# Homework 4.(b)

Modeling Complex Systems, Javier Lobato

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## 1 Problem overview and motivation

After having designed a simple swarm model that implemented the boids with relations of proximity (circular zone of repulsion) and angle of vision (field of view), some questions were still unanswered. Reading the discussion paper about which were the different types of relationships between the birds and which ones were the more realistic, the next step is clearly to code some of those relations and see how the model behaves under a range parameter values. The objective is to see how the flock cohesion is maintained or lost when varying the different variables.

Modeling nature with accuracy is important in order to predict phenomena and to be able to study the processes that take place in nature. Obtaining models that are close representations of the reality is essential before asking more detailed questions to a system. Thus, testing a code under a wide range of parameters and knowing how will it behave is a good point to start.

## 2 Experimental design

Beginning with the MATLAB code of the previous assignment, some modifications were done to implement the different types of relations between birds. A vector called visionMode consisting on three elements was used. Each component turns on and off one type or relation between birds - being possible to combine some of them:

```
visionMode = [zone of repulsion field of view k-nearest]
```

There were some combinations that were not feasible, given that mixing a constant circle with the k closest neighbors is excluyent. Thus, the 8 possible combinations were:

- (0 0 0): a case with no possible combinations makes no sense
- (1 0 0): just the zone of repulsion
- (0 1 0): field of view that extends to the end of the grid
- (0 0 1): k-nearest neighbors
- (1 1 0): mixing zone of repulsion with field of view, as the previous assignment
- (1 0 1): this case was not analyzed given that mixing the radius of the zone of repulsion and the k-nearest individuals are restricting and the one that has the greatest value will dominate the other selection method
- (0 1 1): this is the case more interesting and, at least for me, more realistic. Birds look in their field of view but instead trying to analyze all birds (0 1 0) or restricting themselves to certain metric distance (1 1 0), a topological distance is taken. Thus, the bird will analyze only the k closest birds in its field of view
- (1 1 1): given that (1 0 1) was not implemented, this one was neither considered because it has the same restrictions

The model takes most part of the past homework, but some functions were modified. The modifications done to the next functions are a little explained below, whereas the functions without anything written next to haven't been modified (all code is listed at the end):

- boundary.m: the only modification is the increase of speed when a boid bounces on the limits
- boundposition.m
- dataAnalysis.m: a script to run the test performed and the experiment setup
- HW4b\_driver.m: the whole driver was redesigned
- initialization.m
- limitvelocity.m
- maxDist.m: computes the maximum distance between a list of points in 2D
- pltdistribution.m: plot the current boid with the desired options
- rule1.m
- rule2.m: computes rule 2 with the different possible combinations of boid perception
- rule3.m
- rule4.m
- swarmModel.m: includes all the changes related to function calling and vector manipulation
- updateboid.m: changes of function calling and data manipulation
- updatepred.m

# 3 Quantification of the results

The metric used to know how disperse the flock was consisted of the maximum distance between boids. In order to increase the performance of c theode and reduce computation time, a more efficient approach was carried out. The naive idea of comparing each boid to the others in the flock, taking the maximum of those distances and repeating that process for all boids, having at the end the maximum of the maximums as the metric of the size is computationally slow. A less precise but quicker method was implemented. The idea (taken from https://stackoverflow.com/a/8006849) begins by computing the smaller and biggest possible (x + y) (where x and y are the position of these boid) and the smallest and biggest (x-y). Those values are afterwards combined in cross: the distance between the lower left element (minimum (x+y)) and the upper right element (maximum (x+y)) is computed (d1) and the distance between the upper left element (minimum (x-y)) and the lower right element (maximum (x-y)) that gives d2. The maximum between d1 and d2 will be the value of the metric for a given flock at certain timestep. This metric will be computed and analyzed for each timestep, having its evolution in time.

In the images below it can be seen three example flocks with the value returned from this function. These are randomly chosen timesteps of a simulation with different flock densities:

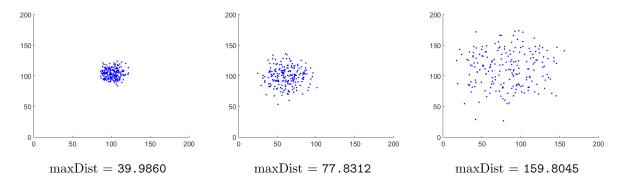


Figure 1: Metric used to measure the cohesion of the flock

The maximum distance was afterward normalized by the maximum possible flock distance. This case will happen if one bird is located at (0,0) and the other is located at  $(\dim,\dim)$ . This distance will be  $\sqrt{2\dim^2}$ , so the values are normalized by that distance in the discussion figures.

