frac{dK}{dA} = \frac{1}{(1+AB)^2}$

Current to voltage



PLL



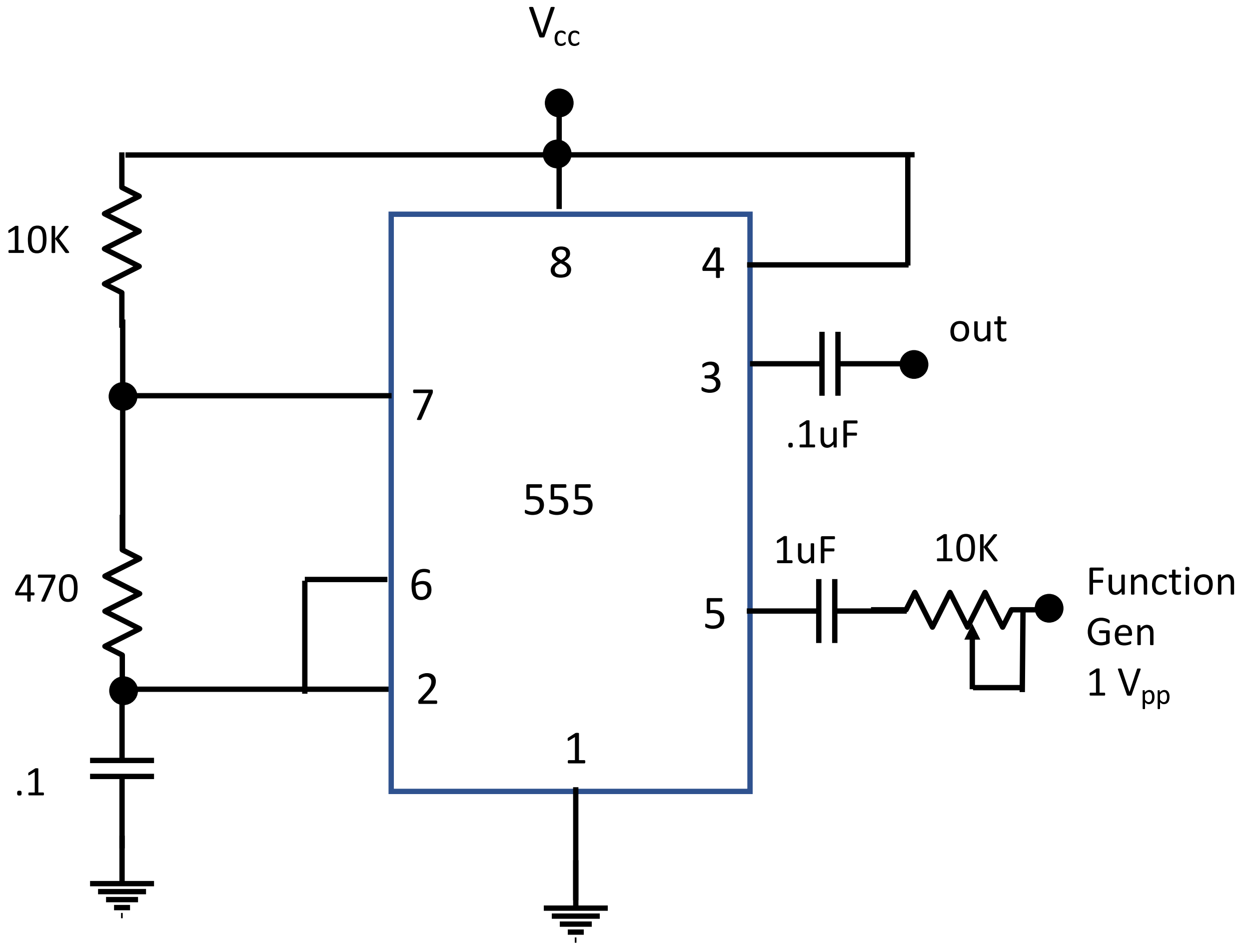


- For general PLL, if $v_{ref}(t) = V_R \cos\big(\omega_0 t + \phi_R(t)\big)$ and $v_{VCO}(t) = V_V \cos(\omega_0 t + \phi_V(t))$, $v_D = k_D(v_{ref}(t) - v_{VCO}(t))$ where v_D is the output of the phase detector.

- $f_0 = .3(RC)^{-1}$
  1. With no input, VCO out is about 1360Hz
  2. For input, set function generator to $1V_{pp}$

PLL FM detector

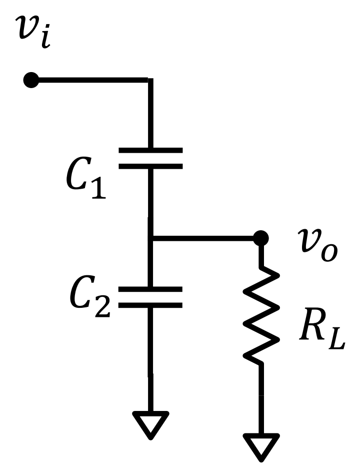
1. With no input, out is about 1360Hz
2. For input, set function generator to 1V_{pp} and connect to previous 565

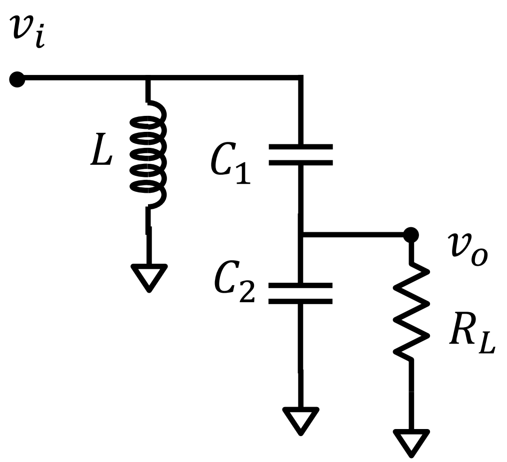


Miscellaneous

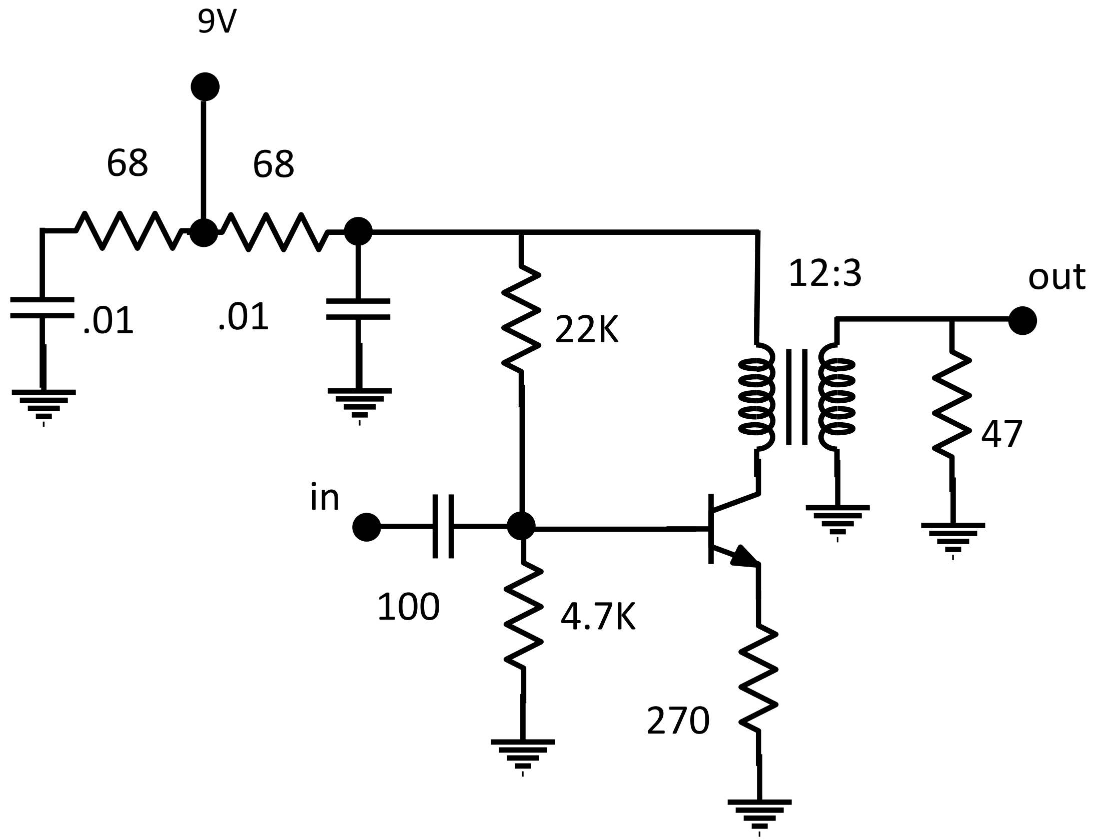
- $C = Blg(1 + SNR)$
- $P_n = 4kTB$
- Diode: $i_D = i_S[\exp\left(\frac{V_{diode}}{nV_T}\right) - 1]$, $V_T = \frac{kT}{q}$

