Electronics of Radio, Part 1

Notes on David Rutledge's book

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Basic concepts

- Potential difference (V, ϕ) : $\phi = \int_a^r E \cdot ds$, energy per charge, 1V = 1 J/s
- Kirhhoff loop: $\sum_{loop} V_i = 0$ (Conservation of energy)
- Kirhhoff node: $\sum_{node} I_i = 0$ (Conservation of charge)
- $V(t) = V_p \cos(\omega t)$, $\omega = 2\pi f$, $I(t) = I_p \cos(\omega t)$, $\omega = 2\pi f$
- Instantaneous power: $P(t) = V(t)I(t) = V_pI_p \cos^2(\omega t)$
- Average power: $P_a = \int_0^{1/f} V(t)I(t)dt = \int_0^{2\pi/\omega} V_p I_p \cos^2(\omega t)dt = \frac{V_p I_p}{2}$
- Band names:

Name	Frequency
VLF	3-30kHz
LW	20-300kHz
MW	300kHz-3MHz
HF	3MHz-30MHz
VHF	30-300MHz

Name	Frequency
UHF	300MHz-1GHz
uW	1-30GHz
milliW	30-300GHz
submilliF	>300GHz

Resistors, capacitors, inductors











- Resistors
 - Analytic model: IR = V
 - Energy dissipated: $E = \int_{t_i}^{t_f} IV \, dt = \int_{t_i}^{t_f} I^2 R dt$
- Capacitors
 - Analytic model: CV = q, $C\frac{dV}{dt} = i$
 - Capacitor Energy stored: $E = \int_{t}^{t} CV \frac{dV}{dt} dt = \frac{1}{2}CV^2$
- Inductors
 - Analytic model: $V = L \frac{di}{dt}$
 - Inductor Energy stored: $E = \int_{t_i}^{t_f} IV \, dt = \int_{t_i}^{t_f} LI \frac{dI}{dt} \, dt = \frac{1}{2} LI^2$
 - Open air: $L(H) = \mu_0 K n^2 \frac{A}{I}$, distances in meters, $\mu_0 = 4\pi \times 10^{-7}$, K = 1

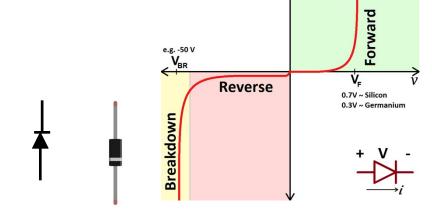




Diodes, transformers

• Diodes

- Devices that allow current to flow only in one direction
- Silicon diodes, for example have, essentially infinite resistance if V_{ac} <0, that is if the cathode is at a higher potential than the anode and very low resistance if V_{ac} > .7V.
- The cathode is usually labelled with a band
- Transformers
 - AC only: $\frac{N_2}{N_1} = \frac{V_2}{V_1}$

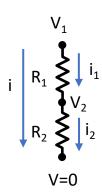


Credit: Make Electronics





Simple circuit analysis with Kirchhoff

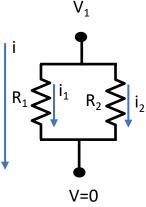


- R_{eq} is the equivalent resistance, replacing the top left circuit with a single resistance.
- By Kirchhoff's node rule, $i_1 = i_2 = i$, so

 By Kirchhoff's node rule, $i_1 = i_2 = i$, so

 $\frac{V_1 V_2}{R_1} = \frac{V_2}{R_2} = \frac{V_1}{R_{eq}}$ thus $\frac{R_1}{R_{eq}} V_1 = V_1 V_2$ and

 $\frac{R_2}{R_2} V_1 = V_2$. Adding, we get $\frac{R_1}{R_{eq}} V_1 + \frac{R_2}{R_{eq}} V_1 = \frac{d(V_1 V_2)}{dt} = \frac{d(V_1 V_2)}{dt}$ and $\frac{C_{eq}}{C_1} \frac{d(V_1)}{dt} = \frac{d(V_2)}{dt}$ V_1 . Dividing by V_1 and solving, we get R_1 + $R_2 = R_{eq}$



- Again let R_{eq} is the equivalent resistance, replacing the bottom left circuit with a single resistance.
- By Kirchhoff's node rule, $i_1 + i_2 = i$, so

$$\bullet \ \frac{V_1}{R_1} + \frac{V_1}{R_2} = \frac{V_1}{R_{eq}}.$$

• Solving, we get. $\frac{R_1R_2}{R_1+R_2}=R_{eq}$

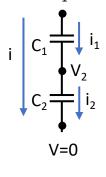
- C_{eq} is the equivalent capacitance, replacing the top right circuit with a single capacitor.

•
$$C_1 \frac{d(V_1 - V_2)}{dt} = C_2 \frac{d(V_2)}{dt} = C_{eq} \frac{dV_2}{dt}$$

•
$$\frac{C_{eq}}{C_1} \frac{d(V_1)}{dt} = \frac{d(V_1 - V_2)}{dt} \text{ and } \frac{C_{eq}}{C_2} \frac{d(V_1)}{dt} = \frac{d(V_2)}{dt}$$



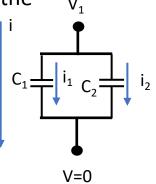
•
$$\frac{C_{eq}}{C_1} + \frac{C_{eq}}{C_2} = 1$$
 and solving, we get. $\frac{C_1C_2}{C_1 + C_2} = C_{eq}$



- C_{eq} is the equivalent capacitance, replacing the bottom right circuit with a single capacitor.
- By Kirchhoff's node rule, $i_1 + i_2 = i$

•
$$C_{eq} \frac{dV_1}{dt} = C_1 \frac{dV_1}{dt} + C_2 \frac{dV_1}{dt}$$
, so

•
$$C_{eq} = C_1 + C_2$$



Simple circuit analysis with Kirchhoff

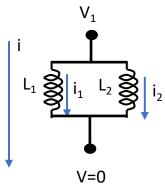


- Let L_{eq} be the equivalent inductance, replacing the top left circuit with a single inductor.
- By Kirchhoff's node rule, $i_1 = i_2 = i$, so

•
$$L_{eq} \frac{di}{dt} = V_1$$
, $L_1 \frac{di_1}{dt} = V_1 - V_2$, $L_1 \frac{di_2}{dt} = V_2$

•
$$V_1 = L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$
 and

•
$$L_{eq} = L_1 + L_2$$



- Let L_{eq} be the equivalent inductance, replacing the bottom left circuit with a $\frac{di}{dt} = \frac{V_1}{L_{eq}}$, $\frac{di_1}{dt} = \frac{V_1}{L_1}$, $\frac{di_2}{dt} = \frac{V_1}{L_2}$,
- single inductor.
- By Kirchhoff's node rule, $i_1 + i_2 = i$, so

•
$$\frac{V_1}{L_{eq}} = \frac{V_1}{L_1} + \frac{V_1}{L_2}$$
 and

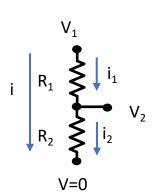
•
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

• The circuit on the right, is useful and is called a *voltage divider*.

•
$$i = i_1 = i_2$$
 so $\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2}$, $V_1 - V_2 = \frac{R_1}{R_2} V_2$

• Thus,
$$V_1 = (1 + \frac{R_1}{R_2})V_2$$
 and so

•
$$V_2 = \frac{R_2}{R_1 + R_2} V_1$$



RC/RL circuit analysis with Kirchhoff



RC behavior: charging

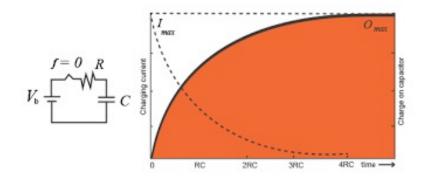
•
$$V_0 - V_2 = i_1 R = V_R, i_1 = \frac{V_R}{R}$$

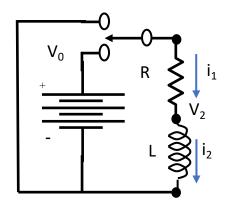
•
$$i_2 = C \frac{dV_2}{dt}, V_C = V_2$$

•
$$i_1 = i_2$$
, $V_C = V_0 - V_R$

•
$$i_1 = i_2$$
, $V_C = V_0 - V_R$
• $\frac{V_R}{R} = C \frac{dV_C}{dt}$, $RC \frac{dV_C}{dt} = V_0 - V_C$, or $RC \frac{dV_C}{dt} + V_C = V_0$







RL behavior: charging

•
$$V_0 - V_2 = i_1 R = V_R$$

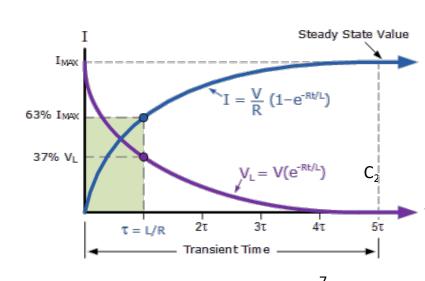
•
$$V_L = V_2 = L \frac{di_2}{dt}$$

•
$$V_0 - V_2 = i_1 R = V_R$$

• $V_L = V_2 = L \frac{di_2}{dt}$
• $i_1 = i_2$, $V_R = V_0 - V_L$, so $L \frac{d}{dt} \frac{V_0 - V_L}{R} = V_L$

$$\bullet \ \frac{L}{R} \frac{d V_L}{dt} + V_L = 0$$

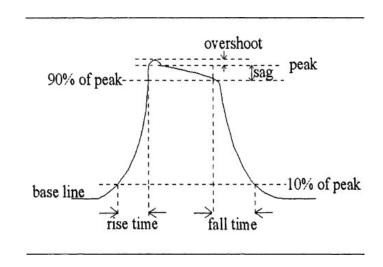
• Solution is $V_L = V_0 e^{-\frac{Rt}{L}}$

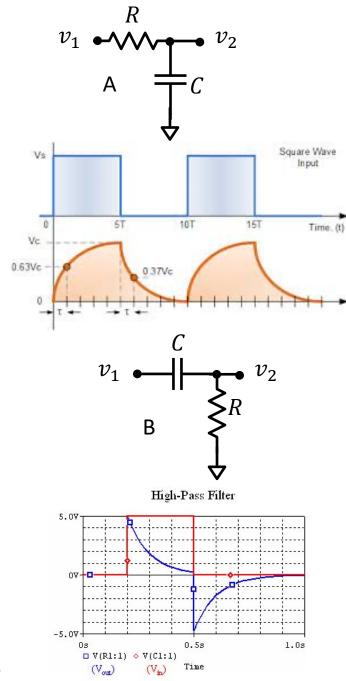


Voltage responses

- Response to square wave with width T
 - A: $\tau = RC$, $\tau = T$
 - B: $\tau = RC$, $\tau = .1T$

Overshoot below





Phasors

- V(t) = RI(t)
- $V(t) = L\dot{I}(t)$
- $I(t) = C\dot{V}(t)$
- Suppose $V(t) = Acos(\omega t + \theta)$ and $I(t) = Bcos(\omega t + \phi)$. If $\phi > \theta$, we say the current leads the voltage.
- $V(t) = Re(Ae^{j(\omega t + \theta)})$, and $I(t) = Re(Be^{j(\omega t + \phi)})$
- Now define $\hat{V} = V = Ae^{j\theta}$ and $\hat{I} = Be^{j\phi}$, so |V| = A, |I| = B, $\angle V = \theta$, and $\angle I = \phi$. \hat{V} and \hat{I} are called phasors and do not include time. Note that $V(t) = Re(\hat{V}e^{j\omega t})$ and $I(t) = Re(\hat{I}e^{j\omega t})$.
- Note that $I = CVj\omega$, for a capacitor and $V = LIj\omega$, for an inductor
- $\hat{V} = Z\hat{I}, Z = R + jX$
- $\hat{I} = Y\hat{V}, Y = G + jB$

Circuit analysis and impedance

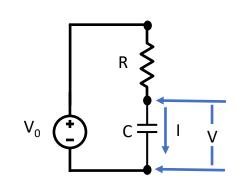
- Impedance unifies the "simple" ohms law with capacitance and inductance.
- Z=R, for resistors, $Z=j\omega L$, for inductors and $Z=\frac{1}{j\omega C}$, for capacitors.
- In general, Z = R + jX and all the ohm like laws hold for resistors, capacitors and inductors .
 - $Z_{eq}=Z_1+Z_2$ for two components with impedance Z_1 , Z_2 connected in series
 - $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$ for two components with impedance Z_1 , Z_2 connected in parallel
- For example, for a resistor and capacitor in series has impedance $Z=R+\frac{1}{j\omega C}$

Phasors, impedance and power

- For the circuit on the right, $Z=R+\frac{1}{j\omega C}$ is the impedance for the resistor and capacitor in series.
- The phasor $I = \frac{V_0}{Z}$ and the phasor $V = \frac{I}{j\omega C} = \frac{V_0}{1+j\omega RC}$
- Further, $|I|=\frac{V_0}{|Z|}$, $\angle I=\angle\frac{V_0}{|Z|}$ and $|V|=\frac{|I|}{|j\omega C|}=|\frac{V_0}{1+j\omega RC}|$



- The average power is $P_a = Re(P) = Re(\frac{V\bar{I}}{2})$. We define the reactive power as $P_r = Im(P)$.
- $P_r = \omega(E_L E_C)$, where E_L and E_C are respectively, the energy stored in the inductor and capacitor respectively.



Q and phasors

- Consider the series resonance on the right. $Z_{LCR} = R + j \left(\omega L \frac{1}{\omega C}\right)$
- The phasor, $I=\frac{V_0}{Z_{LCR}}$, and the phasor $V_R=\frac{V_0}{Z_{LCR}}Z_R$, where $Z_R=R$.
- So $V_R = \frac{RC\omega V_0}{RC\omega + i(LC\omega^2 1)}$.
- $|V_R|$ is maximum when $\omega^2 LC = 1$. Put $\omega_0 = \frac{1}{\sqrt{LC}}$. When $\omega = \omega_0$, $|V_R| = V_R = V_0$.



- Let the frequencies where $R = \pm X$ be denoted ω_u and ω_l , where $\omega_u > \omega_l$.
- We define $Q = \frac{X}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$.
- Solving for ω_u and ω_l , we get $\frac{L\omega_u}{\omega_0} \frac{\omega_0}{c\omega_u} = R$ and $\frac{L\omega_l}{\omega_0} \frac{\omega_0}{c\omega_l} = -R$, or, in terms of Q, $\frac{\omega_u}{\omega_0} \frac{\omega_0}{\omega_u} = \frac{1}{Q}$ and $\frac{\omega_l}{\omega_0} \frac{\omega_0}{\omega_l} = -\frac{1}{Q}$. In fact, $\omega_0 = \sqrt{\omega_u \omega_l}$, and so $\frac{\omega_u}{\omega_0} \frac{\omega_l}{\omega_0} = \frac{1}{Q}$. Thus $Q = \frac{\omega_0}{\omega_u \omega_l} = \frac{\omega_0}{\Delta \omega}$
- From the definition of P_a , earlier, $Q = \omega_0 \frac{E}{P_a}$, where E is the total energy stored in L and C, which is in turn the peak E_L and peak E_C at resonance.