Also, the different cases of the perception of the boids were run to test if they were working correctly:

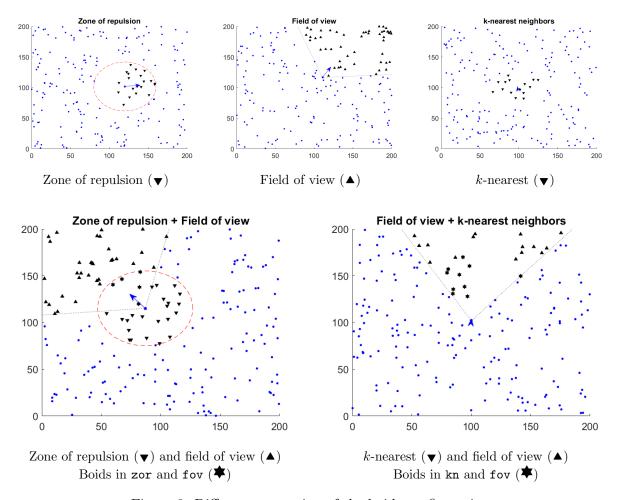


Figure 3: Different perception of the boids configuration

## 4 Results

After having tried with the five cases, the results seen may be outlined as:

- Zone of repulsion alone type give nice results
- Field of view type is not stable by itself: given that all birds in the direction of flight are analyzed, boids will oscillate because they will turn around very quickly without barely moving
- k-nearest used in its own resulted in cohesion and good flock behavior
- Zone of repulsion and field of view combined gave the results discussed in the previous assignment
- Field of view with k-nearest yield the most interesting behavior and is the one that will be analyzed

The different configurations tested may be seen in the next table:

PARAMETER	$\mathbf{VALUE}$
Grid size	$200 \times 200$
Individuals	50
${ m Timesteps}$	100
Predator	No
Velocity coefficients	(1/100, 1, 1/8, 1)
Vision modes tested	[k-nearest, $k$ -nearest & field of vision]
Same simulation runs	10
k-nearest	$[1,\!5,\!10,\!15,\!20]$
Field of vision angle	$[0.2\pi, 0.4\pi, 0.6\pi, 0.8\pi, 1.0\pi, 1.2\pi]$
Zone of repulsion radio	[1,2,3,4,5]

After running 10 times the same setup, the maximum distance of the flock was analyzed. Computing the mean and the standard deviation of the maximum distance the behavior of the system may be analyzed. This setup may be summarized in the three next simulations:

• Parameter sweep of the number of k nearest boids with the k-nearest & field of view mode:

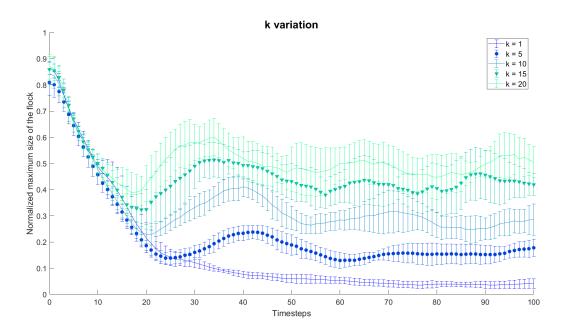


Figure 4: k-nearest parameter sweep (maximum distance mean and standard deviation)

• Parameter sweep of the number of the angle of the field of view with the k-nearest & field of view mode:

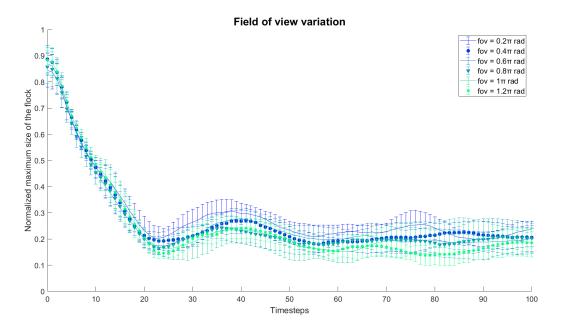


Figure 5: Field of vision angle parameter sweep: maximum distance mean and standard deviation

• Parameter sweep of the radius of the zone of repulsion with k-nearest neighbors (not considering field of view):

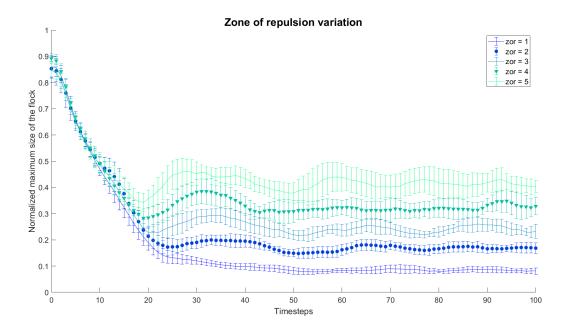


Figure 6: Zone of repulsion parameter sweep: maximum distance mean and standard deviation