- Circuits on right
  - $Y = G + jB$
- Capacitive Divider
  - $G_{in} = \frac{1}{R_L}(\frac{C_1}{C_1+C_2})^2$, $B_{in} = \frac{\omega C_1 C_2}{C_1+C_2}$
- Resonant capacitive divider
  - ${\omega_0}^2 = \frac{C_1+C_2}{LC_1C_2}$
  - $Q = \frac{R_{in}||R_s}{\omega_0 L}$



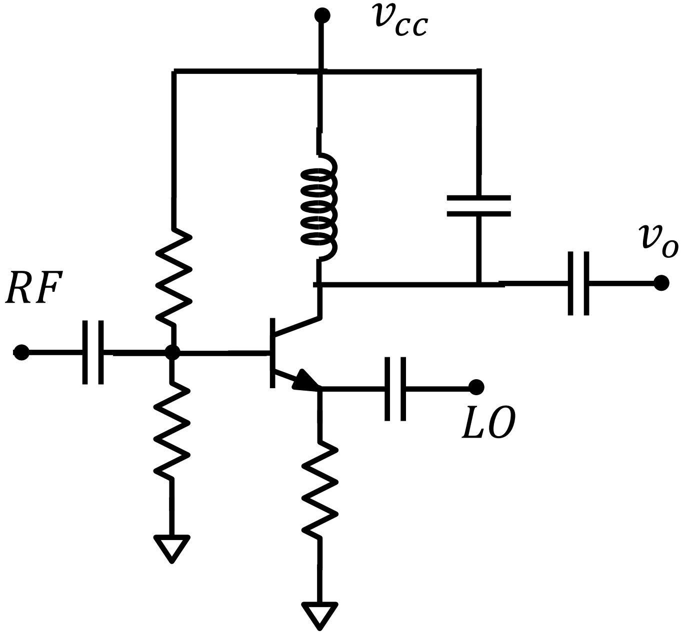


RF Amp



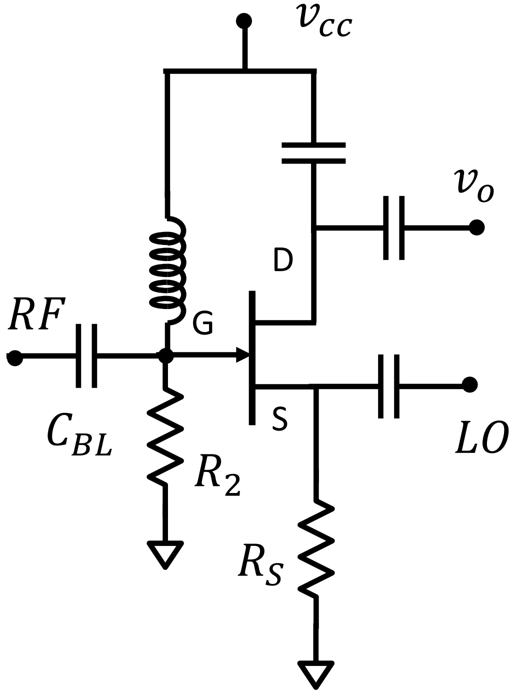
BJT and FET mixers

BJT mixer

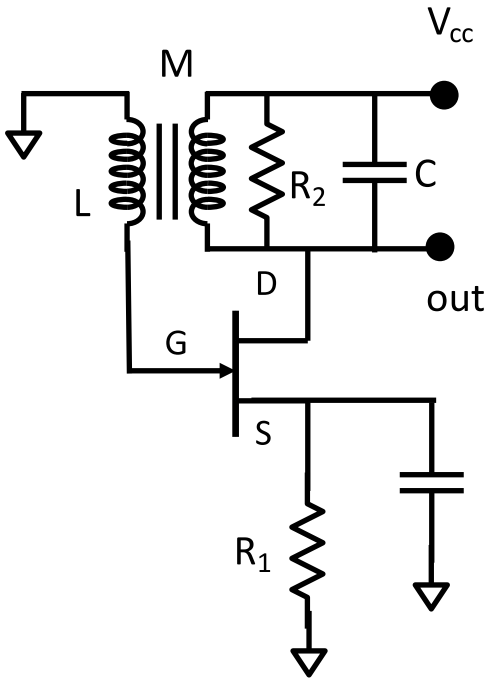


- AM: $v(t) = [1 + ma(t)]\cos(\omega_c t)$
- Phase: $v(t) = A\cos(\omega_c t + \phi)$, $\phi = ma(t)$
- FM: $v(t) = A\cos(\omega_c t + \phi)$, $\phi = \int ma(t)\, dt$

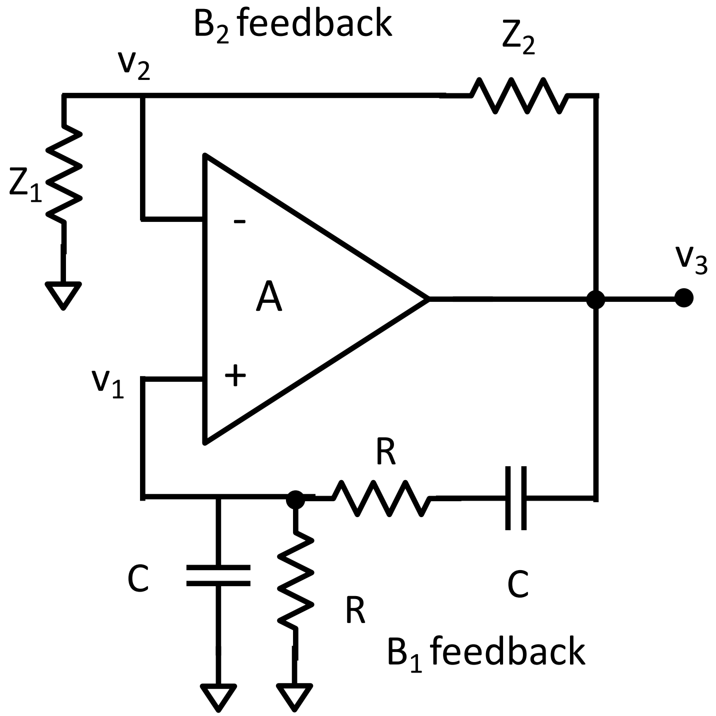
FET mixer



Some real oscillators



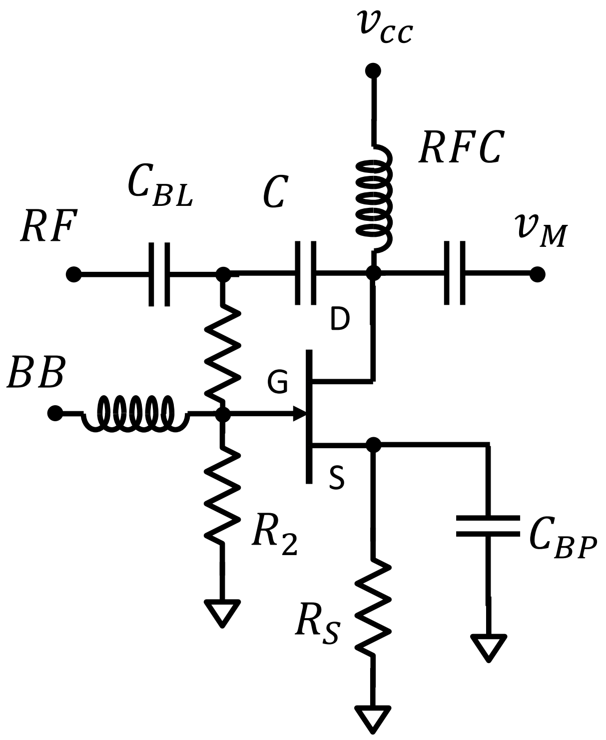
- $B = \frac{M}{L}\left(\frac{R_2}{R_1+R_2}\right)$
- $\omega = \frac{1}{\sqrt{LC}}$