Resonance and Q

Series Resonance

- At ω_u and ω_l , $X=\pm R$ [ω_u is upper 3dB cutoff and ω_l is lower 3dB cutoff]
- $\omega_u L \frac{1}{\omega_u C} = R$, $\omega_l L \frac{1}{\omega_l C} = -R$
- Define $Q = \frac{X}{R}$
- $\frac{\omega_u}{\omega_0} \frac{\omega_0}{\omega_u} = \frac{R}{\omega_0 L} = \frac{1}{Q}$ and $\frac{\omega_l}{\omega_0} \frac{\omega_0}{\omega_l} = -\frac{R}{\omega_0 L} = -\frac{1}{Q}$

•
$$\frac{\omega_u}{\omega_0} - \frac{\omega_0}{\omega_u} = \frac{\omega_0}{\omega_l} - \frac{\omega_l}{\omega_0}$$
, so $\omega_0^2 = \omega_u \omega_l$ and $\frac{\omega_u - \omega_l}{\omega_0} = \frac{1}{Q}$

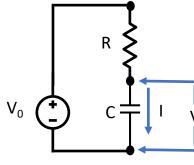
Parallel Resonance

•
$$\frac{\omega_u}{\omega_0} - \frac{\omega_0}{\omega_u} = \frac{G}{\omega_0 C} = \frac{1}{Q_p}$$
 and $\frac{\omega_l}{\omega_0} - \frac{\omega_0}{\omega_l} = -\frac{G}{\omega_0 C} = -\frac{1}{Q_p}$

Phasors, impedance and power

- For the circuit on the right, $Z=R+\frac{1}{j\omega C}$ is the impedance for the resistor and capacitor in series.
 The phasor $I=\frac{V_0}{Z}$ and the phasor $V=\frac{I}{j\omega C}=\frac{V_0}{1+j\omega RC}$ Further, $|I|=\frac{V_0}{|Z|}$, $\angle I=\angle\frac{V_0}{|Z|}$ and $|V|=\frac{|I|}{|j\omega C|}=|\frac{V_0}{1+j\omega RC}|$

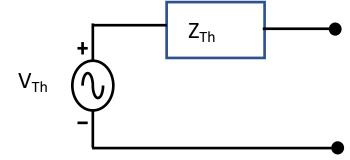
- Complex power: $P = \frac{V\bar{I}}{2} = Z \frac{|I|^2}{2} = P_a + jP_r = R \frac{|I|^2}{2} + jX \frac{|I|^2}{2}$
 - P_a is power delivered to resistor, P_r is power stored in inductor
 - For phasors V, I, define the complex power as $P_{av} = \frac{V\bar{I}}{2} = Z\frac{I\bar{I}}{2} = R\frac{|I|^2}{2} + jX\frac{|I|^2}{2}$; the first term is the real power, the second is called the reactive power.
- $P_r = \omega(E_L E_C)$, where E_L and E_C are respectively, the energy stored in the inductor and capacitor respectively.
- $P_r = \frac{\omega L|I|^2}{2} \frac{\omega C|V_c|^2}{2} = \omega (E_L E_C)$
- $Q = \omega \frac{L|I|^2}{R|I|^2} = \omega \frac{L}{R} = \omega \frac{E_L}{P_R}$



Thevenin and Norton

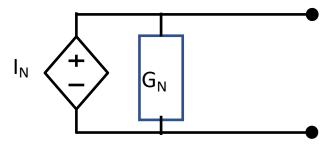
• Thevenin: Any combination of *linear* sources and passive elements terminating in two terminals is

equivalent to a pure voltage source in series with an impedance



• Norton: Any combination of *linear* sources and passive elements terminating in two terminals is

equivalent to a pure current source in parallel with a conductance

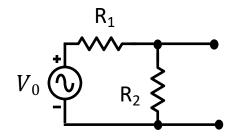


• Above, $G_N = \frac{1}{Z_{Th}}$

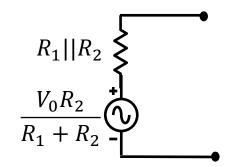
• Similar theorems for *linear* two terminal input and output devices (with transfer function)

Thevenin and Norton

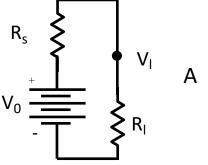
- We can use lookback resistance to calculate the Thevenin equivalent resistance and ideal source.
- To find the lookback resistance, short the source and apply the usual laws.
 - Here $R_s = R_1 || R_2$
- To find the new ideal source, notice R_1 and R_2 form a voltage divider.
 - The new source voltage is $\frac{V_0R_2}{R_1+R_2}$



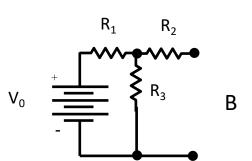
is equivalent to

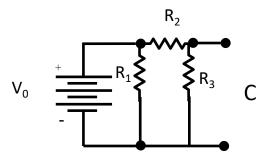


Exercise 1: Resistors



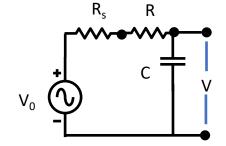
- 1. Consider (A). Find the formula for power in the load. Find the R_l that maximizes the power to the load.
 - $V_l = \frac{R_l}{R_S + R_l} V_0$, $I_l = \frac{V_0}{R_S + R_l}$.
 - $P_l = V_l I_l = \frac{R_l}{(R_S + R_l)^2} V_0^2$, which is maximum when $R_l = R_S$
- 2. Find the Thevenin and Norton parameters for (B).
 - $V_{Th} = \frac{R_3}{R_1 + R_3} V_0$
 - $R_{Th} = R_2 + R_1 || R_3$
- 3. Find the Thevenin and Norton parameters for (C).
 - $V_{Th} = \frac{R_3}{R_2 + R_3} V_0$
 - $\bullet \quad R_{Th} = R_2 || R_3$



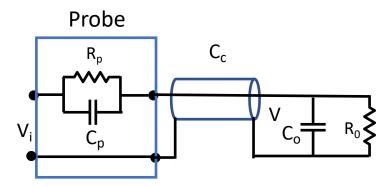


Exercise 3: Capacitors

1. In the circuit on the right, V_0 is a 2 volt pp ideal square wave source of frequency 20Hz, $R_S=50\Omega$, $R=300k\Omega$ and C=10~nF. Period is 50~millisec



- 2. What is the voltage, V, at the output? The scope has an input resistance of $1M\Omega$.
 - About a volt at peak
- 3. Let t_2 , the time to discharge to 0V. Calculate τ and t_2 .
 - $\tau = 3 \times 10^5 \times 10^{-8} \ sec = 3 \ millisec$
 - $t_{12} \approx 1.5 ms$
- 4. Capacitance on the scope prevents the delay from being 0. Measure the new t_2 with these changes.
- 5. Given C_0 and C_p and $R_{p.}$
 - $C_0 = 100pf/m$, $C_o = 50pF$, $C_p = 10pF$
- 6. Now calculate the new t_{12} .
 - $\tau = 6\mu$ -sec

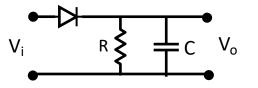


Signals and modulation

- Gain: $G = \frac{P_o}{P_i}$, Loss: $L = \frac{P_o}{P_{max}}$, Rejection: $R = \frac{P_{max}}{P_{pb}}$. Gain (G) expressed in decibels: $G = 10 \log_{10}(\frac{P_{out}}{P_{in}})$
- $P_S = \int \sigma E \cdot E + \epsilon \frac{E \cdot E}{2} + \frac{H \cdot H}{2\mu} dV + \int E \times H dA$
- Mixer: $V(t) = \cos(\omega_1 t) \cos(\omega_2 t) = \frac{1}{2} [\cos(\omega_+ t) + \cos(\omega_- t)], \omega_+ = \omega_1 + \omega_2, \omega_- = \omega_1 \omega_2$
- Modulation

Name	Equation
AM	$V(t) = (1 + am(t))V_c \cos(\omega_c t)$
FM	$V(t) = V_c \cos([\omega_c + am(t)]t)$
Angle	$V(t) = V_c \cos(\omega_c t + \phi(t)), \phi(t) = am(t)$. [Like FM]
FSK	$V(t) = V_c \cos(\omega_1 t)$, if 1 $V(t) = V_c \cos(\omega_0 t)$, if 0
PSK	$V(t) = +V_p \cos(\omega t), \text{ if } 1$ $V(t) = -V_p \cos(\omega t), \text{ if } 0$

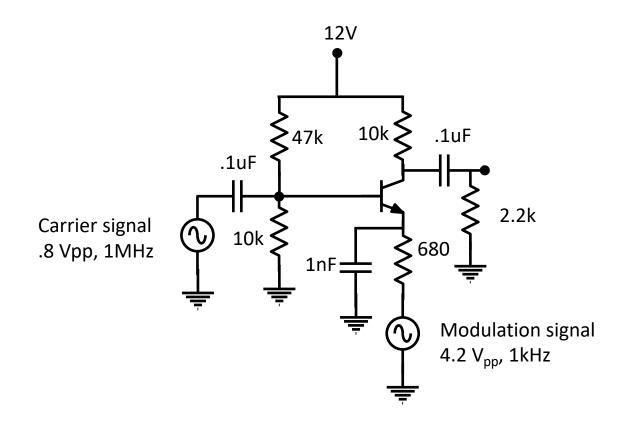
Exercise 4: Diode detectors



- For AM, $V(t) = V_c \cos(\omega_c t) + a(t) \cos(\omega_c t)$, Define the modulation depth $m = \frac{a_p}{V_c}$
- In circuit on the right, $R = 10k\Omega$, C = 10 nF
- Set function generator for $f_c = 1MHz$, $V_{c,pp} = 5V$, $f_m = 1kHz$, m = .7
 - 1. Calculate τ for the RC circuit. $\tau = 10^4 \times 10^{-8} \, \text{sec} = .1 \, \text{ms}$.
 - T_m is period of modulating signal. $T_m=10^{-3}sec=1ms$. So $au\ll T_m$
 - T_c is period of modulating signal. $T_c = 10^{-6}sec = 1\mu s$. $\tau \gg T_c$
 - As you change f_m does the frequency of V_o track it? (It better)
 - 2. Compare the max voltage of the AM signal to the max of V_0 .
 - $V_0, p \approx .8V, V_{i,p} \approx 1.4V$
 - 3. What happens when we make m=1.0

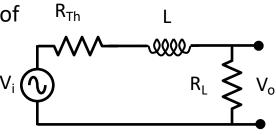
AM Modulator for previous exercise

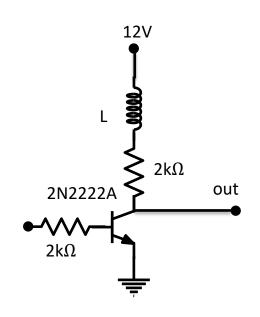
• I didn't have a signal generator that produced an AM signal, so I used the modulator on the right with the indicated inputs to produce the AM needed for the detector in the previous exercise.



Exercise 5: Inductors

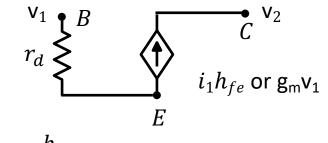
- Set function generator for square wave with 5V V_{pp} , a Thevenin equivalent source resistance of $R_{Th}=50\Omega$ and frequency 1kHz. Connect a load, $R_L=100\Omega$ load, L=1mH
 - Observe square wave with rounded corners, measure the time, t_2 to decay to 0
 - About $20\mu sec$
 - In the top circuit, calculate inductor current and the expected delay, t_2
 - $Z_{eq} = 150 + jL\omega$, $\omega = 2\pi \times 10^3$, $V_i = Re(V_{i,p}e^{j\omega t})$
 - As phasors, $iZ_{eq} = V_i$, $|i|\sqrt{150^2 + (\omega L)^2} = V_{i,p}$, $|i| = \frac{V_{i,p}}{\sqrt{150^2 + (2\pi)^2}}$, $\theta = \angle i = \sqrt{150^2 + (2\pi)^2}$ $\arctan(-\frac{2\pi}{150}), \theta \approx -2.4 \, rad = -15^{\circ}$
 - $V_o = Re\left(\frac{100V_{i,p}}{\sqrt{150^2 + (2\pi)^2}}e^{j(\omega t + \theta)}\right), |V_o| = 1.6V,$ $\tau_{RL} = \frac{10^{-3}}{100}sec \approx 10 \ \mu sec$
 - In the second circuit, use 2 scope channels: one at input, one at output.
 - $1\mu \sec rise \ time$. Ringing at 10MHz. $\frac{1}{\sqrt{LC}} = 62.8 \times 10^6$. $C = \frac{10^3}{(62.8 \times 10^6)^2} \approx .25 pF$
 - Note: I made the pull-up 100K.

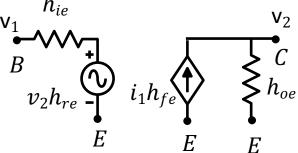


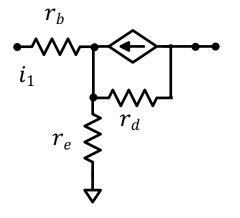


Diodes and bipolar small signal models

- Diode model: $i_d = i_S(\exp\left(\frac{eV}{kT}\right) 1), \frac{e}{kT} = 40V^{-1}$
- $\frac{di_d}{dV} = i_d \frac{e}{kT}$, $r_d = \frac{e}{kTi_d}$
- When i_d is a few nano-Amps, $r_d \approx 5\Omega$
- When i_d is a few μA , $r_d \approx 10^4 \Omega$





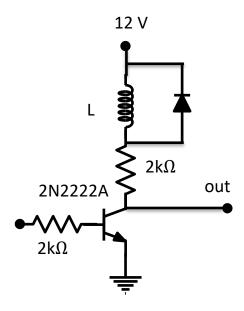


Exercise 6: Diodes and snubbers

- Add indicated snubber diode.
- 1. Swing up is nearly immediate with snubber

2. Ringing at 10MHz.
$$\frac{1}{\sqrt{LC}} = 62.8 \times 10^6$$
. $C = \frac{10^3}{(62.8 \times 10^6)^2} \approx .25 pF$

- 3. What is its effect on ringing?
 - Ringing is uniform at 5 MHz
- 4. Diode should be on when transistor is off.
- Note: I made the pull-up 100K.



Exercise 7: Parallel to Series conversion

- For series: $Z_S = R_S + j\omega L$, $Q_S = \frac{X_S}{R_S}$
- For parallel: $\frac{1}{Z_p}=\frac{1}{R_p}+\frac{1}{j\omega L}$, so $Z_p=\frac{j\omega LR_p}{R_p+j\omega L}$ and $Q_p=\frac{R_p}{X_p}$
- If $Q_p = Q_S$, $X_p X_s = R_p R_S$

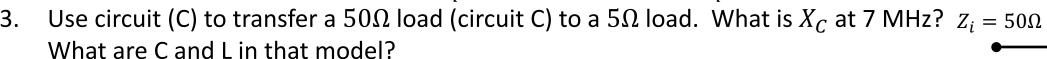




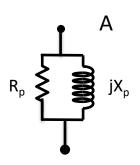
•
$$R_S = \frac{{X_p}^2 R_p}{{R_p}^2 + {X_p}^2}$$
, $X_S = \frac{{R_p}^2 X_p}{{R_p}^2 + {X_p}^2}$, $R_S = X_p \frac{{X_p} R_p}{{R_p}^2 + {X_p}^2}$, $X_S = R_p \frac{{X_p} R_p}{{R_p}^2 + {X_p}^2}$, set $\rho = \frac{{X_p} R_p}{{R_p}^2 + {X_p}^2}$

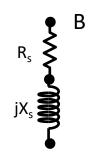
for later reference

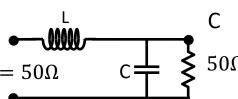
- This shows the Q's must be equal as stated above.
- 2. Find a formula for $X_{\mathcal{S}}$, for large $Q=Q_p=Q_{\mathcal{S}}$ and small $Q=Q_p=Q_{\mathcal{S}}$



• Use the parallel to series conversion to make a series equivalent circuit consisting of C and the 50Ω with $R_S=5\Omega$

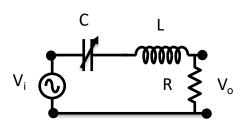






Exercise 8: Series resonance

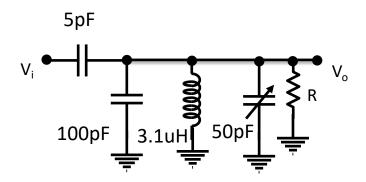
- For the circuit on the right, C= 8-50pf, $L=15\mu H$ forming a bandpass filter. $R=100\Omega$
- If C=34pf, the resonant frequency is $\omega=\frac{1}{\sqrt{35\times10^{-12}\times15\times10^{-6}}}=\frac{10^9}{\sqrt{525}}\approx 44.2$, so the resonant frequency is $\frac{44.2}{2\pi}\approx 7.07MHz$



