# Electronics of Radio, Part 2

## Notes on David Rutledge's book

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# NorCal power levels

# Noise

- $V_{n(rms)} = \sqrt{\frac{1}{\tau} \int_0^\tau V(t)^2 \, dt}$

- Average power: $P_n = \frac{V_{n(rms)}^2}{R}$, $R$ is load resistance

- $SNR = \frac{P}{P_n}$

- $MDS = \frac{P_n}{G}$

- $P_n = NB$, $N$ is noise power density, $B$ is bandwidth

- $NEP = \frac{N}{G}$ (Noise equivalent power)

- Noise phasors
  - $\overline{|V_n|^2} = \int |V_n|^2 p \, dA$
  - $N = \frac{\overline{|V_n|^2}}{2R}$

# Nyquist

- $V_C = \dfrac{1}{j\omega C} \dfrac{V_n}{R + j\omega L + \frac{1}{j\omega C}}$

- $\overline{|V_c|^2} = \dfrac{\overline{|V_n|^2}}{|1 - \omega^2 LC + j\omega RC|^2}$

- Expected the energy stored in C, at resonance is $kT = \dfrac{C}{2}\int_0^\infty |V_c|^2 df$, by equipartition theorem

- $\int_0^\infty \dfrac{1}{|1 - \omega^2 LC + j\omega RC|^2} df = \dfrac{1}{4RC}$

- So, $\overline{|V_n|^2} = 8kTR$, $V_{n(rms)} = \sqrt{4kTR}$

- $N = kT = \dfrac{|\frac{V_n}{2}|^2}{2R}$ (Noise density)

- $T_e = \dfrac{N}{k}$

- $T_n = \dfrac{NEP}{k}$
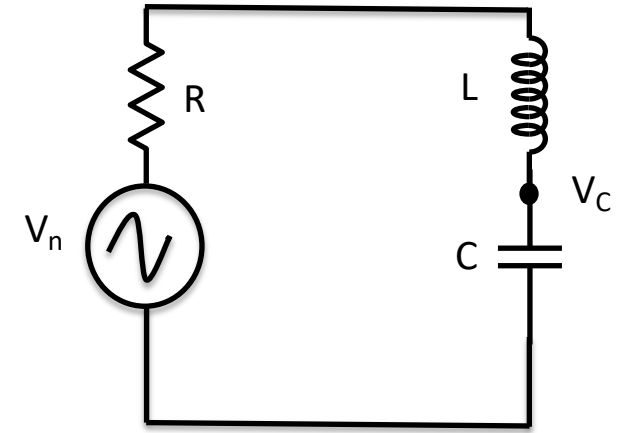
Noise model for R

Matched load

4
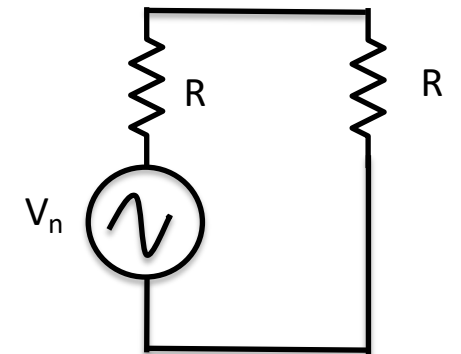
# System noise

- Attenuator (and filter) noise
  - $N_a = kT\left(1 - \frac{1}{L}\right), T_a = T(L - 1)$

$T_a$



G$_1$, N$_1$, T$_1$     G$_2$, N$_2$, T$_2$     G$_3$, N$_3$, T$_3$

Process Cascade

- $N = G_3 G_2 G_1 k T_a + G_2 G_3 N_1 + G_3 N_2 + N_3$
- $T_r = T_a + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}$
- $\frac{T_n}{T_0} = F - 1, T_0 = 290\ K$
- For NorCal, $L = 3.2$, each filter has $T_f = 290(L - 1) = 630K$
- $T_r = T_f + LT_m + \frac{T_f L}{G} + T_m \frac{L^2}{G} = 2780K$

- Want receiver noise << Antenna noise
- For NorCal, antenna noise is at least 30 dB bigger than receiver noise
- To measure noise, inject a signal that doubles the Noise power

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# Intermodulation

- If frequency components are $k_1 f_1 + k_2 f_2 + \cdots + k_n\, f_n$, modulation order is $|k_1| + |k_2| + \cdots + |k_n|$
- Intermodulation response
  - $V = G_v V_i + G_2 {V_i}^2 + G_3 {V_i}^3 + \ldots$
  - $V_i = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)$
- MDS is where input power ($P_i$) is $2P_n$. MDI is where intermodulation power is $2P_n$.
- Dynamic range = MDI-MDS.
- Good receivers have DR $\sim 100 dB$

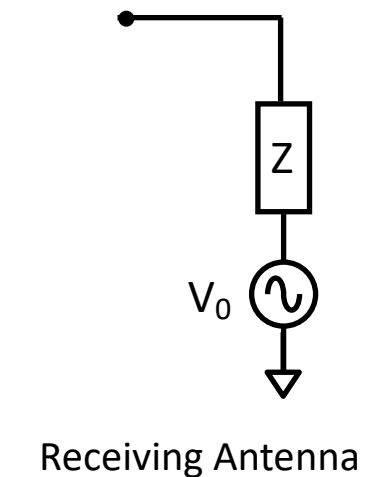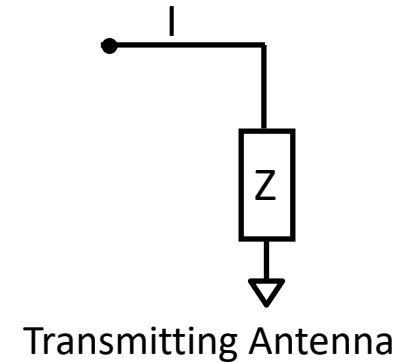

Intermodulation products

# Antennas

- From Maxwell, for a plane wave (E in x direction, H in y direction), wave is of form $\exp(j\omega t - j\beta z)$
- $\nabla \times E = -j\mu_0 \omega H$
- $\nabla \times B = j\epsilon_0 \omega E$
- $\beta \hat{z} \times E = \mu_0 \omega H, \beta E_x \hat{y} = \mu_0 \omega H$
- Substituting and taking the cross product, we get:
  - $\beta E_x = \omega \mu_0 H_y$ and $\beta H_y = -\omega \epsilon_0 E_x$
  - $\eta_0 = \dfrac{E_x}{H_y} = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = 377\Omega$
- Write $\beta = \omega \sqrt{\mu_0 \epsilon_0}$
- Power density: $S = Re\left(\dfrac{E_x \overline{H_y}}{2}\right) = \dfrac{(|E_x|)^2}{2\eta_0}$

# Transmitting Antennas

- Antenna Impedance: $P_t = \dfrac{R|I|^2}{2}$, $R$ is real part of $Z$,

- $R = R_r + R_l, \eta = \dfrac{R_r}{R}$

- Power density for isotropic antenna: $S_i = \dfrac{P_t}{4\pi r^2}$

- Define $G(\theta, \phi) = \dfrac{S(\theta,\phi)}{S_r}$. $S(\theta, \phi)$ is just the Poynting vector

- $G = \dfrac{S}{S_i} = \dfrac{4\pi r^2 S}{P_t}$

- Note: $r^2 d\Omega = dA$, $d\Omega = d\varphi d(\cos(\theta)) = \sin(\theta)d\varphi d\theta$, $\int d\Omega = 4\pi$

- $\int G \, d\Omega = \int S(\theta, \phi)\dfrac{4\pi r^2}{P_t} d\Omega = \dfrac{4\pi}{P_t}\int S(\theta,\phi)\dfrac{r^2}{P_t}d\Omega$. But $S_\Omega(\theta,\phi) = r^2 S(\theta,\phi)$, so

- $\int G \, d\Omega = \dfrac{4\pi}{P_t}\int S_\Omega(\theta, \phi)d\Omega = 4\pi$

I

Z

Transmitting Antenna

Z

$V_0$

Receiving Antenna

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# Receiving Antenna

- $V_0 = hE$, $h$ is effective antenna length ($h = \frac{l}{2}$ for short antenna)

- For dipole: $V_0 = \frac{l}{2} E \sin(\theta)$, $h = \frac{l}{2}\sin(\theta)$

- $A(\theta, \phi) = \frac{P_r}{S(\theta,\phi)}$. This is the definition of the effective area, $A$.

- By reciprocity, $A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi)$

- $P_r = \frac{|V_0|^2}{8R_a} = \frac{|hE|^2}{8R_a}$, so

- $P_r = \frac{h^2 S \eta_0}{4R}$

- $A = \frac{h^2 \eta_0}{4R}$



Receiving dipole



$V_0 = \frac{El}{2}\sin(\theta)$

Dipole Thevenin equivalent circuit

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# Friis

- For transmitting/receiving antenna pairs:
  - $G_1 A_2 = \dfrac{|V|^2 \pi r^2}{|I|^2 R_1 R_2} = G_2 A_1.$
- So $\dfrac{G_1}{A_1} = \dfrac{G_2}{A_2} = \dfrac{4\pi}{\lambda^2}$
- $S = \dfrac{P_t G}{4\pi r^2}$
- $P_r = SA = \dfrac{P_t G A}{4\pi r^2}.$  --- Friis radiation formula
- For us, $G = 1, A = 150\ m^2, r = 2000 km, P_t = 2W$
- $P_r = 6pW$
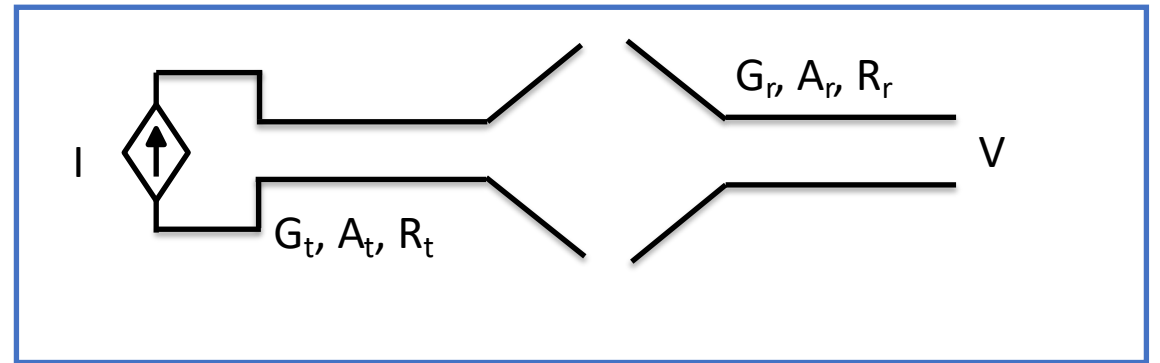
# Antenna Theorem

- Antenna theorem: $\oint A \, d\Omega = \lambda^2$
- For cavity on right, T is constant at thermodynamic equilibrium and the same power is emitted and absorbed, the Johnson noise is $kT$. The energy received is
  - $E = \dfrac{4\pi kT}{c\lambda^2}$.
  - Set $B = \dfrac{kT}{\lambda^2}$.
  - $kT = \oint BA \, d\Omega = \oint A \dfrac{kT}{\lambda^2} \, d\Omega$, which gives the antenna theorem



T      A

Insulated cavity

# Reciprocity

- *Reciprocity:* The position of an ideal voltmeter and ideal current source can be interchanged without changing the voltmeter reading.

- $P_t = |I|^2 \frac{R_1}{2}$

- $P_r = \frac{P_t G_1 A_2}{4\pi r^2}$, *by Friis*

- $P_r = \frac{|V|^2}{8R_2}$

- $\frac{|V|^2}{8R_2} = \frac{|I|^2 R_1 G_1 A_2}{8\pi r^2}$

- $\frac{|V|^2 \pi r^2}{|I|^2 R_1 R_2} = G_1 A_2$, so $G_1 A_2 = G_2 A_1$ or $\frac{G_1}{A_1} = \frac{G_2}{A_2} = \frac{G}{A}$

- $\int G \, d\Omega = 4\pi$ and by the Antenna theorem, $\int A \, d\Omega = \lambda^2$ , so

- $\frac{G}{A} = \frac{4\pi}{\lambda^2}$



$G_2, A_2, R_2$

I

$G_1, A_1, R_1$

V
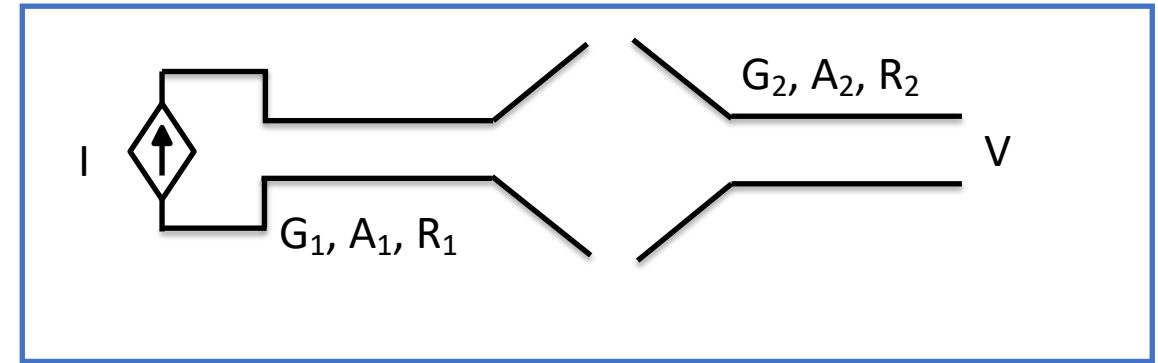
# Reciprocity and dipoles

- For dipole (Length: $l = \frac{\lambda}{2}$)

- $\lambda^2 = \int A \, d\Omega = \int \frac{h^2 \eta_0}{4R_r} \, d\Omega$, so

- $R_r = \frac{l^2 \eta_0}{16\lambda^2} \int sin^2(\theta) d\Omega = \eta_0 \frac{\pi}{6} \left(\frac{l}{\lambda}\right)^2$

- Put $\eta_0 = 120\pi$, $R_r = 20\pi^2 \frac{l^2}{\lambda^2}$ and $A = \frac{3\lambda^2}{8\pi} sin^2(\theta)$

- $G = 1.5 \, sin^2(\theta)$. Note we used $h = \frac{l}{2} \sin(\theta)$

# Exercise 35: Intermodulation

- Only $f_{3\uparrow} = 2f_1 - f_2, f_{3\downarrow} = 2f_2 - f_1, f_{5\uparrow} = 3f_1 - 2f_2 \text{ } and \text{ } f_{5\downarrow} = 3f_2 + 2f_1$ are close enough to the rf frequency to matter for intermodulation

1. Find coefficients and frequencies for $[\cos(\omega_1 t) + [\cos(\omega_2 t)]^5$
2. Find $f_{3\uparrow}, f_{3\downarrow}, f_{5\uparrow}$ and $f_1$
3. Find the MDS and the antenna limited MDR

- $MDS = \frac{P_n}{G}$, $MDI$ is input that gives output tone + $2P_n$
- $MDR = MDI - MDS$
- This needs measurements to measure the minimum detectable signal.

