The Role of Heterogeneity in Coopetition in Professional Road Cycling

Julian Matthes & David Piazolo

December 20, 2023

Motivation

Road cycling has unique strategic components

- Possibility of *drafting* provides the opportunity to **free-ride**.
- Individual winners, but riders organized in **teams**.


Tour de France

Tadej Pogačar Is Vulnerable If Jumbo or Ineos Are Willing To Sacrifice

THE TWO-TIME DEFENDING TOUR CHAMPION HAS A THIN LEAD AND A SHAKY TEAM; ARE HIS RIVALS WILLING TO DO WHAT IT TAKES TO BEAT HIM?

Tadej Pogačar defends his team as firmly as he defends yellow

Pogačar bats aways suggestions his UAE Team Emirates teammates are not up to the challenge of defending yellow jersey.

Bad news for the rest of the peloton as Pogačar's Achilles' heel is fixed

UAE Team Emirates raced a textbook finale up Jebel Hafeet to help Tadej Pogačar to a third consecutive victory at their home race.

For an entertaining Tour de France, thank a weak UAE-Team Emirates

Tadej Pogačar has a weakness, but it's not his own legs.

(a) Teams Matter

(b) Riders arguing

Preview

Potential insights

- Regarding Cycling: Effect of team strength on win probability not entirely clear.
- Off-Cycling: Field evidence for feasibility of cooperation of eventual competitors.

Method

1. Model interactions of riders in groups as dynamic game.
2. Confirm predictions with
   (a) results data from over 40 years of races, and
   (b) manually labeled in-race data from the last 5 years.

Anatomy of a Stage in a Stage Race

1. "Breakaway" forms.
2. Breakaway builds unassailable lead OR is kept on a leash.
3. Groups start splintering and working against each other close to the finish.
4. Sprint to the line (unless solo win).

Video

Existing Literature (Incomplete)

Game-theoretic analyses of competitive sports

- Economic models of behavior in cycling races: Cooperation of the breakaway as a social dilemma (Scelles et al. [2018], Brouwer and Potters [2019]), dynamics of sprint finishes (Dilger and Geyer [2009])

Coopetition / Collective Action

- Collective action and heterogeneity (Gavrilets [2015])
- Simultaneous inter- and intra-group conflict (Münster [2007])

Research Questions

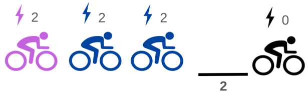
RQ1: Which groups have an advantage in inter-group competition?

- Coordination is supported by heterogeneity in individual capability, which establishes common beliefs about who should take the initiative.

RQ2: When is it advantageous for a team to add a teammate to a group?

- Adding someone's teammate to a group only helps the strongest rider of the group.
- Adding a teammate to another group has strategic benefits.

Modelling the Decisive Part of a Bike Race



- n players (teams), team i has k_i riders
- Each **rider** has, in each period:
  - position (integer)
  - level of remaining energy (integer)
- A group are all riders with the same position.
- Riders can invest energy to be at the front of the race, but wants to keep as much as possible for the sprint **after** the last period.

After period T, each team's utility is its win probability:

1. Take every team's strongest rider in the first group.
2. Tullock contest with remaining energy levels. ▸ Formula

Actions and Strategies

How does energy expenditure translate into positional change?

Video

Energy and Position Updating With Free-Riding

Each **rider** can act in each period as follows:

Action	Energy Cost	Position Impact	
0	0	+0 or +1	Hold wheel
1^-	1	+1	Set a steady pace for the group
1^+	1	+0 or +1 or +2	Focus on attackers
2	2	+2	Attack

Table: Actions

(a) 2 and 1^- (b) 2 and 1^+ (c) 1^- and 0

Chasing as a solo rider

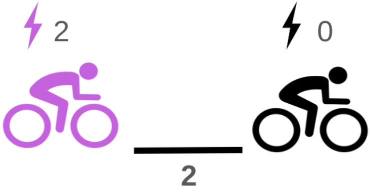
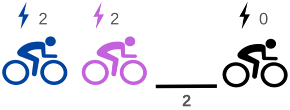


Figure: $n = 2$, pos$=((2),(0))$, energy$=((0),(2))$, $t = T$

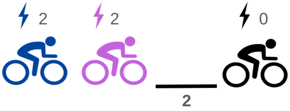
$\Longrightarrow$ Unique equilibrium: $(0, 2)$ leads to $u^* = (\frac{1}{2}, \frac{1}{2})$.