- 1. Tune the resonant frequency to 7MHz and find f_u , f_l and Δf and thus Q.
 - $f_u = 7.67MHz$, $f_l = 6.47MHz$, $Q = \frac{f}{\Delta f} = \frac{7}{1.2} = 5.8$
- 2. Compute what these values should be
 - $Z_{eq} = R + j(\omega L \frac{1}{\omega C})$
 - As phasors, $i = |i|e^{j\theta}$, $|i| = \frac{V_{i,0}}{\sqrt{R^2 + (\omega L \frac{1}{\omega C})^2}}$, $\theta = -arctan(\frac{\omega L \frac{1}{\omega C}}{R})$
 - $V_0=iR$, Power through R at ω is $P(\omega)=|i^2|R=\frac{{V_{i,0}}^2R}{R^2+(\omega L-\frac{1}{\omega C})^2}$. At resonance, $P(\omega_r)=\frac{{V_{i,0}}^2}{R}$. To find half power, $\frac{1}{2}=(\frac{{V_{i,0}}^2R}{R^2+(\omega L-\frac{1}{\omega C})^2})/(\frac{{V_{i,0}}^2}{R})$, or $R=\omega L-\frac{1}{\omega C}$.
 - Solving gives $f_u = 7.67MHz$, $f_l = 6.53MHz$, Q = 6.1
 - General formulas: $\omega_u = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$, $\omega_l = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$

Exercise 9: Parallel resonance

- $L=A_l N^2$, $A_l=4\frac{nH}{turn^2}$ for T37-2 core so for 28 turns, $L=3.1\mu H$
- 1. Find the resonant frequency, the frequencies corresponding to a 3db falloff, the bandwidth and the Q of this circuit. This circuit is in the transmit oscillator.
 - At tuned resonance (7MHz), effective capacitance is about 167 pF
 - $Q_p = \omega_0 RC$
 - For $R = 1500\Omega$, network: $Q = 1500 \times 44 \times 10^6 \times 1.67 \times 10^{-10} = 11$
 - $BW = \frac{f_r}{Q} = \frac{7MHZ}{11} = .636MHz$. $f_u = f_r + \frac{BW}{2} = 7.318MHz$, $f_l = f_r \frac{BW}{2} = 6.682MHz$. This is 3dB cutoff.
- General formulas: $BW = \frac{f_r}{Q}$, $f_u = f_r + \frac{BW}{2}$, $f_l = f_r \frac{BW}{2}$



Norcal 40A



NorCal power levels

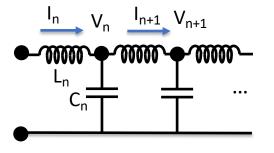


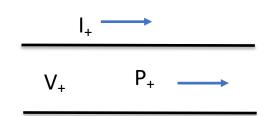
Transmission Lines

•
$$V_{n+1} - V_n = -L_l \frac{\partial I_{n+1}}{\partial t}$$
, $L = \frac{L_l}{l}$, so $\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$

•
$$I_{n+1} - I_n = -C_l \frac{\partial V_n}{\partial t}$$
, $C = \frac{C_l}{l}$, so $\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}$

- Thus, $\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2}$ and $\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2}$, whose solution is is $V(z \pm vt)$
- $v = \frac{1}{\sqrt{LC}}$ for the forward wave and $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$ and $\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$ implies
- V'=vLI' and $\frac{V}{I}=\sqrt{\frac{L}{C'}}$, so $Z_0=\sqrt{\frac{L}{C'}}$, where Z_0 is the forward impedance
- Another solution is V(z+vt), with the same velocity for the reverse wave
- $Z_0 = \frac{V_+}{I_+}$, $-Z_0 = \frac{V_-}{I_-}$, $V = V_+ + V_-$, $-Z_0$ is the backwards looking impedance
- $P_+(t) = \frac{V_+^2}{Z_0}$, $P_-(t) = -\frac{V_-^2}{Z_0}$ (the negative sign implies energy flows to the left)







Transmission Lines - continued

- $V(z-vt)=Acos(\omega t-\beta z)$, $v=\frac{\omega}{\beta}$. The phasor is $\hat{V}=Ae^{-j\beta z}$ although we drop the cap below.
- Now the forward and backward voltage phasors are $V_+ = Ae^{-j\beta z}$, $V_- = Ae^{j\beta z}$
- The complex power is $P_{av} = \frac{V\bar{I}}{2}$, $P_{+} = \frac{V_{+}\bar{I}_{+}}{2} = \frac{|V_{+}|^{2}}{2Z_{0}}$, $P_{-} = \frac{V_{-}\bar{I}_{-}}{2} = -\frac{|V_{-}|^{2}}{2Z_{0}}$, with $\frac{V}{I} = Z_{0}$
- Suppose over a transmission line, Z is the distributed impedance/m, Y is the distributed admittance/m and suppose the forward wave is $Ae^{j(\omega t-jk)}$, with phasor is $V=Ae^{-jkz}$. Let $Z=\frac{V}{I}$ then $\frac{dV}{dz}=-ZI$, $\frac{dI}{dt}=-YV$.
- Put $jk = \alpha + \beta j$ (to account for attenuation), then $jk = \sqrt{ZY}$ and the forward phasor becomes $e^{(-\alpha z j\beta z)}$. $\alpha_{dB/m}$ is a transmission loss. $\alpha_{dB/m} = 8.686\alpha_{nepers/m}$.
- By differentiating , we get jkV=ZI, -jkI=YV. Solutions are $jk=\sqrt{ZY}$, $Z_0=\frac{V}{I}=\sqrt{\frac{Z}{Y}}$, all complex
- So, if $Z=R+j\omega L$, $Y=j\omega C+G$ for the transmission line, then $jk=\sqrt{(j\omega L+R)(j\omega C+G)}$ and $Z_o=\sqrt{\frac{(j\omega L+R)}{(j\omega C+G)}}$ (positive real root)

Transmission Lines - dispersion

- α and v can vary with frequency; this is dispersion.
- Heaviside: Adjust parameters so $\frac{R}{L} = \frac{G}{C}$, then α doesn't depend on v and we get:

•
$$jk = j\omega\sqrt{LC}(1 + \frac{R}{j\omega L})$$
 and $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$, $\alpha = \sqrt{RG}$

- We also get $Z_0 = \sqrt{\frac{L}{C}}$ as with a lossless line.
- If $\omega L \gg R$

•
$$G = 0$$
 and $Z_0 = \sqrt{\frac{(j\omega L + R)}{j\omega C}} \approx \sqrt{\frac{L}{C}}$

• If $R \gg \omega L$

•
$$jk = \sqrt{\frac{(j\omega L + R)}{j\omega C}} \approx \sqrt{j\omega RC}$$
, and $\alpha = \sqrt{\frac{\omega RC}{2}}$, $\alpha = \sqrt{\frac{2\omega}{RC}}$

- For first transatlantic cable, $L = 460 \frac{nH}{m}$, $C = 75 \frac{pF}{m}$, f = 12Hz, $R = 7 \frac{m\Omega}{m}$, $l = 3600 \ km$, $\alpha = \sqrt{\frac{\omega RC}{2}} = 4.4 \times 10^{-3} \frac{nepers}{m}$, $\alpha l = 140 dB$
 - $\alpha l \approx 140 dB$ and highly dispersive

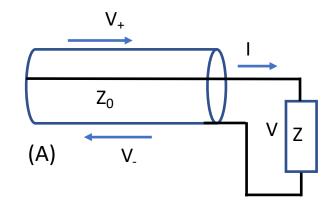
Transmission Lines-reflections

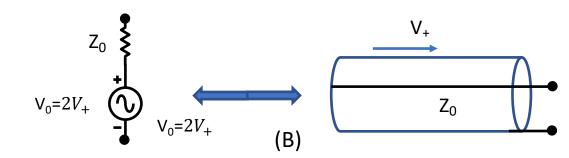
- Now we look at the end of the transmission line and define $\rho=\frac{V_-}{V_+}$, and $\rho_i=\frac{i_-}{i_+}=-\rho$
- $V = V_+ + V_- = (1 + \rho)V_+$
- $\tau = \frac{V}{V_{+}} = 1 + \rho = \frac{2Z}{Z + Z_{0}}, V = 2V_{+}$
- Consider the circuit in the upper right (A): $V = V_+ + V_-$, $I = I_+ + I_-$, $Z = \frac{V}{I}$

•
$$Z = \frac{V}{I} = \frac{V_+ + V_-}{I_+ + I_-}$$

•
$$\frac{Z}{Z_0} = \frac{1+\rho}{1-\rho}, \ \rho = \frac{Z-Z_0}{Z+Z_0}, \ \rho_{open-circuit} = 1.$$

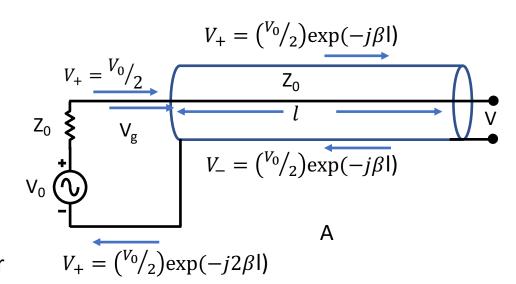
- For (B):
- Lookback resistance is $R_S=Z_0$, short circuit for (B) is $i_S=\frac{v_0}{Z_0}$
- Thevenin equivalent for open circuit is (B)
- $P_+ = \frac{{V_+}^2}{2Z_0} = \frac{{V_0}^2}{8R_S}$. This is the total available power.

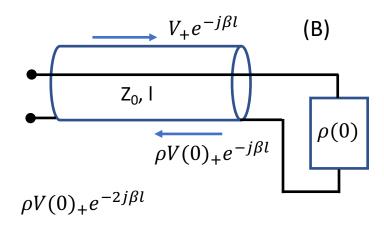




Transmission Lines — resonance and Q

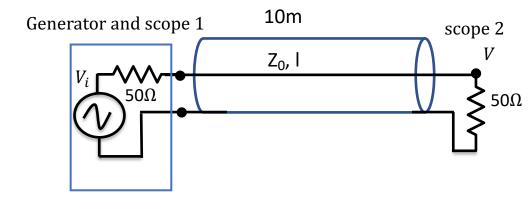
- For (A) on right, $V_+ = \frac{V_0}{2}$, $V = V_+ + V_- = V_0 e^{-j\beta l}$, $V_- = \frac{V_0}{2} e^{-2j\beta l}$
 - $V_g = V_0 e^{-j\beta l} \cos(\beta l) = \frac{V_0}{2} (1 + e^{-2j\beta l}), V_g(\frac{\lambda}{4}) = 0$
 - $I_g = \frac{V_+}{Z_0} \frac{V_-}{Z_0} = jI_S e^{-j\beta l} \sin(\beta l).$ $X = \frac{V_g}{jI_g} = \frac{Z_0}{\tan(\beta l)}$
- $Q = \omega \frac{E}{P_a}, E = \frac{lP_+}{v}, P_a = P_+ P_+ e^{-2\alpha l} \rho(0) \approx 2\alpha l P_+, Q = \frac{\beta}{2\alpha}$
- In (B) to the right, the coefficient of reflection is $\rho(0)$ and the generator absorbs the reverse wave. $V = V_+ + V_- = V_0 e^{-j\beta l}$.
 - $V_f = \rho(0)V_+e^{-j\beta l}, V_r = \rho(0)V_+e^{-2j\beta l}$
 - $\rho(l) = \frac{V_-}{V_-} = \rho(0)e^{-2j\beta l}$ is the reflection coefficient at generator.
 - $\rho\left(\frac{\lambda}{2}\right) = \rho(0), \, \rho\left(\frac{\lambda}{4}\right) = -\rho(0)$
 - $\frac{Z(\lambda/4)}{Z_0} = \frac{Z_0}{Z(0)}, Z = \frac{Z}{Z_0}, y = \frac{1}{Z}, Z(\frac{\lambda}{4}) = -\frac{1}{Z(0)}$
 - Normalized: $Z(^{\lambda}/_4) = \frac{1}{z(0)}$
 - $Z_0 = \sqrt{Z(^{\lambda}/_4)Z(0)}, Z_0 = \sqrt{R_S R_L}$



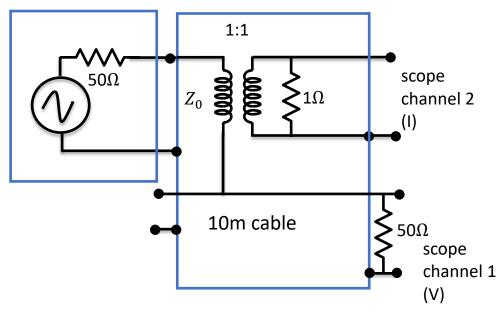


Exercise 10: Coax

- We'll measure the velocity of waves in RG58U by connecting one channel of the scope to the input and one to the output.
- 1. Measure the velocity, v, in 10m coax at 7MHz. Try different frequencies. Use 50ns, $5V_{pp}$ using square waves at 20kHz. Ans: about $\frac{2}{3}c$
- 2. Do the same with an antenna.
- 3. Calculate Z_0 with 50Ω termination for the circuit on the right.
- 4. Remove the 50Ω and measure the V and use it and Z_0 to calculate L, and C for the coax
 - Measured speed is $v=2\times 10^8$ m/s. $Z_0=50\Omega$. For high impedance, $Z_0=\sqrt{\frac{L}{c}}$ and $v=\frac{1}{\sqrt{LC}}$. So, $Z_0{}^2C=L$ and $v^2=\frac{1}{LC}$, so $Z_0{}^2C^2v^2$ =1. $C=\frac{1}{Z_0v}=10^{-10}F$. $2500\times 10^{-10}F=L=250$ nH, which is what we use in the next problem.