# Exercise 37: Antennas

1. Use the relation between gain and effective area to rewrite the Friis transmission formula in terms of gain only. Consider UHF for airplanes. If the frequency makes the quarter length stub antenna have gain 2, find the maximum possible LOS at 10km height. Required receiver power is –90 dBm. Find the minimum transmission power.

   - Friis radiation formula is $P_r = \frac{P_t G A}{4\pi r^2}$, $\lambda = \frac{c}{f}$ and $\frac{G}{A} = \frac{4\pi}{\lambda^2}$, so $P_r = P_t(\frac{Gc}{4\pi r f})^2$

   - $P_r$ requires $-90 dBm$ power. $-90 = 10\log\left(\frac{P_r}{1mW}\right)$, so $P_r$ must be at least $10^{-12}\ W$.

   - $P_t(f) = (\frac{4\pi r f}{Gc})^2 \times 10^{-12} W$. $P_t(10^8) = 4.4mW$, $P_t(2\times10^8) = 19.24mW$, $P_t(3\times10^8) = 40mW$ and $P_t(10^9) = 440mW$.

2. Find the inductance to resonate with a 3m whip. Assuming the Q of the coil is 200, find the turns ratio required to give a transceiver a 50 ohm load. What is the radiation efficiency?

   - For whip, $R_r = 160\pi^2(\frac{l}{\lambda})^2 \approx 7.8\Omega$. $Q = 200 = \frac{L}{7.8}$. $L = 1551\Omega$.

   - $Z_s = 7.8 + 1551j$. $L = 3.6\mu H$

   - $\frac{1551}{50} = \frac{Z_p}{Z_s} = (\frac{N_p}{N_s})^2$, $\frac{N_p}{N_s} = \sqrt{31} = 5.2$.

   - Radiation efficiency is $\frac{P_{radiated}}{P_{input}} = \frac{R_r}{R_r + R_L} = \frac{7.8}{57.8} \approx .14$
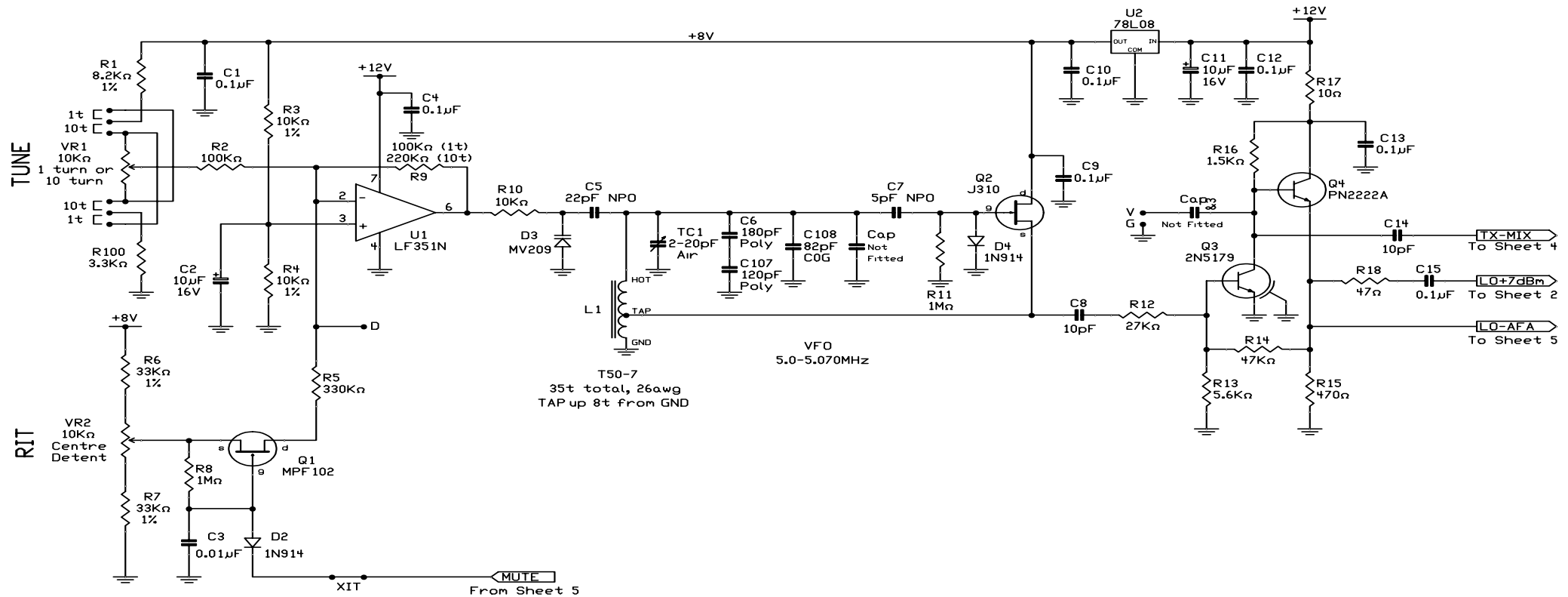
Project: **Norcal 20 CW Transceiver**

Title: **VFO**   Revision: **D**

Author: **Dave Fifield, AD6A**   Size: **A**

Date: **January 2, 1999**   Sheet **1** of **5**

# Norcal circuit diagram, 2

# Norcal circuit diagram, 3

# Norcal circuit diagram, 4

# Norcal circuit diagram, 5

# Appendix

# Exercise 17: Tuned Speaker

- Connect speaker to function generator 600Hz, 25mVrms.

1. Sound peaks at resonance.  Find resonant frequency $L_p$.
2. Measure $f_l$, $f_u$ by noting the 3dB loss.  Calculate Q.
3. Use voltmeter to find resonance with speaker (nominally 8ohm) to calculate impedance
4. Calculate the resonant frequency from a transmission line equivalent circuit.

# Morse

| Symbol | Code | Symbol | Code | Symbol | Code |
|---|---|---|---|---|---|
| a | ._ | m | __ | y | _.__ |
| b | _... | n | _. | z | __.. |
| c | _._. | o | ___ | 0 | _____ |
| d | _.. | p | .__. | 1 | .____ |
| e | . | q | __._ | 2 | ..___ |
| f | .._. | r | -_. | 3 | ...__ |
| g | __. | s | ... | 4 | ...._ |
| h | .... | t | _ | 5 | ..... |
| i | .. | u | .._ | 6 | _.... |
| j | .___ | v | ..._ | 7 | __... |
| k | _._ | w | .__ | 8 | ___.. |
| l | ._.. | x | _.._ | 9 | ____. |

# Color codes



## RESISTOR COLOR CODE GUIDE

4- Band Code

1.0 KΩ ±5%

1st 2nd 3rd 4th
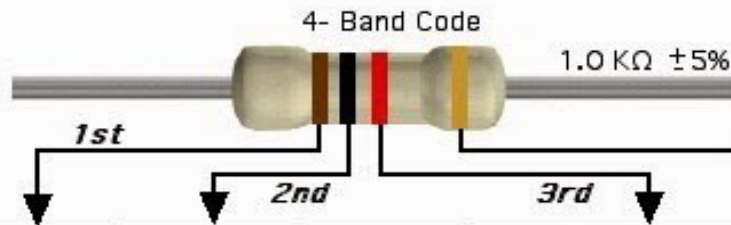
| Color | 1st Band | 2nd Band | 3rd Band | Decimal Multiplier | | Tolerance | |
|-------|----------|----------|----------|--------------------|--------|-----------|------|
| Black | 0 | 0 | 0 | 1 | 1 | | |
| Brown | 1 | 1 | 1 | 10 | 10 | ± | 1 % |
| Red | 2 | 2 | 2 | 100 | 100 | ± | 2 % |
| Orange | 3 | 3 | 3 | 1K | 1,000 | | |
| Yellow | 4 | 4 | 4 | 10K | 10,000 | | |
| Green | 5 | 5 | 5 | 100K | 100,000 | | |
| Blue | 6 | 6 | 6 | 1M | 1,000,000 | | |
| Violet | 7 | 7 | 7 | 10M | 10,000,000 | | |
| Gray | 8 | 8 | 8 | | 100,000,000 | | |
| White | 9 | 9 | 9 | | 1,000,000,000 | | |
| Gold | | | | | 0.1 | ± | 5 % |
| Silver | | | | | 0.01 | ± | 10 % |
| None | | | | | | ± | 20 % |

1st 2nd 3rd 4th 5th

254 Ω ±1 %

5- Band Code
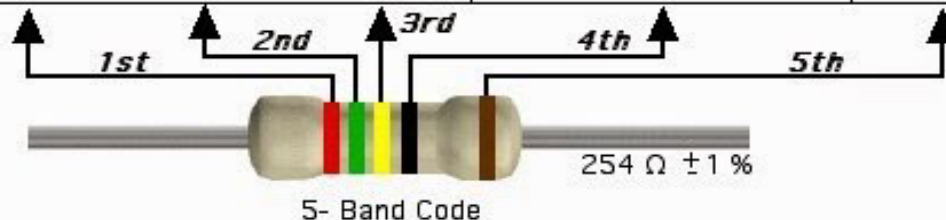
- Resistors markings in ohms
- Capacitors markings in picoFarads
- Inductors markings in microHenries

# Component data



5P  10P  20P
30P  40P  50P
60P  70P  120P



SOME IRON POWDER TOROIDS

| Core Size | 26 | 3 | 15 | 1 | 2 | 6 | 10 | 12/17 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| T-12-( ) | * | 60 | 50 | 48 | 20 | 17 | 12 | 7.5 | 2.4 |
| T-16-( ) | 145 | 61 | 55 | 44 | 22 | 19 | 13 | 8 | 3 |
| T-20-( ) | 185 | 76 | 65 | 52 | 25 | 22 | 16 | 10 | 3.5 |
| T-25-( ) | 245 | 100 | 85 | 70 | 34 | 27 | 19 | 12 | 4.5 |
| T-30-( ) | 335 | 140 | 93 | 85 | 43 | 36 | 25 | 16 | 6 |
| T-37-( ) | 285 | 120 | 90 | 80 | 40 | 30 | 25 | 15 | 4.9 |
| T-44-( ) | 370 | 180 | 160 | 105 | 52 | 42 | 33 | 18.5 | 6.5 |
| T-50-( ) | 330 | 175 | 135 | 100 | 49 | 40 | 31 | 18 | 6.4 |
| T-68-( ) | 435 | 195 | 180 | 115 | 57 | 47 | 32 | 21 | 7.5 |
| T-80-( ) | 460 | 180 | 170 | 115 | 55 | 45 | 32 | 22 | 8.5 |
| T-94-( ) | 600 | 248 | 200 | 160 | 84 | 70 | 58 | * | 10.6 |
| T-106-( ) | 930 | 450 | 345 | 325 | 135 | 116 | * | * | 19 |
| T-130-( ) | 810 | 350 | 250 | 200 | 110 | 96 | * | * | 15 |
| T-157-( ) | 1000 | 420 | * | 320 | 140 | 115 | * | * | * |
| T-184-( ) | 1690 | 720 | * | 500 | 240 | 195 | * | * | * |
| T-200-( ) | 920 | 425 | * | 250 | 120 | 100 | * | * | * |
| T-200A-( ) | 1600 | * | * | * | 218 | * | * | * | * |

***IRON POWDER TOROIDS - A$_l$ Values ****
* size not available in this material
** L= µH/100 turns

# In the beginning...

- The laws of EM according to Clerk Maxwell are:

1. $\nabla \cdot \boldsymbol{E} = \dfrac{\rho}{\epsilon_0}$

2. $\nabla \cdot \boldsymbol{B} = 0$

3. $\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$

4. $c^2 \, \nabla \times \boldsymbol{B} = \dfrac{\boldsymbol{j}}{\epsilon_0} + \dfrac{\partial \boldsymbol{E}}{\partial t}$, $\epsilon_0 = 8.854 \times 10^{-12} \dfrac{C^2}{N-m^2}, \dfrac{1}{c^2} = \epsilon_0 \mu_0$

5. $\nabla \cdot \boldsymbol{j} = -\dfrac{\partial \rho}{\partial t}$

   - Here **E** is the electric field, **B** is the magnetic field, **j** is the current density through a closed surface, c is the speed of light and $\rho$ is the charge density at a point.
   - In non-dispersive matter, $B = \mu H = \mu_0(H + M), \mu = \mu_0(1 + \chi_m),$
   - $D = \epsilon E = \epsilon_0 E + P, \epsilon = \kappa \epsilon_0$ and (1) becomes $\nabla \cdot \boldsymbol{D} = \rho_f$, (4) becomes $\nabla \times \boldsymbol{H} = \boldsymbol{j_f} + \dfrac{\partial \boldsymbol{D}}{\partial t}$

- The rest of classical physics, including special relativity, is:
   - Newton-Einstein: $\boldsymbol{p} = m\boldsymbol{v}, m = \dfrac{m_0}{\sqrt{1-(\frac{v}{c})^2}}, \boldsymbol{F} = m\dfrac{d\boldsymbol{p}}{dt}.$
   - Gravity: $\boldsymbol{F} = -\dfrac{Gm_1 m_2}{r^2} \boldsymbol{u}_r$ where **u$_r$** is the unit vector from $m_1$ to $m_2$ and **F** is the force on $m_2$.

# Solutions to the wave equation

- The solution of $\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = -s$ is $\psi(x, y, z, t) = \frac{S(t-\frac{r}{c})}{4\pi r}$ where $S = \int_V s\, dV$

- Later, we will use this to find the "general" solution to Maxwell's equations

  - $\phi(r_1, t) = \int_{V_2} \frac{\rho(r_2, t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0|r_1-r_2|} dV_2$ and $\mathbf{A}(r_1, t) = \int_{V_2} \frac{\mathbf{j}(r_2, t-\frac{|r_1-r_2|}{c})}{4\pi\epsilon_0 c^2|r_1-r_2|} dV_2$, where

  - $B = \nabla\times A$, $E = -\nabla\phi - \frac{\partial A}{\partial t}$, and $c^2\nabla \cdot A = -\frac{\partial \phi}{\partial t}$

- You are not expected to have guessed this answer

- To do this, we'll need the "BAC-CAB" identity: $A\times(B\times C) = B(A \cdot C) - C(A \cdot B)$

- When we apply this to $\nabla\times(\nabla\times\mathbf{A})$, we get $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla \cdot A) - \nabla^2 A$

# General solution to Maxwell's equations

- Returning to the general Maxwell equations, from $\nabla \cdot B = 0$, we get $B = \nabla \times A$

- Substituting into $c^2 \nabla \times \mathbf{B} = \frac{j}{\epsilon_0} + \frac{\partial \mathbf{E}}{\partial t}$, we get $c^2 \nabla \times (\nabla \times A) = \frac{j}{\epsilon_0} + \frac{\partial E}{\partial t}$

- Applying "BAC-CAB", we get $\nabla (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0} + \frac{1}{c^2} \frac{\partial E}{\partial t}$ (Equation 1)

- Now, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, so substituting for B, we get $\nabla \times \left( \mathrm{E} + \frac{\partial A}{\partial t} \right) = 0$ and so $E = -\nabla \phi - \frac{\partial A}{\partial t}$

- Substituting into equation 1, $\nabla (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0} + \frac{1}{c^2} \frac{\partial}{\partial t} (-\nabla \phi - \frac{\partial A}{\partial t})$, or

- $\nabla (\nabla \cdot A) - \nabla^2 A = \frac{j}{c^2 \epsilon_0} - \frac{1}{c^2} \nabla \frac{\partial \phi}{\partial t} - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2}$.