Coordinating a chase as a group



	1^+	2
1^+	$0, 0$	$1, 0$
2	$0, 1$	$\frac{1}{3}, \frac{1}{3}$

Coordinating a chase as a group

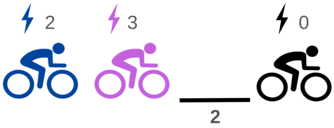


	1^+	2
1^+	$\boxed{0,0}$	1,0
2	0,1	$\frac{1}{3},\frac{1}{3}$

$\Longrightarrow$ Unique THPE: $(0, 1^+, 1^+)$ leading to $u^* = (1, 0, 0)$.

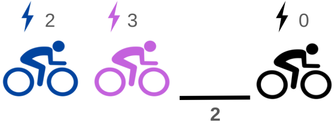
$\Longrightarrow$ **R1**: The introduction of homogeneous competition (with no teammates) into the pursuing group increases the probability of 'failure to coordinate on catching up with a (empty) break'.

Heterogeneity in energy levels solves the coordination problem



	1^+	2
1^+	$0,0$	$\frac{1}{2},\frac{1}{2}$
2	$0,1$	$0,1$

Heterogeneity in energy levels solves the coordination problem

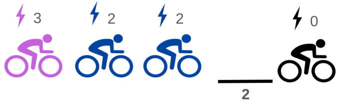


	1^+	2
1^+	$0, 0$	$\boxed{\frac{1}{2}, \frac{1}{2}}$
2	$0, 1$	$0, 1$

$\implies$ Unique THPE: $(0, 1^+, 2)$ leading to $u^* = (0, \frac{1}{2}, \frac{1}{2})$.

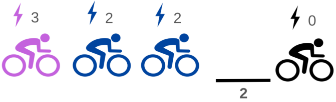
$\implies$ **R2**: A group is more likely to catch up or stay in front of a competing group if there is heterogeneity in the group: If some rider is stronger than the rest or some rider in the group has a teammate by his side.

The shared value of helpers



	1^+	2
$1^+\|1^+$	$0,0$	$1,0$
$1^+\|2$	$\frac{1}{3},\frac{2}{3}$	$1,0$

The shared value of helpers

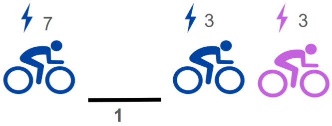


	1^+	2
$1^+\|1^+$	$0,0$	$1,0$
$1^+\|2$	$\boxed{\frac{1}{3},\frac{2}{3}}$	$1,0$

$\Longrightarrow$ Unique THPE: $(0, 1^+|2, 1^+)$ leading to $u^* = (0, \frac{1}{3}, \frac{2}{3})$.

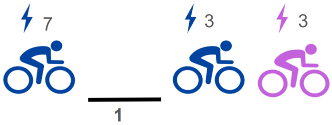
$\Longrightarrow$ **R3**: Helpers help the strongest rider in the group and might be harmful to weaker riders (for Lemma see here).

The (indirect) value of having a teammate behind



	1^-	1^+	2
0	$\frac{5}{7},\frac{2}{7}$	$\frac{5}{7},\frac{2}{7}$	$0,1$
1^+	$\frac{5}{7},\frac{2}{7}$	$\frac{5}{7},\frac{2}{7}$	$\frac{2}{3},\frac{1}{3}$
2	$1,0$	$\frac{1}{3},\frac{2}{3}$	$\frac{1}{2},\frac{1}{2}$

The (indirect) value of having a teammate behind



	1^-	1^+	2
0	$\frac{5}{7},\frac{2}{7}$	$\frac{5}{7},\frac{2}{7}$	$0,1$
1^+	$\frac{5}{7},\frac{2}{7}$	$\frac{5}{7},\frac{2}{7}$	$\boxed{\frac{2}{3},\frac{1}{3}}$
2	$1,0$	$\frac{1}{3},\frac{2}{3}$	$\frac{1}{2},\frac{1}{2}$

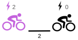
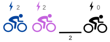
$\implies$ Teammate behind attacks and therewith forces rider to attack (if rider behind is credible threat).