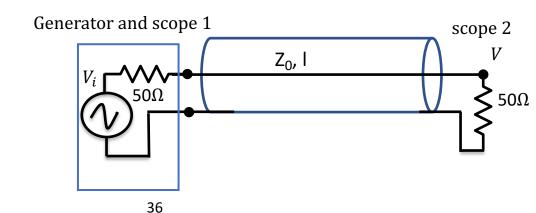


Function generator



Exercise 11: Waves

- Suppose we want to send voice over 100km of coax, $Z_L = 50\Omega$, l = 100km.
- 1. Measure the SWR which is the ratio of the maximum to minimum output
 - $V = V_+ + V_-$, $\rho = \frac{Z Z_0}{Z + Z_0}$, $Z = 50\Omega$, we get Z_0 from the previous exercise.
 - $|V_{max}| = |V_f| + |V_r| = (1+\rho)|V_f|, |V_{min}| = (1-\rho)|V_f| = |V_f|. SWR = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho},$
- 2. If $L = 250 \frac{nH}{m}$, C = 100 pf/m and the distributed resistance at voice is $50 m\Omega/m$, calculate total dB loss at 500, 1000 and 2000Hz using the high frequency approximation.
 - $Z(f) = j\omega L + R = 50 \times 10^{-3} + j \cdot 2\pi f \cdot 250 \times 10^{-9}$
 - $Y(f) = j\omega C + \frac{1}{R} = \frac{1}{50 \times 10^{-3}} + j \ 2\pi f \cdot 10^{-10}$
 - $Z_0(f) = \sqrt{\frac{Z(f)}{Y(f)}}$
 - $Z_0(500) = 400\Omega$, $Z_0(1000) = 282\Omega$, $Z_0(500) = 200\Omega$,
 - High resistance approximation: $\alpha(f) = \sqrt{\frac{\omega RC}{2}}$,
 - $\alpha(500) = 8.8 \times 10^{-5}, \alpha(1000) = 12.6 \times 10^{-5}$
 - $\alpha(2000) = 17.6 \times 10^{-5} \times 10^{5}$
 - For 100km, loss is $\alpha \times 10^5$



Exercise 11: Waves

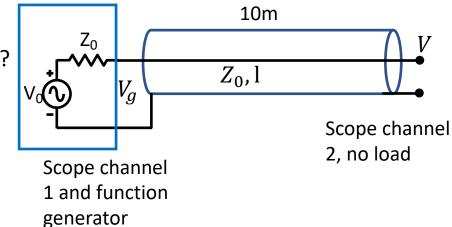
- 3. Add a 100mH inductor every 1km. Now what's the loss?
 - $Z(f) = j\omega L + R = 50 \times 10^{-3} + j \cdot 2\pi f \cdot 10^{-4}, Z_0(f) = \sqrt{\frac{Z(f)}{Y(f)}}$
 - $Z_0(f) = \sqrt{\frac{Z(f)}{Y(f)}}$
 - $Z_0(500) = 318\Omega$, $Z_0(1000) = 317\Omega$, $Z_0(2000) = 316\Omega$

 - High reactance approximation: $\alpha(f) = \frac{R}{2Z_0}$, $Z_0 = \sqrt{\frac{L}{c}} = 1000\Omega$ $\alpha(f) = \frac{R}{2Z_0(f)}$, $\alpha(500) = \alpha(1000) = \alpha(2000) = \frac{5\times10^{-2}}{2000} = 5.5\times10^{-5}$ nepers/m
 - For 100km, loss is $\alpha \times 10^5 = 5.5$ or $5.5 \times 8.868 \approx 49dB$

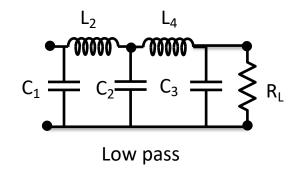
Exercise 12: Resonance

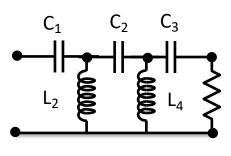
- RG58U has a capacitance of about $100\frac{pF}{m}$. Let α be the attenuation constant and β be the phase
- Derive an expression for $|\frac{V_g}{V}|$ and assuming α is small by finding the first resonance where V_g is minimum.
 - $V_g = V_0 e^{-j\beta l} \cos(\beta l)$, $V = V_0 \exp(-j\beta l)$. $\left| \frac{V_g}{V} \right| = \cos(\beta l)$. So, at $l = \frac{\lambda}{2}$, $\left| V_g \right| = |V|$
- Find lpha and the wave velocity by finding the resonant frequency (without the load, $1V_{pp}$) and noting the time delay with a scope on the input and output. Use $|\frac{V_g}{V}|$ to calculate α .
 - |V| will be maximum at resonant frequency with unterminated line.
 - $|V_g(l)|$ is minimum when $l=\frac{\lambda_r}{4}$ and $\beta l=\frac{\pi}{4}$. This gives β .
 - At $l = \frac{\lambda}{2}$, $\left| \frac{V_g}{V} \right| = e^{-\alpha(\lambda/2)}$
- Use this to calculate the velocity, v. How large is the frequency shift caused?
 - $v = \frac{\omega_r}{\beta}$. [v should be about $2 \times 10^8 \ m/s$]
- Find, as usual, f_u , f_l , and Q.

 - $Q=rac{eta}{2lpha}$ $Q=rac{f_r}{BW}$, so $BW=rac{f_r}{O}$. $f_u=f_r+rac{BW}{2}$, and $f_l=f_r-rac{BW}{2}$

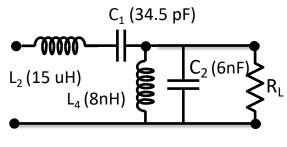


Filters





High pass



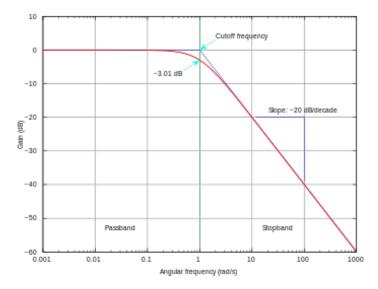
7 MHz bandpass

- Circuits on the left are called ladder filters.
- Low pass (Butterworth equivalent): Tabled values for inductors and capacitors based on frequency and dB drop-off.
- Can convert low pass into bandpass.
- For low pass to high pass
- Butterworth: $L = \frac{P_i}{P} = 1 + (\frac{f}{f_c})^{2n}$, f_c is 3dB cutoff
- Chebyshev: $L = 1 + \alpha C_n^2 (\frac{f}{f_c})^{2n}$, f_c is 3dB cutoff
- Normalized reactance's: $a_i = \sin(\frac{(2i-1)\pi}{2n})$
- Ripple loss: $1 + \alpha = 10^{L_r/_{10}}$
- $\beta = \sinh(\frac{\tanh^{-1}(1/\sqrt{1+\alpha})}{n}), c_i = \frac{a_i a_{i-1}}{c_{i-1}(\beta^2 + \sin^2((i-1)\pi/n))}$
- Example: cutoff at 10MHz, 4th order, 50ohm output, 3dB cutoff, L(20MHz)=6n=24dB, $a_1=0.765$, $X_1=x_1Z_0=38\Omega$, $L_1=\frac{X_1}{\omega_c}=610nH$, $b_2=a_2=1.848$, $B_2=\frac{b_2}{Z_0}$, $C_2=\frac{B_2}{\omega_c}$





Bandpass - WIkipedia



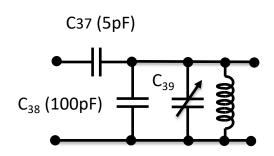
Lowpass - Wlkipedia

Norcal transmit bandpass filter

- $C_{39} = 50pF$,
- L_6 is 36 turns #28 on T37-2 which has $A_l = 4 \frac{nH}{turn^2}$, $L_6 = A_l \cdot 36^2 = 3.1 \mu H$

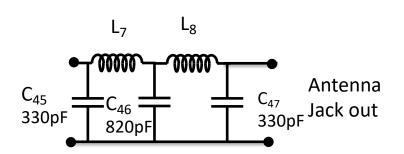
•
$$Z_2 = -\frac{j}{(C_{38} + C_{39})\omega_o}$$
, $Z_3 = jL_6\omega_o$, $Z_1 = \frac{j}{C_{37}\omega_o}$

- $Z_{2,3-eq} = \frac{jL_6\omega_0}{L_6(C_{38}+C_{39})\omega_0^2-1}$ L₆
- Resonance is when $Z_{2,3-eq} \rightarrow \infty$, $\omega_o^2 = \frac{1}{(C_{38}+C_{30})L_6} \approx \frac{10^{18}}{465}$, when almost all the voltage drop is across $Z_{2,3-eq}$ $\omega_o = \frac{10^9}{\sqrt{465}} \approx 50.8 \times 10^6$, $f_0 = \frac{\omega_o}{2\pi} \approx 7.1$ MHz
- Q of filter is: $Q_S = \frac{X_S}{R_S}$. R_S comes from the other components and must be measured
- Note that $Z_{2,3-eq}$ is small for the other modulation product



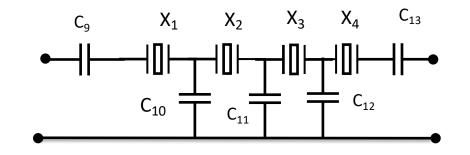
Exercise 13: Norcal Harmonic Filter

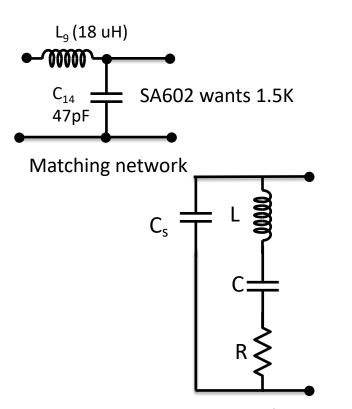
- L₇, L₈ use T37-2 core, 18 turns, 1.3uH. Use 50Ω termination and set function generator at $10V_{pp}$.
- 1. Compute and compare loss at 7MHz and 14MHz.
- 2. From $A_l = 5nH/turn^2$, calculate L_7 and L_8 .
- 3. What is the spur strength at 7, 14 and 28MHz? Measure and calculate.
- Need Puff (a simulator) to get losses. However, answer is there is a 6dB drop-off at every frequency doubling



Exercise 14: Norcal IF Cohn Filter

- X₁ through X₄ are 4.91 MHz
- C₁₀, C₁₁, C₁₂ are 270 pF
- Set function generator to 50mV_{pp} from function generator
- Calculate R and X for filter
- 1. Measure the resonant frequency of one of the crystals
 - Duh
- 2. Calculate the parameters of the crystal. Omitting C_S
 - $f_r=\frac{1}{2\pi\sqrt{LC}}$ and $Q=\frac{1}{R}\sqrt{\frac{L}{C}}$. We can measure f_r and find Q using the 3dB bandwidth. R is the resistance at resonance.
 - $Q \approx 80$
 - $25\Omega < R < 100\Omega$
 - If R = 50, C = 8.1pF, $L = 130\mu H$





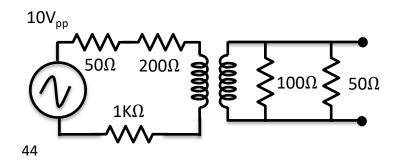
Transformers

- For solenoid, $\oint B \cdot ds = \mu_0 nI$ inside
- $LI = \Phi_B$. Since there are n turns in the solenoid, over the solenoid, $LI = \mu_0 n^2 I$, so $L = \mu_0 n^2$.
- This is the source of $L = A_l n^2$
- $V_S = \frac{N_S}{N_p} V_p$ $Z_p = (\frac{N_p}{N_S})^2 Z_S$

Exercise 15: Norcal Driver Transformers (1)

• T₁ uses FT 37-43.
$$L(\mu H) = \frac{A_L t^2}{1000}$$
, $A_L = 350$. $f_r = 7 \times 10^6 MHz$, $n_p = 14$, $n_s = 4$, $\omega_r = 2\pi \times 7 \times 10^6 MHz = 4.4 \times 10^7$

- 1. Measure the output V_{out} .
- 2. Calculate V_{out}
 - $L_p = 68.6 \mu H$, $L_s = 5.6 \mu H$
 - $Z_{eq,in}(\omega) = 1250 + j(\omega L_p), Z_{eq,in}(\omega_r) = 1250 + 3016j, |Z_{eq,in}(\omega_r)| = 3264$
 - $Z_{eq,out}(\omega) = 33 + j\omega L_s$, $Z_{eq,out}(\omega_r) = 33 + j246$, $|Z_{eq,out}(\omega_r)| = 248$
 - $V_{t,in} = \frac{3016}{3264} V_{in}$
 - $V_{out} = V_{t,out} = \frac{n_s}{n_p} V_{t,in} = .29 V_{t,in} = .29 \times \frac{3016}{3264} \times 5 = 1.3 V$
 - $i_p(\omega) = \frac{V_{in}}{|Z_{eq,in}|} e^{j\dot{\theta}_p(\omega)}$, $\theta_p(\omega) = \arctan\left(\frac{\omega L_p}{1250}\right)$; $i_s(\omega) = \frac{V_{out}}{|Z_{eq,out}|} e^{j\theta_s(\omega)}$, $\theta_s(\omega) = \arctan\left(\frac{\omega L_s}{33}\right)$.
 - $P_{in,a} = Re\left(\frac{V_{in}\overline{I_{in}}}{2}\right) = Re\left(\frac{V_{in}^2}{2|Z_{ea,in}(\omega)|}e^{j\theta_p(\omega)}\right)$
 - $P_{out,a} = Re\left(\frac{V_{out}\overline{I_{out}}}{2}\right) = Re\left(\frac{V_{out}^2}{2|Z_{eq,out}(\omega)|}e^{j\theta_s(\omega)}\right)$



T₁, 14:4

Exercise 15: Norcal Driver Transformers (2)

- $\cos(\theta_s(\omega_r)) = .13, \cos(\theta_p(\omega_r)) = .38,$
- $\frac{P_{out,a}(\omega_r)}{P_{in,a}(\omega_r)} = \left(\frac{V_{out}}{V_{in}}\right)^2 \frac{|Z_{eq,in}(\omega_r)|}{|Z_{eq,out}(\omega_r)|} \frac{\cos(\theta_s(\omega_r))}{\cos(\theta_p(\omega_r))} = \left(\frac{1.3}{5}\right)^2 \times \frac{3264}{248} \times \frac{.13}{.38} = .3$
- 3. Measure the 3dB cutoff, f_c .
- $\frac{P_{out,a}(\omega)}{P_{in,a}(\omega)} = \left(\frac{V_{out}}{V_{in}}\right)^2 \frac{|Z_{eq,in}(\omega)|}{|Z_{eq,out}(\omega)|} \frac{\cos(\theta_s(\omega))}{\cos(\theta_p(\omega))} = .15$

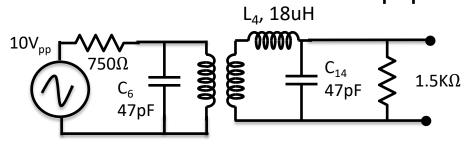
Exercise 16: Norcal Tuned Transformers

 $\begin{array}{c|c}
 & C_4 \\
 & 5pF \\
 & 1V_{pp} \\
 & 1.5K\Omega
\end{array}$

 T_2 , 1:20

T₃, 23:6

- T_2 , T_3 are IF matchers using FT 37–61. $A_L = 55 \, nH/turn^2$.
- 1. Measure 3dB bandwidth
- 2. Find P/P₊
- T₂
 - $P_a = Re\left(\frac{V\bar{I}}{2}\right)$, $V = V_+ + V_-$, $\rho = \frac{V_+}{V_-} = \frac{Z Z_0}{Z + Z_0}$, Z is look forward, Z_0 is look back resistance. $P_+ = \frac{{V_+}^2}{2Z_0}$. $L_{in} = 55nH$, $L_{out} = 22\mu H$, $\omega = 4.4 \times 10^7$
 - Z = 50 + 2.4j, $Z_0 = 203 4030j$, $\rho = 1$, so $V_+ = V_-$, $V = 2V_+$
 - $\bullet \qquad \frac{P}{P_+} = \frac{1}{4}$
- Similar calculation for T₃.