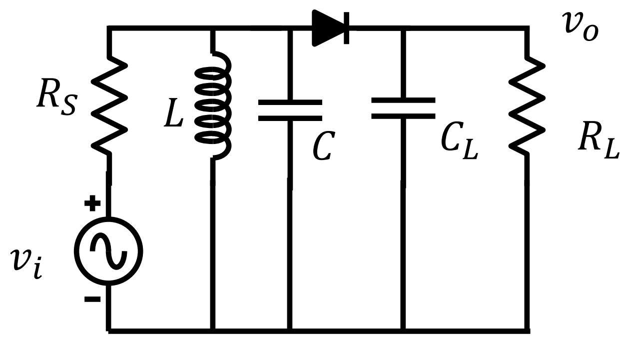
- $v_3 = A(v_1 - v_2)$
- $K = \frac{1}{B_1}$

Modulators and detectors

Phase modulator

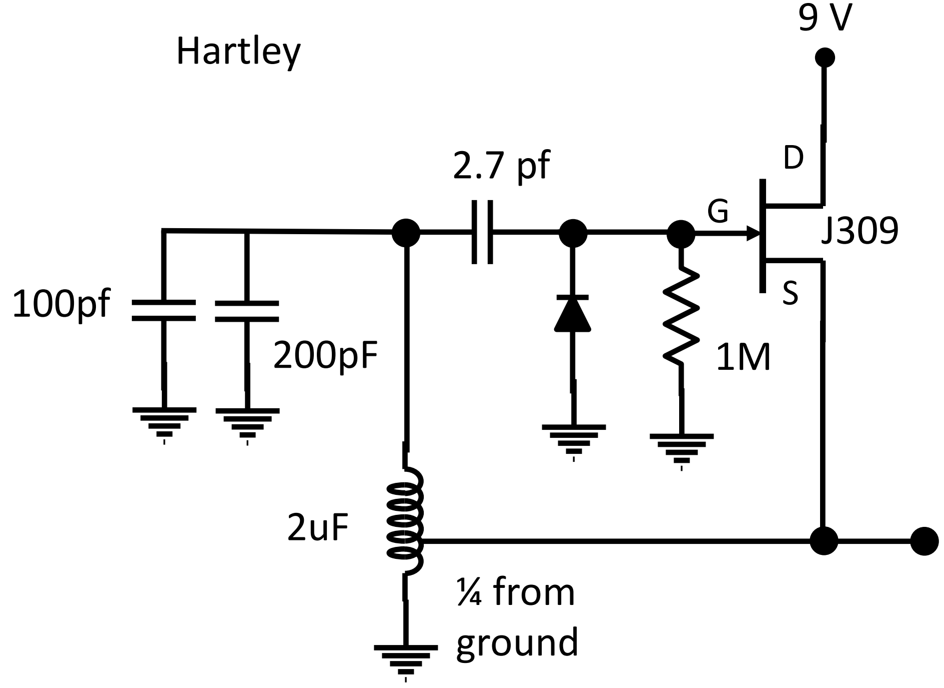


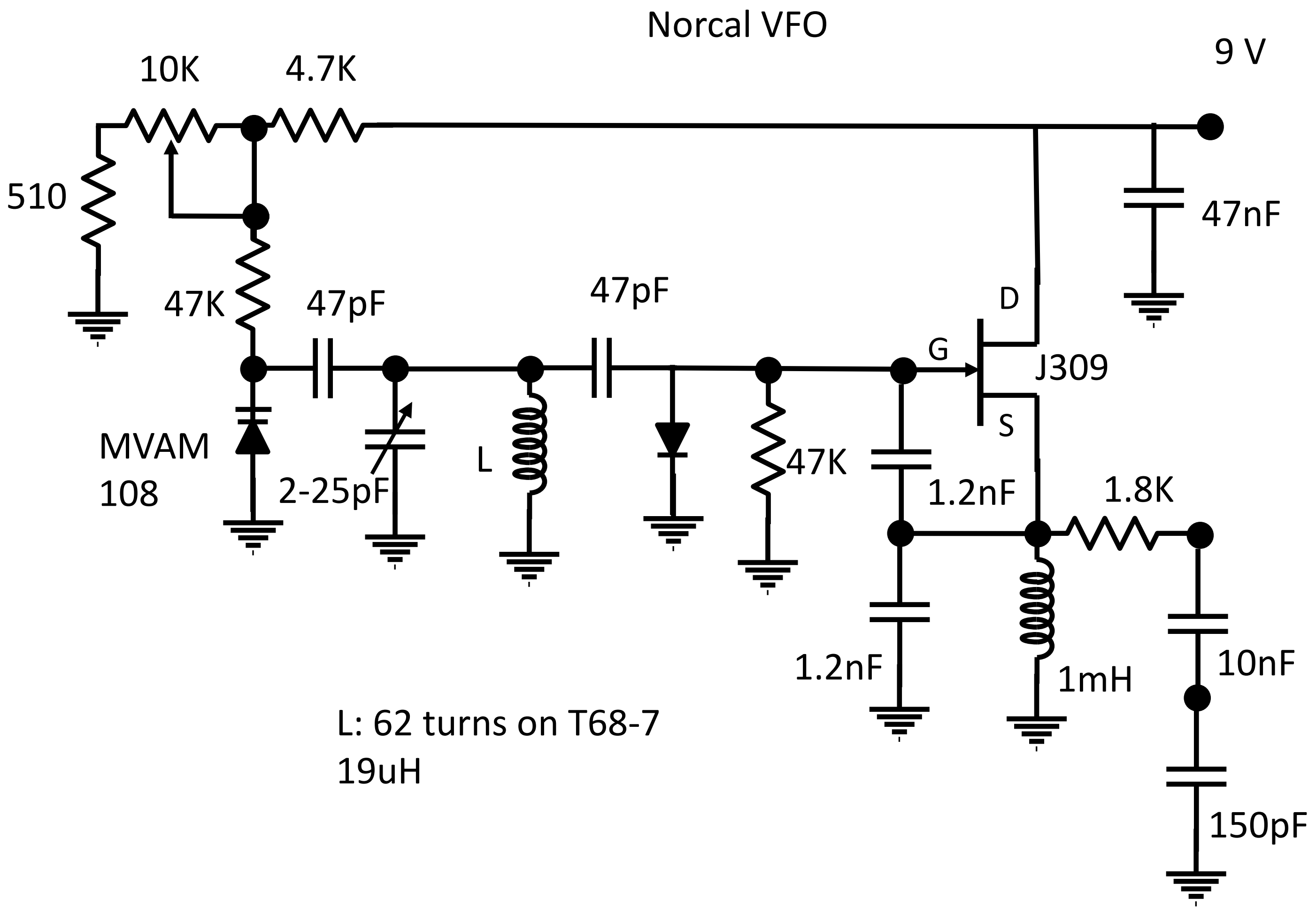
Simple FM detector



- $v_o = \frac{R}{(R+R_S)[1+\frac{4Q^2}{\omega_0{}^2}(\omega-\omega_0)^2]} v_i$
- $Q = \frac{R||R_S}{\omega_0 L}$