$\implies$ Rider in front follows and hence profits from the introduced **heterogeneity** (w/o teammate behind EU would be $\frac{1}{2}$).

$\implies$ Similarly, a Satellite rider induces heterogeneity in the pursuing group in favor of a teammate behind.

$\implies$ **R4**: Riders profit from having teammates in competing groups.

Our theoretical results

Situation	EU	Result
	$\left(\frac{1}{2}, \frac{1}{2}\right)$	R1
	$(1, 0, 0)$	R1
	$\left(0, \frac{1}{2}, \frac{1}{2}\right)$	R2
	$\left(0, \frac{1}{2}, \frac{1}{2}\right)$	R2
	$\left(0, \frac{1}{3}, \frac{2}{3}\right)$	R2, R3
	$\left(\frac{2}{3}, \frac{1}{3}\right)$	R4
	$(1, 0)$	R4

R1: Homogeneous competition increases coordination failure.

R2: Heterogeneity decreases coordination failure.

R3: Helpers only help the strongest rider of a group.

R4: Placing riders in competing groups is always beneficial.

Data

Race results 1981-2023 of the following stage races:

- 3 week: Tour de France, Giro d'Italia, and Vuelta à Espana.
- 1 week: Criterium du Dauphiné, Tour de Romandie, Itzulia Basque Country, Tour de Pologne, Tour de Suisse, Paris-Nice, and Tirreno Adriatico.
- Total number: 3717 stages

Data obtained:

- Top 15 riders with teams and time gaps for each race.
- Group: Riders that have less than 5 seconds between each other. Group sizes
- Drop races where the third group (riders arriving within 5 seconds) is not complete after the 14th rider.
- Total number: 1449 stages

Hypotheses

H1 Individual strength is beneficial. (R0)

We use a "Star-dummy" as a measure for individual strength. "Star" riders are riders which in the previous season reached a score that puts them in the top 20%*.*

H2 In-group heterogeneity makes it more likely that a group sticks together and does well in the competition with other groups. (R1, R2)

We use the existence of teammates and/or stars as a measure for heterogeneity.

H3 Having a teammate in the same group is beneficial (especially for strong riders). (R3) update!

H4 Having a teammate in the group behind is beneficial. (R4)

Making the first group

Table: Being in Group 1

	G1 if in G1/G2 (1)	G1 if in G1/G2/G3 (2)
Star	0.043***	0.064***
	(0.015)	(0.011)
Underdog	-0.003	-0.032**
	(0.019)	(0.014)
Other Star around	-0.014	-0.014
	(0.015)	(0.011)
Teammates behind	0.055**	
	(0.022)	
Observations	5052	7766
R^2	0.186	0.143
Adjusted R^2	0.175	0.136
Residual Std. Error	0.453 (df=4986)	0.426 (df=7701)
F Statistic	17.540*** (df=65; 4986)	20.085*** (df=64; 7701)

Note: *p<0.1; **p<0.05; ***p<0.01
We do OLS regressions with non-clustered standard errors.
The endogenous variables are dummies indicating whether riders are part of Group 1.
We additionally control for year, race, stage type, gap sizes between groups, and group sizes.