- $\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2 \epsilon_0} + \nabla [\frac{1}{c^2} \frac{\partial \phi}{\partial t} + (\nabla \cdot A)]$

- Now if $A$ and $\phi$ give $B = \nabla \times A$ and $E = -\nabla \phi - \frac{\partial A}{\partial t}$, then $A' = A + \nabla \varphi$ and $\phi' = \phi - \frac{\partial \varphi}{\partial t}$ give $B = \nabla \times A'$ and $E = -\nabla \phi' - \frac{\partial A'}{\partial t}$, for any function $\varphi$

# General solution to Maxwell's equations

- Thus, we can pick a solution $(A, \phi)$ with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$. Then we get

- $\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2 \epsilon_0}$

- Substituting for $E = -\nabla \phi - \frac{\partial A}{\partial t}$ into $\nabla \cdot \mathrm{E} = \frac{\rho}{\epsilon_0}$, we get

- $\nabla \cdot (\nabla \phi) + \frac{\partial\, \nabla \cdot A}{\partial t} = -\frac{\rho}{\epsilon_0}$, or $\nabla \cdot (\nabla \phi) - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$, or $\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$

- The solutions are $\phi(r_1, t) = \int_{V_2} \frac{\rho(r_2, t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 |r_1 - r_2|} dV_2$ and

- $\mathbf{A}(r_1, t) = \int_{V_2} \frac{\mathbf{j}(r_2, t - \frac{|r_1 - r_2|}{c})}{4\pi\epsilon_0 c^2 |r_1 - r_2|} dV_2$ with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$, with $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$.

# Solution to Maxwell's equations in free space

- Free space is defined by $\rho = 0$ and $j = 0$, so our potentials satisfy

- $\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$ and $\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0$

- These have the usual wave equation solutions $\phi(x, y, z, t) = f(k \cdot r - \omega t)$, etc

- Thus, in free space $\phi$ and **A** and hence E and B propagate as waves.

# Solution to Maxwell's equations in conductors

- In conductors, $\mathbf{j} = \sigma\mathbf{E}$
  - $c^2\,\nabla\times\mathbf{B} = \dfrac{j}{\epsilon} + \dfrac{\partial\mathbf{E}}{\partial t} = \dfrac{\sigma}{\epsilon}\mathbf{E} + \dfrac{\partial\mathbf{E}}{\partial t}$
  - This becomes $c^2\,\dfrac{\partial(\nabla\times\mathbf{B})}{\partial t} = \dfrac{\sigma}{\epsilon}\dfrac{\partial\mathbf{E}}{\partial t} + \dfrac{\partial^2\mathbf{E}}{\partial t^2}$
  - $\nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$, so we get $c^2\nabla\times\dfrac{\partial B}{\partial t} = -c^2\nabla\times(\nabla\times E) = \dfrac{\sigma}{\epsilon}\dfrac{\partial\mathbf{E}}{\partial t} + \dfrac{\partial^2\mathbf{E}}{\partial t^2} = -c^2[\nabla(\boldsymbol{\nabla}\cdot\mathbf{E}) - \nabla^2\mathbf{E}] = c^2\boldsymbol{\nabla^2}\mathbf{E}$ (since $\rho = 0$ in a conductor)
  - Applying the trial solution E= $E_0$ exp($\omega$t-kr), we get $-k^2-i\omega\mu\sigma + \omega^2\mu\epsilon$ = 0.
  - Putting k = $\alpha$-$\beta$i, $\alpha = \dfrac{\omega}{2}\sqrt{\mu\epsilon}\,(1 + \sqrt{1 + \dfrac{\sigma^2}{\omega^2\epsilon^2}})$ and $\beta = \dfrac{\omega\mu\sigma}{2\alpha}$.
  - For copper, $\sigma$ = 5.78 × 107 $\boldsymbol{\Omega}$-m.  This explains the "skin effect" in conductors.

# Radiation, antennas

- Accelerating charges radiate energy in the form of electromagnetic waves (companion E and B fields).

- The radiation from accelerating charge q is $\mathbf{E}_{rad} = -\frac{1}{4\pi\epsilon_0 c^2}\frac{q}{r}\boldsymbol{a}_\perp(t - \frac{r_{12}}{c})$.

- Here, $\mathbf{a}_\perp$ is the acceleration $\perp$ to the line from $r_1$ to $r_2$.

- For example, applying a time varying potential $V_0\sin(\omega t)$ to an antenna will cause the antenna to radiate power since the voltage and hence charges affected accelerate within the antenna, that is, their positions have a non-zero second derivative.  That's how a transmitter "couples" to the antenna of a receiver.  In the receiver, the radiated wave accelerates charges in the antenna replicating the original wave (at much reduced power).

- These simple radio waves are carrier waves of frequency $\frac{\omega}{2\pi}$.  To transfer information (voice, images, binary data), we modulate carrier waves combining them with an "information source" signal.  Receivers demodulate the incoming wave and recreate the original "information source" signal.

# Maxwell's equations in a non-dispersive media

- $B = \mu H, D = \epsilon E$
- $\nabla \cdot D = \rho$
- $\nabla \cdot B = 0$
- $\nabla \times E = -\dfrac{\partial B}{\partial t}$
- $\nabla \times H = j + \dfrac{\partial E}{\partial t}$
- $\nabla \cdot j = -\dfrac{\partial \rho}{\partial t}$

# Radiation from a small dipole

- $A(1,t) = \int \frac{\vec{j}\left(2, t - \frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}}\, dV_2$.  From figure on right,  $j_z\, dV_2 = I\, dz'$.

- Note $\vec{p}(t) = q(t)l\vec{k}$, $k = \hat{z}$ and $\dot{q}(t) = I$.  $\phi$ is the angle from the x axis to the projection of r on the x-y plane.

- $\vec{r'} + z'\vec{k} = \vec{r}$ and $|\vec{r} - z'\vec{k}| = r - z'\cos(\theta)$

- If $l \ll cT = \lambda$, $I\left(\vec{z'}, t - \frac{r'}{c}\right) \approx I\left(0, t - \frac{r'}{c}\right)$ and we get:

- $A_z(r,t) = \frac{\mu_0}{4\pi} \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{I\left(\vec{z'}, t - \frac{r'}{c}\right)}{4\pi\epsilon_0 c^2 r'}\, dz' = \frac{\mu_0}{4\pi} \frac{l}{r} I\left(0, t - \frac{r'}{c}\right)$

- Choosing gauge, $\nabla \cdot A + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$

- $\frac{\partial\varphi}{\partial t} = -\frac{l}{4\pi\epsilon_0}\frac{\partial}{\partial z}\left(\frac{1}{r} I\left(t - \frac{r}{c}\right)\right) = \frac{z}{r^2}\left(\frac{q\left(t - \frac{r}{c}\right)}{r} - \frac{I\left(\left(t - \frac{r}{c}\right)\right)}{c}\right)$

- Pick and oscillating dipole: $q\left(t - \frac{r}{c}\right) = q_0 \cos\left(\omega\left[t - \frac{r}{c}\right]\right)$

- $I\left(t - \frac{r}{c}\right) = I_0 \sin\left(\omega\left[t - \frac{r}{c}\right]\right) = -\omega q_0 \sin\left(\omega\left[t - \frac{r}{c}\right]\right)$



$q(t)$        $1$ is where $A(1,t)$ is being measured

$\frac{\ell}{2}$   $z'$   $\vec{r'}$   $\vec{r}$   $1$

$\theta$

y

$\frac{\ell}{2}$

z

$-q(t)$

# Radiation from a small dipole

- $\nabla^2 H - \epsilon\mu \dfrac{\partial^2 H}{\partial t^2} - \sigma\mu \dfrac{\partial H}{\partial t} = 0$

- $\nabla^2 E - \epsilon\mu \dfrac{\partial^2 E}{\partial t^2} - \sigma\mu \dfrac{\partial E}{\partial t} = 0$

- $A_r = \dfrac{\mu_0}{4\pi} \dfrac{I_0 l}{r} \cos(\theta) \sin(\omega \left[ t - \dfrac{r}{c} \right])$

- $A_\phi = 0, \ A_\theta = -\dfrac{\mu_0}{4\pi} \dfrac{I_0 l}{r} \cos(\theta) \sin(\omega \left[ t - \dfrac{r}{c} \right])$

- $B_\phi = \dfrac{1}{r} \dfrac{\partial}{\partial r}(r A_\theta) - \dfrac{1}{r} \dfrac{\partial A_r}{\partial \theta} = \dfrac{\mu_0}{4\pi} \dfrac{I_0 l}{r} \sin(\theta)[\dfrac{\omega}{r}\cos\left(\omega \left[ t - \dfrac{r}{c} \right]\right) + \dfrac{1}{r}\sin(\omega[t - \dfrac{r}{c}])]$

- $E_r = -\dfrac{\partial\phi}{\partial t} - \dfrac{\partial A_r}{\partial t} = \dfrac{2 l I_0 \cos(\theta)}{4\pi\epsilon_0} [\dfrac{\sin(\omega[t - \frac{r}{c}])}{r^2 c} - \dfrac{\cos(\omega[t - \frac{r}{c}])}{\omega r^3}]$

- $E_\theta = \dfrac{-I_0 \, l \sin(\theta)}{4\pi\epsilon_0} (\left[ \dfrac{1}{r^3\omega} - \dfrac{\omega}{rc^2} \right] \cos\left(\omega \left[ t - \dfrac{r}{c} \right]\right) - \dfrac{1}{cr^2}\sin\left(\omega \left[ t - \dfrac{r}{c} \right]\right))$

- $E_\phi = -\dfrac{1}{r\sin(\theta)} \dfrac{\partial\varphi}{\partial\phi} - \dfrac{\partial A_\phi}{\partial t} = 0$

# Radiation from a small dipole

- $\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\mathbf{H} \dfrac{\partial \mathbf{B}}{\partial t} - \mathbf{E} \dfrac{\partial \mathbf{D}}{\partial t} - \mathbf{E} \cdot \mathbf{j}$

- $\mathbf{S} = \mathbf{E} \times H, \nabla \cdot \mathbf{S} + \dfrac{\partial u}{\partial t} = -\mathbf{E} \cdot \mathbf{j}$ (u is energy density)

- $\int S \cdot dA = \dfrac{(l\, I_0\, \omega)^2}{6\pi\epsilon_0 c^3} \cos(\omega \left[ t - \dfrac{r}{c} \right])^2$

- $P_{av} = \dfrac{(l\omega)^2}{6\pi\epsilon_0 c^3} \dfrac{I_0^{\,2}}{2} = \dfrac{2\pi}{3} \sqrt{\dfrac{\mu_0}{\epsilon_0}} \, (\dfrac{l}{\lambda})^2 \dfrac{I_0^{\,2}}{2}$

- $R_r = \dfrac{2\pi}{3} \sqrt{\dfrac{\mu_0}{\epsilon_0}} \, (\dfrac{l}{\lambda})^2$

# Large half wave dipole

- For large half wave, add small dipoles to produce half wave antenna.

- $dE_\theta = I_0 \frac{\sin(\theta)}{4\pi\epsilon_0 Rc^2} \omega \cos(\omega) \cos\left(\frac{2\pi z'}{\lambda}\right) dz'$

- $dB_\phi = I_0 \frac{\mu_0 \omega}{4\pi Rc} \omega \cos\left(\omega\left[\left[t - \frac{r}{c}\right]\right]\right) \cos\left(\frac{2\pi z'}{\lambda}\right) dz'$

- Suffices to find: $K = \int_{[-\frac{\pi}{2},\frac{\pi}{2}]} \frac{1}{R} \cos\left(t - \frac{R}{c}\right) \cos(u)\, du = \frac{1}{2\pi\epsilon_0 rc} \cos\left(\omega\left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)}, u = \frac{2\pi z'}{\lambda}$

- $E_\theta = I_0 \frac{1}{2\pi\epsilon_0 rc} \cos\left(\omega\left[t - \frac{r}{c}\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)}$

- $B_\phi = I_0 \frac{\mu_0}{2\pi r} \omega \cos\left(\omega\left[\left[t - \frac{r}{c}\right]\right]\right) \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)}$

- $P_{av} = \frac{1}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} I_0^2 \int_{[0,\pi]} \frac{\cos^2\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)} \sin(\theta) d\theta = 73.1\Omega \frac{I_0^2}{2}$

# Radiation from charge distribution

- $r' + R = r, \ R = |r - r'|$

- $\varphi(r, t) = \frac{1}{4\pi\epsilon_0} \int_{V_1} \frac{\rho(r', t - \frac{R}{c})}{|r - r'|} dv' = \frac{1}{4\pi\epsilon_0} \left[ \frac{Q}{r} + \frac{r \cdot p(t - \frac{r}{c})}{r^3} + \frac{r \cdot \frac{dp}{dt}(t - \frac{r}{c})}{cr^2} \right]$

- $\boldsymbol{A}(r, t) = \frac{\mu_0}{4\pi} \int_{V_1} \frac{\boldsymbol{j}(r', t - \frac{R}{c})}{|r - r'|} dv' = \frac{\mu_0}{4\pi r} \frac{d}{dt} \boldsymbol{p}(t - \frac{r}{c})$

- $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \varphi$

- $\boldsymbol{B}(r, t) = \frac{-\mu_0}{4\pi c r^2} \boldsymbol{r} \times \ddot{\boldsymbol{p}}(t - \frac{r}{c})$

- $\boldsymbol{E}(r, t) = -\frac{c}{r} \boldsymbol{r} \times \boldsymbol{B}(r, t)$

- $\frac{d\boldsymbol{p}}{dt} = q \frac{dr'}{dt} = qv, \ \frac{d^2}{dt^2} \boldsymbol{p}(t - \frac{r}{c}) = q \frac{dv}{dt}$

- $S = \frac{1}{\mu_0} E \times B = \frac{\ddot{p}^2 \sin(\theta)^2}{16\pi^2 \epsilon_0 c^3 r^r} \frac{\vec{r}}{r}$