## 5 Conclusion and discussion

The conclusions that can be drawn may be separated in three bullet items:

- When dealing with a relation between boids of the k-nearest neighbors that are located in the field of view of each boid, flock dispersion is very sensitive to the number of individuals k (see Fig. 4). The behavior that has been analyzed appears beginning from the timestep 20 the decline of normalized maximum size from 0 to 20 is accounted to the random dispersion of boids at first until they form a flock. If k is low, flock distance is small, having that the normalized maximum distance goes even beyond 0.1. Increasing the number of boids which form the neighborhood of certain boid, rise the maximum size of the flock and also give more instabilities, having a size varying flock. Given that there are 50 individuals, when the number k is big enough, the increase in normalized maximum distance in smaller (the increase between the means of k = 1 and k = 10 is larger than the increase between k = 10 and k = 20).
- However, the same system as before with fixed k = 6 (as suggested from the paper) with a variation of the angle of the field of view, showed that the system is less susceptible to the variation of the fov angle than the variation of k (see Fig. 5).
- Finally, taking the system that only depends on the k-nearest neighbors and varying the minimum radius (see Fig. 6) it can be seen a predictable result: as the minimum distance between boids increases, the maximum size of the flock also increases. A less expected result is that the oscillations also increase when the radius of repulsion is bigger.

Further developments in this study can consist of a parameter sweep of multiple variables at the same time and the inclusion of other possible neighbors types to get a realistic model.

# Code listing

initialization.m file

```
function [boid, pred] = initialization(dim,n,prd)
    % INITIALIZATION: function to create random initial boid (and predator)
    % positions and velocity
    %
    % INPUTS:
    % n = number of boids
    % dim = size of the domain
    % prd = if True, include the predator in the simulation
    % OUTPUTS:
    % boids = nx4 with n boids position and velocity in both components
    % pred = 1x4 with predator position and velocity in both components
13
    % Xing Jin and Javier Lobato, created on 2018/04/03
14
    % Loop over boids and fill in random position and velocity inside limits
16
17
    boid(:,1:2) = rand(n,2)*dim;
    boid(:,3:4) = (rand(n,2)*2-1)*dim*0.02;
18
19
    % Initialize predator location and velocity if desired
20
    if prd
21
        pred(:,1:2) = rand(1,2)*dim;
22
        % Contrary to the boids, initialize the predator with zero velocity
23
        pred(:,3:4) = [0 0];
24
    else
25
        % Case where no predator is desired
26
        pred = [];
27
    end
28
29
30
    end
```

### rule1.m file

```
function v1 = rule1(n,boid)

% RULE1: function that calculates the effect of Rule 1: boids try to fly

% towards the center of mass of the neighboring boids.

%

%

NOUTPUTS:

OUTPUTS:

VII = correction of the velocity for rule 1

%

Xing Jin and Javier Lobato, created on 2018/04/03

Javier Lobato, last modified on 2018/04/15

Given that the test will be carried out without any predator, this rule
```

```
% will not be updated with the different possible neighborhood cases
16
17
    % Preallocation of the output matrix
18
    v1 = zeros(n,2);
19
20
    % Loop over n boids
21
    for i = 1:n
22
        % Copy the position of all boids
23
        tem = boid(:,1:2);
24
        % Erase the row of the current i-th boid
25
        tem(i,:) = [];
        % Compute the mean of the position (relative center of gravity)
27
        xave = mean(tem(:,1));
        yave = mean(tem(:,2));
29
         % Direct the current boid position to the center of gravity
30
         v1(i,1:2) = [xave, yave] - boid(i,1:2);
31
    end
32
33
34
     end
```