$\Longrightarrow$ H1 ✓ & H4 ✓

Winning from the first group

Table: Winning the Race

	Win if in G1		
	(1)	(2)	(3)
Star	0.012	0.014	0.032
	(0.018)	(0.030)	(0.022)
Underdog	0.001	0.023	0.027
	(0.024)	(0.038)	(0.032)
Other Star in Group	-0.223***	-0.051*	-0.155***
	(0.021)	(0.030)	(0.025)
Helper in Group	0.125***	0.152***	0.096*
	(0.042)	(0.057)	(0.052)
Teammates behind	0.056**	0.120***	0.010
	(0.027)	(0.043)	(0.032)
Solos included	YES	NO	YES
Hypothetical teams	NO	NO	YES
Observations	2335	1303	2986
R^2	0.415	0.095	0.092
Adjusted R^2	0.398	0.048	0.072
Residual Std. Error	0.376 (df=2270)	0.455 (df=1238)	0.458 (df=2921)
F Statistic	25.149*** (df=64; 2270)	2.030*** (df=64; 1238)	4.642*** (df=64; 2921)

Note: *p<0.1; **p<0.05; ***p<0.01
We do OLS regressions with non-clustered standard errors.
The endogenous variables are dummies indicating whether riders have won.
We additionally control for year, race, stage type, gap sizes between groups, and group sizes.
In the hypothetical teams treatment we randomly assign teammates.

$\Longrightarrow$ H3 (✓) & H4 ✓

Solos

More than half of the wins are solos. Individual strength or coordination failure?

- Throw out mountain top finishes.

Compare a winning group (with 3 to 6 riders) with

- A group of similar size that comes second after a solo winner.
    - At least one helper is present in 16% of the groups that lose against a solo rider (versus 24% in winning groups[1]).
    - Stars are present in 31% of the respective groups (versus 41%[2]).

$\implies$ H2 (✓)

[1] χ^2-tests yields $p = 0.2$.
[2] χ^2-tests yields $p = 0.14$.

Solos

Compare a winning group (with 3 to 6 riders) with

- Riders that win or are in short distance to winner but do not finish as one group.

Table: Staying together as a Group

	versus Group behind Solo winner	versus riders not finishing as Group
Star in Group exists	0.064	-0.004
	(0.074)	(0.026)
Helper in Group exists	0.085	0.082**
	(0.088)	(0.042)
Observations	231	827
R^2	0.026	0.054
Adjusted R^2	-0.005	0.046
Residual Std. Error	0.498 (df=223)	0.348 (df=819)
F Statistic	0.851 (df=7; 223)	6.656*** (df=7; 819)

Note: *p<0.1; **p<0.05; ***p<0.01
We do an OLS regression with non-clustered standard errors.
The endogenous variables are dummies indicating whether Group finishes first.
We additionally control for the number of teams per group and stage type.

$\Longrightarrow$ Existence of at least one Helper has significant and positive impact on being in a group that finishes together. $\Longrightarrow$ H2 ✓

Takeaways

1. **Heterogeneity in individual capability positively impacts a group's success rate by establishing common beliefs about who should take the initiative.**
2. **Splitting resources can have strategic benefits** (beyond diversification).

Applications outside cycling?

- Joint research ventures
- Political agendas

Potential Extensions

Theoretical

- Applying our model to more general team competition with "local" free-riding incentives.
- Model energy levels as private information:
  Mimicking behavior of low energy riders might be beneficial.

Empirical

- Currently: Document in-race data on development of gaps between groups and relate to group characteristics.
- Maybe: Compare impact of teammates in cycling to impact of teammates in running, where there is no drafting.
- Use more of available data (different race types, different finish types, uploaded GPS and power data).

Thanks!

Energy and Position Updating: Modelling Free-Riding

Rider j's available actions depend on the amount of energy they have remaining:

$$A^t_{i,j} = \begin{cases} \{0, 1^-, 1^+, 2\} & \text{if } e^t_{i,j} \geq 2 \\ \{0, 1^-, 1^+\} & \text{if } e^t_{i,j} = 1 \\ \{0\} & \text{if } e^t_{i,j} = 0. \end{cases}$$