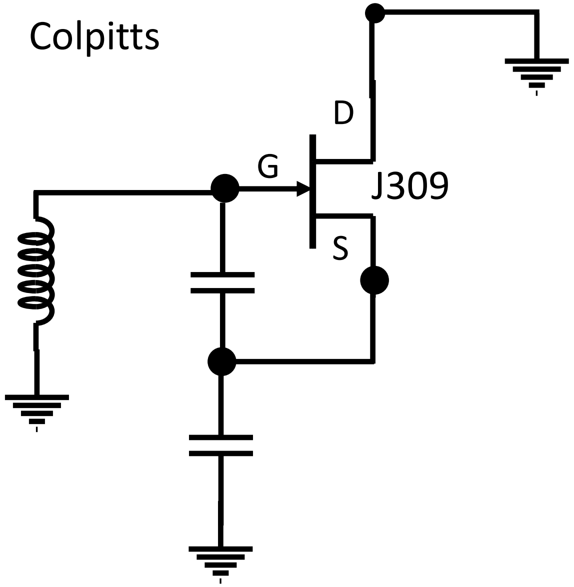
More Oscillators



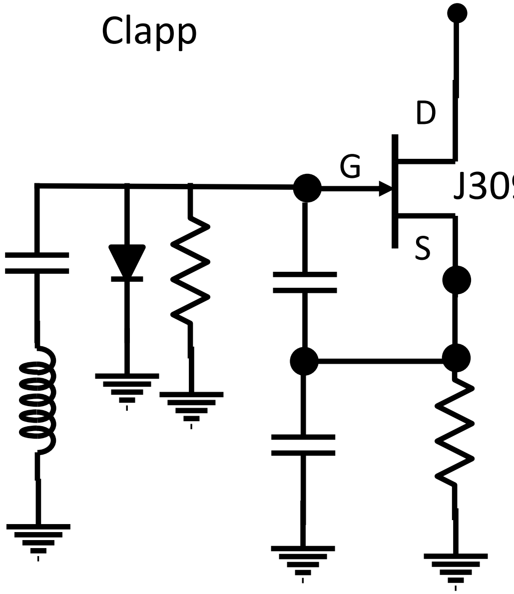


More Oscillators

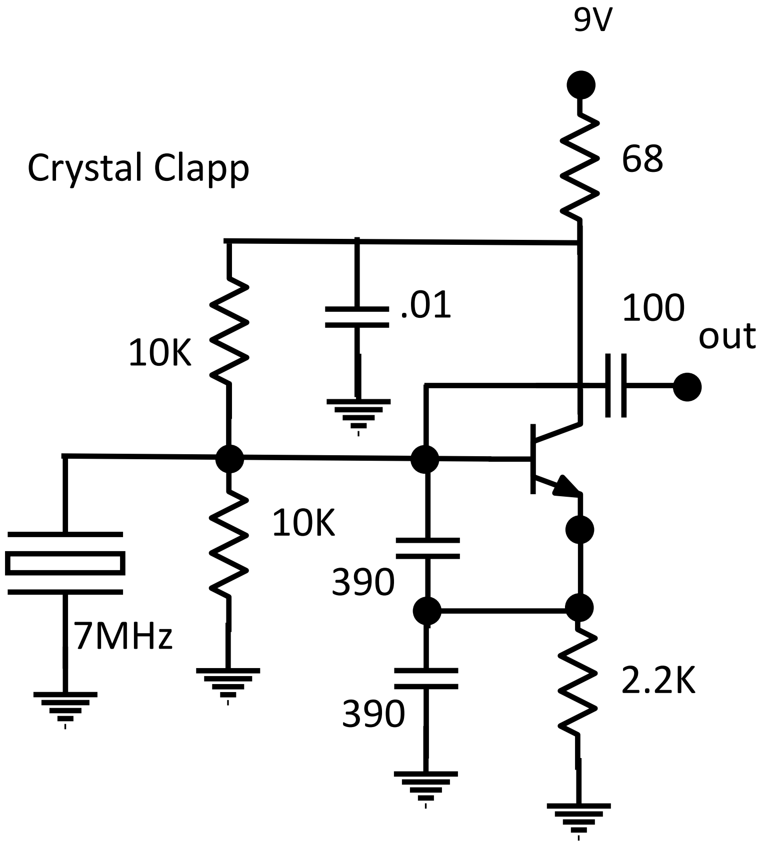
Colpitts



Clapp



Crystal Clapp



GPS

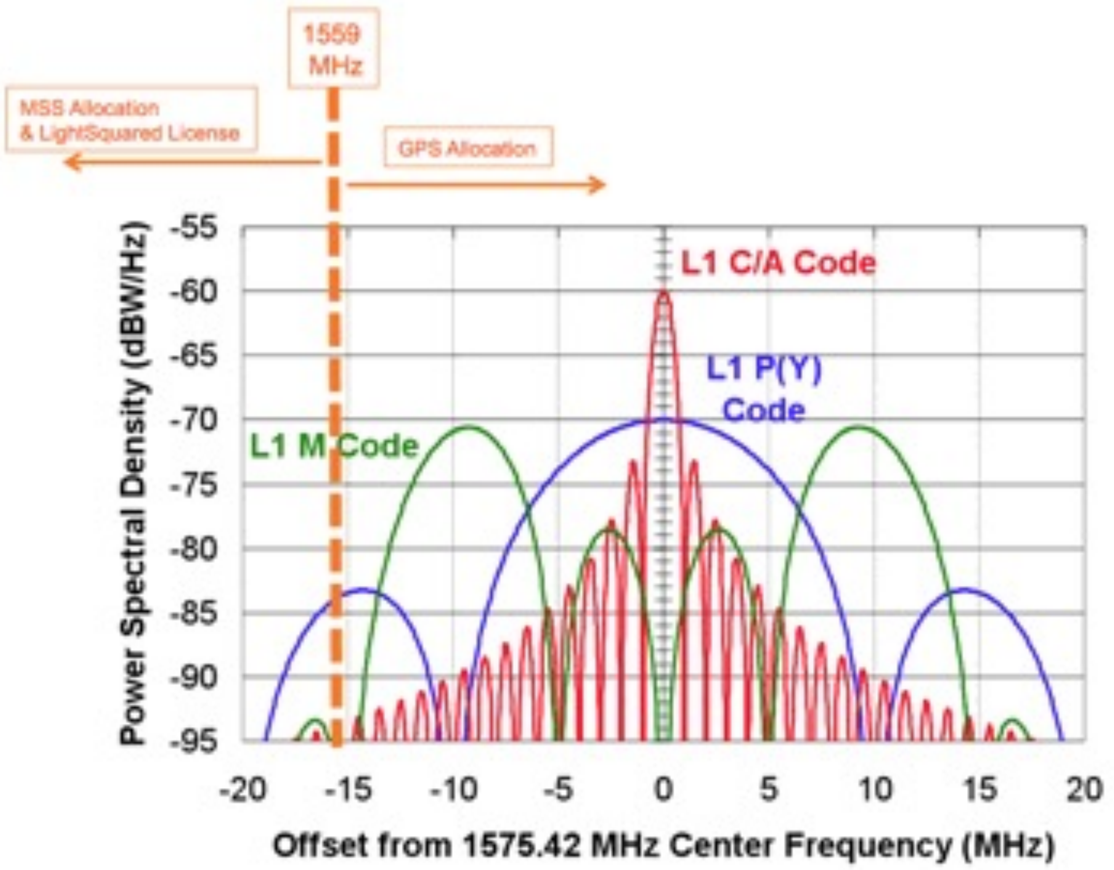
- Space Segment
  - 31 operating satellites (Current generation: block IIIA – 2018)
  - 12 hour orbits MEO (21000km), 8-12 satellites in view
  - Transmitter power: 44.8* W, $G_t = 12dBi$. Clocks accurate to better than 10ns (no leap seconds).
- Control segment master at Schriever AFB (plus alternative master, 4 ground antennas and 6 ground stations). Ephemeris updates daily.
- User segment
  - Two bands: L1 (1575.42 MHz [2.046MHz BW]) civilian and military, L2 (1227.6MHz)], military only
  - Two signals: C/A (civilian) and P(Y) (military). C/A signal is Direct Sequence Spread Spectrum
  - Satellite identified by PRN (1-32).
  - L1 signal: $s(t) = A_c C(t) D(t) \cos(\omega_c t + \theta_1) + A_p P(t) D(t) \sin(\omega_c t + \theta_2)$
    - $C(t)$ - coarse ranging code
    - $D(t)$ - navigation signal consists of (clock, ephemeris, almanac corrections) 37,500 bits. 50 bps transmission.
    - $P(t)$ - precision ranging (Y is encrypted)