### rule2.m file

```
function v2=rule2(n,boid,zor,fov,k,vM)
    % RULE2: function that calculates the effect of Rule 2: boids try to keep
    % certain distance with their neighbors
3
    % INPUTS:
    % n = number of boids
    % boids = boids position and velocity
    % zor = (circular) zone of repulsion / minimum distance (see below)
    % for = angle (in radians) of the boid field of view
9
    % k
            = number of nearest boids
            = string with the vision mode
    % vM
11
    %
12
    % OUTPUTS:
13
    % v2
          = correction of the velocity for rule 2
14
15
    % Xing Jin and Javier Lobato, created on 2018/04/03
16
17
    % Javier Lobato, last modified on 2018/04/15
18
    % Preallocation of the output matrix
19
    v2 = zeros(n,2);
20
21
    % Preallocate space for the boids that are inside the field of view
23
    fovInd = zeros([length(boid),1]);
    fovCount = 1;
24
25
    % Loop over n boids
26
    for i = 1:n
27
28
        % Copy the position of all boids
        tem = boid(:,1:2);
29
        \% Instead of erasing the row of the current i-th boid, a comparison
30
        % inside the loop will be done. This is done in order to mantain the
31
```

```
32
         % indexes of the original matrix the same
         % Get the current i-th boid
33
         bi = boid(i,1:2);
34
         % Loop over the other n-1 boids for zor, for and zf
35
         for j = 1:n
36
             if j ~= i
37
                  \mbox{\%} Store the relative position vector between boid j and boid i
38
                  dis = tem(j,:) - bi;
39
                  switch vM
40
                      case 'zor'
41
                          % If the modulus of the distance is smaller than the
                          % specified radius
43
                          if abs(norm(dis)) < zor</pre>
                              % Repel the i-th bird with respect the j-th bird
45
                              % and accumulate the consecutive repulsion of all
46
                              % the n-1 birds
47
                              v2(i,1:2) = v2(i,1:2) - dis;
48
                          end
                      case 'fov'
50
                          % If the angle is included within the field of vision
51
                          if acos(dot(dis,boid(i,3:4))/(norm(dis)*norm(boid(i,3:4)))) < fov/2</pre>
52
                              % Repel the i-th bird with respect the j-th bird
53
                              % and accumulate the consecutive repulsion of all
54
                              % the n-1 birds
55
                              v2(i,1:2) = v2(i,1:2) - dis;
56
57
                      case 'kn'
                          % Do nothing inside this for loop
59
                      case 'zf'
60
                          % If the modulus of the distance is smaller than the
61
                          % specified radius and the angle is included within the
62
                          % field of vision
63
                          if abs(norm(dis)) < zor &&
64
                           \rightarrow acos(dot(dis,boid(i,3:4))/(norm(dis)*norm(boid(i,3:4)))) < fov/2
                              % Repel the i-th bird with respect the j-th bird
65
                              % and accumulate the consecutive repulsion of all
66
                              % the n-1 birds
67
                              v2(i,1:2) = v2(i,1:2) - dis;
68
                          end
69
                      case 'fk'
70
                          % If the angle is included within the field of vision
71
                              if acos(dot(dis,boid(i,3:4))/(norm(dis)*norm(boid(i,3:4)))) < fov/2</pre>
72
                                   % Just save the birds that are inside for
                                   fovInd(fovCount) = j;
74
                                   fovCount = fovCount + 1;
75
76
                              end
77
                          error('Bad vision mode selection')
78
                  end
79
             end
         end
81
         % Remove the zeros of the preallocated matrix
82
         fovInd = fovInd(1:fovCount-1);
83
         % Don't enter the loop if there is no birds in the field of view
84
         if sum(fovInd) \sim = -2
85
             switch vM
86
                  case 'zor'
87
```

```
% Everything done
88
                   case 'fov'
89
                       % Everything done
90
                   case 'kn'
91
                       disXY = boid(:,1:2) - boid(i,1:2);
92
                       dist = sqrt(disXY(:,1).^2+disXY(:,2).^2);
93
                       [~,I] = sort(dist);
94
                       % ind will have a distance of zero so let's remove it
95
                       for j=1:length(I)
96
                           dis = tem(I(j),:) - bi;
97
                           % Given that these modes can't be combined with zone of
                           % repulsion, that input value will be used as the minimum
99
                           % distance between boids
100
                           if norm(dis) < zor</pre>
101
                               v2(i,1:2) = v2(i,1:2) - dis;
102
                           end
103
                       end
104
                   case 'zf'
105
                       % Everything done
106
                   case 'fk'
107
                       % If field of view is also used, only the birds inside of
108
                       % it will be considered to compute the closest ones
109
                       disXY = boid(fovInd,1:2) - boid(i,1:2);
110
                       dist = sqrt(disXY(:,1).^2+disXY(:,2).^2);
111
                       [~,I] = sort(dist);
112
                       I = fovInd(I);
113
                       if k > length(I)
                           I = I(1:length(I));
115
                       else
                           I = I(1:k);
117
                       end
118
                       for j=1:length(I)
119
120
                           dis = tem(I(j),:) - bi;
121
                           % Given that these modes can't be combined with zone of
                           % repulsion, that input value will be used as the minimum
122
                           % distance between boids
123
                           if norm(dis) < zor</pre>
124
125
                               v2(i,1:2) = v2(i,1:2) - dis;
                           end
126
                       end
127
                   otherwise
128
                       error('Bad vision mode selection')
129
130
              end
          end
131
      end
132
133
134
      end
```

