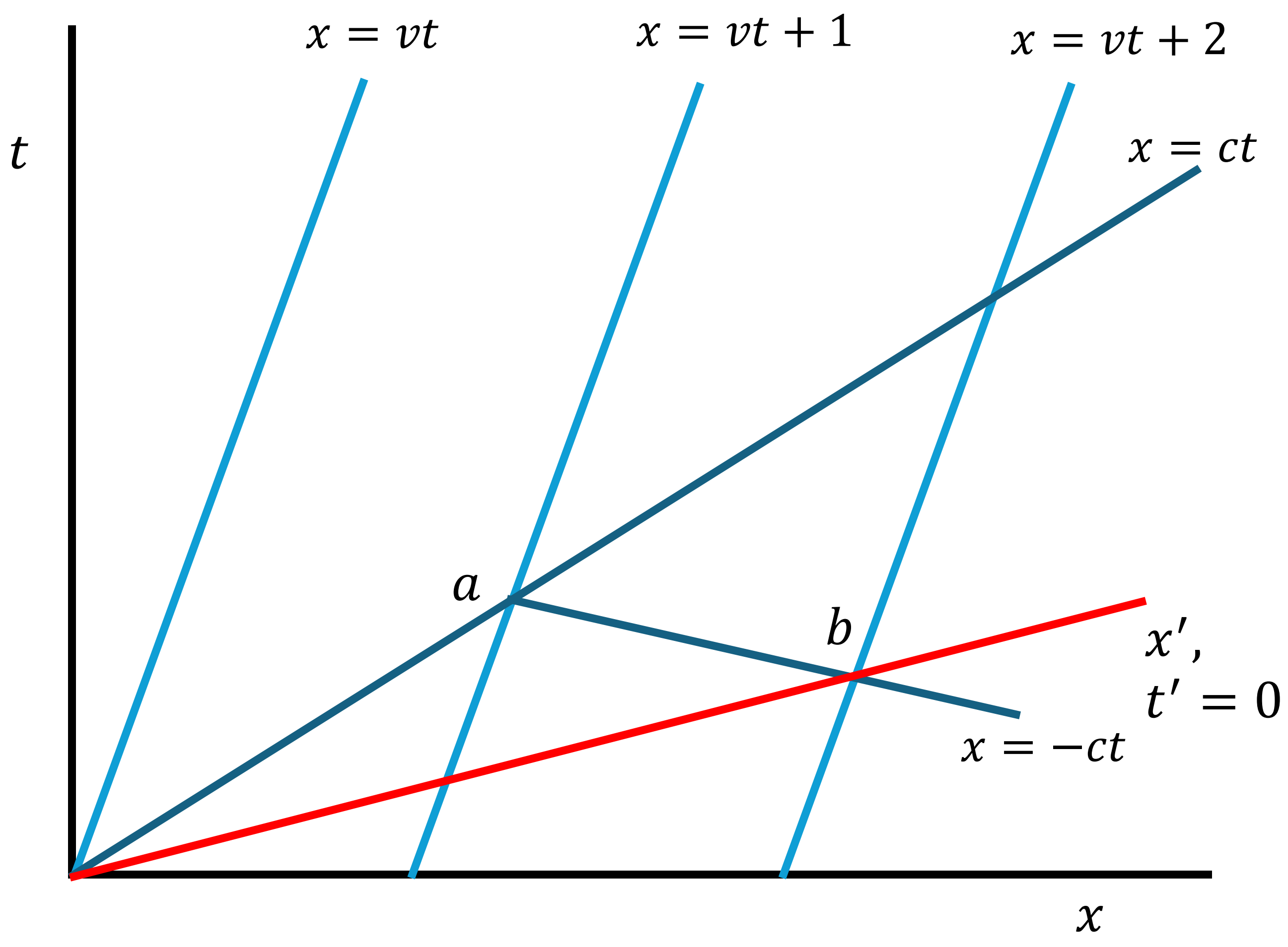
Proof of Lorentz Transformation



(x, t) frame is stationary
(x', t') frame moves to the right at speed v with respect to (x, t) frame
Light emitted at $t = 0, x = x' = 0$. Speed of light is same in both frames
Light leaves b at same time (in moving frame). Both arrive at a simultaneously.

- $ct_a = vt_a + 1$, so $t_a = \frac{1}{c-v}$
- $\frac{2c}{c-v} = x_a + ct_a = x_b + ct_b$
- Since $x_b = vt_b + 2$, $\frac{2c}{c-v} = vt_b + 2 + ct_b$
- Thus, $\frac{2c}{c-v} - 2 = (c+v)t_b$, so $t_b = \frac{2v}{c^2-v^2}$
- Now, $\frac{2c}{c-v} = x_b + ct_b$, so $\frac{2c}{c-v} - ct_b = x_b = \frac{2c^2}{c^2-v^2}$
- Finally, we get $t_b = \frac{v}{c^2} x_b$
- $x' = (x - ct) f(v^2)$, $t' = \left(t - \frac{vx}{c^2}\right) g(v^2)$
- Since $ct - x = ct' - x'$, substituting into the above equations, we get $f(v^2) = g(v^2)$
- Because these equations hold for both frames

1. $x' = (x - vt) f(v^2)$, $t' = \left(t - \frac{vx}{c^2}\right) f(v^2)$ and
2. $x = (x' + vt') f(v^2)$, $t = \left(t' + \frac{vx'}{c^2}\right) f(v^2)$

- Substituting 2 into 1, we get
- $x' = f(v^2)^2 \cdot x' \frac{1}{1-\frac{v^2}{c^2}}$ or $f(v^2) = \sqrt{1 - \frac{v^2}{c^2}}^{\,-1}$
- This yields the Lorentz transformation