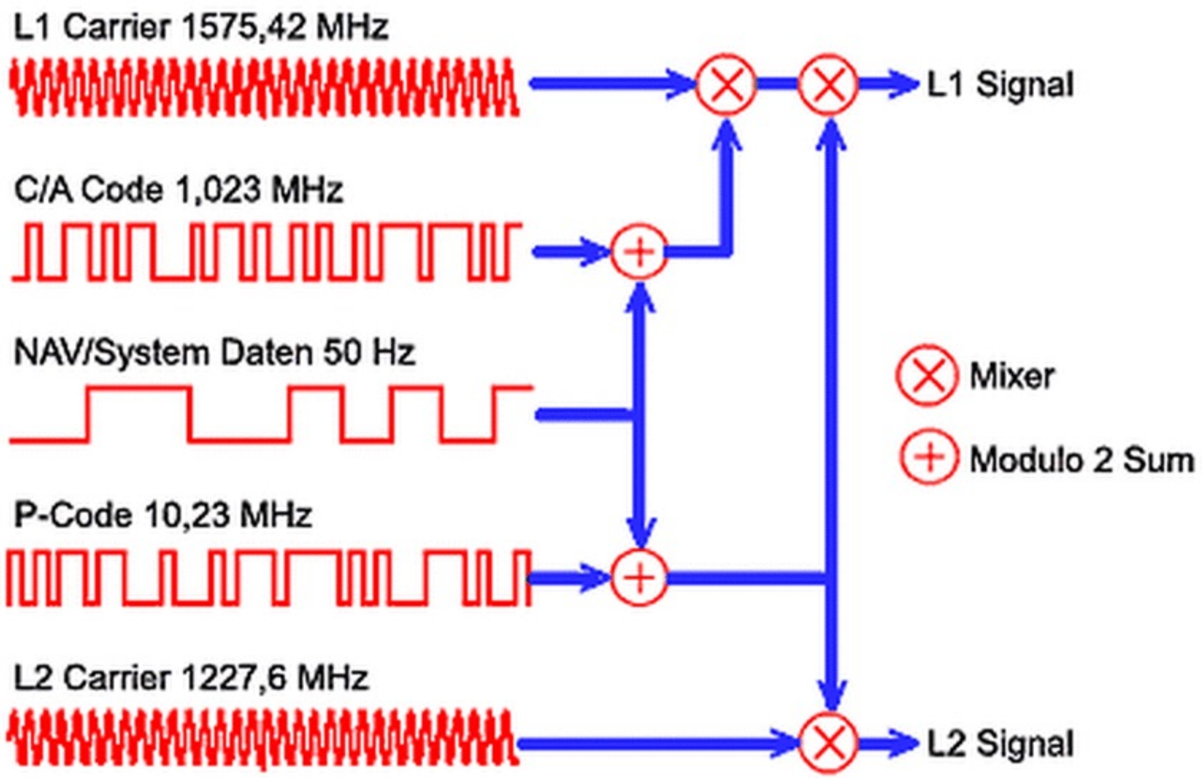
***** This is total transmitter power, 27 W devoted to ranging code. Newer satellites have higher power and can surge power

Some data and rough calculations

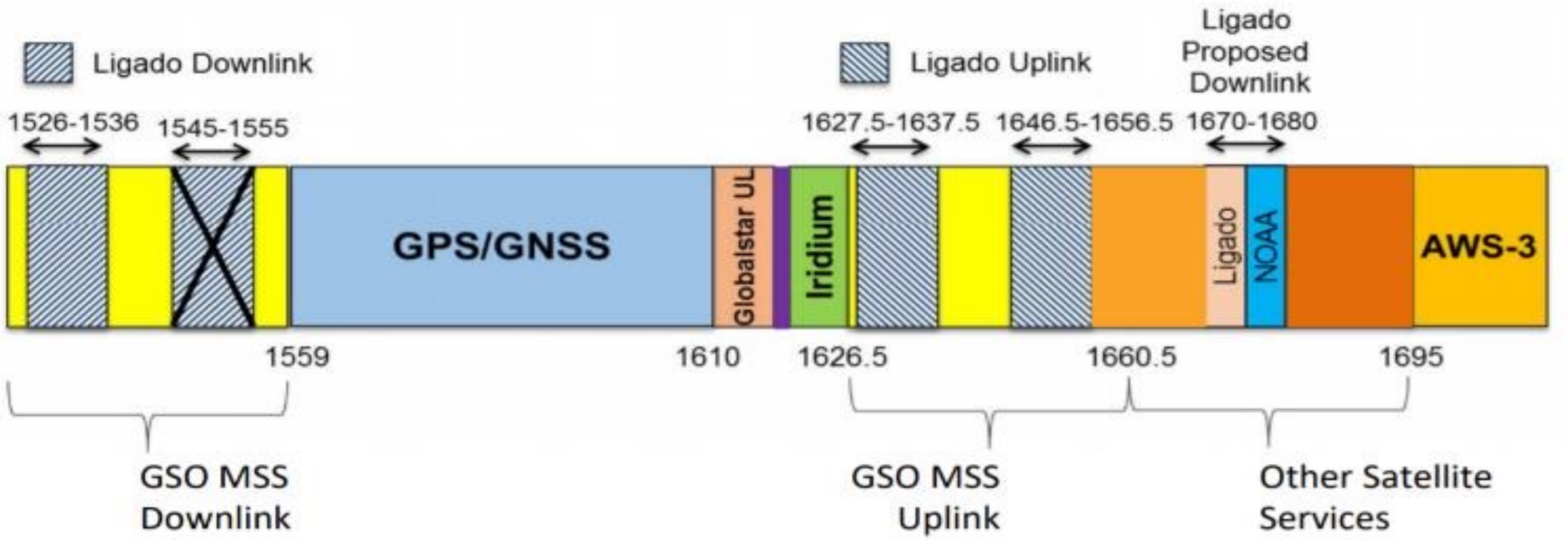
- $\mathrm{N} = \mathrm{kTB}, \mathrm{k} = 1.38 \times 10^{-23}\ J/K$. BW: $2.046 MHz$
- For comparison: laptop emission noise density is capped at $10^{-7}\ \frac{W}{MHz}$
- Noise: $1.38 \times 10^{-23} \frac{J}{K} \times 100 \times 2 \times 10^{6} = 2.76 \times 10^{-15} W$, $(-145.6\ dB)$
- Free space loss (@100K): $P_r = \frac{P_t G A}{4\pi r^2}$. $\int A d\Omega = \lambda^2$. $P_r = \frac{(44.7)10^{1.24}}{4\pi(21 \times 10^{6})^{2}} \times \frac{3 \times 10^{8}}{1.575 \times 10^{9}} = 4 \times 10^{-15} W(-144 dB)$
- Final SNR: $-144 + 145.6 + 43 = 44.6 dB$.
  - Lower in practice since satellite not directly overhead, and there are shadowing and multipath losses. In addition, we have receiver loss (and receiver antenna gain) so P_r could be as small as $-170 dB$ and final SNR is approximately $-170 + 144 + 43 = 17 dB$
- Receiver modeled at $G_r = 2 dBi$

Signal Structure





GPS spectrum neighborhood



Signal Summary

GNSS System	GPS	GPS		GPS	GPS
Service Name	C/A	L1C		P(Y) Code	M-Code
Centre Frequency	1575.42 MHz	1575.42 MHz		1575.42 MHz	1575.42 MHz
Frequency Band	L1	L1		L1	L1
Access Technique	CDMA	CDMA		CDMA	CDMA
Signal Component	Data	Data	Pilot	Data	N.A.
Modulation	BPSK(1)	TMBOC(6,1,1/11)		BPSK(10)	$BOC_{sin}(10,5)$
Sub-carrier frequency [MHz]	-	1.023	1.023 & 6.138	-	10.23
Code frequency	1.023 MHz	1.023 MHz		10.23 MHz	5.115 MHz
Primary PRN Code length	1023	10230		$6.19 \cdot 10^{12}$	N.A.
Code Family	Gold Codes	Weil Codes		Combination and short-cycling of M-sequences	N.A.
Secondary PRN Code length	-	-	1800	-	N.A.
Data rate	50 bps / 50 sps	50 bps / 100 sps	-	50 bps / 50 sps	N.A.
Minimum Received Power [dBW]	-158.5	-157		-161.5	N.A.
Elevation	5°	5°		5°	5°

L1 Signal Structure

- $s_m(t) = C(t) \oplus D(t)$. $s(t) = A_c s_m(t)\cos(\omega_c t + \theta_1)$ [bspk]
  - D(t) - 50 bps transmission (20ms).
  - C(t) – 1 Mbps chipping rate. 300 meters, $1\mu s/chip$. Chipping code repeats 20 times for single navigation bit.
- C/A code is generated from two Gold codes and the PRN identifier (5 bits).
  - $g_1(x) = 1 + x^3 + x^{10}$, $g_2(x) = 1 + x^2 + x^3 + x^8 + x^9 + x^{10}$
  - Phase selector takes PRN and uses it to select bits of the second Gold code.