#### rule3.m file

```
function v3 = rule3(n,boid)
    \mbox{\it \%} RULE3: function that calculates the effect of Rule 3: boids try to match
2
    % the velocity with the mean velocity
3
    % INPUTS:
    % n = number of boids
    % boids = boids position and velocity
    % OUTPUTS:
9
10
    % v3 = correction of the velocity for rule 3
    % Xing Jin and Javier Lobato, created on 2018/04/03
12
    % Javier Lobato, last modified on 2018/04/15
13
14
    % Given that the test will be carried out without any predator, this rule
15
    % will not be updated with the different possible neighborhood cases
16
17
    % Preallocation of the output matrix
18
    v3 = zeros(n,2);
19
20
    % Loop over n boids
21
22
    for i = 1:n
        % Copy the velocity of all boids
23
        tem = boid(:,3:4);
24
        % Erase the row of the current i-th boid
25
        tem(i,:) = [];
26
        % Compute the mean of the veloicty
27
        vxave = mean(tem(:,1));
28
        vyave = mean(tem(:,2));
29
         % Match the mean velocity with the i-th boid's velocity
30
        v3(i,1:2) = [vxave, vyave] - boid(i,3:4);
31
32
    end
33
34
    end
```

#### rule4.m file

```
function v4 = rule4(n,boid,pred,zop,fov)
    % RULE4: function that calculates the effect of Rule 4: boids try to avoid
2
    % a possible incoming predator
3
    % INPUTS:
    % n = number of boids
    % boids = boids position and velocity
    % pred = current predator position and velocity
    % zop = (circular) zone of predator avoidance
10
    % for = angle (in radians) of the boid field of view
    % OUTPUTS:
12
    % 04
           = correction of the velocity for rule 4
13
14
    % Xing Jin and Javier Lobato, created on 2018/04/03
15
    % Javier Lobato, last modified on 2018/04/15
16
17
    % Given that the test will be carried out without any predator, this rule
18
    \% will not be updated with the different possible neighborhood cases
19
20
    % Preallocation of the output matrix
21
22
    v4 = zeros(n,2);
23
    % Loop over n boids
24
    for i = 1:n
25
        % Get the distance of the i-th boid to the predator
26
27
        dis = boid(i,1:2) - pred(1:2);
        % Get the current boid velocity and transpose
28
        vel = (boid(i,3:4))';
29
        % If both the predator is inside the field of view and the radius of
30
        % the zone of predator avoidance
31
        if acos(dot(dis,vel)/norm(dis)/norm(vel))<fov/2 && abs(norm(dis))<zop
32
            % Modify the velocity of the i-th boid to avoid the predator
33
            v4(i,1:2) = (boid(i,1:2)-pred(1:2));
34
        end
35
     end
36
37
38
     end
```

### boundposition.m file

```
function v5 = boundposition(boids, limits, correction)
     %BOUNDPOSITION: function that returns the boids into the limits
2
3
    % INPUTS:
    % boids
                  = boids position (do NOT include velocity)
     % limits
                  = limits in which the position will be bound
     % correction = velocity that will be used to revert the boid inside bounds
    % OUTPUTS:
9
    % v5
10
                  = correction of the velocity for rule 5
11
    % Xing Jin and Javier Lobato, created on 2018/04/03
12
13
     % Preallocation of the output matrix
14
     v5 = zeros(length(boids),2);
15
16
17
     % Loop over n boids
     for i=1:length(boids)
18
         % Minimum horizontal axis position
19
         if boids(i,1) < limits(1)</pre>
20
             % Positive velocity increment
21
22
             v5(i,1) = correction;
         % Maximum horizontal axis position
23
         elseif boids(i,1) > limits(2)
24
             % Negative velocity increment
25
             v5(i,1) = - correction;
26
27
         end
28
         % Minimum vertical axis position
29
         if boids(i,2) < limits(3)</pre>
30
             % Positive velocity increment
31
             v5(i,2) = correction;
32
         % Maximum vertical axis position
33
         elseif boids(i,2) > limits(4)
34
             % Negative velocity increment
35
             v5(i,2) = - correction;
         end
37
38
     end
39
40
```

## limitvelocity.m file

```
function vel = limitvelocity(boidVel, vlim)
    % LIMITVELOCITY: function that limitates the maximum possible velocity of
2
    % the boids
3
    % INPUTS:
    % boidVel = boids velocity (do NOT include position)
    % vlim = maximum allowable boid velocity
    % OUTPUTS:
9
10
    % vel = new boid velocity
11
    % Xing Jin and Javier Lobato, created on 2018/04/03
12
13
    % Preallocation of the output matrix
14
    vel = zeros(length(boidVel),2);
15
16
17
    % Loop over all the n boids
    for i = 1:length(boidVel)
18
        \% If the modulus of the velocity of the i\text{--}th boid is over the limit
19
        if norm(boidVel(i,:)) > vlim
20
             % Set the velocity to keep the same direction but the magnitude
21
             % established in the limit velocity
             vel(i,:) = vlim*(boidVel(i,:)/norm(boidVel(i,:)));
23
         else
24
             % If it is under the limit, don't modify the velocity
25
             vel(i,:) = boidVel(i,:);
26
27
         end
    end
28
29
    end
30
```