- Playing 1^- costs 1 energy, leads to position $+1$
- Playing 2 costs 2 energy, leads to position $+2$
- Playing 0 costs 0 energy, leads to position $+1$ (if someone in group plays 1^-) or position $+0$ (else)
- Playing 1^+ costs 1 energy, leads to position $+2$ (if someone in group plays 2) or position $+1$ (if someone in group plays 1^- or in group that is one position behind plays 2) or 0 (else)

back

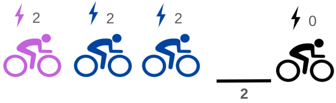
Winning probabilities

Payoff: After period T, each team's payoff is their winning probability

$$u_i = \begin{cases} \frac{\max_{j \in B_i} e_{i,j}^{T+1}}{\sum_{k=1}^{n} \max_{j \in B_k} e_{k,j}^{T+1}} & \text{if } \sum_{k=1}^{n} \max_{j \in B_k} e_{k,j}^{T+1} \neq 0 \\ \frac{1}{|\{k \mid B_k \neq \emptyset\}|} & \text{if } \sum_{k=1}^{n} \max_{j \in B_k} e_{k,j}^{T+1} = 0 \end{cases}$$

where $B_i = \{1 \leq j \leq k_i \mid p_{i,j}^{T+1} = \max_l p_l^{T+1}\}$. back

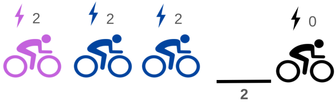
Adding a helpers solves the coordination problem



	1^+	2
$1^+\|1^+$	0,0	1,0
$1^+\|2$	$\frac{1}{2}, \frac{1}{2}$	1,0

back

Adding a helpers solves the coordination problem



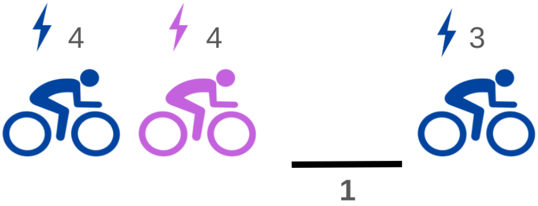
	1^+	2
$1^+\|1^+$	$0,0$	$1,0$
$1^+\|2$	$\boxed{\frac{1}{2},\frac{1}{2}}$	$1,0$

$\implies$ Unique THPE: $(0, 1^+|2, 1^+)$ leading to $u^* = (0, \frac{1}{2}, \frac{1}{2})$.

back

Satellite riders - The value of having a teammate in front

Consider 2 periods to play.



$\Longrightarrow$ In unique THPE: Attack in $t = T - 1$
(if not then Satellite wins with certainty by playing 2 and 1^-).

$\Longrightarrow$ Should satellite fall back to "help" his captain or stay up front?

back

Satellite riders - The value of having a teammate in front

Attack in $t = T - 1$.

Option 1: Satellite falls back (1^-) into his captain's (1^+) group.

	0	1^-	1^+	2
$2 \vert 1^+$	$1,0$	$1,0$	$\frac{2}{3},\frac{1}{3}$	$1,0$
$1^+ \vert 0$	$\frac{3}{5},\frac{2}{5}$	$\frac{3}{4},\frac{1}{4}$	$\frac{3}{4},\frac{1}{4}$	$1,0$

Satellite riders - The value of having a teammate in front

Attack in $t = T - 1$.

Option 1: Satellite falls back (1^-) into his captain's (1^+) group.

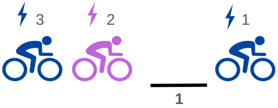
	0	1^-	1^+	2
$2\|1^+$	$\boxed{1,0}$	$1,0$	$\boxed{\frac{2}{3},\frac{1}{3}}$	$1,0$
$1^+\|0$	$\boxed{\frac{3}{5},\frac{2}{5}}$	$\frac{3}{4},\frac{1}{4}$	$\boxed{\frac{3}{4},\frac{1}{4}}$	$1,0$

$\Longrightarrow$ Satellite falling back leads to unique THPE (in mixed strategies) with positive winning probabilities for both teams.

Satellite riders - The value of having a teammate in front

Attack in $t = T - 1$.

Option 2: Satellite stays in front (2) of his captain's (1^+) group.

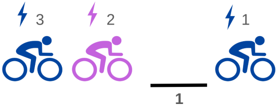


	1^-	2
$1^- \| 1^+$	$1,0$	$1,0$
$1^- \| 0$	$1,0$	$\frac{1}{2},\frac{1}{2}$
$0 \| 1^+$	$\frac{2}{3},\frac{1}{3}$	$1,0$
$0 \| 0$	$\frac{3}{4},\frac{1}{4}$	$0,1$

Satellite riders - The value of having a teammate in front

Attack in $t = T - 1$.