- Signal Characteristics
  - Received power: -130dBm, Noise power: -111dBm
  - Spread spectrum contributes 43 dB to processing. Want SNR greater than 14 dB
  - SNR for C/A: -20dB

Acquisition, tracking and navigation

- Acquisition
  - Receiver generates known C/A code and attempts to correlate with signal.
  - To generate C/A code, receiver needs to know which satellite it's attempting to acquire to determine code
- Tracking via delay loop to maintain C/A code alignment
- The Navigation Message includes the Ephemeris parameters, the Time parameters and Clock Corrections, the Service Parameters with satellite health information, Ionospheric parameters model, and the Almanacs, allowing the computation of the position of ”all satellites in the constellation”. The ephemeris and clocks parameters are usually updated every two hours, while the almanac is updated at least every six days.

Navigation Message

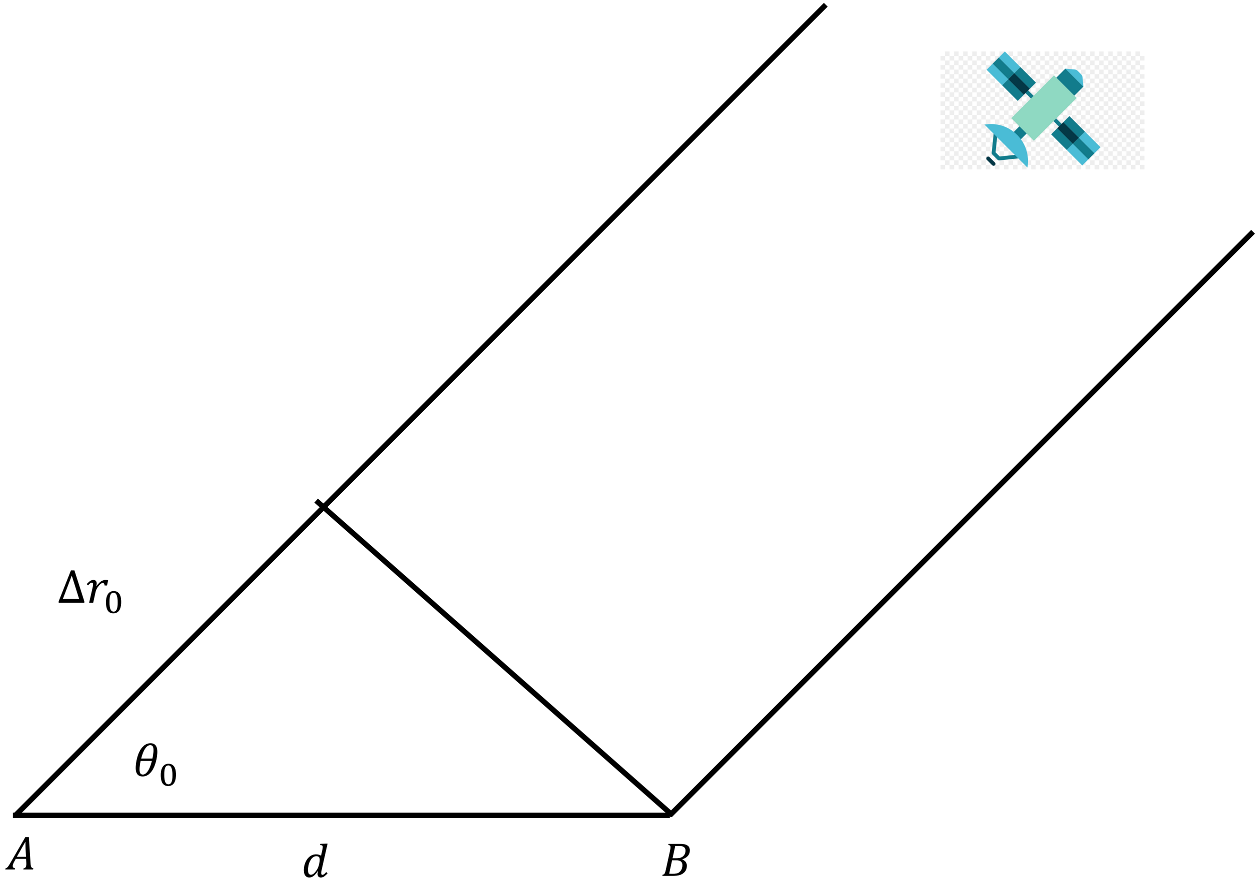
- The navigation message contains 25 pages (’frames’) of 30 seconds each. Entire message takes 12.5 minutes to be transmitted. Every frame is subdivided into 5 sub-frames of 6 seconds each; every sub-frame consists of 10 words, with 30 bits per word.
- Every sub-frame starts with the telemetry word (TLM), needed for synchronism. Next, the transference word (HOW) which provides time information (seconds of the GPS week), allowing the receiver to acquire the week-long P(Y)-code segment.
  - Sub-frame 1: contains information the satellite clock. It also has information about satellite health condition.
  - Sub-frames 2 and 3: contain satellite ephemeris.
  - Sub-frame 4: provides ionospheric model parameters, UTC information (Universal Coordinate Time), part of the almanac, and indications whether the Anti-Spoofing, A/S, is activated.
  - Sub-frame 5: contains data from the almanac and the constellation status. It allows to quickly identify the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.
  - **Sub-frames 1, 2 and 3 are transmitted with each frame.** The content of sub-frames 4 and 5 is common for all satellites. So, the almanac data for all in orbit satellites can be obtained from a single tracked satellite.

