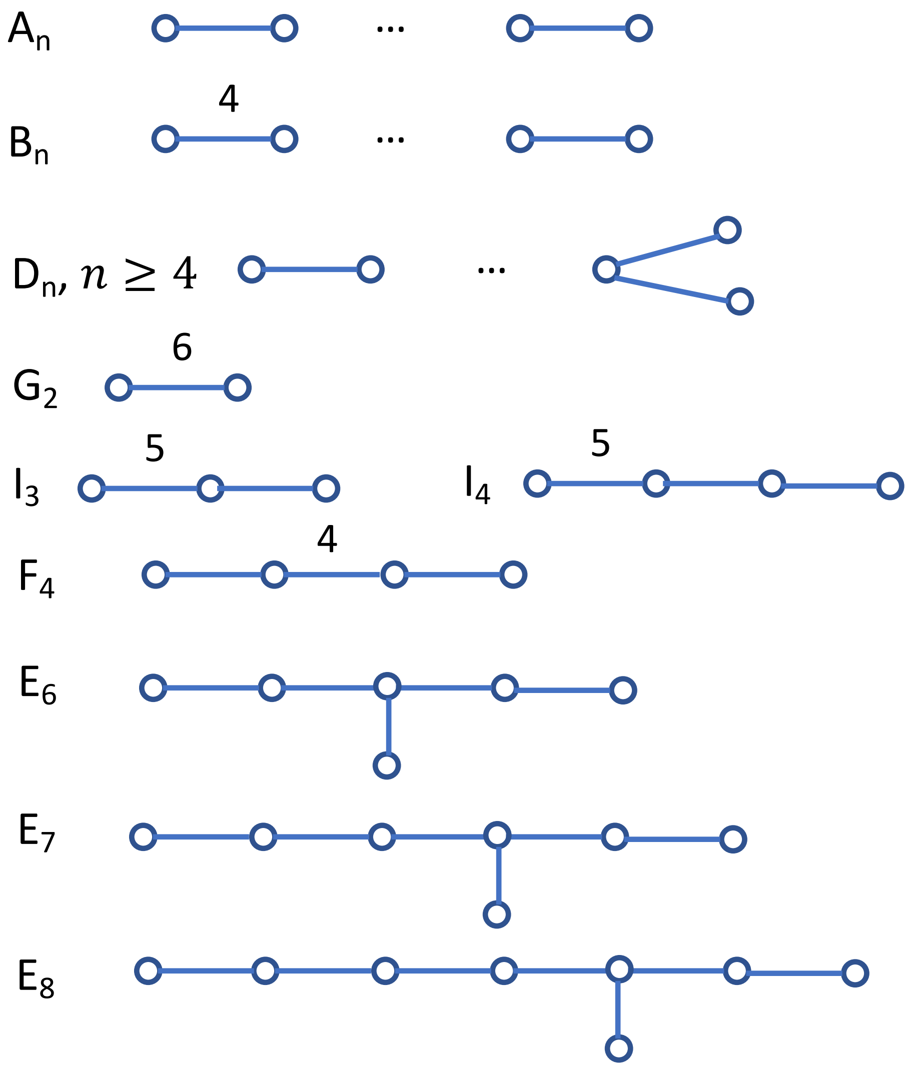
Coxeter groups



- If the root system Π is not a union of non-empty orthogonal sets, it is irreducible
- The elements of Π are called fundamental roots.
- G is connected iff G is irreducible
- If G is a connected positive definite Coxeter graph, it has one of the graphs $A_n, B_n, D_n, H_2^n, G_2, I_3, I_4, F_4, E_6, E_7, E_8$.
- G associated with $A_n, B_n, D_n, G_2, F_4, E_6, E_7, E_8$, satisfies the crystallographic condition, so $p_{ij} = 1,2,3,4,6$.
- Quadratic form for a graph is $P = (c_{ij})$ where $c_{ij} = -\cos(\frac{\pi}{p_{ij}})$ where $p_{ij} = 3$ if two nodes are connected by unlabeled edge and the label if labelled. $c_{ii} = 1$ while $c_{ij} = 0$ if nodes i and j are not connected.
- If $r_i, r_j \in \Pi$, $\frac{(r_i, r_j)}{||r_i|| \cdot r_j||} = -\cos(\frac{\pi}{p_{ij}})$. If s_i, s_j are the reflections associated with r_i, r_j, $|s_i s_j| = p_{ij}$.
- $S_r(x) = x - 2\, \frac{(x,r)}{(r,r)}\, r$