Option 2: Satellite stays in front (2) of his captain's (1^+) group.



	1^-	2
$1^-\|1^+$	$\boxed{1,0}$	$\boxed{1,0}$
$1^-\|0$	$1,0$	$\frac{1}{2},\frac{1}{2}$
$0\|1^+$	$\frac{2}{3},\frac{1}{3}$	$1,0$
$0\|0$	$\frac{3}{4},\frac{1}{4}$	$0,1$

$\Longrightarrow$ Satellite staying up front leads to certain win. back

Lemma: Helpers only help the strongest rider in the group.

Let $k_1 = 1$, $k_i = k > 0$ for $i \in \{2, ..., n\}$. Further, let $e_1^{T-1} = 0$, $e_i^{T-1} = (\hat{e}_i, \tilde{e}_i, ..., \tilde{e}_i)$ with $\hat{e}_i \geq \tilde{e}_i$ for $i \in \{2, ..., n\}$, and $p^{T-1} = (2, 0, ..., 0)$.
Then

- $u_1^* = 0$ in any Nash equilibrium if $\hat{e}_i \geq 2$ for any $i > 1$
- if $\tilde{e}_i = \tilde{e}_j$ and $\hat{e}_i = \hat{e}_j$ for all $i, j \neq 1$, all players except 1 obtain the same equilibrium payoff, denoted by u^*
- Let $\hat{e}_i > \hat{e}_j$ for all $j \neq i$. Denote the risk-dominant equilibrium payoff of i in the case of $\tilde{e}_j < 2$ for all j by u_i' and the risk-dominant equilibrium payoff of i in the case of $\tilde{e}_j \geq 2$ for some j by u_i''. Then $u_i'' > u_i'$.
- Let $\hat{e}_i < \hat{e}_j$ for some j. Denote the risk dominant equilibrium payoff in the case of $\tilde{e}_j < 2$ for all j by u_i' and the risk-dominant equilibrium payoff of i in the case of $\tilde{e}_j \geq 2$ for some j by u_i''. Then $u_i'' < u_i'$.

back

Equilibrium Multiplicity

- Uniqueness of equilibria hinges on absolute energy levels.
- Equilibrium selection criterion with equivalent results: Risk-dominance
  - no risk-dominant equilibrium without heterogeneity
  - experimentally suggested (see Schmidt et al. [2003])
  - QRE seems to select the risk-dominant equilibrium in our cases, but no proof yet

Stylized Results

R1 Individual strength and team strength (in the sense of the presence of helpers) have a positive impact on the team's winning probability.

R2 A group is more likely to outlast a competing group if there is heterogeneity in the group: If some rider is much stronger than the rest or some rider in the group has a teammate by his side.

R2.1 The probability of a solo attack being caught increases, if there is heterogeneity in the chasing group.

R2.2 The probability of a breakaway group being caught decreases, if there is heterogeneity in the breakaway group.

R3 Riders profit from having teammates in competing groups.

R4 Strong riders profit from weaker riders with teammates in the same group.

Group Composition Results

	Group 1 size	Group 2 size	Group 3 size	Star	Star (G1)	Star (G2)
mean	1.67	2.16	2.18	0.11	0.18	0.15

Table: Group Composition

There are significantly more stars in Group 1 than in Group 2 (or the top 15). back

Dummies

	Underdog	Star	Helper in group	Teammates behind
Mean	0.185	0.224	0.057	0.078

Table: Summary Statistics Dummies

- **Star**: "Star" riders are riders which in the previous season reached a score that puts them in the top 20%.
- **Underdog**: "Underdogs" are riders that were ranked in the bottom 20% of the last season's score ranking (or not ranked at all).

back

Empirical Questions

First Group

- Which riders make the first group?
- How does the first group differ from the second?

Winning

- Conditional on being in the first group, do teammates increase the probability of winning?
- Conditional on being in the first group, does individual capability increase the probability of winning?