Processing and Accuracy

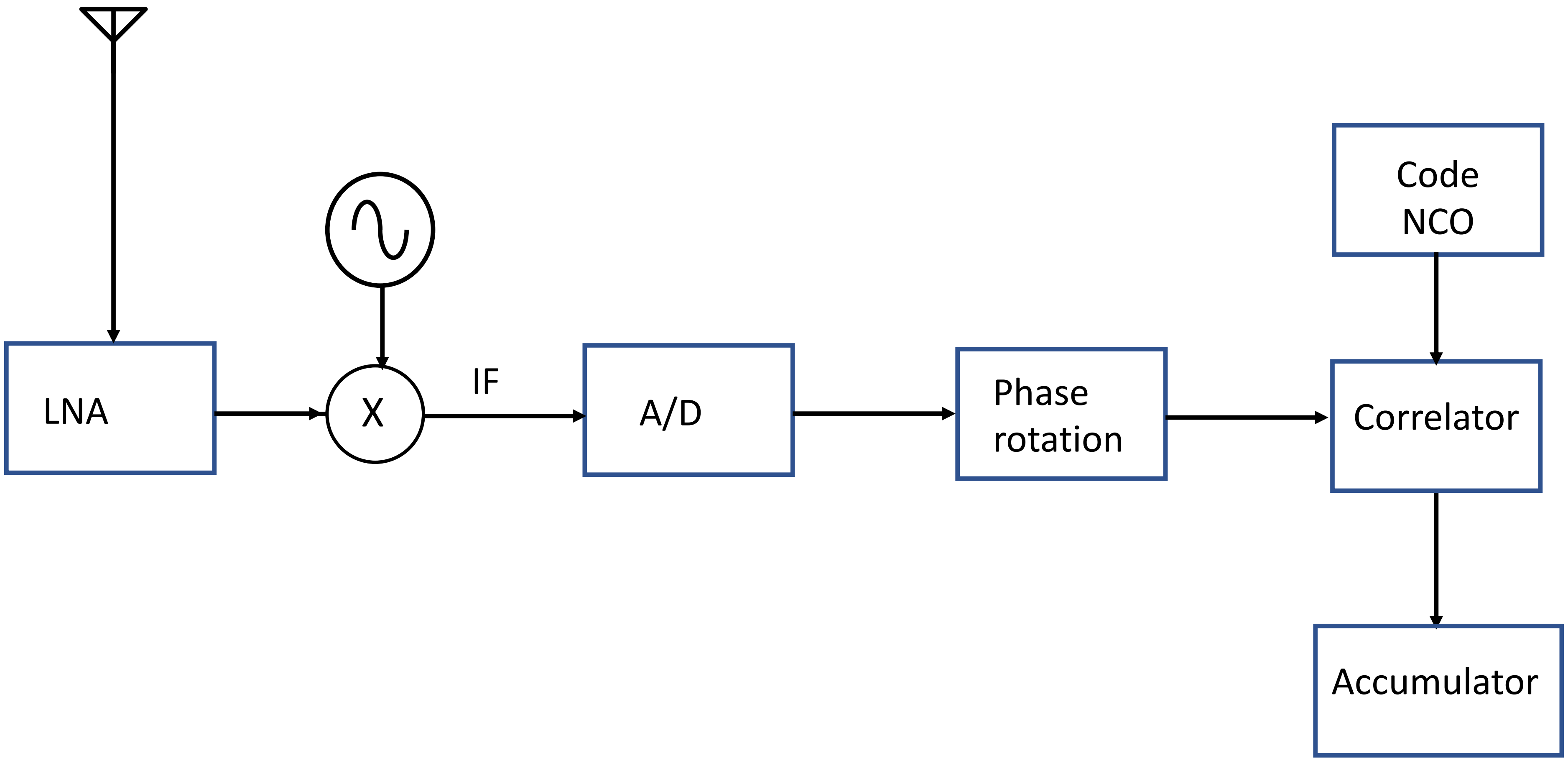
- For satellite k: $\rho = \sqrt{(x^{(k)} - x)^2 + (y^{(k)} - y)^2 + (z^{(k)} - z)^2}$
  - Need to account for skew between satellite clocks and receiver clock. b is receivers clock bias
  - Four satellites required. More satellites increase precision with least squares.
- Doppler correction improves accuracy
- *Carrier-Phase Enhancement* corrects timing errors caused by non-zero PRN pulse transition.
  - It uses the L1 carrier wave, which has a period about one-thousandth of the C/A Gold code bit providing an additional clock.
  - The phase difference error in the normal GPS amounts to 2–3 m error. *Carrier-Phase Enhancement* reduces this to 3 cm (1.2 in).
- *Differential GPS*

Carrier phase

- $\Delta r = d\cos(\theta_0)$
- Wave arrives at B first. Arrives at A with phase delay $\phi_0 = \lfloor \phi_0 \rfloor + \Delta_0$, so
- $d\cos(\theta_0) = \lambda(\phi_0)$
- Satellite moves in time t_1 changing angle to θ_1
- Suppose $\lfloor \phi_0 \rfloor = \lfloor \phi_1 \rfloor = N$, and put $d' = \frac{d}{\lambda}$
  - $d'\cos(\theta_0) - N = \Delta_0$
  - $d'\cos(\theta_1) - N = \Delta_1$
- We can solve for N, d'
- Now, $\phi = \lambda^{-1}[r - I + T] + f \cdot (\delta t_u - \delta t^s) + N + \epsilon_\phi$
- I is ionospheric delay, T is tropospheric delay which we can estimate.
- $\epsilon_\phi \approx .025\ cycles$ (5mm), so we can recover r to mm precision



GPS receiver



More careful estimates*

Elevation	5^o	40^o	90^o
Power at sat input	14.3dB	14.3dB	14.3dB
Sat antenna gain	12.1dB	12.9dB	10.2dB
EIRP	26.4dB	27.2dB	24.5dB
Range	25240km	22020km	20190km
Path loss	-159 dB/m^2	-157.8 dB/m^2	-157.1 dB/m^2
Atmo loss	.5dB	.5dB	.5dB
Received power density	-133.1dB/m^2	-131.1dB/m^2 7.8×10^{-14} W/m^2	-133.1dB/m^2 4.9×10^{-14} W/m^2

*Misra and Enge, GPS

More careful estimates*

Elevation	5^o	40^o	90^o
Received power density	$4.9x10^{-14}W/m^2$	$7.8x10^{-14}W/m^2$	$4.9x10^{-14}W/m^2$
Effective area of receiver	$2.7 \times 10^{-3}m^2$ $-24.5dBm^2$	$2.7 \times 10^{-3}m^2$	$2.7 \times 10^{-3}m^2$
Isotropic receiver power	-158.5 dB	-156.5dB	-158.5dB
G_r	-4 dBic	2 dBic	4dBoc
C/A received power	-162.5dB	-154.5dB	-154.5dB

*Misra and Enge, GPS

More careful estimates*

- $-162.5dB \leq C \leq -154.5dB$
- $P_N = N_0(2\times 10^7)$
- $\sigma_{\Delta\tau} = cT_c\sqrt{\frac{d}{4T^C/_{N_0}}}$ where τ is the noise sample interval, d is the correlator interval, T is the averaging time, d is measured in chips and so dT_c is the time sampling

Noise loss

Elevation	Ant cable	LNA	cable
Gain	-1dB	20dB	-10dB
F	1dB	2-3dB	10dB
T	75.4	290	2610

SNR

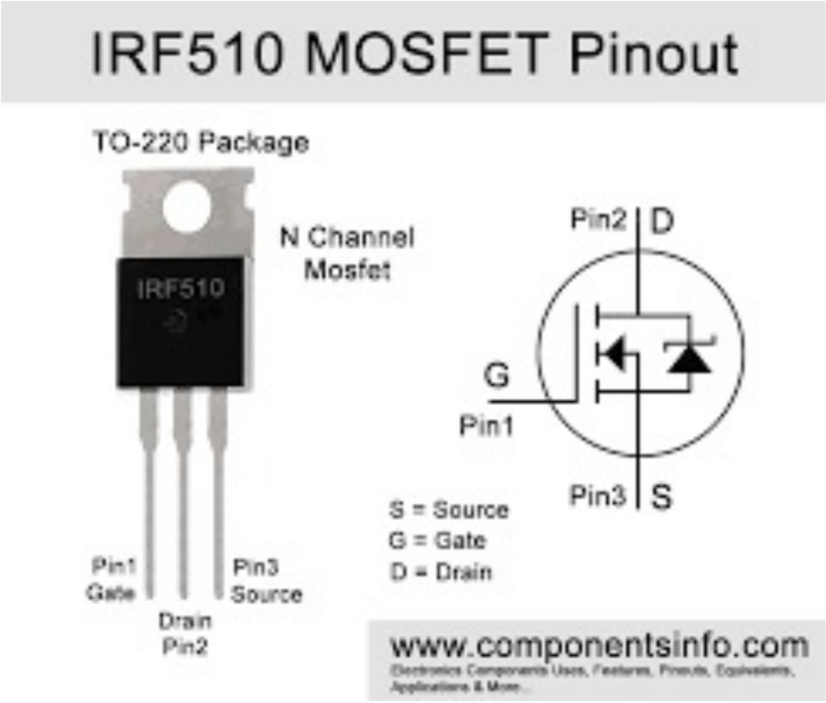
Elevation	5º	90º
Received C/A power (C)	-162.5dB	-154.5dB
N_0 for 3dB LNA	-201 W/Hz	-201W/Hz
C/N_0	34.5 db-Hz	46.5 dBHz
C/P_n 20MHz	-34.6 dB	-26.5dB
C/P_n 2MHz	-24.5dB	-16.5dB