### boundary.m file

```
function boid = boundary(dim, boid)
    % BOUNDARY: function that bounces the boids inside the grid if they get to
2
    % the walls with the same velocity it hit the wall
3
    % INPUTS:
5
    % dim = size of the domain
    % boid = boids position and velocity
    % OUTPUTS:
9
10
    % boid = boids position and velocity
11
    % Xing Jin and Javier Lobato, created on 2018/04/03
12
    % Javier Lobato, last modified on 2018/04/15
13
14
    % The changes made in this function are related to an increase in the
15
    % velocity of the boids when they hit the wall. This was done to reduce the
16
    % possibility that a boid hits the wall too slow and the bouncing velocity
17
    % was also too slow (taking too much time to move towards the flock)
18
19
    % Loop over all the boids
20
    for j = 1:size(boid,1)
21
        % Bounce on the horizontal direction
22
         % If the boid moves to negative x-axis
23
         if boid(j,1) < 0
24
            boid(j,1) = -boid(j,1);
25
            boid(j,3) = -10*boid(j,3);
26
         % If it leaves the grid on the right side
27
         elseif boid(j,1) > dim
28
            boid(j,1) = 2*dim - boid(j,1);
29
             boid(j,3) = -10*boid(j,3);
30
31
         % Bounce on the vertical direction
         % If the boid moves to the negative y-axis
33
         if boid(j,2) < 0
34
            boid(j,2) = -boid(j,2);
35
            boid(j,4) = -10*boid(j,4);
         % If the boid moves upper a distance of dim
37
         elseif boid(j,2) > dim
38
             boid(j,2) = 2*dim - boid(j,2);
39
             boid(j,4) = -10*boid(j,4);
40
41
         end
42
    end
43
44
     end
```

### updateboid.m file

```
function boid = updateboid(dim,n,boid,prd,pred,coe,zor,zop,fov,k,velLim,velCorr,vM)
    % UPDATEBOID: updates the position of each boid using the different rules
2
3
    % INPUTS:
    % dim
              = size of domain
    % n
              = number of boids
    % boids = boids position and velocity
    % prd = predator flag
    % coe
             = weighting of the velocity from each rule
9
              = radius of the zone of repulsion
    % zop
              = radius of the zone of predator avoidence
    % for
             = field of view angle
12
             = number of nearest neighbors
    % k
    % velLim = limit on the boid velocity
14
    % velCorr = correction velocity if the boid moves outside bounds
15
             = vision mode of the boids
    % υM
16
17
    % OUTPUTS:
    % boid = array with updated position and velocities
19
20
    % Xing Jin and Javier Lobato, created on 2018/04/03
21
    % Javier Lobato, last modified on 2018/04/15
22
23
    % Compute the variations in velocity for each rule
24
    v1 = rule1(n,boid);
    v2 = rule2(n,boid,zor,fov,k,vM);
26
    v3 = rule3(n,boid);
27
28
    % The fourth rule will one be computed if the predator flag is True
    if prd
30
        v4 = rule4(n,boid,pred,zop,fov);
31
32
    else
33
        v4 = 0;
34
    end
35
    % Update the velocity with the variations and their respective weights
    boid(:,3:4) = boid(:,3:4) + coe(1)*v1 + coe(2)*v2 + coe(3)*v3 + coe(4)*v4;
37
    % If the velocity if over the limit, correct it
38
    boid(:,3:4) = limitvelocity(boid(:,3:4), velLim);
39
40
    % Update the position with the new velocity
41
42
    boid(:,1) = boid(:,1) + boid(:,3);
    boid(:,2) = boid(:,2) + boid(:,4);
43
44
    % Correct velocity if the boid is going out of bounds (bounds = +-0.1*dim)
45
    boid(:,3:4) = boid(:,3:4) + boundposition(boid(:,1:2), ...
46
47
         [dim/10, 9*dim/10, dim/10, 9*dim/10], velCorr);
48
    % If boids are in the boundary, bounce them
49
    boid = boundary(dim,boid);
50
51
```

### updatepred.m file

```
function pred = updatepred(dim, n, boid, pred, prdSpeed)
    % UPDATEPRED: updates the position of the predator
    %
3
    % INPUTS:
4
    % dim = size of domain
    % n
              = number of boids
    % boid = boids position and velocity
    % pred
              = predator position and velocity
    % prdSpeed = predator maximum speed
10
    % OUTPUTS:
11
    % pred = array with updated position and velocity of the predator
13
    % Xing Jin & Javier Lobato 2018/04/03
14
15
16
    % Get the true center of gravity of the flock
    c = sum(boid(:,1:2))/n;
17
18
    % Move the predator with the desired speed towards the center of gravity
19
    pred(3:4) = prdSpeed*(c-pred(1:2))/norm(c-pred(1:2));
20
^{21}
    % Update the position of the predator with the new velocity
22
    pred(1) = pred(1)+pred(:,3);
    pred(2) = pred(2)+pred(:,4);
24
    % If the predator is over the boundaries of the grid, it must also bounce
26
    pred = boundary(dim, pred);
27
28
    end
29
```