Solos

- Which group 2 characteristics make solos successful?

Potential Extensions II

Abundance of data available

- Hundreds of highest-level race days per year
- Results data of over 100 years of large races easily accessible on PCS.com
- Different race types (e.g. stage races, time trials, one-day races) and incentives (stage wins, different classifications)
- Solo wins, small group sprints, large group sprints all common, depending on race terrain
- Some riders even upload in-race GPS and power data on social networks

Future Analyses

We currently hand-label in-race data to answer:

How do groups split?

- Identify races won by small groups
- Go back in race until the final split happens - compare group characteristics

Which breakaways are caught?

- Identify races won by small groups
- Collect group composition and gaps at 10km, 5km and 1km to go

References I

Kyung Hwan Baik. Contests with group-specific public-good prizes. *Social choice and Welfare*, 30:103–117, 2008.

Michael Bar-Eli, Alex Krumer, and Elia Morgulev. Ask not what economics can do for sports - ask what sports can do for economics. *Journal of Behavioral and Experimental Economics*, 89:101597, 2020. ISSN 2214-8043. doi: https://doi.org/10.1016/j.socec.2020.101597. URL https://www.sciencedirect.com/science/article/pii/S2214804320305954.

References II

Thijs Brouwer and Jan Potters. Friends for (almost) a day: Studying breakaways in cycling races. *Journal of Economic Psychology*, 75:102092, 2019. ISSN 0167-4870. doi: https://doi.org/10.1016/j.joep.2018.08.001. URL `https://www.sciencedirect.com/science/article/pii/S0167487017307596`.

Bertrand Candelon and Arnaud Dupuy. Hierarchical organization and inequality in an economy with an implicit market for productive time. IZA Discussion Papers 5391, Institute of Labor Economics (IZA), 2010. URL `https://ideas.repec.org/p/iza/izadps/dp5391.html`.

Todd L Cherry, Stephen J Cotten, and Stephan Kroll. Heterogeneity, coordination and the provision of best-shot public goods. *Experimental Economics*, 16:497–510, 2013.

References III

Luis C Corchón and Marco Serena. “contest theory: A survey.”. *Handbook of Game Theory and Industrial Organization*, 2016.

Arie-Willem de Leeuw, Mathieu Heijboer, Mathijs Hofmijster, Stephan van der Zwaard, and Arno Knobbe. Time series regression in professional road cycling. In Annalisa Appice, Grigorios Tsoumakas, Yannis Manolopoulos, and Stan Matwin, editors, *Discovery Science*, pages 689–703, Cham, 2020. Springer International Publishing.

Emmanuel Dechenaux, Dan Kovenock, and Roman M Sheremeta. A survey of experimental research on contests, all-pay auctions and tournaments. *Experimental Economics*, 18:609–669, 2015.

References IV

Alexander Dilger and Hannah Geyer. The dynamic of bicycle finals: A theoretical and empirical analysis of slipstreaming. *Economic Analysis and Policy*, 39(3):429–442, 2009. ISSN 0313-5926. doi: https://doi.org/10.1016/S0313-5926(09)50037-9. URL https://www.sciencedirect.com/science/article/pii/S0313592609500379.

Christopher Ferrall and Anthony A Smith Jr. A sequential game model of sports championship series: theory and estimation. *Review of Economics and Statistics*, 81(4):704–719, 1999.

Sergey Gavrilets. Collective action problem in heterogeneous groups. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 370(1683):20150016, 2015.

References V

Robert G Hammond and Xiaoyong Zheng. Heterogeneity in tournaments with incomplete information: An experimental analysis. *International Journal of Industrial Organization*, 31(3): 248–260, 2013.

Arye L Hillman and John G Riley. Politically contestable rents and transfers. *Economics & Politics*, 1(1):17–39, 1989.

Hao Jia, Stergios Skaperdas, and Samarth Vaidya. Contest functions: Theoretical foundations and issues in estimation. *International Journal of Industrial Organization*, 31, 05 2013. doi: 10.1016/j.ijindorg.2012.06.007.