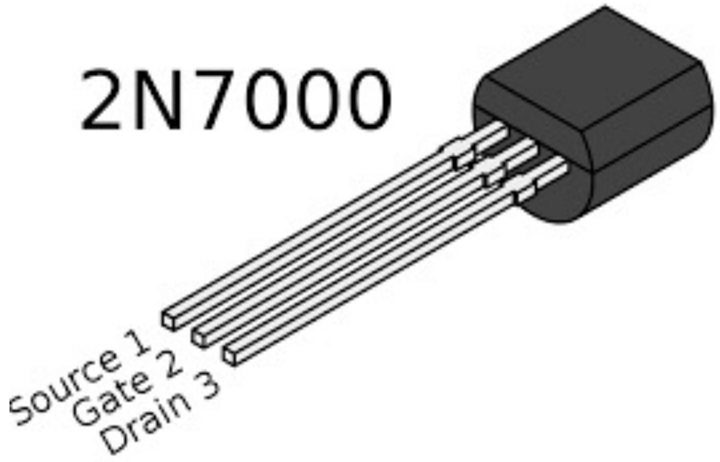
*Misra and Enge, GPS

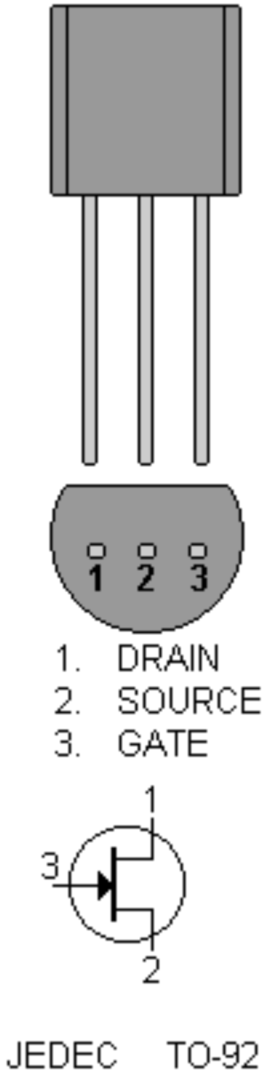
Morse

Symbol	Code	Symbol	Code	Symbol	Code
a	._	m	__	y	_.__
b	_...	n	_.	z	__..
c	_._.	o	___	0	_____
d	_..	p	.__.	1	.____
e	.	q	__._	2	..___
f	.._.	r	-_.	3	...__
g	__.	s	...	4	...._
h	....	t	_	5	.....
i	..	u	.._	6	_....
j	.___	v	..._	7	__...
k	_._	w	.__	8	___..
l	._..	x	_.._	9	____.

Pinouts







J309

Component data

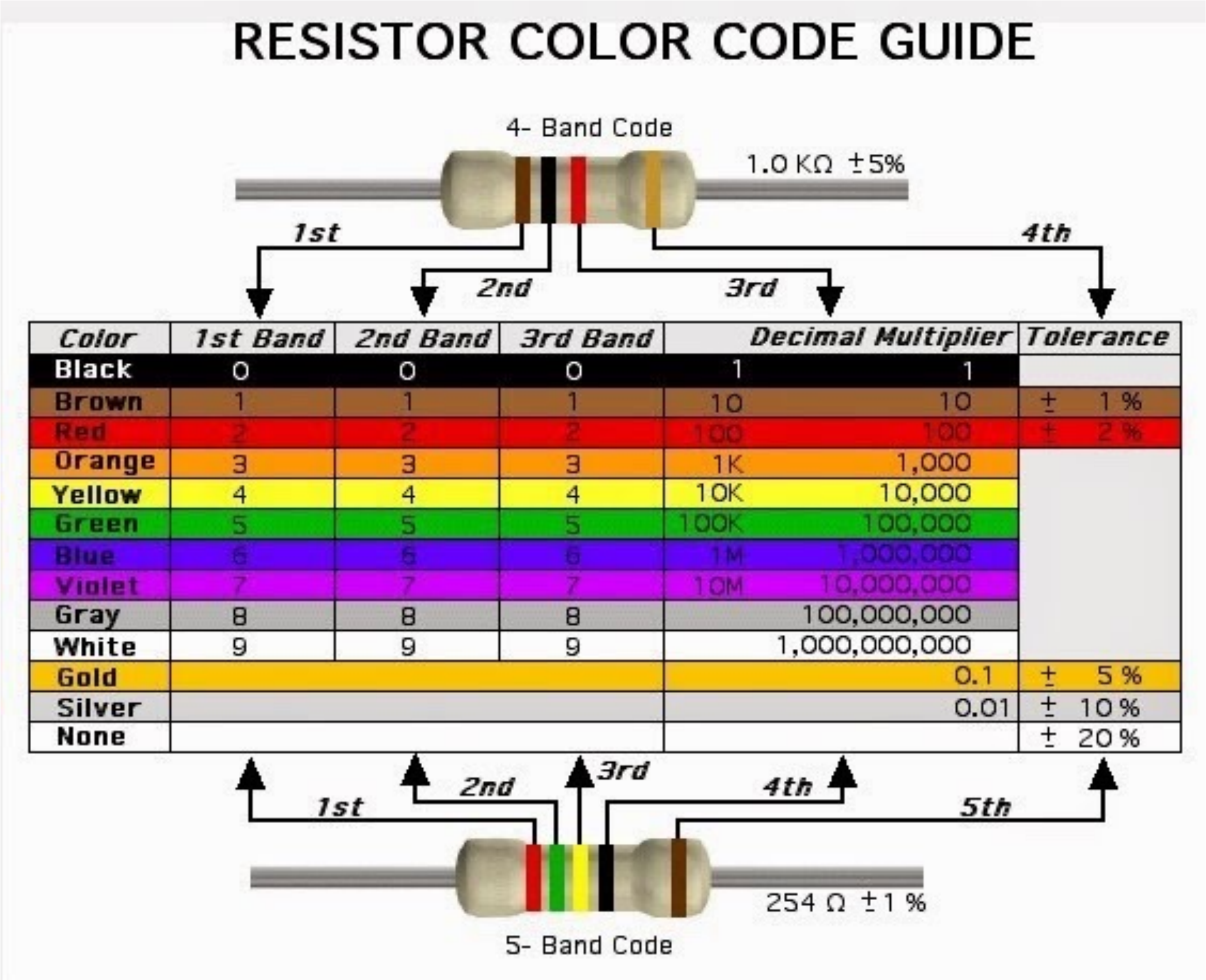
Core Size	26	3	15	1	2	6	10	12/17	0
T-12-( )	*	60	50	48	20	17	12	7.5	2.4
T-16-( )	145	61	55	44	22	19	13	8	3
T-20-( )	185	76	65	52	25	22	16	10	3.5
T-25-( )	245	100	85	70	34	27	19	12	4.5
T-30-( )	335	140	93	85	43	36	25	16	6
T-37-( )	285	120	90	80	40	30	25	15	4.9
T-44-( )	370	180	160	105	52	42	33	18.5	6.5
T-50-( )	330	175	135	100	49	40	31	18	6.4
T-68-( )	435	195	180	115	57	47	32	21	7.5
T-80-( )	460	180	170	115	55	45	32	22	8.5
T-94-( )	600	248	200	160	84	70	58	*	10.6
T-106-( )	930	450	345	325	135	116	*	*	19
T-130-( )	810	350	250	200	110	96	*	*	15
T-157-( )	1000	420	*	320	140	115	*	*	*
T-184-( )	1690	720	*	500	240	195	*	*	*
T-200-( )	920	425	*	250	120	100	*	*	*
T-200A-( )	1600	*	*	*	218	*	*	*	*

IRON POWDER TOROIDS - A_l Values **

* size not available in this material

** L= µH/100 turns

Color codes



Color	1st Band	2nd Band	3rd Band	Decimal Multiplier	Tolerance
Black	0	0	0	1 1	
Brown	1	1	1	10 10	± 1 %
Red	2	2	2	100 100	± 2 %
Orange	3	3	3	1K 1,000	
Yellow	4	4	4	10K 10,000	
Green	5	5	5	100K 100,000	
Blue	6	6	6	1M 1,000,000	
Violet	7	7	7	10M 10,000,000	
Gray	8	8	8	100,000,000	
White	9	9	9	1,000,000,000	
Gold				0.1	± 5 %
Silver				0.01	± 10 %
None					± 20 %

- Resistor markings in ohms
- Capacitor markings in picoFarads
- Inductor markings in microHenries

Blank