### pltdistribution.m file

```
function pltdistribution(dim,boid,prd,pred,arrow,plotFov, zor, fov,k,ind,visionMode)
    % PLTDISTRIBUTION: function that plots the position of the boids with a set
    % of flags for customization
4
    % INPUTS:
5
    % dim
                 = dimension of the grid
    % boid
                 = boids with position and velocity
    % prd
                 = predator flag
                 = predator position and velocity
    % pred
                 = arrow plotting flag
    % arrow
    % plotFov
                 = field of vision/zone of repulsion plotting
    % zor
                 = zone of repulsion radius
12
                 = field of vision angles
    % for
13
   % k
                 = number of closest neighbors
    % ind = index of the individual to plot the 'zor' and 'fov' over
15
    % visionMode = vision mode configuration to plot
16
17
    % OUTPUTS:
18
    % Plot with the results
19
```

```
20
    % Xing Jin and Javier Lobato, created on 2018/04/03
21
    % Javier Lobato, last modified on 2018/04/15
22
23
    % Delete previous arrows (i.e. annotations)
24
    delete(findall(gcf, 'type', 'annotation'))
    % Plot all points with blue circles
26
     scatter(boid(:,1),boid(:,2),...
27
         10,'o','filled','MarkerFaceColor','b')
28
     % Get the position of the current figure for the arrows
29
    pos = get(gca, 'Position');
30
31
    % Preallocate space for the boids that are inside the field of view
    fovInd = zeros([length(boid),1]);
33
34
    fovCount = 1;
35
    % If the zor & fov is wanted to be shown
36
    if plotFov
37
         % Get a list with all index from 1 to n
38
         listInd = linspace(1,length(boid),length(boid));
         % Set as empty the index of the individual ind
40
         listInd(ind) = [];
41
         hold on
42
         % Plot that individual as a blue square to make it bigger
43
         scatter(boid(ind,1),boid(ind,2),...
44
             30, 's', 'filled', 'MarkerFaceColor', 'b')
45
         hold off
         % Get the velocity vector of the individual ind
47
         vel=(-boid(ind,3:4))';
48
         % Create and arrow pointing in the direction of individual ind movement
49
         annotation('arrow', [boid(ind,1)/dim*pos(3) + pos(1),...
50
          (boid(ind,1)+boid(ind,3))/dim*pos(3) + pos(1)],...
51
          [boid(ind,2)/dim*pos(4) + pos(2),...
52
          (boid(ind,2)+boid(ind,4))/dim*pos(4) + pos(2)],...
          'Color', 'b');
54
         % Draw a circle on the zone of repulsion
55
         if visionMode(1) == 1
56
57
             th = 0:pi/50:2*pi;
             xunit = zor * cos(th) + boid(ind,1);
             yunit = zor * sin(th) + boid(ind,2);
59
             hold on
             h = plot(xunit, yunit, '--r');
61
             hold off
         end
63
         if visionMode(2) == 1
64
             % Draw two lines that will define the field of vision of the boid
65
             arc = atan(boid(ind,4)/boid(ind,3));
66
             % Taking into account the possible returns of the atan function
67
             if boid(ind,3) > 0
68
                 hold on
                 plot([boid(ind,1),boid(ind,1)-dim^2*cos(pi+arc+fov/2)],...
70
                      [boid(ind,2),boid(ind,2)-dim^2*sin(pi+arc+fov/2)],':k')
71
                 plot([boid(ind,1),boid(ind,1)-dim^2*cos(pi+arc-fov/2)],...
72
                      [boid(ind,2),boid(ind,2)-dim^2*sin(pi+arc-fov/2)],':k')
73
                 hold off
74
             else
75
                 hold on
76
```

```
plot([boid(ind,1),boid(ind,1)-dim^2*cos(arc+fov/2)],...
77
                       [boid(ind,2),boid(ind,2)-dim^2*sin(arc+fov/2)],':k')
78
                  plot([boid(ind,1),boid(ind,1)-dim^2*cos(arc-fov/2)],...
79
                       [boid(ind,2),boid(ind,2)-dim^2*sin(arc-fov/2)],':k')
 80
                  hold off
81
              end
82
          end
 83
          % Loop over all the possible individuals except ind
84
          for i=listInd
 85
              \% Compute the vectorial distance from ind to the i-th individual
86
              dis=boid(ind,1:2)-boid(i,1:2);
              if visionMode(1) == 1
88
                  % If the i-th boid is inside the zor
                  if abs(norm(dis)) < zor</pre>
90
                       % Plot a downwards pointing triangle
91
                       hold on
92
                       scatter(boid(i,1),boid(i,2),...
93
                           25, 'v', 'filled', 'MarkerFaceColor', 'k')
                       hold off
95
                  end
              end
97
              if visionMode(2) == 1
98
                  % If the i-th boid is inside the field of view
aa
                  if acos(dot(dis,vel)/(norm(dis)*norm(vel))) < fov/2</pre>
100