Mark Joseph. Cycling analysis: Bicycle stage racing is chess on wheels. *The Aspen Times, August 21, 2011*, 2011. URL https://www.aspentimes.com/news/cycling-analysis-bicycle-stage-racing-is-chess-on-wheels

References VI

Leonid Kholkine, Tom De Schepper, Tim Verdonck, and Steven Latré. *A Machine Learning Approach for Road Cycling Race Performance Prediction*, pages 103–112. Springer International Publishing, 12 2020. ISBN 978-3-030-64911-1. doi: 10.1007/978-3-030-64912-8_9.

Leonid Kholkine, Thomas Servotte, Arie-Willem Leeuw, Tom De Schepper, Peter Hellinckx, Tim Verdonck, and Steven Latré. A learn-to-rank approach for predicting road cycling race outcomes. *Frontiers in Sports and Active Living*, 3:714107, 10 2021. doi: 10.3389/fspor.2021.714107.

Kai A Konrad*. Dynamic contests and the discouragement effect. *Revue d'économie politique*, 2:233–256, 2012.

References VII

Stephan Kroll, Todd L Cherry, and Jason F Shogren. The impact of endowment heterogeneity and origin on contributions in best-shot public good games. *Experimental Economics*, 10: 411–428, 2007.

Chester R. Kyle. Reduction of wind resistance and power output of racing cyclists and runners travelling in groups. *Ergonomics*, 22: 387–397, 1979.

Daniel J Larson and Joel Maxcy. Uncertainty of outcome and radio policy in professional road cycling. *Journal of Sport Management*, 28(3):311–323, 2014.

David A Malueg and Andrew J Yates. Testing contest theory: evidence from best-of-three tennis matches. *The Review of Economics and Statistics*, 92(3):689–692, 2010.

References VIII

Jean-François Mignot. *Strategic Behavior in Road Cycling Competitions*, pages 207–231. Springer International Publishing, Cham, 2016.

Johannes Münster. Simultaneous inter-and intra-group conflicts. *Economic Theory*, 32(2):333–352, 2007.

T. S. Olds, K. I. Norton, E. L. Lowe, S. Olive, F. Reay, and S. Ly. Modeling road-cycling performance. *Journal of Applied Physiology*, 78(4):1596–1611, 1995. doi: 10.1152/jappl.1995.78.4.1596. URL https://doi.org/10.1152/jappl.1995.78.4.1596. PMID: 7615475.

Tim Olds. The mathematics of breaking away and chasing in cycling. *European journal of applied physiology and occupational physiology*, 77(6):492–497, 1998.

References IX

Josep Maria Raya. The effect of strategic resting in professional cycling: evidence from the tour de france and the vuelta a españa. *European Sport Management Quarterly*, 15:1–20, 03 2015. doi: 10.1080/16184742.2015.1021825.

Luca Rebeggiani and Davide Tondani. Organizational forms in professional cycling: An examination of the efficiency of the uci pro tour. *International Journal of Sport Finance*, 3:19–41, 02 2008.

Nicolas Scelles, Jean-Francois Mignot, Benjamin Cabaud, and Aurélien Francois. Temporary organizational forms and coopetition in cycling: What makes a breakaway successful in the tour de france? *Team Performance Management*, 24, 06 2018. doi: 10.1108/TPM-03-2017-0012.

References X

David Schmidt, Robert Shupp, James M Walker, and Elinor Ostrom. Playing safe in coordination games:: the roles of risk dominance, payoff dominance, and history of play. *Games and Economic Behavior*, 42(2):281–299, 2003.

Roman M Sheremeta. Behavior in group contests: A review of experimental research. *Journal of Economic Surveys*, 32(3): 683–704, 2018.

Stefan Szymanski. The economic design of sporting contests. *Journal of economic literature*, 41(4):1137–1187, 2003.

Stefan Szymanski and Tommaso M Valletti. Incentive effects of second prizes. *European Journal of Political Economy*, 21(2): 467–481, 2005.

References XI

Gordon Tullock. Efficient rent seeking. In *Efficient rent-seeking: Chronicle of an intellectual quagmire*, pages 3–16. Springer, 2001.