- $P_R = -\frac{dW}{dt} = \frac{2}{3} \frac{q^2}{4\pi\epsilon_0 c^3} (\frac{dv}{dt})^2$

- Compare to Feynman later



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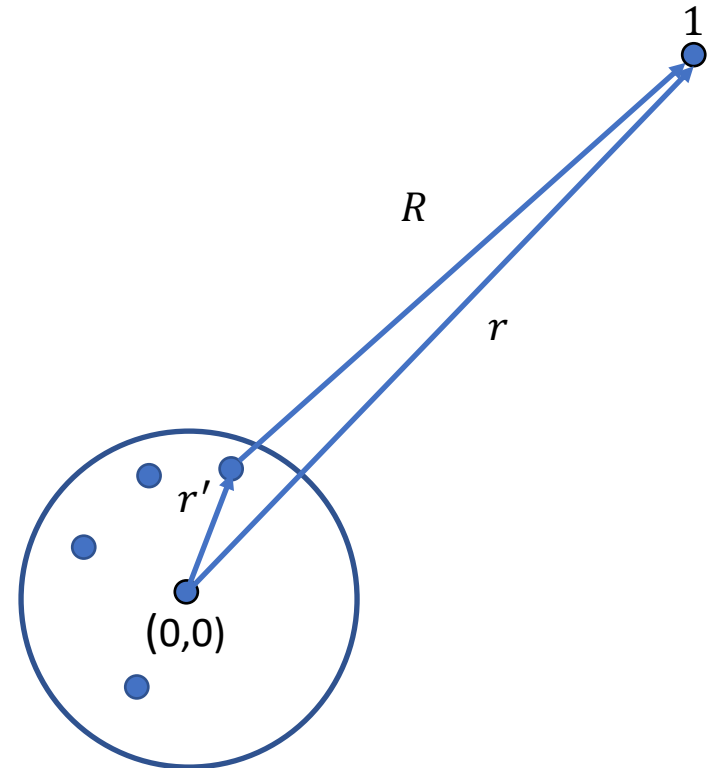
# Radiation from a single accelerating charge

- Near zone

  - $\varphi(r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R'(1+\frac{v\cdot n'}{c})}\right]$

  - $R^* = R' - \frac{v}{c}(x_0 - x'_1)$

  - $E(r,t) = \frac{1}{4\pi\epsilon_0}\frac{1}{R^{*3}}\left[\left(R' - \frac{R'\,v'}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \frac{R'}{c^2}\left(R' - \frac{R'v'}{c}\right)\times\frac{dv'}{dt}\right]$

  - $B = \frac{R'\times E}{R'c}$

  - $S = \frac{q^2}{16\pi^2 c^3 \epsilon_0}\frac{R'(R'\times v')^2}{(R')^5}$

# Feynman's Treatment - 1

- Want to derive the Heaviside-Feynman equation:

    - $E = \frac{q}{4\pi\epsilon_0}\left[\frac{e_{r\prime}}{r\prime^2} + \frac{r\prime}{c}\frac{d}{dt}\frac{e_{r\prime}}{r\prime^2} + \frac{1}{c^2}\frac{d^2 e_{r\prime}}{dt^2}\right]$

    - $cB = e_{r\prime} \times E$

- Solution to Maxwell:

    - $E = -\nabla\phi - \frac{\partial A}{\partial t}, B = \nabla\times A$

    - $\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}, \nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{j}{c^2\epsilon_0}$

    - With gauge $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$

- Consider solution to $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = -s$

- First solve $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0, \psi(x,t,z,t) = \frac{f(t-\frac{r}{c})}{r}$

- As $r \to 0, \psi(x,t,z,t) = \frac{f(t)}{r}$.

- Consider the electrostatic case where $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$ where

    $\phi = \frac{q}{4\pi\epsilon_0 r}$; by analogy:

    - $f\left(t - \frac{r}{c}\right) = \frac{S(t-\frac{r}{c})}{4\pi}$

    - $S = \int s \, dV$

- So, solution is: $\psi(x,t,z,t) = \frac{1}{4\pi}\frac{S(t-\frac{r}{c})}{r}$. Thus,

    - $A(1,t) = \int\frac{\vec{j}\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}} dV_2$

    - $\phi(1,t) = \int\frac{\rho\left(2,t-\frac{r_{12}}{c}\right)}{4\pi\epsilon_0 r_{12}} dV_2$

    - $E = -\nabla\phi - \frac{\partial A}{\partial t}, B = \nabla\times A$

    - With gauge $\nabla \cdot A = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$

# Feynman's Treatment - 2

- For small blob of charge moving with velocity v, $j = v\rho$.

- $A(1,t) = \int \frac{\vec{j}\left(2, t - \frac{r_{12}}{c}\right)}{4\pi\epsilon_0 c^2 r_{12}} dV_2 = \frac{1}{r}\int v\rho dV_2 = \frac{qv}{r}$

- So, for a dipole with $\vec{p}(r,t) = qd\vec{k}$, $\frac{qv}{r} = \frac{\partial \vec{p}}{\partial t}$

- $A(1,t) = \frac{\dot{p}\left(t - \frac{r}{c}\right)}{4\pi\epsilon_0 c^2 r}$, $B = \nabla \times A$ and $B_z = 0$. $A$ is due to dipole

  current. If the dipole charges oscillate $p(t) = p_0 \sin(\omega t)$,

  $A_z(1,t) = \frac{1}{4\pi\epsilon_0 c^2 r} p_0 \omega \cos\left(\omega\left[t - \frac{r}{c}\right]\right)$

- $B_x = \frac{\partial A_z}{\partial y} = \frac{1}{4\pi\epsilon_0 c^2 r} \frac{\partial \dot{p}\left(t - \frac{r}{c}\right)}{\partial y}$. $B_y = -\frac{\partial A_z}{\partial x} = -\frac{1}{4\pi\epsilon_0 c^2 r} \frac{\partial \dot{p}\left(t - \frac{r}{c}\right)}{\partial x}$

- $B_x = \frac{1}{4\pi\epsilon_0 c^2 r}\left[-\frac{y\left(t - \frac{r}{c}\right)}{r^3} - \frac{y\ddot{p}\left(t - \frac{r}{c}\right)}{cr^2}\right]$, $B_y = \frac{1}{4\pi\epsilon_0 c^2 r}\left[\frac{x\dot{p}\left(t - \frac{r}{c}\right)}{r^3} + \frac{x\ddot{p}\left(t - \frac{r}{c}\right)}{cr^2}\right]$

- Rewrite this as

  - $\vec{B} = \frac{1}{4\pi\epsilon_0 c^2 r^3}\left[\dot{p}\left(t - \frac{r}{c}\right) + \frac{r}{c}\ddot{p}\left(t - \frac{r}{c}\right)\right] \times r$

  - Compare this to Biot Savart: $dB = \frac{1}{4\pi\epsilon_0 c^2} \frac{j \times r}{r^3} dV$

- Now suppose the dipole charges oscillate $q(t) = q_0 \sin(\omega t)$

- When $\frac{r}{c}$ is small, $\dot{p}\left(t - \frac{r}{c}\right) = \dot{p}(t) - \frac{r}{c}\ddot{p}(t) + \dots$

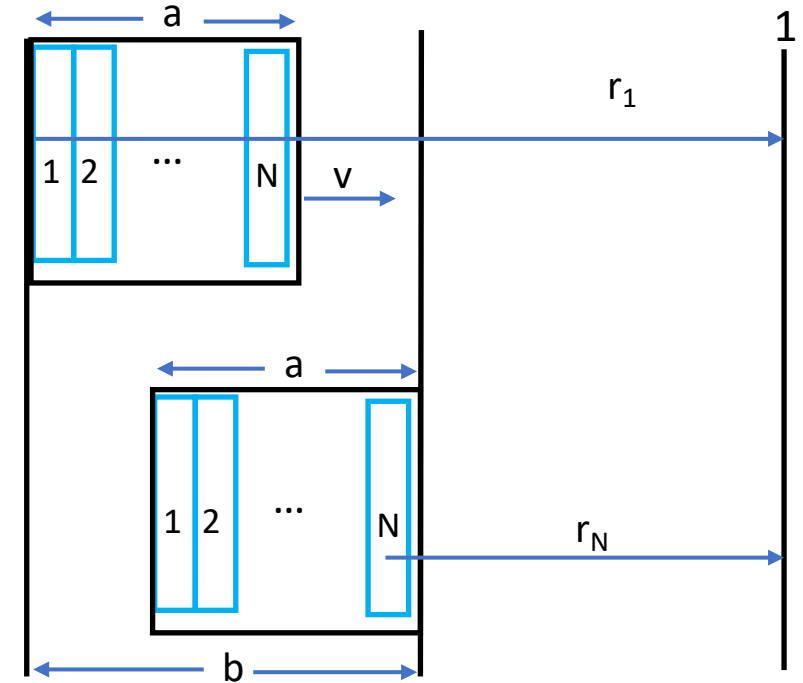- $E \perp B$, $E = cB$ and from $\frac{\partial \phi}{\partial t} = -\nabla \cdot A$, we get

- $\phi = \frac{1}{4\pi\epsilon_0 r^3}\left[p\left(t - \frac{r}{c}\right) + \frac{r}{c}\dot{p}\left(t - \frac{r}{c}\right)\right] \cdot r$, and

- $E = -\nabla\phi - \frac{\partial A}{\partial t} = \frac{1}{4\pi\epsilon_0 c^2 r^3}\left[\frac{3(p^* \cdot r)\vec{r}}{r^2} + \frac{1}{c^2}\left(\ddot{p}\left(t - \frac{r}{c}\right) \times r \times r\right)\right]$ where

- $p^* = p\left(t - \frac{r}{c}\right) + \frac{r}{c}\dot{p}\left(t - \frac{r}{c}\right)$

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# Lienard-Wierchert

- $\phi(1,t) = \int \frac{\rho\left(2, t - \frac{r_{12}}{c}\right)}{4\pi\epsilon_0 r_{12}} dV_2$

- $\phi(1,t) = \sum \frac{\rho_i \Delta V_i}{r_i'} = \sum \frac{\rho w a^2}{r'} = \frac{\rho a^3}{r'} \frac{Nw}{a} = \frac{q}{r'} \frac{b}{a}$

- $b = a + \frac{v}{c} b$

- So, we get

  - $\phi(1,t) = \frac{q}{4\pi\epsilon_0 [r - \frac{v \cdot r}{c}]_{ret}}$ and

  - $A(1,t) = \frac{q v_{ret}}{4\pi\epsilon_0 c^2 [r - \frac{v \cdot r}{c}]_{ret}}$

- Now we can compute $E$ as above and verify the Heaviside formula

# Radiation loss and antenna aperture

- Spreading loss: $L_s = 32 + \log(\text{d}) + 20 \log(\text{f})$
  - d in kilometers
  - F in megahertz
- $W = A_e P_e, A_e = \dfrac{\lambda^2}{4\pi}$

# Antenna aperture

- Deriving antenna aperture uses thermodynamic argument: black body equilibrium

- $P_A = \frac{A_e}{2} B_v \Delta v \int_{[0,4\pi]} d\Omega = 2\pi \, A_e B_v \Delta v$

- $B_v = \frac{2v^2 kT}{c^2} = \frac{2kT}{\lambda^2}$ (Rayleigh-Jeans)

- $P_R = kT\Delta v$

- $P_A = P_R$

- $A_e = \frac{\lambda^2}{4\pi}$



Filter$[v, v+\Delta v]$
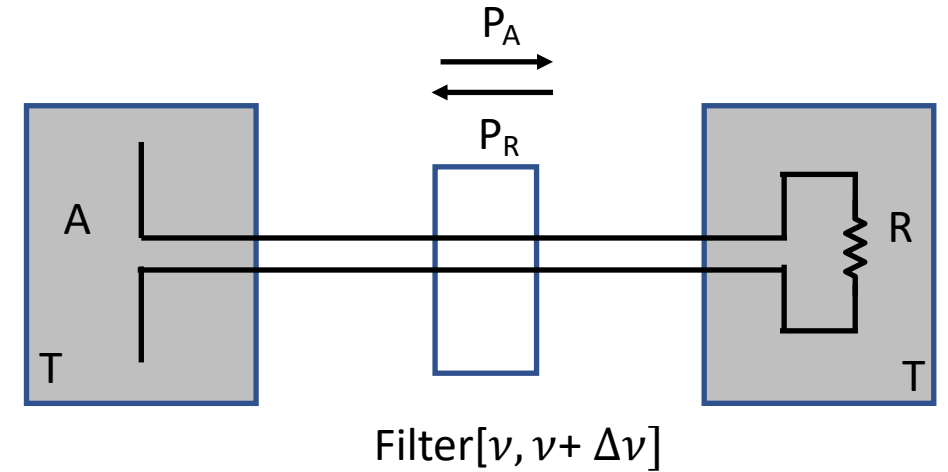
# Impedance matching

- L networks, $\pi$ and T networks

- Impedance and reflection coefficients

# GPS

- Space Segment
  - 31 operating satellites (Current generation: block IIIA – 2018)
  - 12 hour orbits MEO (21000km), 8-12 satellites in view
  - Transmitter power: 44.8 W, $G_t = 12 dBi$. Clocks accurate to better than 10ns (no leap seconds).
- Control segment master at Schriever AFB (plus alternative master, 4 ground antennas and 6 ground stations). Ephemeris updates daily.
- User segment
  - Two bands: L1 (1575.42 MHz [2.046MHz BW]) civilian and military, L2 (1227.6MHz)], military only
  - Two signals: C/A (civilian) and P(Y) (military). C/A signal is Direct Sequence Spread Spectrum
  - Satellite identified by PRN (1-32).
  - L1 signal: $s(t) = A_c C(t) D(t) \cos(\omega_c t + \theta_1) + A_p P(t) D(t) \sin(\omega_c t + \theta_2)$
    - $C(t)$ - coarse ranging code
    - $D(t)$ - navigation signal consists of (clock, ephemeris, almanac corrections) 37,500 bits. 50 bps transmission.
    - $P(t)$ - precision ranging (Y is encrypted)

# Some data and rough calculations

- $N = kTB, k = 1.38 \times 10^{-23} \, J/K$. BW: $2.046 MHz$
- For comparison: laptop emission noise density is capped at $10^{-7} \frac{W}{MHz}$
- Noise: $1.38 \times 10^{-23} \frac{J}{K} \times 100 \times 2 \times 10^6 = 2.76 \times 10^{-15} W, (-145.6 \, dB)$
- Free space loss (@100K): $P_r = \frac{P_t G A}{4\pi r^2} \cdot \int A d\Omega = \lambda^2 . P_r = \frac{(44.7) 10^{1.24}}{4\pi (21 \times 10^6)^2} \times \frac{3 \times 10^8}{1.575 \times 10^9} = 4 \times 10^{-15} W (-144 dB)$
- Final SNR: $-144 + 145.6 + 43 = 44.6 dB$.
  - Lower in practice since satellite not directly overhead, and there are shadowing and multipath losses. In addition, we have receiver loss (and receiver antenna gain) so $P_r$ could be as small as $-170 dB$ and final SNR is approximately $-170 + 144 + 43 = 17 dB$
- Receiver modeled at $G_r = 2 dBi$