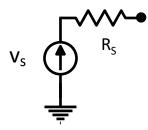


Acoustics

- $P = P_0 + P_e$, $\rho = \rho_0 + \rho_e$
- Gas moves and changes density: Displacement of undisturbed air is x. At time t, it's at $x + \chi(x, t)$, so $\rho_0 \Delta x = \rho(x + \Delta x + \chi(x + \Delta x, t) - x - \chi(x, t))$, or $\rho_0 \Delta x = \rho(\frac{\partial \chi(x,t)}{\partial x} \Delta x + \Delta x)$. So, $\rho_e = -\rho_0 \frac{\partial \chi}{\partial x}$
- 2. Change in density causes change in pressure: $P = f(\rho)$, $P_0 + P_e = f(\rho_0 + \rho_0)$ $(\rho_e) = f(\rho_0) + \rho_e f'(\rho_0), f'(\rho_0) = \kappa = (\frac{dP}{d\rho_0})_0, \text{ or } P_e = \kappa \rho_e$
- Pressure differences cause motion: $P(x,t) P(x + \Delta x, t) = -\frac{\partial P_e}{\partial x} \Delta x$, Newton's law gives $\rho_0 \frac{\partial^2 \chi}{\partial t^2} = -\frac{\partial P_e}{\partial x} = -\kappa \frac{\partial \rho_e}{\partial x}$ Substituting (1) into (3) gives $\frac{\partial^2 \chi}{\partial t^2} = \kappa \frac{\partial^2 \chi}{\partial x^2}$, put $\kappa = \frac{1}{C_2^2}$
- Solution is $\chi(x,t) = f(x-vt)$ [Different f than above].
- To find, $\kappa = (\frac{dP}{d\rho})_0$, note that the flow is adiabatic so $PV^{\gamma} = C'$ and ρ varies inversely as V, so $P = \rho^{\gamma} C$, and finally, using PV = Nkt, $\kappa = (\frac{dP}{dQ})_0 = \frac{\gamma kT}{r}$
- $L_p = 20 \log(\frac{P}{P_0}), P_0 = 20 \,\mu Pa$

Sound	L _p	Power density
rustling leaves	10dB	1pW/m²
broadcast studio	20dB	1pW/m²
classroom	50dB	10nW/m²
heavy truck	90dB	1nW/m²
Shout at 1m	100dB	10mW/m²
jackhammer	110db	100mW/m ²
jet takeoff at 50m	120dB	1W/m ²

Bipolar Transistors - I



Bipolar source model

$$r_b = (1+\beta)r_e$$

$$r_o = \beta i_b$$

Bipolar equivalent circuit

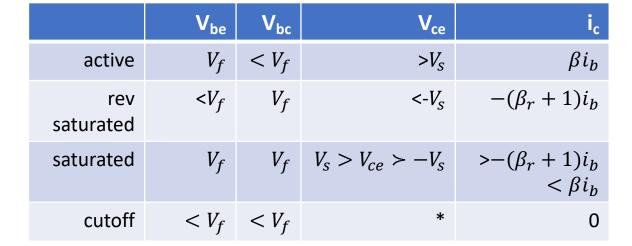
NPN Model

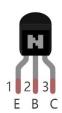
- $V_f \approx .7V, V_S \approx .2V$
- Conducts when $V_{be} > V_f$
- $i_c = \beta i_b$
- $i_c = \alpha i_e$
- $\beta = \frac{\alpha}{1-\alpha} [= h_{fe}$, small signal]
- $\beta \approx 100, \beta_r \approx 10$
- $r_e i_e = 25 mV$, $r_b = (1 + \beta) r_e$, $r_e \approx 33 \Omega$
- $i_b = \frac{v_{be}}{(1+\beta)r_e}$
- $g_m v_{be} = g_m r_b i_b$

Switch

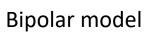
•
$$G_S = \frac{i_b}{15mV}$$

•
$$R_S = 2\Omega$$



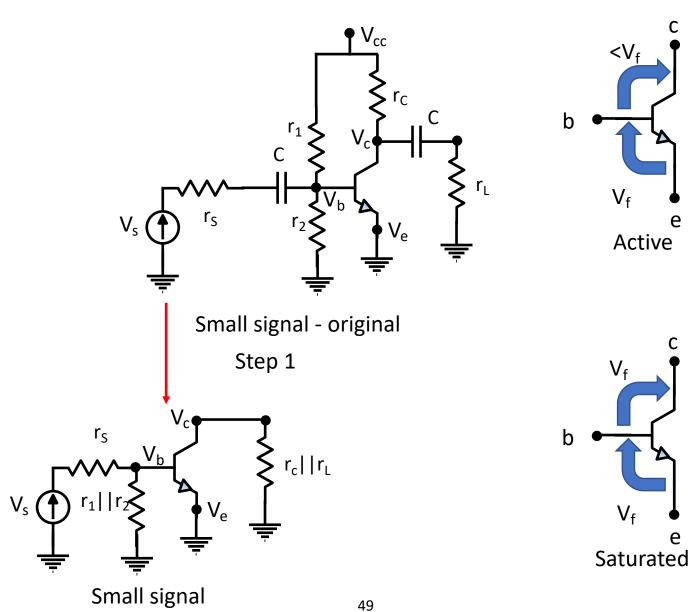






Bipolar Transistors - II

- NPN Mode
 - $V_f = .7V$
 - $\beta = g_m r_{\pi}$
 - $g_b = \frac{i_b}{V_t}$, $V_t \approx 25mV$, $g_m = \frac{i_c}{V_t}$
- DC
 - $\frac{V_{cc}-2V_f}{R_C} < i_c, \beta i_b = i_c$
 - $V_c = V_{cc} i_c R_C$
 - $\bullet \quad \frac{V_{cc} V_b}{R_B} = i_b$
- Small signal
 - 1. Convert to AC only and simplify
 - 2. Thevenize circuit
 - 3. Replace transistor with model



Bipolar Transistors - III

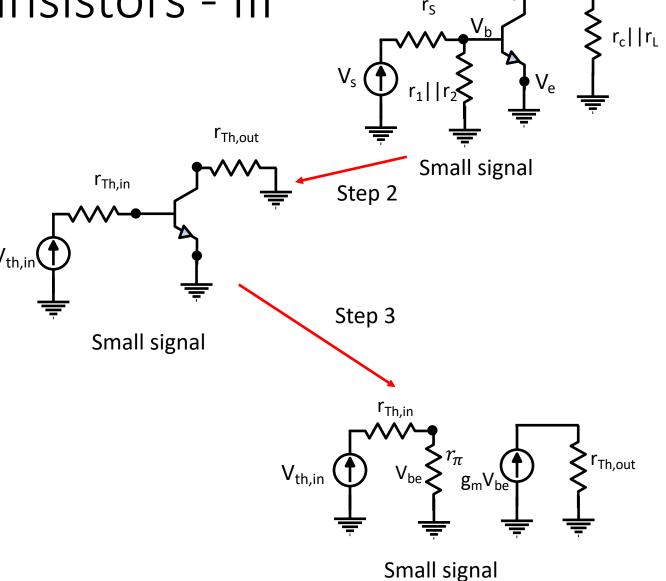
- Small signal
 - 1. Convert to AC only and simplify
 - 2. Thevienize

•
$$V_{th,in} = V_S \frac{r_1||r_2|}{r_1||r_2+r_S|}$$

- $r_{th,in} = r_s ||r_1||r_2|$
- $r_{th,out} = r_C || r_L$
- 3. Replace transistor with model

$$\bullet \quad \frac{V_{be}}{V_{Th,in}} = \frac{r_{\pi}}{r_{\pi} + r_{Th,in}}$$

- r_0 is the transistor model resistance between b and c
- $A_{gail} = \frac{V_{out}}{V_s}$



Bipolar transistors - IV

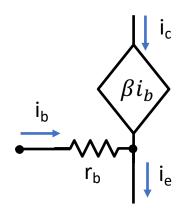
- At saturation, $v_{bc} < V_f$, so there is conduction from the collector to the base.
- $i_b = i_{bs} \exp(\frac{V_b}{V_t})$, V_t is the thermal voltage, $V_t = 25mV$, i_{bs} is the base saturation current.
- $i_c = i_{cs} \exp\left(\frac{V_c}{V_t}\right)$. Note $i_{cs} = \beta i_{bs}$. Both increase rapidly with temperature
- Base resistance

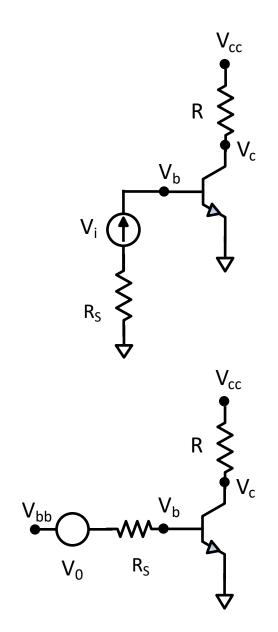
$$\bullet \quad g_b = \frac{i_b}{V_t} = \frac{di_b}{dV_b}$$

•
$$r_b = \frac{25mV}{i_b}$$

•
$$g_m = \frac{i_c}{V_t} = \frac{di_c}{dV_h}$$

•
$$i_b = r_b V_b$$



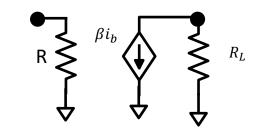


More on bipolar transistor model

- $v_{R_2} = v_{be} + (i_c + i_b)R_E$
- $\bullet \quad i_{R_2} = \frac{v_{R_2}}{R_2}$
- $i_{R_1} = i_{R_2} + i_b$
- $v_{cc} = R_1 i_b + (R_1 + R_2) i_2$
- For $R_B = R_1 || R_2$, $v_{cc}R_B v_{be}R_1 = R_1R_2i_b + (i_c + i_b)R_1R_E$, $i_c = \beta i_b$
- $i_c = \frac{v_{cc} \frac{R_B}{R_1} v_{be}}{R_E + \frac{(R_C + R_E)}{\beta}}$
- If $R_E \gg \frac{(R_C + R_E)}{\beta}$, $\frac{\partial i_C}{\partial v_{be}} = -\frac{1}{R_E}$, be acts like diode so $i_C = i_S \beta \exp(\frac{V_{be}}{V_T})$. Want $V_E \approx 2v$



Small signal equivalent



$$R = R_1 ||R_2||r_{\pi}$$

$$r_e = \frac{V_T}{i_c}, \quad r_{\pi} = r_e(\beta + 1)$$

More on bipolar transistor model

•
$$v_{be} = v_b - v_e$$
, $v_{ce} = v_c - v_e$, $v_{bb} = \frac{R_2}{R_1 + R_2} v_{cc}$, $v_f \approx .6$ (for Si)

•
$$v_b = v_e + v_f$$
, $i_b = \frac{v_{bb} - v_b}{R_b}$

•
$$i_c = \beta i_b + i_{ceo}, \frac{\partial i_b}{\partial v_{be}} = \frac{1}{r_d}$$

•
$$\frac{\partial i_c}{\partial v_{be}} = g_s = -\frac{1}{R_B}$$
, and $\frac{\partial i_c}{\partial \beta}$ measure stability

•
$$i_b = \frac{v_{bb} - v_b}{R_b}$$

•
$$i_c = \frac{\beta(v_{bb}-v_{be})}{R_b+(1+\beta)R_e} + \frac{(R_b-R_e)}{R_b+(1+\beta)R_e} i_{ceo}$$
, i_{ceo} is leakage current

• So, if
$$\beta R_e \gg R_b + R_e$$
, $i_c = \frac{v_{bb} - v_{be}}{R_e} = \frac{v_{cc} - v_c}{R_c}$

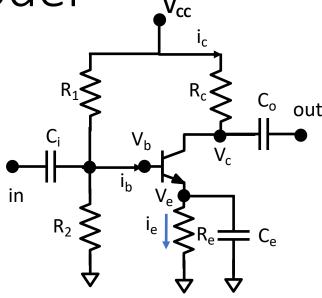
•
$$Z_{in} = R_1 ||R_2|| (\beta + 1) R_e, Z_{out} = R_c$$

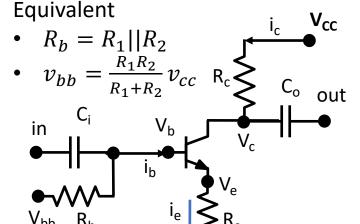
•
$$v_{bb} - v_{be} = \frac{R_e}{R_c} (v_{cc} - v_c)$$

• Typical for Si: $v_{be}=v_f\approx .6V$, $\beta\approx 200$, $v_{cc}=9V$, $v_{bb}=3V$, $R_b=10^4\Omega$, $R_c=10^3\Omega$, $R_e=270\Omega$

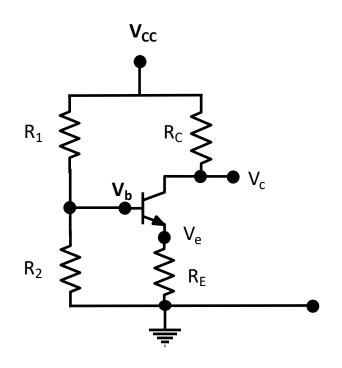
•
$$3 - .6 = \frac{270}{1000}(9 - v_c)$$
, so $v_c = .2V$

• For voltage divider: $R_1 = 2 \times 10^4 \Omega$, $R_2 = 10^4 \Omega$,





Transistor experiment



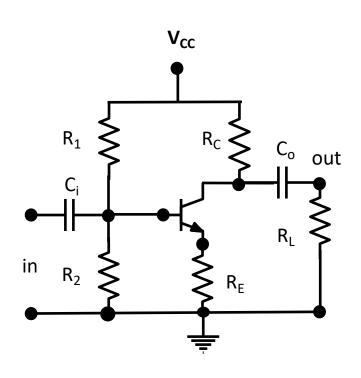
Experiment A

- $V_{cc}=9V$, $R_1=22.8k\Omega$, $R_2=7.2k\Omega$, $R_c=1k\Omega$, $R_E=220k\Omega$. 2n3904 transistor, $\beta=150$.
- With no transistor, R_2 adjusted so $V_b=2.36V$. $V_b=2.24V$, $V_e=1.54V$, $V_c=1.89V$. $i_c=7mA$, $i_b=46\mu A$.

Experiment B

- Again, $V_{cc}=9V$, $R_1=20k\Omega$, $R_2=10k\Omega$, $R_c=1k\Omega$, $R_E=220k\Omega$. 2n3904 transistor, $\beta=150$. With no transistor, R_2 adjusted so $V_b=5.8V$. Put transistor in and $V_b=2.4V$.
- With transistor, $V_b = 2.4 V$, $V_e = 1.7 V$, $V_c = 1.74 V$. $i_c = 7 mA$, $i_b = 46 \mu A$.
- Analyze these with our transistor model.
- Now use the Thevenin equivalents to analyze them.

Turn the transistor experiment into a CE amplifier



• Add C_i and C_o. Component values are:

•
$$C_i = C_o = 1 \mu F$$

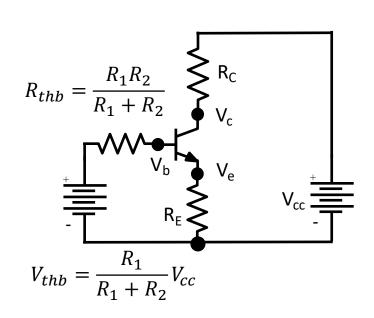
•
$$R_1 = 20k\Omega, R_2 = 10k\Omega$$

•
$$R_C = 1k\Omega$$
, $R_E = 220\Omega$

•
$$V_{cc} = 9V$$

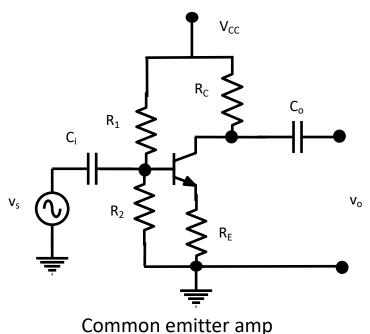
- 1. Use a function generator to generate a $V_{pp} = 800 mV$, 10kHz.
- 2. The input impedance is $Z_{in} = R_1 ||R_2|| (\beta + 1) R_E$, and the output impedance is $Z_{out} = R_C$. Add a load R_L whose value is Z_{out} .
- 3. Now connect a scope to the output and measure the gain. Calculate what it should be an compare them. How do the input and output waveforms compare?

Transistor experiment - Thevenin equivalent DC



- In Experiment A
 - $R_1 = 22.8k\Omega$, $R_2 = 7.2k\Omega$, $V_{thb} = 2.16V$, $R_{th} = 5.5k\Omega$.
 - If $r_e \approx 33\Omega$, $r_b \approx 5k\Omega$, $i_b = \frac{2.16 1.54}{11500} = 53\mu A$, which is close.
- In Experiment B
 - $R_1 = 22k\Omega$, $R_2 = 8k\Omega$, $V_{thb} = 2.4V$, $R_{th} = 5.9k\Omega$.
 - If $r_e \approx 33\Omega$, $r_b \approx 5k\Omega$, $i_b=\frac{2.4-1.7}{10900}=64\mu A$, which is also close, but a little high.
- Turn this into a CE amplifier by adding 1uF input and output capacitors.
 Measure and calculate the voltages and gains.

BJT common emitter amplifier

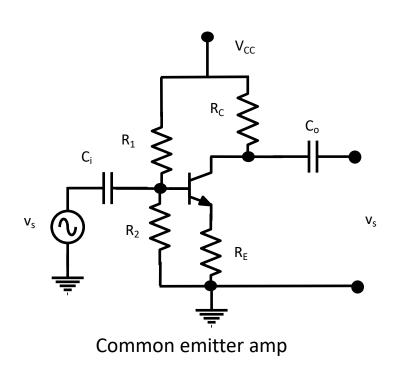


• Here's how to design a common emitter amplifier. We use a 2n3904 transistor with β =150. This circuit will work! Build it.

