

Exponential distribution and comparison with the Central Limit Theorem

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Overview

We will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. Set `lambda = 0.2` for all of the simulations. We will investigate the distribution of averages of 40 exponentials.

Simulation

A thousand random numbers from the exponential distribution, with the given `lambda`, will be generated. The sample mean will be compared with the theoretical mean and the sample variance with the theoretical variance

```
lambda <- 0.2
n <- 40 #Number is exponentials to average

num.sim <- 1000

set.seed(123456)
data <- rexp(num.sim * n, lambda)

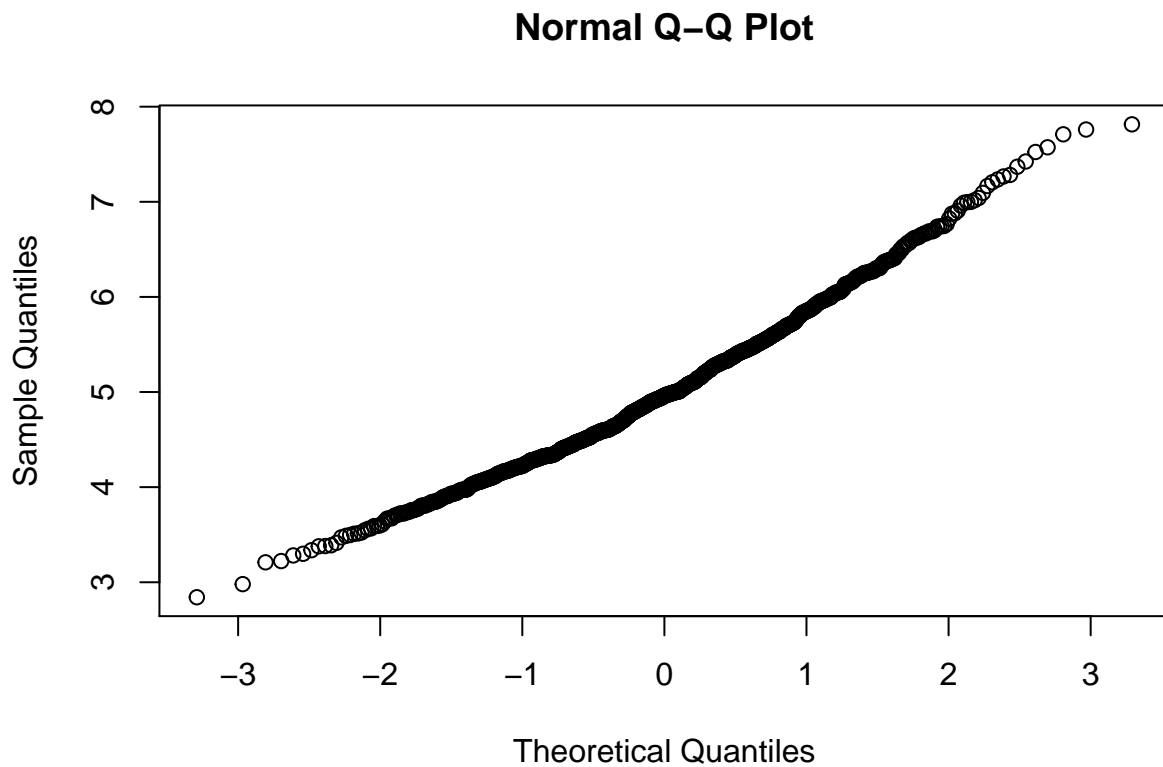
distribution <- matrix(data, nrow = num.sim)
mean.distribution <- apply(distribution, 1, mean)
mean.df <- data.frame(mean = mean.distribution)
```

Check normality with `qqnorm` plot.

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.2.2
```

```
qqnorm(mean.df$mean)
```



The plot suggests the mean distribution is normal.

Next the sample mean from the distribution is compared against the theoretical mean of the exponential distribution.

```
#Sample mean  
mean(mean.df$mean)
```

```
## [1] 5.022915
```

```
#theoretical mean of the exponential distribution  
1/lambda
```

```
## [1] 5
```

Comparison of the variance.

```
#theoretical variance of the exponential distribution  
#Standard error of mean for the sample  
sem_sd <- (1/lambda)/sqrt(n)  
  
#variance is  
sem_sd ^2
```

```
## [1] 0.625
```

```
#Variance from the sample  
var(mean.df$mean)
```

```
## [1] 0.6557463
```

Here we can see the theoretical variance of the exponential distribution is the same as the variance of the means of the sample.

The sample distribution of the sample means

```
hist_plot <- ggplot(mean.df, aes(x=mean))  
hist_plot <- hist_plot + geom_histogram(binwidth=.2, aes(y=..density..),  
                                       colour="black", fill="white")  
hist_plot <- hist_plot + geom_density(alpha=0.2, fill="#FF6666")  
print(hist_plot)
```