# Signal Structure
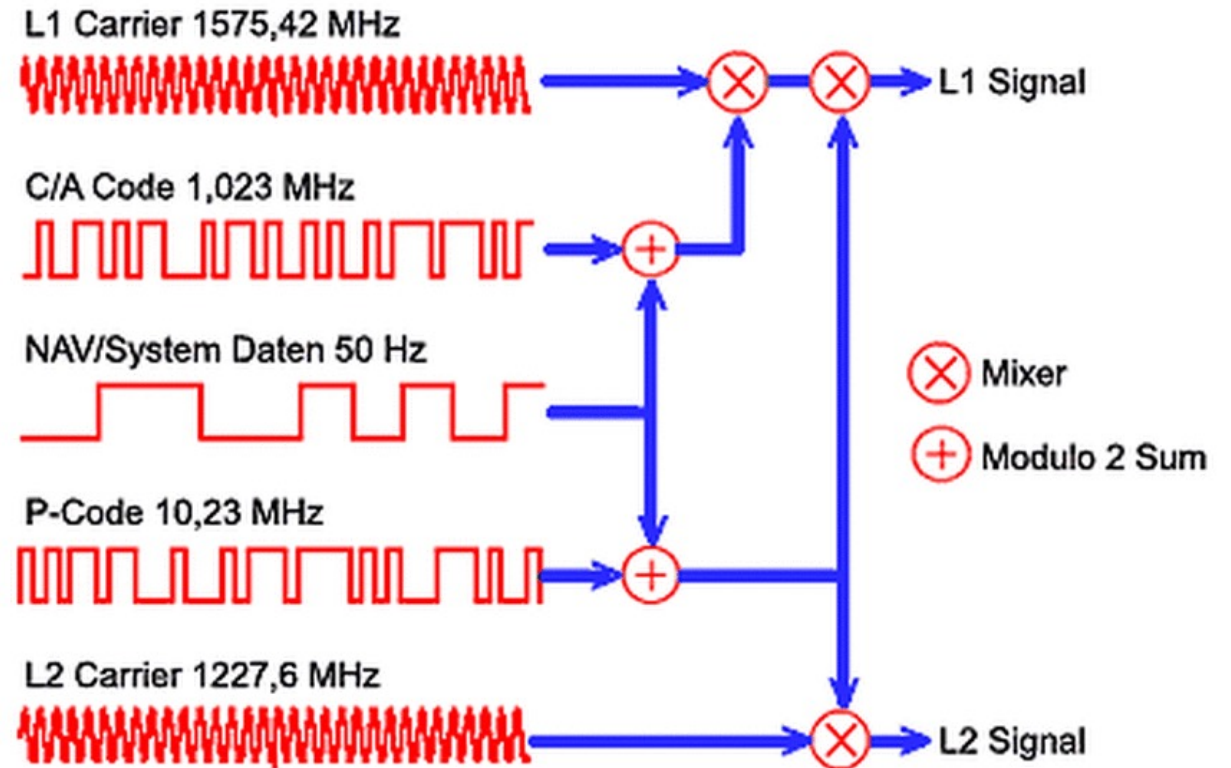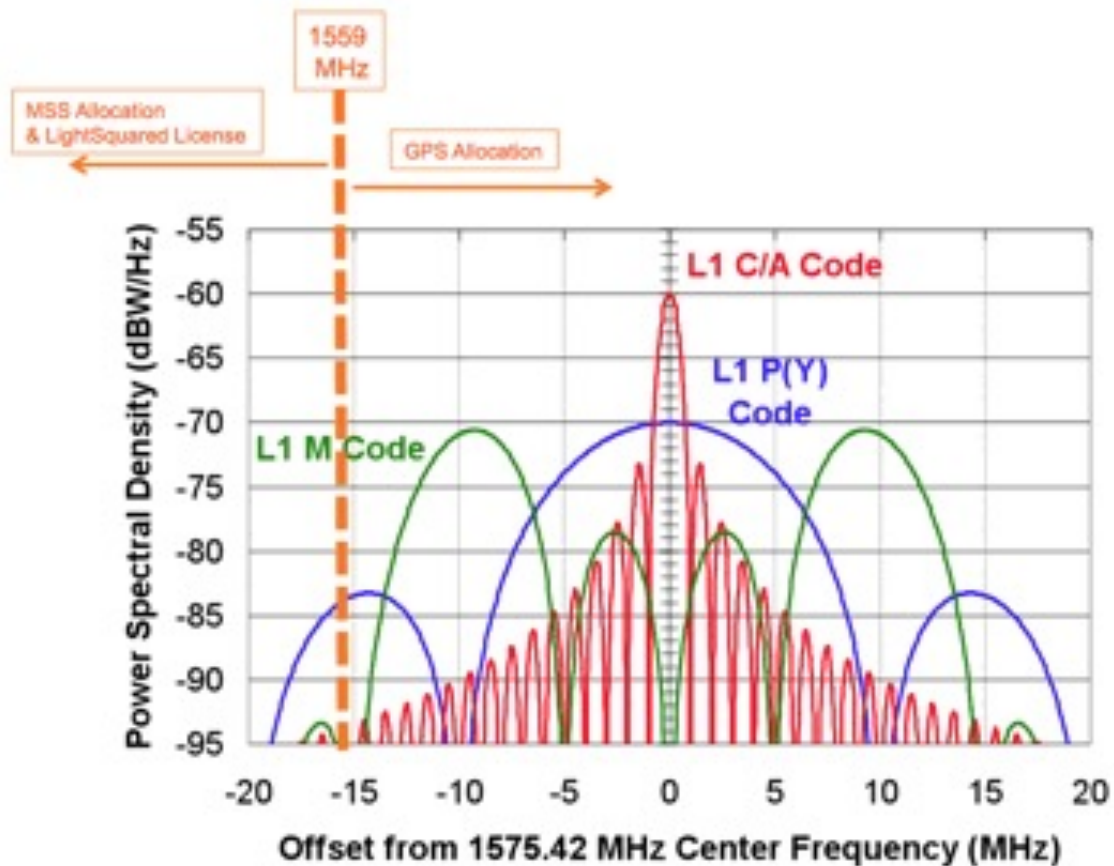
# Signal Summary

| GNSS System | GPS | GPS | | GPS | GPS |
|---|---|---|---|---|---|
| Service Name | C/A | L1C | | P(Y) Code | M-Code |
| Centre Frequency | 1575.42 MHz | 1575.42 MHz | | 1575.42 MHz | 1575.42 MHz |
| Frequency Band | L1 | L1 | | L1 | L1 |
| Access Technique | CDMA | CDMA | | CDMA | CDMA |
| Signal Component | Data | Data | Pilot | Data | N.A. |
| Modulation | BPSK(1) | TMBOC(6,1,1/11) | | BPSK(10) | $BOC_{sin}(10,5)$ |
| Sub-carrier frequency [MHz] | - | 1.023 | 1.023 & 6.138 | - | 10.23 |
| Code frequency | 1.023 MHz | 1.023 MHz | | 10.23 MHz | 5.115 MHz |
| Primary PRN Code length | 1023 | 10230 | | $6.19 \cdot 10^{12}$ | N.A. |
| Code Family | Gold Codes | Weil Codes | | Combination and short-cycling of M-sequences | N.A. |
| Secondary PRN Code length | - | - | 1800 | - | N.A. |
| Data rate | 50 bps / 50 sps | 50 bps / 100 sps | - | 50 bps / 50 sps | N.A. |
| Minimum Received Power [dBW] | -158.5 | -157 | | -161.5 | N.A. |
| Elevation | 5° | 5° | | 5° | 5° |

# L1 Signal Structure

- $s_m(t) = C(t) \oplus D(t).\ s(t) = A_c s_m(t)\cos(\omega_c t + \theta_1)$ [bspk]
  - D(t) - 50 bps transmission (20ms).
  - C(t) $- 1$ Mbps chipping rate.  300 meters, $1\mu s/chip$.  Chipping code repeats 20 times for single navigation bit.
- C/A code is generated from two Gold codes and the PRN identifier (5 bits).
  - $g_1(x) = 1 + x^3 + x^{10}, g_2(x) = 1 + x^2 + x^3 + x^8 + x^9 + x^{10}$
  - Phase selector takes PRN and uses it to select bits of the second Gold code.

- Signal Characteristics
  - Received power: -130dBm, Noise power: -111dBm
  - Spread spectrum contributes 43 dB to processing.  Want SNR greater than 14 dB
  - SNR for C/A: -20dB

# Acquisition, tracking and navigation

- Acquisition
  - Receiver generates known C/A code and attempts to correlate with signal.
  - To generate C/A code, receiver needs to know which satellite it's attempting to acquire to determine code
- Tracking via delay loop to maintain C/A code alignment
- The Navigation Message includes the Ephemeris parameters, the Time parameters and Clock Corrections, the Service Parameters with satellite health information, Ionospheric parameters model, and the Almanacs, allowing the computation of the position of "all satellites in the constellation". The ephemeris and clocks parameters are usually updated every two hours, while the almanac is updated at least every six days.

# Navigation Message

- The navigation message contains 25 pages ('frames') of 30 seconds each. Entire message takes 12.5 minutes to be transmitted. Every frame is subdivided into 5 sub-frames of 6 seconds each; every sub-frame consists of 10 words, with 30 bits per word.
- Every sub-frame starts with the telemetry word (TLM), needed for synchronism. Next, the transference word (HOW) which provides time information (seconds of the GPS week), allowing the receiver to acquire the week-long P(Y)-code segment.
  - Sub-frame 1: contains information the satellite clock. It also has information about satellite health condition.
  - Sub-frames 2 and 3: contain satellite ephemeris.
  - Sub-frame 4: provides ionospheric model parameters, UTC information (Universal Coordinate Time), part of the almanac, and indications whether the Anti-Spoofing, A/S, is activated.
  - Sub-frame 5: contains data from the almanac and the constellation status. It allows to quickly identify the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.
  - **Sub-frames 1, 2 and 3 are transmitted with each frame.** The content of sub-frames 4 and 5 is common for all satellites. So, the almanac data for all in orbit satellites can be obtained from a single tracked satellite.

# Processing and Accuracy

- For satellite k:  $\rho = \sqrt{(x^{(k)} - x)^2 + (y^{(k)} - y)^2 + (z^{(k)} - z)^2}$
  - Need to account for skew between satellite clocks and receiver clock. b is receivers clock bias
  - Four satellites required.  More satellites increase precision with least squares.
- Doppler correction improves accuracy
- *Carrier-Phase Enhancement* corrects timing errors caused by non-zero PRN pulse transition.
  - It uses the L1 carrier wave, which has a period about one-thousandth of the C/A Gold code bit providing an additional clock.
  - The phase difference error in the normal GPS amounts to 2–3 m error. *Carrier-Phase Enhancement* reduces this to 3 cm (1.2 in).
- *Differential GPS*

# GPS receiver
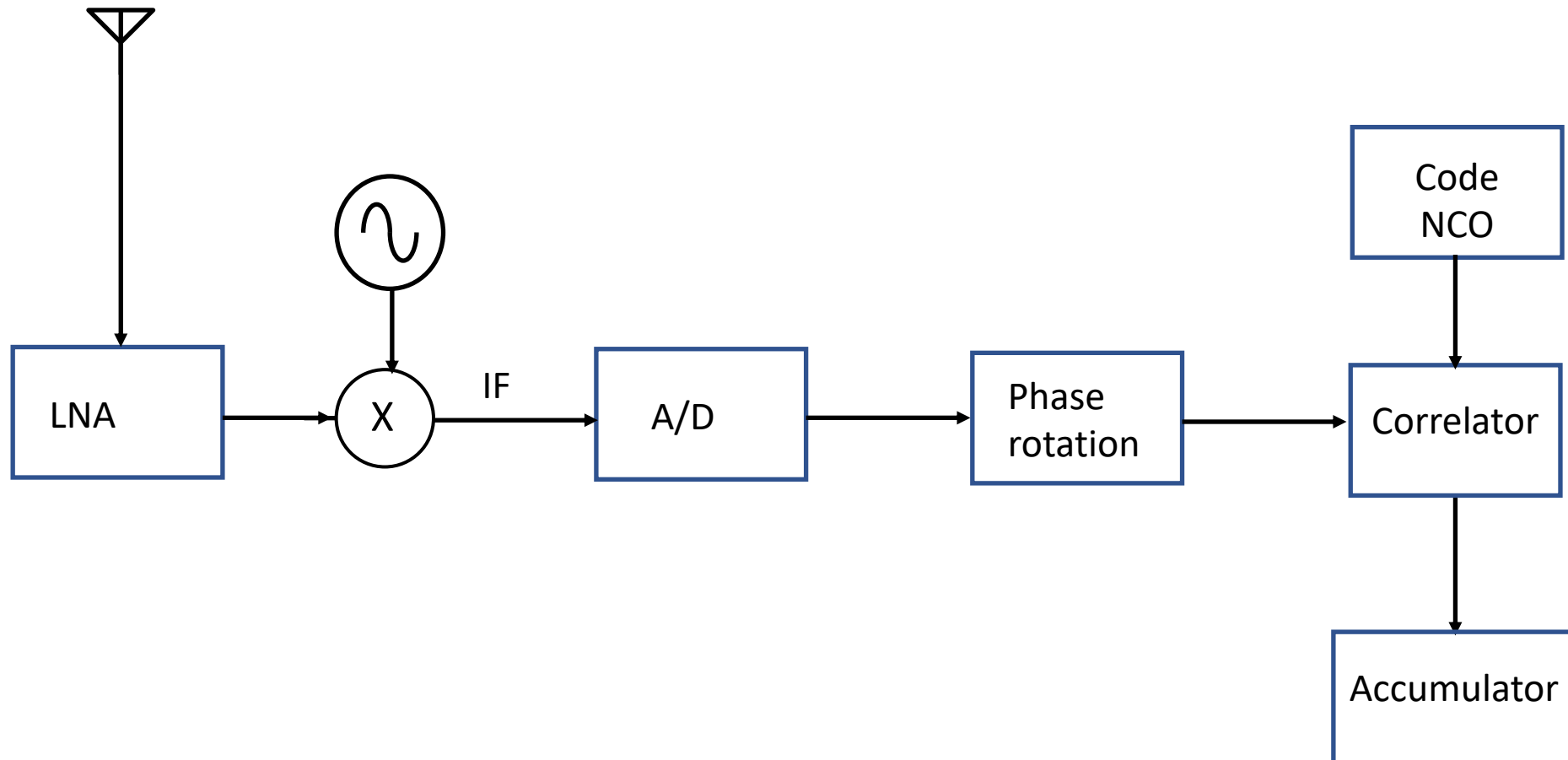
# More careful estimates*

| Elevation | 5° | 40° | 90° |
|---|---|---|---|
| Power at sat input | 14.3dB | 14.3dB | 14.3dB |
| Sat antenna gain | 12.1dB | 12.9dB | 10.2dB |
| EIRP | 26.4dB | 27.2dB | 24.5dB |
| Range | 25240km | 22020km | 20190km |
| Path loss | $-159 \ dB/m^2$ | $-157.8 \ dB/m^2$ | $-157.1 \ dB/m^2$ |
| Atmo loss | .5dB | .5dB | .5dB |
| Received power density | $-133.1dB/m^2$ | $-131.1dB/m^2$ | $-133.1dB/m^2$ |