- 1. Pick the supply voltage V_{cc} =12V.
- 2. Choose a gain (amplification factor), A = 5.
- 3. Choose the "Q point" of the conducting transistor (4mA) and $V_{ce,q} = 5v$.
- 4. $V_{cc} = (i_c \cdot R_C) + V_{ce} + i_e R_E \sim i_e \cdot (R_C + R_E) + V_{ce}$ with $i_c = 4mA$. We get $(R_C + R_E) = (V_{cc} V_{ce})/(4mA) = 1.75 \text{ k} \Omega$.
- 5. Since A = 5 and A=R_C/R_E, R_C= 5 R_E so R_E \sim 270 Ω (this is a standard resistor value) and R_C= 1.5k Ω .

Credit: Ward, Hands on Radio.

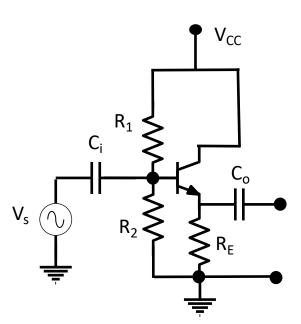
BJT common emitter amplifier continued



Credit: Ward, Hands on Radio.

- 6. $i_b = 4\text{mA}/\beta = 27 \,\mu\text{A}$.
- 7. Since V_{be} must be greater than .7V throughout the input signal range, we want the voltage across R_2 to satisfy $V_{be} + i_c R_E = 1.8V$.
- 8. Rule of thumb is current through R₁ and R₂ is $10i_b$. We insert a voltage divider consisting of R₁ and R₂, so that R₁= (12-1.8)/270 μ A \sim 39 k Ω . $R_2=6.7k\Omega$
- 9. C_o and C_i are picked to offer small resistance to the frequency range we're interested in and $C_o = C_i = 5 \mu F$.
- I haven't explained why we want R_E but it provides thermal stability for the transistor over the range we care about. The fact that $A=R_C/R_E$ can be calculated using Kirchhoff's laws.

BJT common collector amplifier



1.
$$\beta = 150, A_V = 1, V_{CC} = 12v$$

2. Q-pt:
$$i_{ce} = 5mA$$
, $V_{ce,q} = 6v$ (rule of thumb), $v_{be} = .7V$.

3.
$$i_{R_1 \to R_2} = 10i_b$$
 (ROT), $V_{ce} = v_{be} + i_{ce,q}R_E$, $R_E = 1.2k\Omega$, $i_b = \frac{v_{ce,q}}{\beta} = 33\mu A$

4.
$$V_{R_2} = V_{be} + i_C R_E = 6.7V, V_{R_1} = 5.3V$$

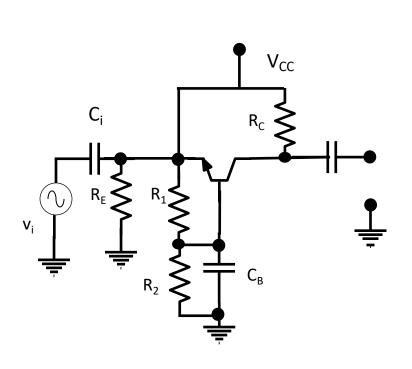
5.
$$R_2 = \frac{6.7}{330\mu A} = 20k\Omega, R_1 = \frac{5.3}{330\mu A} = 16k\Omega$$

6.
$$Z_{in} = R_1 ||R_2|| (\beta + 1) R_E, R_{in} = 50 \Omega, Z_{out} = 5 \Omega$$

Common collector amp (Emitter Follower)

Credit: Ward, Hands on Radio.

BJT common base amplifier



•
$$A_I = \frac{i_C}{i_E} = \frac{\beta}{\beta + 1}$$
, $A_V = \frac{R_C || R_L}{r_e}$, $Z_{out} \approx R_C$

1.
$$V_{CC} = 12, V_{be} = .7V, R_E = 50\Omega, R_L = 1k\Omega, i_{ce,q} = 5mA, V_{ce,q} = 6V$$

2.
$$i_b = \frac{i_{ce,q}}{\beta} = 33\mu A, i_{R_1 \to R_2} = 10 i_b = 330\mu A \text{ (ROT)}$$

3.
$$V_{R_2} = V_{be} + i_C R_E = 6.7V, V_{R_1} = 5.3V$$

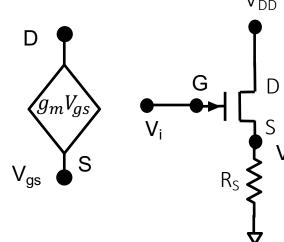
3.
$$V_{R_2} = V_{be} + \iota_C R_E = 6.7V$$
, $V_{R_1} = 5.3V$
4. $R_1 = \frac{5.3}{330\mu A} = 16k\Omega$, $R_C = \frac{V_{cc} - i_{c,Q} R_E - V_{ce,Q}}{i_{c,Q}} = 1.35k\Omega$

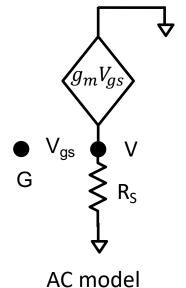
5.
$$A_V = \frac{R_C||R_L}{^{26}/i_e} = 115$$

Common base amp

JFETs

- JFET circuit model (active region):
 - $I_d=I_{dss}\,(1-\frac{v_{gs}}{v_c})^2$, provided $0< v_{gs}< V_c$ and $V_{ds}>V_{gs}-V_c$. i_{dss} is drain to source current when gate is at 0. $v_{gs}\leq 0$
- $g_m = \frac{dI_d}{dV_{gs}} = \frac{\Delta i_{ds}}{\Delta v_{gs}} \approx -\frac{2I_{dss}}{V_c} (1 \frac{V_{gs}}{V_c})$
- For circuit on right, $g_m \Delta v_{gs} = \Delta i_{ds}$ and so $g_m R_S \Delta v_{gs} = V$
- V_c is cutoff voltage. When $v_{gs} < V_C$ there's no channel conduction. Some people call this V_T or V_P . JFET input impedance is high $(10^{10}\Omega)$.
- For J309, $V_c \approx -2.6V$, $i_{dss} \approx 23mA$, $g_m \approx 12$.
- DC: $V_b = -i_b R_S$, AC: $V = R_S g_m V_{gS}$, $v_{gS} = V_g V$
- $V = R_S g_m v_{gs}$
- $v_{gs} = V_i V$, $V = \frac{RV_i}{R + \frac{1}{g_m}}$, $G_v = \frac{V}{V_i} = \frac{Rg_m}{1 + Rg_m} \approx 1$
- $Z_0 = \frac{1}{g_n}$





FETS

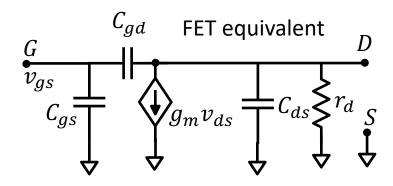
•
$$\Delta i_d = \frac{\partial i_d}{\partial v_{GS}} \Delta v_{GS} + \frac{\partial i_d}{\partial v_{DS}} \Delta v_{DS}$$

• $\frac{\partial i_d}{\partial v_{GS}} = g_d, \frac{\partial i_d}{\partial v_{DS}} = g_m$
• $Z_{in} = 10^9 \Omega$

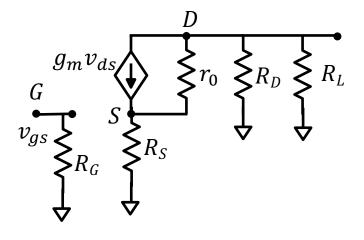
•
$$\frac{\partial i_d}{\partial v_{GS}} = g_d$$
, $\frac{\partial i_d}{\partial v_{DS}} = g_m$

•
$$Z_{in} = 10^9 \Omega$$

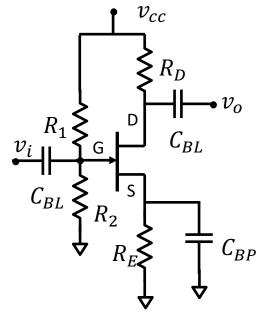
•
$$Z_{out} = R_D ||r_d|| \frac{1}{j\omega(C_{GS} + C_{DS})}$$



FET CS small signal equivalent



FET common source



- R_E provides bias
- $\frac{v_o}{v_i} = -g_m \frac{R_D||r_d}{j\omega(C_{GS} + C_{DS})}$

•
$$A_0 = -g_m(R_D||r_d)$$

• $Z_{in} = \frac{1}{j\omega(C_{GS} + (1 - A_0)C_{GD})}$

More on FETs

For common source

•
$$R_o = [r_d + R_S(1 + g_m r_d)] || R_D, R_o \approx R_D$$

•
$$R_{in} = R_G = R_1 || R_2$$

•
$$v_{in} = v_{gs} + i_d R_S = v_{gs} (1 + g_m R_S)$$

•
$$v_{out} = -i_d(R_D||R_L) = -g_m v_{gs}(R_D||R_L)$$

•
$$A_v = -\frac{R_D||R_L}{R_S + 1/g_m}, A_i = -\frac{R_G}{R_S + 1/g_m} \frac{R_D}{R_D + R_L}$$

•
$$R_1 = 10^6 \Omega$$
, $R_2 = 50 K\Omega$, $R_S = 10^4 \Omega$, $R_D = 25 k\Omega$, $g_m = 2000 \mu S$, $g_d = 20 \mu S$,

• For common drain

•
$$v_{test} = -g_s$$
 , $i_{test} = \frac{v_{test}}{R_S} + g_m v_{test}$

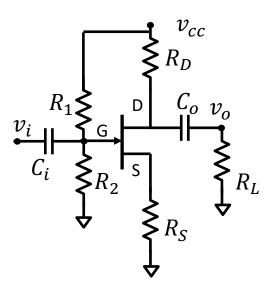
•
$$R_o = \frac{v_{test}}{i_{test}} = R_S || \frac{1}{g_m}, R_{in} = R_G = R_1 || R_2$$

•
$$v_{in} = v_{gs} + g_m v_{gs}(R_S||R_L)$$

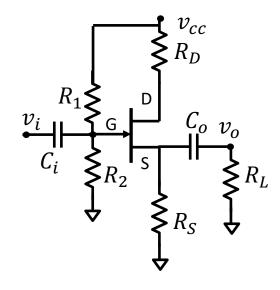
•
$$A_v = -\frac{g_m(R_S||R_L)}{[1+g_m(R_S||R_L)]}, A_i = -\frac{R_G}{R_S+R_L} \frac{R_S}{[(R_S||R_L)+1/g_m]}$$

•
$$R_o = \frac{1}{\frac{1}{r_d} + \frac{1}{R_3} + g_m} \approx 196\Omega$$
, $r_d = 100k$, $R_S = 10^4 \Omega$, $R_L \approx 10^6 \Omega$

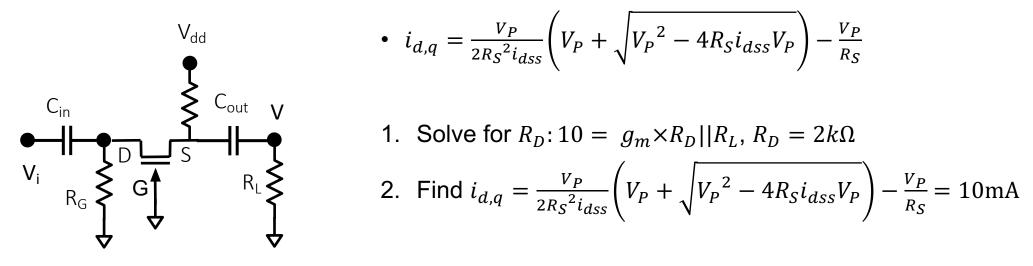
FET common source



FET common drain



JFET common gate amplifier



•
$$A_V = g_m(R_D||R_L), Z_{out} \approx r_0(g_m R_S + 1)||R_D, Z_{in} = R_S||\frac{1}{g_m}|$$

•
$$V_{DD} = 12V$$
, $i_{dss} = 60mA$, $V_P = -6$, $A_V = 10$, $R_L = 1k\Omega$, $R_S = 50\Omega$

•
$$i_{d,q} = \frac{V_P}{2R_S^2 i_{dss}} \left(V_P + \sqrt{{V_P}^2 - 4R_S i_{dss} V_P} \right) - \frac{V_P}{R_S}$$

1. Solve for
$$R_D$$
: $10 = g_m \times R_D || R_L, R_D = 2k\Omega$

2. Find
$$i_{d,q} = \frac{V_P}{2R_S^2 i_{dss}} \left(V_P + \sqrt{V_P^2 - 4R_S i_{dss} V_P} \right) - \frac{V_P}{R_S} = 10 \text{m/s}$$

Mosfet

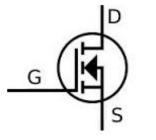
•
$$r_{out} = \frac{1}{\lambda i_d} = \frac{\partial i_{ds}}{\partial V_{ds}}$$
, $g_m = \frac{\partial i_d}{\partial V_{qs}}$

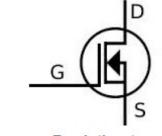
- Weak Inversion ($V_{gs} < V_{th}$)
 - $i_d = i_0 \exp(\frac{V_g V_{th}}{nV_t})$, $n = 1 + \frac{C_{th}}{C_{ox}}$
- Linear ($V_{gs} > V_{th}$, $V_{ds} < V_{gs} V_{th}$)

•
$$i_d = \mu_n C_{ox} \frac{W}{L} [(V_{gs} - V_{th})V_{ds} - \frac{V_{ds}^2}{1}] (1 + \lambda V_{ds})$$

- Saturation ($V_{gs} > V_{th}$, $V_{ds} \ge V_{gs} V_{th}$)
 - $i_d = \mu_n C_{ox} \frac{W}{2L} (V_{gs} V_{th})^2 [1 + \lambda (V_{ds} V_{dsat})]$