*Misra and Enge, GPS

# More careful estimates*

| Elevation | 5° | 40° | 90° |
|---|---|---|---|
| Received power density | $4.9 \times 10^{-14} W/m^2$ | $7.8 \times 10^{-14} W/m^2$ | $4.9 \times 10^{-14} W/m^2$ |
| Effective area of receiver | $2.7 \times 10^{-3} m^2$ | $2.7 \times 10^{-3} m^2$ | $2.7 \times 10^{-3} m^2$ |
| Isotropic receiver power | -158.5 dB | -156.5dB | -158.5dB |
| $G_r$ | -4 dBic | 2 dBic | 4dBoc |
| C/A received power | -162.5dB | -154.5dB | -154.5dB |

*Misra and Enge, GPS

# More careful estimates*

Noise loss

| Elevation | Ant cable | LNA | cable |
|-----------|-----------|------|-------|
| Gain | -1dB | 20dB | -10dB |
| F | 1dB | 2-3dB | 10dB |
| T | 75.4 | 290 | 2610 |

SNR

| Elevation | 5° | 90° |
|-----------|-----|-----|
| Received C/A power (C) | -162.5dB | -154.5dB |
| $N_0$ | -201 W/Hz | -201W/Hz |
| $C/N_0$ | 34.5 db-Hz | 46.5 dBHz |
| $C/P_n$ 20MHz | -34.6 dB | -26.5dB |
| $C/P_n$ 2MHz | -24.5dB | -16.5dB |

*Misra and Enge, GPS

# Supplement

# Modulation

- AM: $V(t) = a(t)\cos(\omega_c t) + V_c \cos(\omega_c t)$

- FM: $V(t) = V_c \cos([\omega_c + a(t)]t)$

- FSK: $V(t) = V_c \cos(\omega_1 t)$, if 1 [mark]; $V_c \cos(\omega_0 t)$, if 0 [space]

- PSK: $V(t) = V_p \cos(\omega_c t)$, if 1; $-V_p \cos(\omega_c t)$, if 0 [space]

- Gain: $G = \dfrac{P_o}{P_i}$, Loss: $L = \dfrac{P_o}{P_{max}}$, Rejection: $R = \dfrac{P_{max}}{P_{pb}}$,

# Direct conversion receivers

- Mixer
  - $V(t) = \cos(\omega_1 t)\cos(\omega_2 t) = \frac{1}{2}[\cos((\omega_1 + \omega_2)t) + \cos((\omega_1 - \omega_2)t)]$
- Image frequency
  - $\omega_{vi} = \omega_{LO} + \omega_a$
  - $\omega_{rf} = \omega_{LO} - \omega_a$
- RF filter removes image
- Downside:
  - Not tunable

RF Filter

LO

RF Mixer

Audio Amp

Direct conversion

audio        RF    LO        image
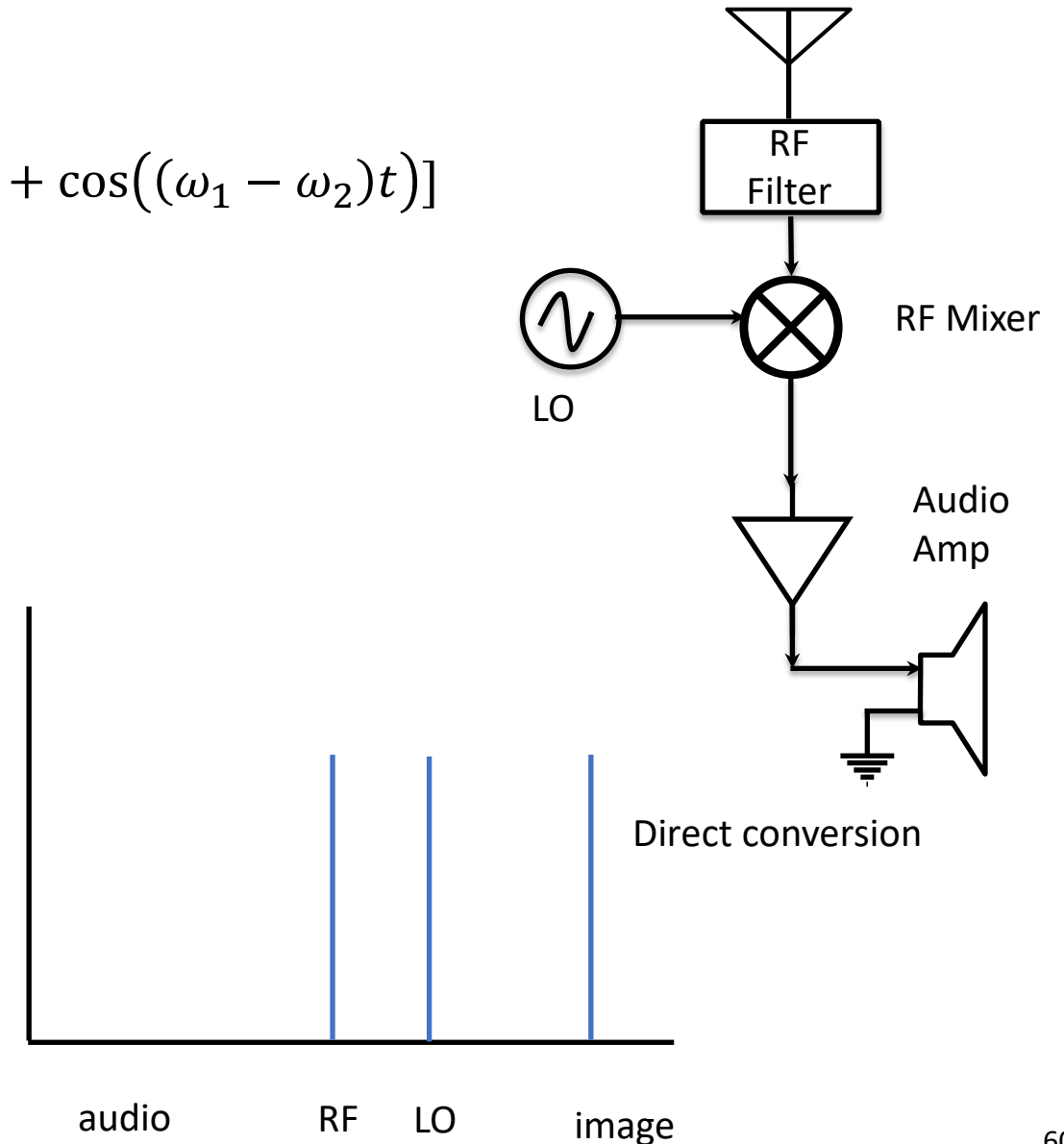
# Norcal 40A
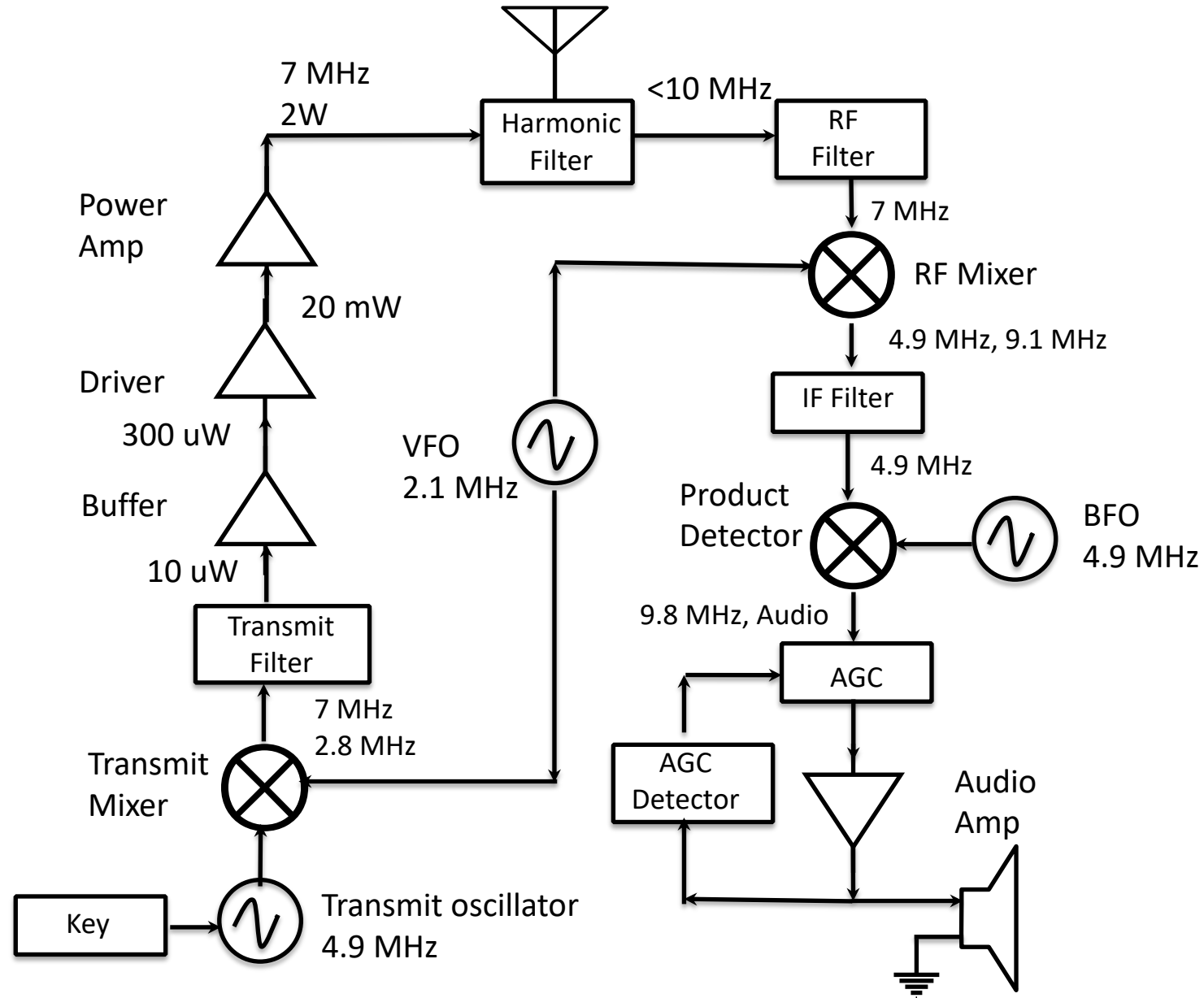
# Mixers

- $V_{lo}(t)$ is a square wave with period $\omega_{lo}$. Expanding this in a Fourier series, we get:

- $V_{lo}(t) = \frac{4}{\pi}(\cos(\omega_{lo}t) - \frac{\cos(3\omega_{lo}t)}{3} + \frac{\cos(5\omega_{lo}t)}{5} \ldots), V_{rf}(t) = V_{rf}\cos(\omega_{rf}t)$

- $V_{lo}(t)V_{rf}(t) = \frac{2V_{rf}}{\pi}(\cos(\omega_-t) - \frac{\cos(3\omega_-t)}{3} + \frac{\cos(5\omega_-t)}{5} \ldots) + \frac{2V_{rf}}{\pi}(\cos(\omega_+t) - \frac{\cos(3\omega_+t)}{3} + \frac{\cos(5\omega_+t)}{5} \ldots)$

- $\omega_+ = \omega_{lo} + \omega_{rf}$ and $\omega_- = |\omega_{lo} - \omega_{rf}|$

- We define $\omega_{k+} = (k\omega_{lo}+\omega_{rf})$ and $\omega_{k-} = |k\omega_{lo} - \omega_{rf}|$ and $V_{k+}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k+}t)$ and $V_{k-}(t) = \frac{2V_{rf}}{k\pi}\cos(\omega_{k-}t)$

- $\omega_i = \omega_{if} - \omega_{lo}$ and $\omega_{if} = \omega_{if} + \omega_i$ , $\omega_i$ is a spurious signal. $\omega_{k+}$ and $\omega_{k-}$ are the spurs from the $k$th harmonic



| LO | Image | IF | RF | 3↑ | 5↑ |
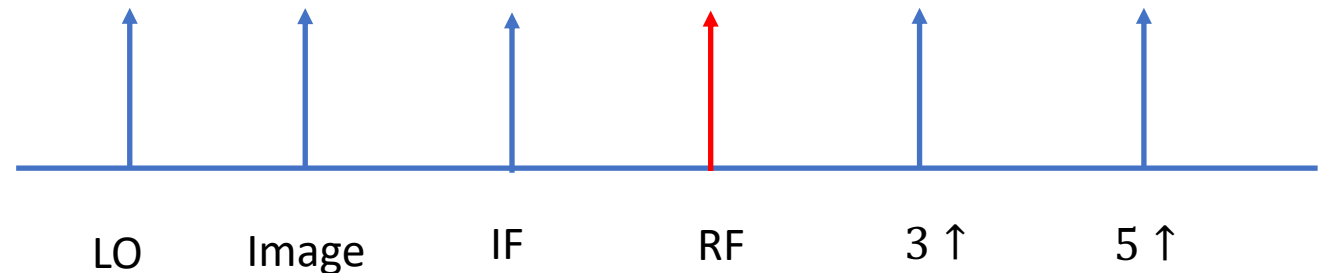
# Phasors

- $V(t) = RI(t)$
- $V(t) = L\dot{I}(t)$
- $I(t) = C\dot{V}(t)$
- Suppose $V(t) = A\cos(\omega t + \theta)$ and $I(t) = B\cos(\omega t + \phi)$ . If $\phi > \theta$, we say the current leads the voltage.
- $V(t) = Re(e^{j(\omega t + \theta)})$, and $I(t) = Re(e^{j(\omega t + \phi)})$
- Now define $V = Ae^{j\theta}$ and $I = Be^{j\phi}$, so $|V| = A$, $|I| = B$, $\angle V = \theta$, and $\angle I = \phi$. $V$ and $I$ are called phasors and do not include time.  Note that $V(t) = Re(Ve^{j\omega t})$ and $I(t) = Re(Ie^{j\omega t})$.
- Note that $I = CVj\omega$, for a capacitor and $V = LIj\omega$, for an inductor
- $\hat{V} = Z\hat{I}, Z = R + jX$
- $\hat{I} = Y\hat{V}, Y = G + jB$