N channel MOSFET

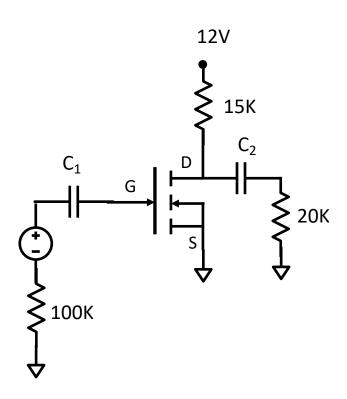


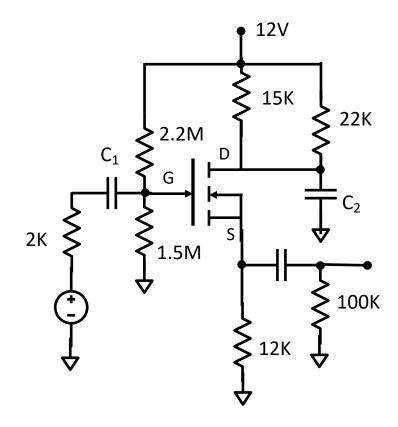


Enhancement type

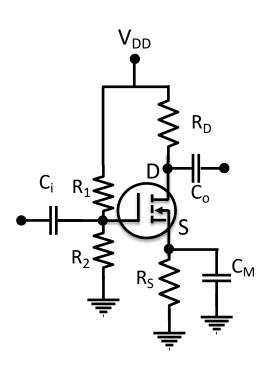
Depletion type

Mosfet amps



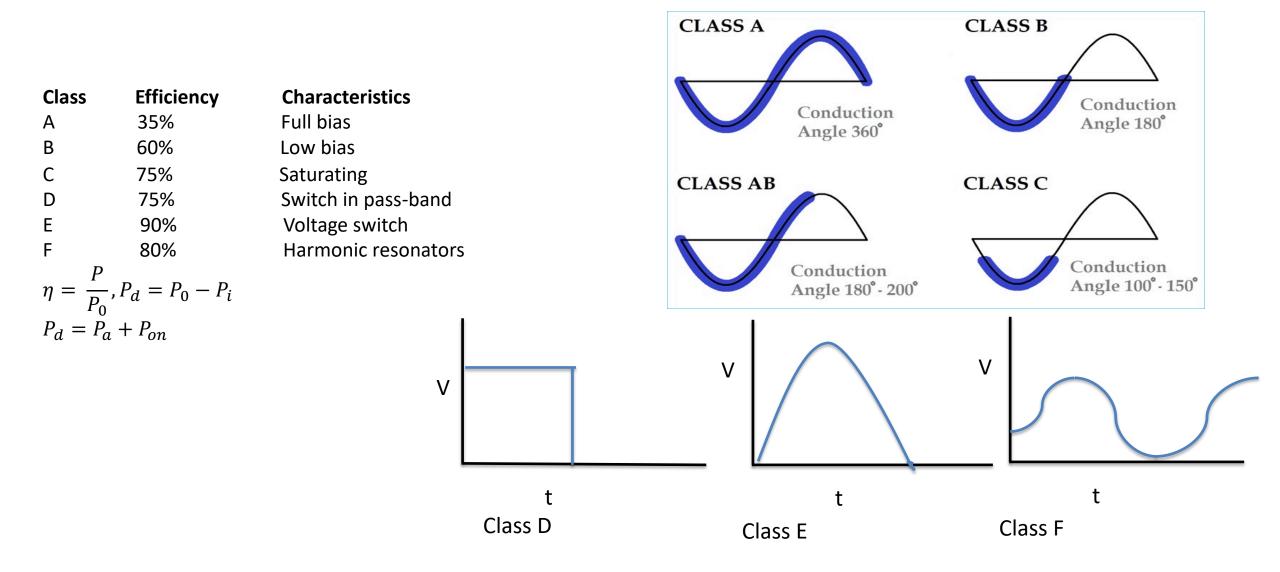


CMOS common emitter amplifier



- Pick power
- $\bullet \quad V_{DD} = i_D R_D + V_{DS} + i_D R_S$
- $V_{GS} = V_G i_S R_S$ $V_G = V_{DD} \frac{R_1}{R_1 + R_2}$
- $i_D = k(V_G V_{TH})^2$
- Bias around $\frac{V_{DD}}{3}$
- Pick gain, $A = \frac{R_D}{R_S + \frac{1}{a_{sol}}}$
- For 2N7000, $g_m \approx 200$

Amplifier classes

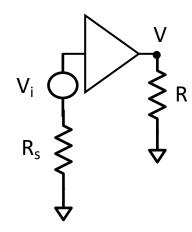


Efficiency of class A amplifiers

- Here, P_o is the power from the supply, P_d is the dissipated power, P is the output power. R is the collector resistor.
- $\eta = \frac{P}{P_0}$, P_0 is DC power
- $P_0 = V_{cc}I_0$, where $I_0 = \langle i_c \rangle$, so $I_0 = \frac{V_{cc}}{2R}$ (R is the collector resistance). Thus, $P_o = \frac{V_{cc}^2}{2R}$.
- AC load power is $P = \frac{V_{pp}I_{pp}}{8} = \frac{V_{cc}^2}{8R}$. So maximum efficiency $\eta = \frac{P}{P_o} = 25\%$.
- DC load power is $\frac{V_{cc}^2}{4R}$ and so is transistor power.
- Half the power in a class A is lost to load resistance. If we replace resistance with transformer, $P_0 = \frac{{V_{cc}}^2}{R'}$, where R' is the effective load resistance and $P = \frac{{V_{pp}I_{pp}}}{8} = \frac{{V_{cc}}^2}{2R'}$, giving 50% efficiency. Transformer turns ratio controls peak-to-peak current. Maximum current is $I_m = \frac{2V_{cc}}{R'} = \frac{2V_{cc}}{n^2R'}$, where n is the turns ratio.

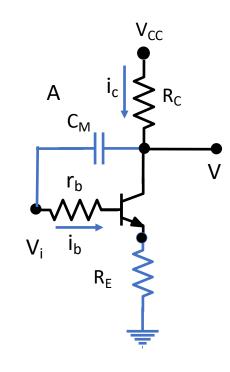
Amplifier gain

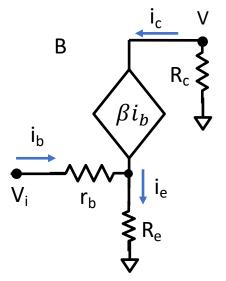
- Let P_+ be the maximum input power (when load is matched) and V_+ is the voltage at maximum power. $V_+ = \frac{V_0}{2}$
- $G = 10\log(\frac{P}{P_+})$
- $\bullet \quad P = \frac{V_{pp}^2}{8R}$
- $P_+ = \frac{V_{+,pp}^2}{8R_S}$
- $G_S = 10\log(\frac{V}{V_+})$



Emitter degeneration

- To the usual transistor circuit (A), on the right, we add R_E . (B) is an equivalent circuit.
- $V_{bb} \approx V_f + i_c R_E$. Let V be the output AC and V_i be the input AC.
- The gain is $G_v = \frac{v}{v_i}$.
- $V_i = i_b r_b + i_E R_E \approx i_C R_E$, $Z_i = \frac{V_i}{i_b}$,
- $V = -i_c R_C$.
- So $G_v = -\frac{R_C}{R_E}$ (Doesn't depend on β).
- $V_i \approx \beta i_b R_E$
- $Z_i = \frac{V_i}{i_b}$, so $Z_i = \beta R_E$.





Emitter degeneration

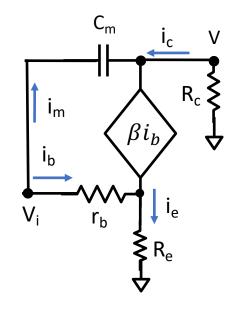
- C_M is called a Miller capacitor and it arises from a capacitance, C_C , between the collector and emitter. $i_m = j\omega C_C(V_i V) = j\omega C_M(1 + |G_v|)V_i$. i_m is the maximum current between base and emitter in the equivalent circuit on the right.
- With the Miller capacitor, $Z_i = \beta R_E ||(1 + |G_v|)C_M$
- $r_c \approx \frac{V_{early}}{i_c}$, r_c is the collector resistanc.e
- $R_S' = R_S + r_b$, r_b is the base resistance. R_S' is the combined source resistance.
- z_C is called the collector impedance and $z_C = r_C || C_C$, C_C is specified in data sheet (8pF).

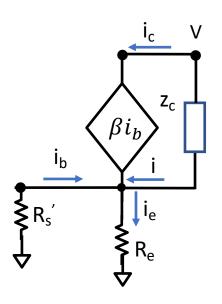
•
$$Z_o = \frac{V}{i_C}$$
, $i = i_C - \beta i_b$,

$$\bullet \quad i_b = -\frac{i_C R_S}{R_{S'} + R_E},$$

•
$$i = i_c (1 + \frac{\beta R_E}{R_{S'} + R_E})$$

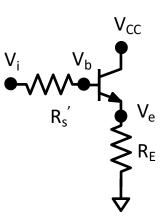
- $V = iz_C + i_C (R_S'||R_E)$
- $Z_o = \frac{V}{i_C} = z_C \left(1 + \frac{\bar{\beta} R_E}{R_{S'} + R_E} \right) + R_{S'} || R_E.$
- $|z_c| \gg R_E$, so $Z_o = z_C \left(1 + \frac{\beta R_E}{R_S' + R_E}\right)$





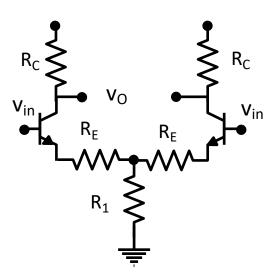
More on emitter follower

- $Z_0 = \frac{v_e}{i_e}$ $v_b = -R_S'i_b$, $R_S' = R_S + r_b$ $i_e \approx \beta i_b$ $Z_0 \approx \frac{R_S'}{\beta}$



Differential Amplifier

- Two port model

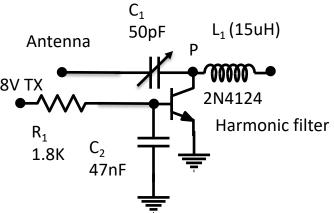


Differential amplifier

- Pick power 12
- Choose collector current (2mA) by picking R_1
- Pick gain, $A = \frac{R_C}{2R_E}$
- $G_d = -\frac{R_c}{R_e}$
- $Z_d = 2R_c$

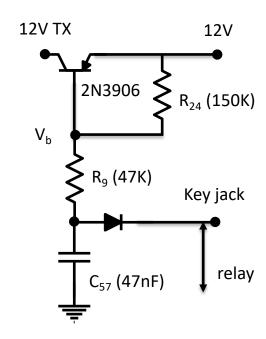
Exercise 19: Norcal receiver switch

- 1. Consider the rising part of the base voltage waveform. Calculate slope.
- 2. Do the same for the falling part for voltage below .6V. Calculate t_2 .
- 3. Measure the switch attenuation
 - When the transistor is saturated, the drop across ce is 1.4V. At full power, $P = \frac{V_m^2}{8R}$ and $V_m = 33.9V$. $\frac{P_{new}}{P_{original}} = \frac{1}{33.9^2}$, so $loss = 10 \log \left(\frac{1}{33.9^2}\right) = -31 dB$.
- 4. Measure the voltage with the switch on. Measure output voltage and calculate on-off rejection ratio $R=20 \log(V_{off}/V_{on})$



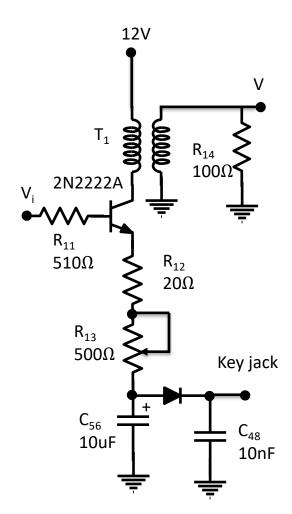
Exercise 20: NorCal transmitter switch

- For transmitter switch, saturation resistance is $\approx 2\Omega$, $G_S = \frac{\iota_b}{15mV}$.
- i is current into the load. In Norcal, i=7mA. For 2n3906, to ensure saturation, $i_b=\frac{2i}{100}=140\mu A$.
- 1. Calculate voltage on C_{57} . Measure time for capacitor to charge half-way. Calculate what the time should be.
 - $\tau = 197 \times 10^4 \times 47 \times 10^{-9} = 9.2 \times 10^{-2} sec = 92 msec.$
- 2. Calculate the approximate current i_c when Q_4 is on. Assume base voltage on Q_1 is 700 mV. Neglect saturation voltage on Q_4 . Calculate base current i_b required to produce this collector current assuming $\beta=100$.
- 3. Calculate i_b at key down assuming a 700 mV drop-in base-emitter of Q_4 and at 600mV at D_{11}
 - $V_b = \frac{R_9}{R_9 + R_{24}} (12) \approx 3V, i_b = i_{bs} \exp(\frac{V_b}{V_t}),$
- 4. Sketch collector voltage at Q_4 showing where transistor is saturated. What is the delay in going active?
- 5. Use the delay to measure eta .



Exercise 21: Norcal Driver

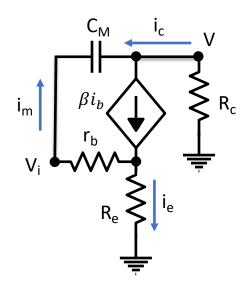
- 1. Measure the output voltage and calculate the power, P.
- 2. Calculate the power from the power supply.
- 3. Measure the voltage gain $G_v = \frac{v}{v_i}$ with R13 at minimum and maximum gain.
- $\omega = 4.4 \times 10^7$, $L_{p,T1} = 68.6 \mu H$. This is a class A amplifier.
- $R' = n^2 R, n = \frac{14}{4}, R' = 1225\Omega$
- $Z_{eq}(R) = (20 + R) + j\omega L_{p,T1}, 0 \le R \le 500. R = R_{13}$
- $Z_{eq}(0) = 20 + 2992j$,
- $i_c = \frac{V_{cc} V_{ce}}{20 + R'}$, $i_c = \beta i_b$, $V_e = 20i_c$
- From text, $P = \frac{(V_{cc} V_e)^2}{2R'}$
- $P_o(R) = \frac{{V_{cc}}^2}{(20+R+R')}$
- Gain is between 2.5 and 60



Exercise 22: Emitter degeneration

- In Driver amplifier, add probe to R_{11} , this allows us to measure the AC voltage, V_i
- 1. Measure $G_v = \frac{V}{V_i}$ with R_{13} turned fully counterclockwise and then fully clockwise.
 - $R' = 1225\Omega$
 - When R_{13} is fully counter-clockwise $R_{E,effective} = 520\Omega$, $G_v = \frac{1225}{520} = 2.36$
 - When R_{13} is fully clockwise $R_{E,effective}=20\Omega$, $G_v=\frac{1225}{20}=61$
 - Calculate the expected voltage gain for each setting of R₁₃
- 2. Measure V_i at the maximal gain setting
- 3. The open circuit voltage is $V_0=2V$, calculate V_i in terms of \mathcal{C}_M

•
$$Z_i = \beta R_E ||(G_v + 1)C_M, V_i = Z_i i_b, V = -1225 i_c \text{ so } \frac{V}{V_i} = -\frac{1225}{Z_i} \cdot \frac{i_c}{i_b} = -\beta \frac{1225}{Z_i}$$



Exercise 23: Norcal Buffer amplifier

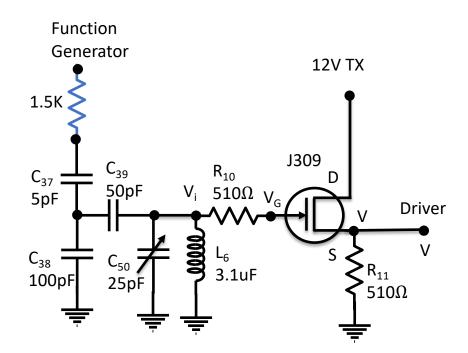
- 1. R_{11} determines the bias. Measure the DC voltage at source of the JFET (V).
- 2. Calculate the drain bias current. Calculate the source and drain voltages you should expect $(R = R_{11})$

•
$$V_{gs} = V_i - V$$
, $i_d = i_{dss} (1 - \frac{V_{gs}}{V_C})^2$, $V = g_m V_{gs} R$

- $g_m \approx 12$, $i_{dss} = 23mA$, $V_C = -2.6V$
- $V_{gs} = \frac{V_i}{1 + g_m R'}$, substitute into $i_d = i_{dss} (1 \frac{V_{gs}}{V_C})^2$ to get i_d . $i_S = \frac{V}{R}$
- 3. Calculate and measure the voltage gain of the buffer.

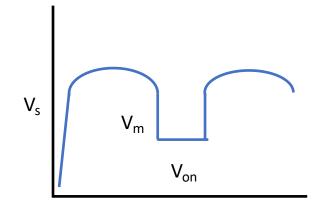
•
$$G_V = \frac{V}{V_i} = \frac{1}{1 + \frac{1}{g_m R}}$$
, or about 1 since $g_m \approx 12$

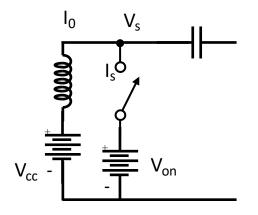
- 4. Find the transconductance using the measured voltage gain.
- 5. Calculate the available power P $_+$ from the function generator through a $1.5k\Omega$ load. Calculate gain in dB.