# Resonance and Q

- Series Resonance
  - At $\omega_u$ and $\omega_l$, $X = \pm R$ [$\omega_u$ is upper 3dB cutoff and $\omega_l$ is lower 3dB cutoff]
  - $\omega_u L - \dfrac{1}{\omega_u C} = R$, $\omega_l L - \dfrac{1}{\omega_l C} = -R$
  - Define $Q = \dfrac{X}{R}$
  - $\dfrac{\omega_u}{\omega_0} - \dfrac{\omega_0}{\omega_u} = \dfrac{R}{\omega_0 L} = \dfrac{1}{Q}$ and $\dfrac{\omega_l}{\omega_0} - \dfrac{\omega_0}{\omega_l} = -\dfrac{R}{\omega_0 L} = -\dfrac{1}{Q}$
  - $\dfrac{\omega_u}{\omega_0} - \dfrac{\omega_0}{\omega_u} = \dfrac{\omega_0}{\omega_l} - \dfrac{\omega_l}{\omega_0}$ , so $\omega_0^2 = \omega_u \omega_l$ and $\dfrac{\omega_u - \omega_l}{\omega_0} = \dfrac{1}{Q}$
- Parallel Resonance
  - $\dfrac{\omega_u}{\omega_0} - \dfrac{\omega_0}{\omega_u} = \dfrac{G}{\omega_0 C} = \dfrac{1}{Q_p}$ and $\dfrac{\omega_l}{\omega_0} - \dfrac{\omega_0}{\omega_l} = -\dfrac{G}{\omega_0 C} = -\dfrac{1}{Q_p}$

# Power

- $P(t) = I(t)V(t)$

- Complex power: $P = \dfrac{V\bar{I}}{2} = Z\dfrac{|I|^2}{2} = P_a + jP_r = R\dfrac{|I|^2}{2} + jX\dfrac{|I|^2}{2}$
  - $P_a$ is power delivered to resistor, $P_r$ is power stored in inductor and capacitor

- $P_r = \dfrac{\omega L|I|^2}{2} - \dfrac{\omega C|V_c|^2}{2} = \omega(E_L - E_C)$

- $Q = \omega\dfrac{L|I|^2}{R|I|^2} = \omega\dfrac{L}{R} = \omega\dfrac{E_L}{P_a}$

# Exercises

- Calculate wave form through bridge and after "smoothing capacitor"

# More Oscillators



Hartley

9 V

2.7 pf

D

G

J309

S

100pf

200pF

1M

2uF

¼ from ground

Norcal VFO

9 V

10K

4.7K

510

47nF

47K

47pF

47pF

D

G

J309

S

MVAM 108

2-25pF

L

47K

1.2nF

1.8K

1.2nF

1mH

10nF

L: 62 turns on T68-7
19uH

150pF

# More Oscillators



Colpitts

Clapp

Crystal Clapp

J309

9V

68

.01

100

out

10K

10K

390

390

2.2K

7MHz

D

G

S

D

G
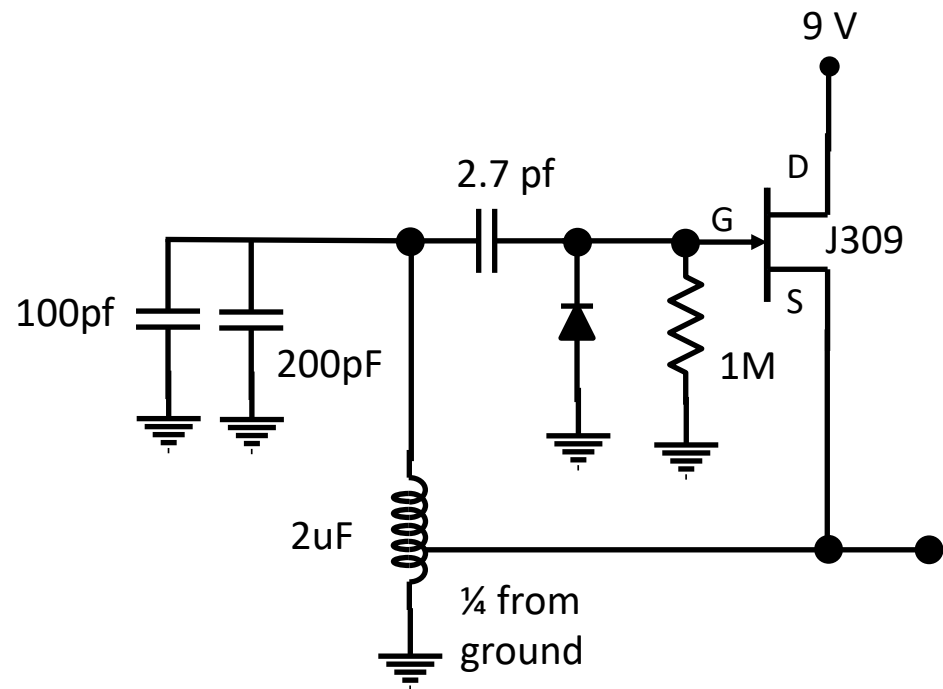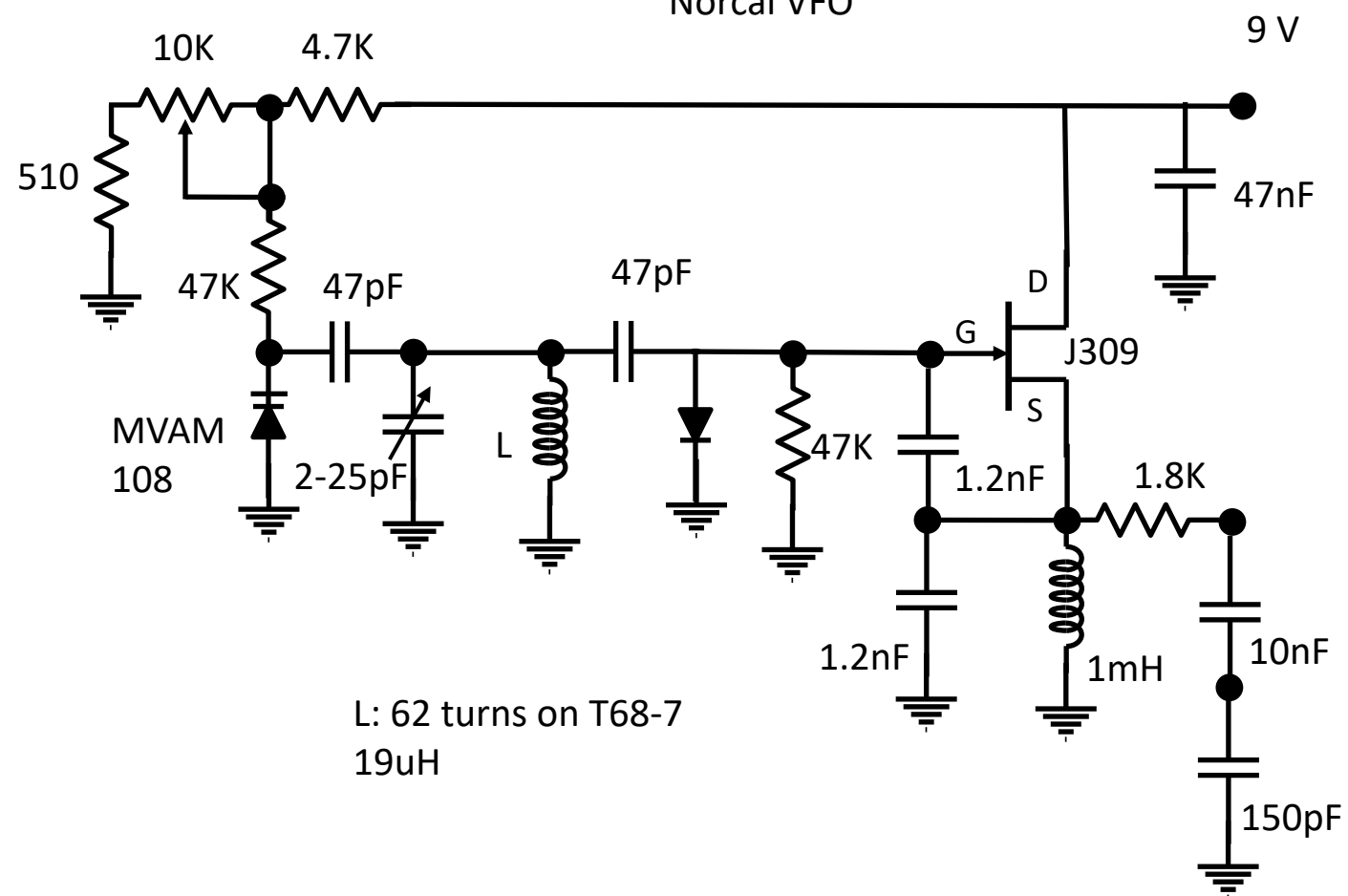
S
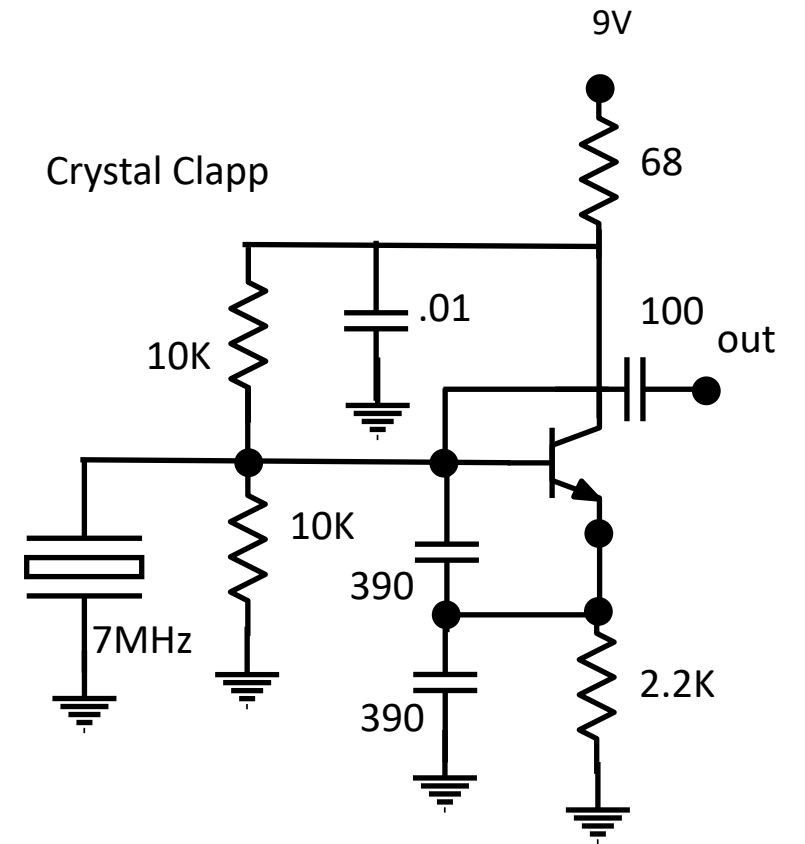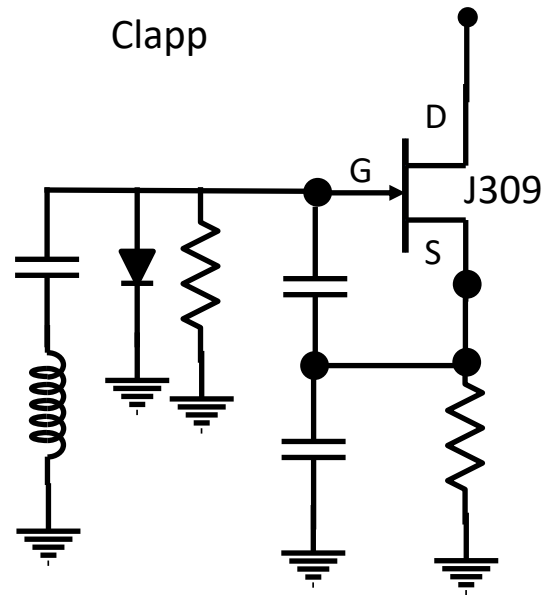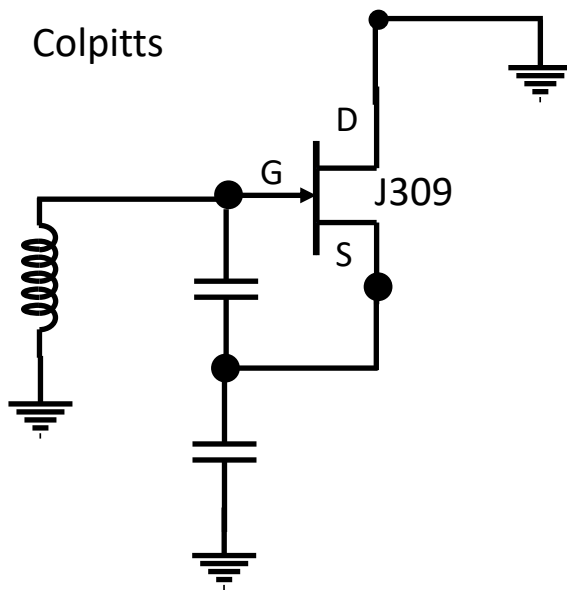
# RF Amp
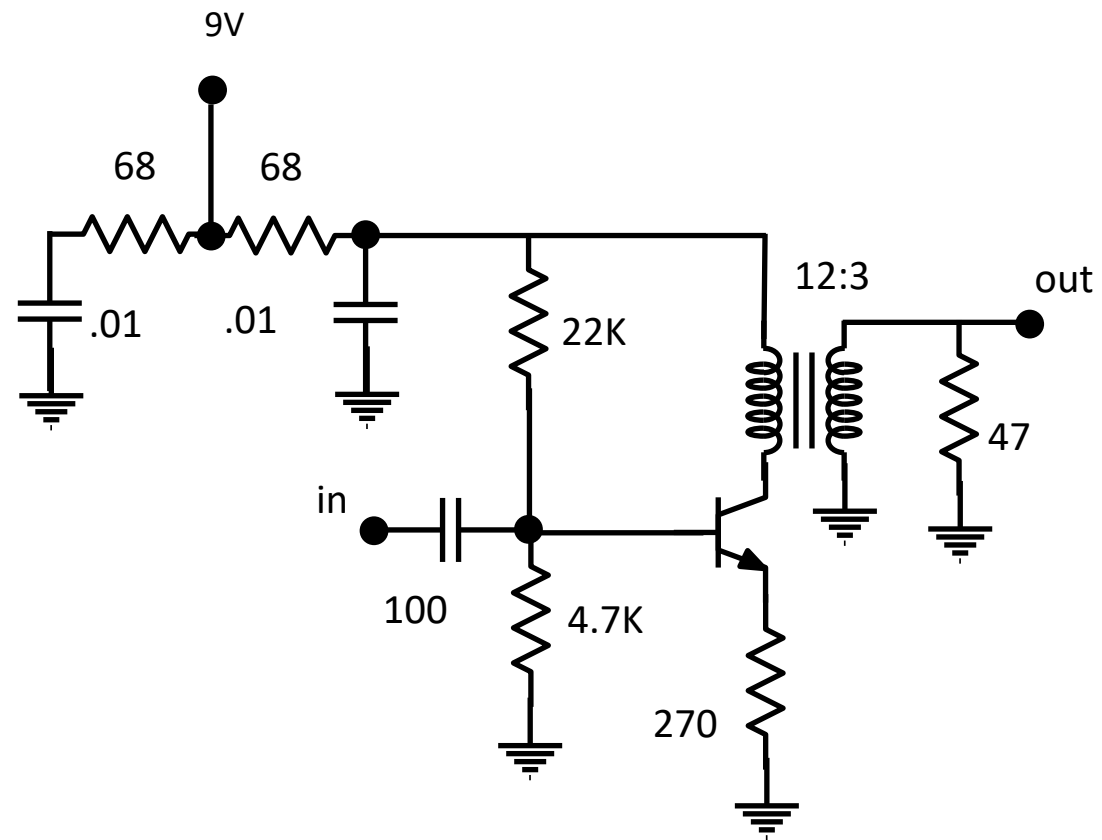
# PLL
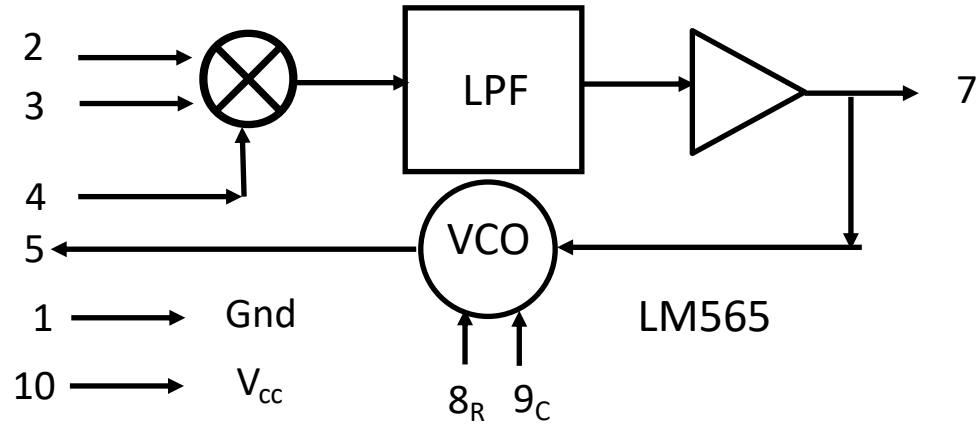


- $f_0 = .3(RC)^{-1}$

1. With no input, VCO out is about 1360Hz
2. For input, set function generator to $1V_{pp}$
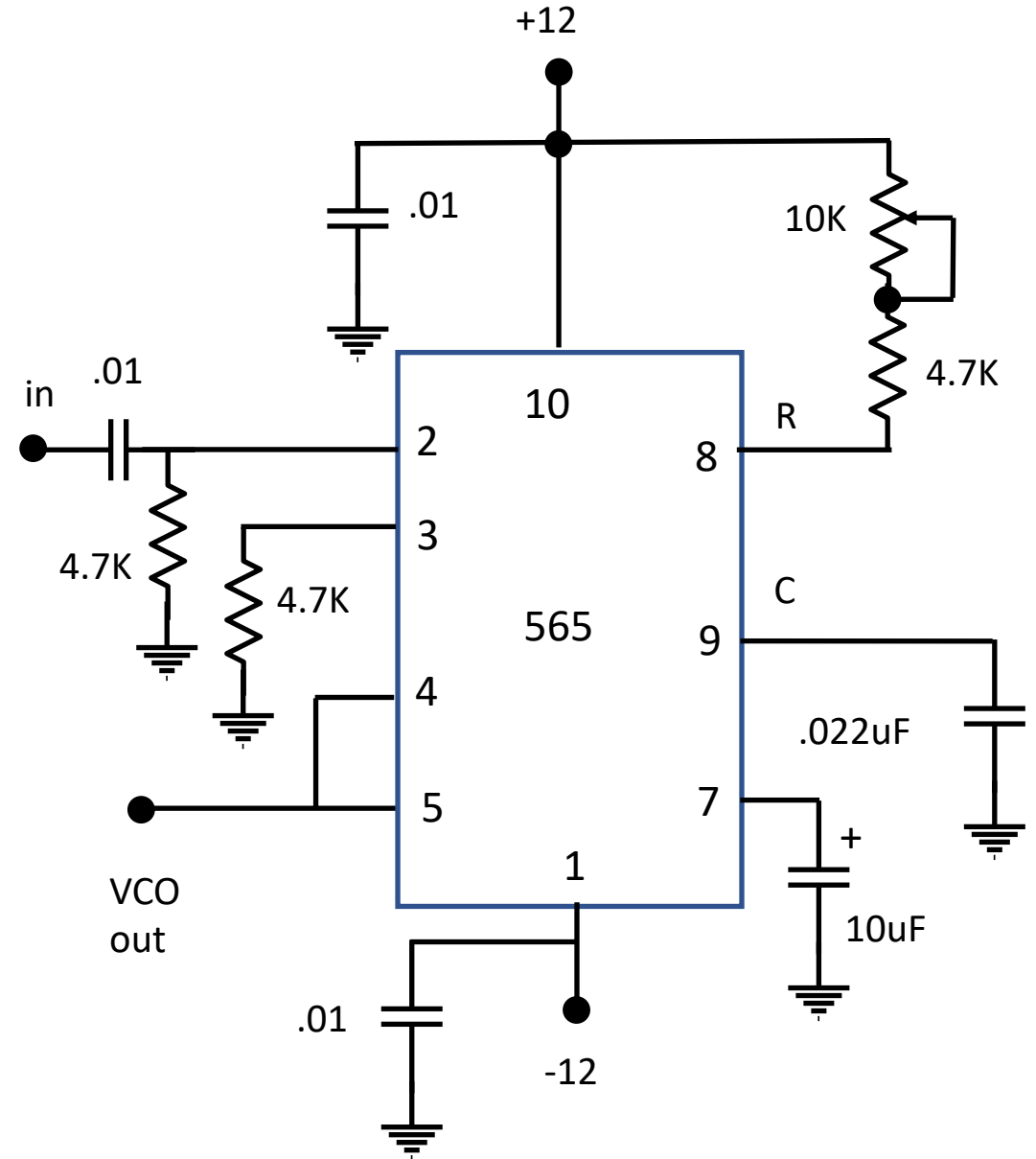
# PLL FM detector

1. With no input, out is about 1360Hz
2. For input, set function generator to $1V_{pp}$ and connect to previous 565
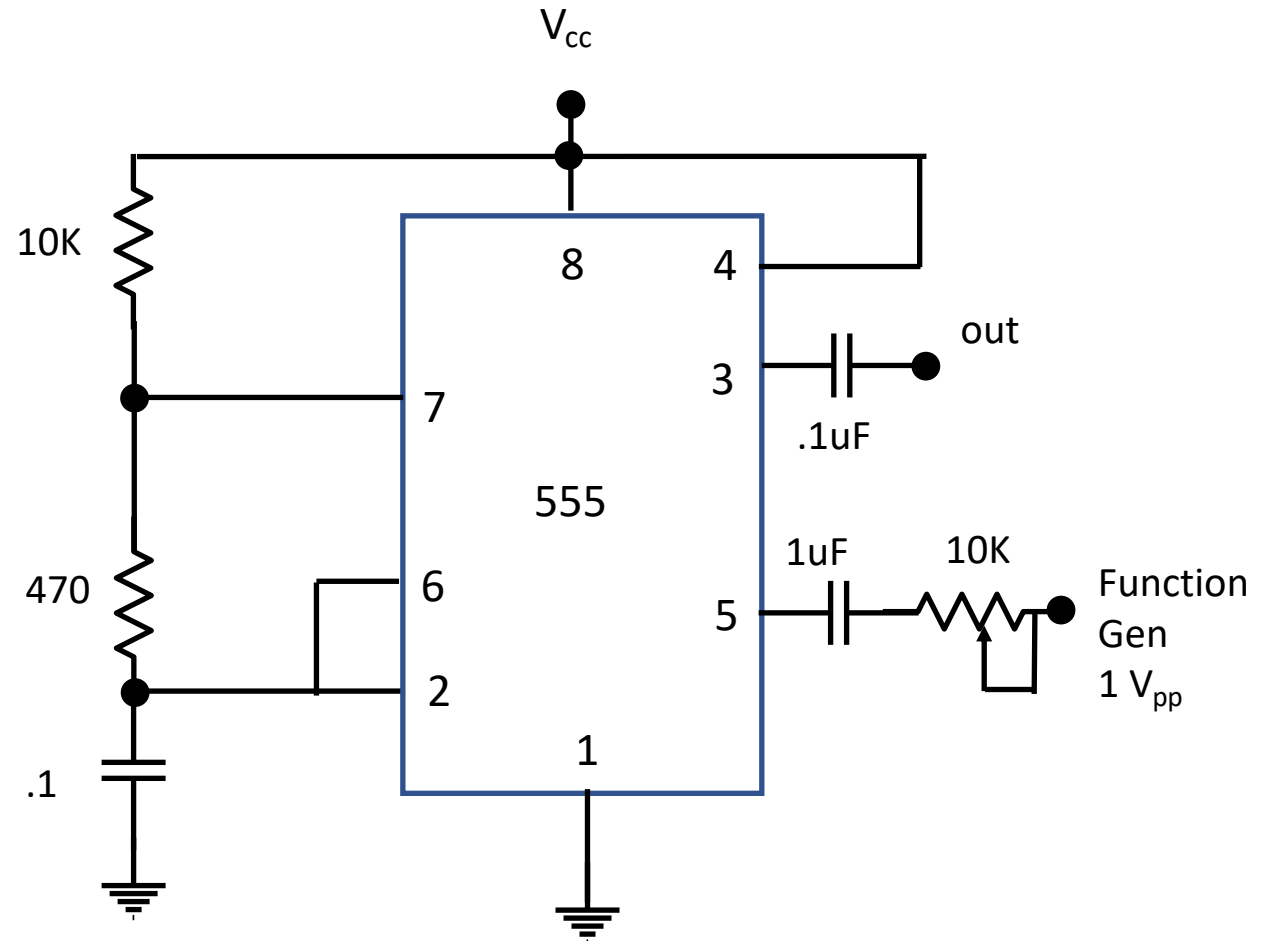
# Pinouts

## IRF510 MOSFET Pinout

TO-220 Package

N Channel Mosfet

IRF510

Pin1 Gate

Pin2 Drain

Pin3 Source

Pin2 | D

G Pin1

Pin3 | S
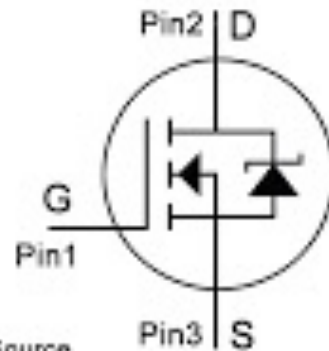
S = Source
G = Gate
D = Drain

www.componentsinfo.com
Electronics Components Uses, Features, Pinouts, Equivalents, Applications & More...

## 2N7000

Source 1
Gate 2
Drain 3
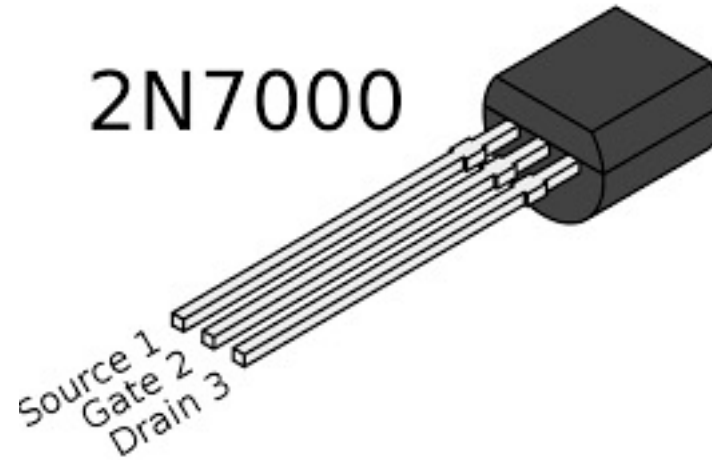
## 2N3904 pinout

1. Emitter
2. Base
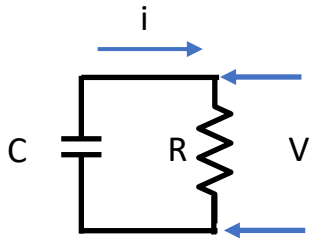3. Collector
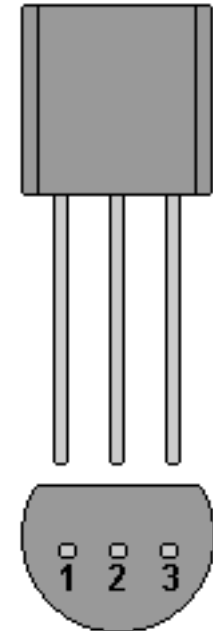
1. DRAIN
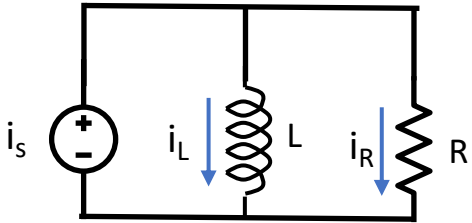2. SOURCE
3. GATE
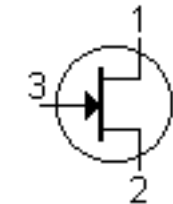
JEDEC    TO-92

J309

# Misc-1
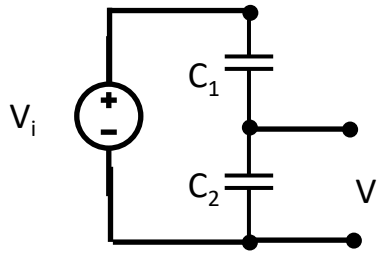


- $\dfrac{V}{V_i} = \dfrac{C_1}{C_1 + C_2}$



- $I = \dfrac{V}{R} = -CV', \tau = RC,$



- $\dfrac{L}{R} I_R' + I_R = 0$
- $i_s = i_r + i_L$

# Blank