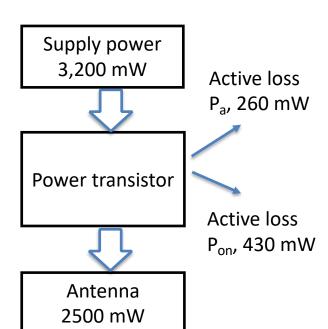
Class C amplifiers and Norcal 40 Power amp

- Here P_o is the power from the supply, P_d is the dissipated power, P is the output power. The Norcal Power amplifier is a class C amplifier. For switch model, switch represents the transistor, when the transistor is on, the switch is open.
- $V_s = V_{on} + V_m \cos(\omega t)$, (switch off), V_{on} (switch on)
- $V_{cc} = V_{on} + \frac{V_m}{\pi}, V_m = \pi(V_{cc} V_{on})$
- $P_0 = V_{cc}I_0$, $P_d = V_{on}I_0$
- $P = P_o P_d = \frac{(V_{cc} V_{on})}{\pi}$
- $\eta = \frac{P}{P_0} = \frac{(V_{cc} V_{on})}{V_{cc}}$
- $P = \frac{V_m^2}{8R}$, R is input filter impedance
- $P_d = P_0 P = 3.2W 2.5W = 700mW$
- Cap energy: $E = \frac{CV^2}{8R} = 37nJ$
- $P_a = Ef = 260mW$
- $i_c = i_0 i_c = 215mA$
- $P_{on} = V_{on}i_{on} = 430mW$
- $P_d = P_a + P_{on} = 690mW$



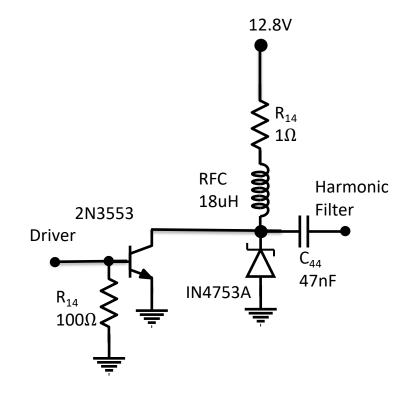


Switch model for class C amp



Exercise 24: Norcal Power Amp

- 1. Measure the peak-to-peak voltage across 50ohm load required for output of 2W. Calculate it and compare. Calculate the gain in dB
 - $V_{cc} = 12.8V$, R $\approx 50\Omega$, $I_0 = 250mA$
 - $P_{on} = V_{on}I_{on} = 430$ mW, $P_a = Ef = 260$ mA, $P_d = P_a + P_{on} = 690$ mW
 - $P_o = V_{cc}I_0 = 3.2W$.
 - $P = \frac{(\pi(V_{cc} V_{on}))^2}{8R} = 2.6W$
- 2. Find pp output voltages or 5, 10, 15, 20, 25 and 30V. Calculate power supply current subtracting 2mA for regulator
- 3. Calculate the output power, efficiency and and dissipation power.
 - $P_d = P_0 P = 3.2W 2.5W$
 - $\eta = \frac{P}{P_0} = \frac{2.5}{3.2} = .78$



Thermal modelling

- T is heat sink temperature, T_0 is ambient temperature, P_d is power dissipated.
- $R_t = \frac{T T_0}{P_d}$, R_t is the thermal resistance
- $C_t \dot{T} = P_d$, C_t is the thermal capacitance
- $R_j = \frac{T_j T}{P_d}$, T_j is the junction temperature

•
$$f(t) + \tau f(t) = f_{\infty}, f(t) = f_0 e^{-\frac{t}{\tau}}$$

•
$$P_d = \frac{T(t) - T_0}{R_t} + C_t T(t), \tau = C_t R_t, T_\infty = P_d R_t + T_0$$

•
$$T(t) + \tau T(t) = T_{\infty}, \tau = C_t R_t$$
.

•
$$T_{\infty} = P_d R_t + T_0$$

•
$$T(t) = T_{\infty} - P_d R_t e^{-\frac{t}{\tau}}$$

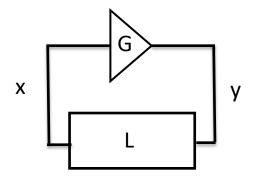
•
$$T_j(t) = T(t) + R_j P_d$$

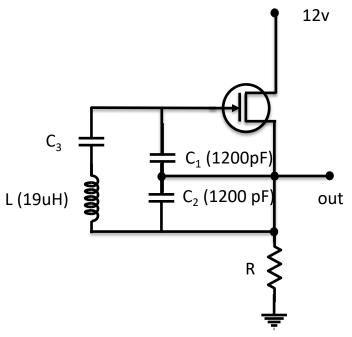
Exercise 25: Thermal modelling

- For Motorola 2N3553, $T_i = 25 \, ^{C}/_{W}$
- 1. Measure ambient temperature
- 2. Turn function generator until output is 30V_{pp}
- 3. After 20 minutes, measure T_{∞} . Use this to calculate R_t and T_i
- 4. Plot heat sink temperature vs time. Measure t_2 and calculate \mathcal{C}_t
- Need measurements

Clapp oscillator

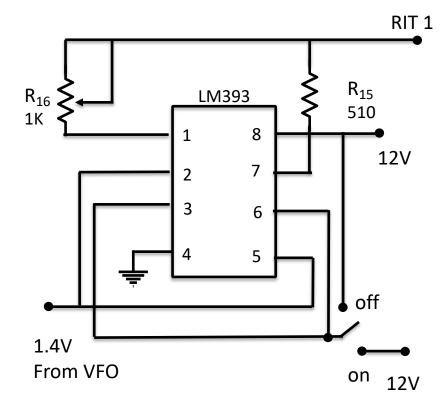
- Oscillation condition
 - Gx = y, Ly=x
 - |G| = |L| and $\angle G = \angle L$
- Clapp (circuit on right)
 - $i_d = g_m v_{gs}$
 - Resonance: $-\frac{1}{j\omega_0 c_2} = j\omega_0 L + \frac{1}{j\omega_0 c_3} + \frac{1}{j\omega_0 c_1}$
 - $\omega_0 = \frac{1}{\sqrt{LC}}, C = C_1 ||C_2||C_3$
 - At resonance, $v_{gs} = Ri_d \frac{c_1}{c_2}$, $L = \frac{c_1}{Rc_2}$
 - Oscillation continues if $g_m > \frac{c_1}{RC_2}$
 - $v_{gs} = 2v_s$





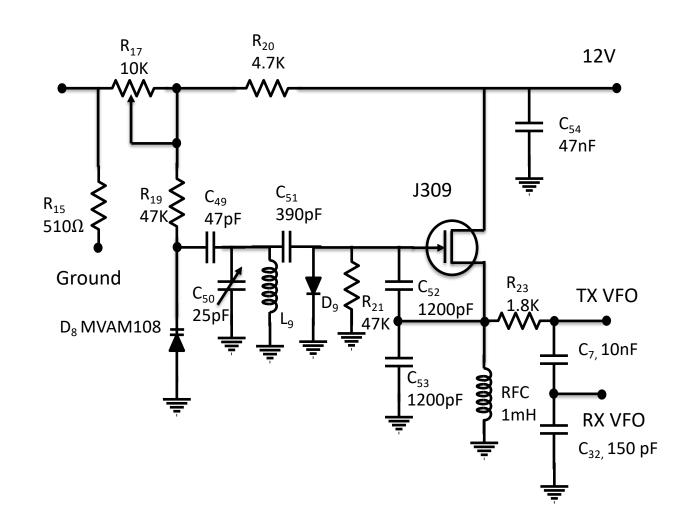
Norcal Receiver Incremental Tuning (RIT)

- LM393 is a comparator
- RIT allows transmit and receive frequency to be offset.
- If transmitter is on, TX will be 8V and the left comparator will be off, the right one on and R_{15} will be grounded.
- For receiving, TX is <1.4V, disconnecting R_{15} and shorting R_{16} to ground.



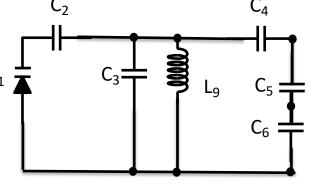
Exercise 26: Norcal VFO

- Check MVAM108 capacitor when R₁₇ is high and low
- Start resistor (R₂₁) pulls gate to ground at start
- When gain limiting diode (D₉) conducts, it pulls gate negative
- Oscillator keeps growing as long as g_m>1/R
- 1. Measure p-p voltage, V. What should you expect?
- 2. Measure DC voltage across wiper in R₁₇
- 3. Calculate expected V for large signal oscillation
- 4. How does the frequency change as R₁₇ changes?
- 5. Calculate the oscillation frequency and the loss ratio $|V/V_1|$
- 6. How would this change if you took when L₉ is turned off?



VFO Problem

- The figure on the right is an equivalent circuit for the oscillator for the purpose of calculating resonant frequency. The varactor, C_1 varies from 30 to 600 pF depending on the voltage.
- $C_2 = 47pF$, $C_3 = 7pF$, $C_4 = 390pF$, $C_5 = C_6 = 1200pF$, $L_9 = 19.2\mu H$
- The equivalent capacitance for $C_4 C_5 C_6$ is $C_{R,eq} = 236pF$.
- When $C_1=187pF$, the equivalent capacitance for $C_1-C_2-C_3$ is $C_{L,eq}=46$. 2pF and $C_{osc}=282pF$. At the resonant frequency, $\omega L_9=\frac{1}{\omega C_{osc}}$. $\omega^2=\frac{1}{L_9C_{osc}}$. $f_r=2.16MHz$
- When $C_1=54pF$, the equivalent capacitance for $C_1-C_2-C_3$ is $C_{L,eq}=32pF$. $C_{osc}=268pF$ At the resonant frequency, $\omega L_9=\frac{1}{\omega C_{osc}}$. $\omega^2=\frac{1}{L_9C_{osc}}$. $f_r=2$. 22MHz.
 - These values are what we want for the tunable VFO.



Gain Limiting in Norcal 40

- The gain here is limited by the diode. $C_1 = C_2$. $V_g = 2V_s$
- $V_g = V_f V$, V_f is the forward voltage of the diode.

•
$$V_m = V_g + \frac{V}{2}$$
, or $V_m = V_g - \frac{V}{2}$

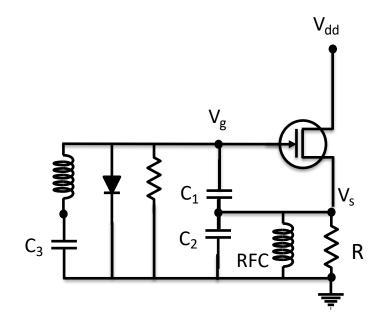
•
$$G_m = \frac{I}{V} = \frac{1}{R}$$

•
$$I_d = I_{dss} (1 - \frac{V_{gs}}{V_c})^2$$

•
$$I_o = \frac{I_m}{4}$$
, $I \approx I_m$

•
$$G_m = \frac{I_m}{V}$$

• Oscillation condition is $G_m = \frac{1}{R}$



Exercise 27: Gain limiting

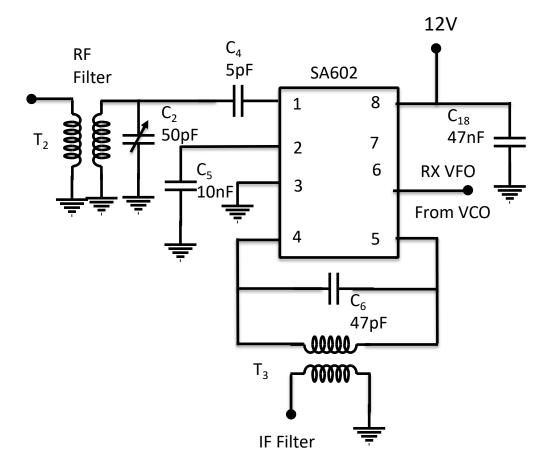
- 1. Measure the voltage, V, on R_{23}
- 2. In deriving the oscillation condition, we neglected the inductor resistance and drain source resistance, r_d . How does this affect the conditions? L₉ has a Q of 250 and $r_d = 5k\Omega$, now what is the predicted V?
- 3. Calculate the loss ratio $|\frac{V}{V_i}|$.
- 4. Measure the temperature dependence of the VFO.
- 5. How much does the temperature have to change to cause a 100Hz shift?
- 6. What is the oscillation change if we remove one turn of the inductor?
- 7. What is the RIT tuning range?

Gain limiting

Need measurements

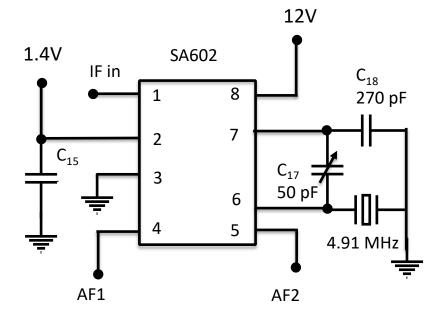
Exercise 28: Norcal RF Mixer

- 1. Measure conversion gain of the Mixer.
- 2. How much attenuation is provided by pot?
- 3. By how many dB is the image response suppressed. Look at the spur $f_{\downarrow 5}$
- Need measurements



Exercise 29: Norcal Product Detector (1)

- 1. Adjust C17 for minimum oscillation frequency and record it.
- 2. Calculate the minimum oscillation frequency you'd expect.
- 3. Measure the temperature coefficient for the BFO.
- 4. Measure the gain through the receiver from the antenna through the product detector.
- 5. Find the f5 spur calculate the expected f3.
- 6. By how much is the if spur suppressed?
- Need measurements



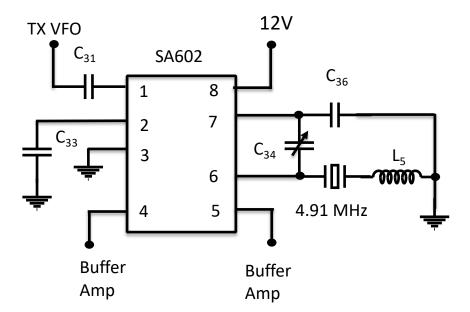
620 Hz output through AF1 and AF2

Norcal Product Detector (2)

Χ.

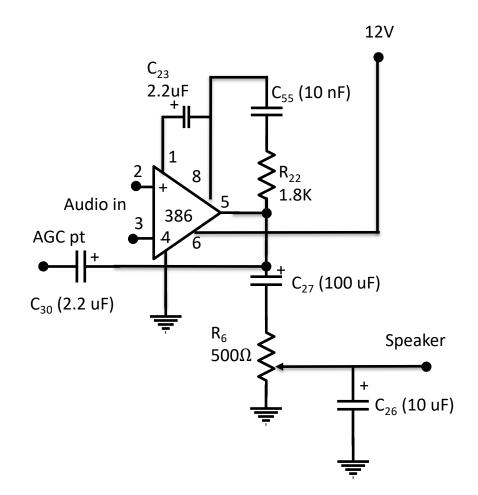
Exercise 30: Norcal transmit mixer and oscillator

- How much would you expect the inductor to lower the oscillation frequency
- 2. Use the TX VFO and the voltage attenuation to calculate the input power from the transmit mixer. Calculate the gain through the entire chain
- 3. Measure the rise and fall time of keying response
- 4. There is a spurious $f_{mn}=mf_{vfo}+nf_{to}$.
- Need measurements



Exercise 31: Norcal Audio Amp

- 1. Calculate input V_i assuming very high input impedance
- 2. Measure the voltage gain G_v at high frequency and 3dB roll-off
- Input impedance is high.
- The 386 acts like a non-inverting op amp. The internal feedback resistor is $R_f=15k\Omega$. $G=2\frac{R_f}{R_e}$. With pins 1, 8 open, $R_e=1.5k\Omega$, so $G=2\frac{15}{1.5}=20$. pins 1 and 8 go across $1.35k\Omega$ of R_e . So, shorting them (using the non-inverting gain circuit) results in a gain of $G=2\frac{15}{.15}=200$.



Exercise 32: Norcal AGC

- Connect function generator through 300K resistor to AF2 (620Hz sine, R₆ fully counterclockwise) and oscilloscope to audio output. Adjust input so output is 1V rms. Connect multimeter to P.
- 1. Plot audio output v dc control
- 2. What is the maximum control voltage we can measure? Infer cutoff voltage V_c
- 3. What is the minimum control voltage?
- Need measurements