                       % Save the birds that are inside the field of view
101
                       fovInd(fovCount) = i;
102
                       fovCount = fovCount + 1;
                       % Plot an upwards pointing triangle
104
105
                       hold on
                       scatter(boid(i,1),boid(i,2),...
106
                           25,'^','filled','MarkerFaceColor','k')
107
                       hold off
108
109
                  end
              end
110
          end
111
          % Remove the zeros of the array to avoid index problems
112
          fovInd = fovInd(1:fovCount-1);
113
114
          % Plotting of the k-nearest boids
          if visionMode(3) == 1
115
              % If field of view is also used, only the birds inside of it will
116
              % be considered to compute the closest ones
117
              if visionMode(2) == 1
118
                  disXY = boid(fovInd,1:2) - boid(ind,1:2);
                  dis = sqrt(disXY(:,1).^2+disXY(:,2).^2);
120
                  [~,I] = sort(dis);
                  I = fovInd(I);
122
                  I = I(1:k);
123
                  if k > length(I)
124
                       I = I(1:length(I));
125
                  else
                       I = I(1:k);
127
128
              % Otherwise all birds will be analyzed
129
130
                  disXY = boid(:,1:2) - boid(ind,1:2);
131
                  dis = sqrt(disXY(:,1).^2+disXY(:,2).^2);
132
133
                   [~,I] = sort(dis);
```

```
134
                   % ind will have a distance of zero so let's remove it
                  I = I(2:k+1);
135
                  if k > length(I)
136
                       I = I(1:length(I));
137
                   else
138
                       I = I(1:k);
139
                   end
140
              end
141
              % Plot a downwards pointing triangle for the k nearest boids
142
143
                  hold on
                   scatter(boid(i,1),boid(i,2),...
145
                           25, 'v', 'filled', 'MarkerFaceColor', 'k')
146
                  hold off
147
              end
148
          end
149
150
      end
151
152
153
      % If the arrows are desired
154
      if arrow
155
          % Loop over all possible boids
156
          for i=1:length(boid)
157
              % Plot the arrow as an annotation object
              annotation('arrow', [boid(i,1)/dim*pos(3) + pos(1),...
159
                    (boid(i,1)+boid(i,3))/dim*pos(3) + pos(1)],...
                    [boid(i,2)/dim*pos(4) + pos(2),...
161
                    (boid(i,2)+boid(i,4))/dim*pos(4) + pos(2)],...
162
                    'Color', 'b');
163
164
          end
165
      end
166
167
      % If there is a predator
      if prd
168
          % Plot it as a red square
169
          hold on
170
171
          scatter(pred(1),pred(2),30,'s','filled','MarkerFaceColor','r')
          hold off
172
          % Plot the arrow of the predator to know where it is pointing
173
          annotation('arrow', [pred(1)/dim*pos(3) + pos(1),...
174
                (pred(1)+pred(3))/dim*pos(3) + pos(1)],...
175
                [pred(2)/dim*pos(4) + pos(2),...
                (pred(2)+pred(4))/dim*pos(4) + pos(2)],...
177
                'Color', 'r');
178
      end
179
180
      % Fix axis of the plot
181
      axis([0 dim 0 dim])
182
      % Set the fontsize of the axis as 16
      set(gca, 'FontSize',14)
184
185
```

#### swarmModel.m file

```
function [max_dist] =
     swarmModel(n,dim,ts,zor,zop,fov,k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arr,figNo,visionMode)
     %SWARMMODEL: simulation of a BOIDS swarm model based on the pseudo code
2
    %taken from http://www.vergenet.net/~conrad/boids/pseudocode.html
4
    % INPUTS:
    % n
                 = number of boids
    % dim
                 = size of the domain
    % ts
                 = simulation time
    % zor
                  = radius of the zone of repulsion
    % zop
                 = radius of the zone of predator avoidance
    % for
                 = field of view angle
11
                 = number of closest neeighbors
    % k
    % coe
                 = weighting of the velocity from each rule
13
    % prd
                  = predator flag
14
                 = predator speed
    % prdSpd
15
    % vLim
                  = velocity limit for the boids
16
                 = velocity correction for the boids leaving bounds
    % vCorr
                 = plotting options, true to plot, false to not
    % plop
18
                 = zor and fow plotting flag
    % pltFov
                 = arrows plotting flag
    % arr
20
               = number of figure to avoid superposition
    % visionMode = vector with three components for different neighborhoods
22
23
    % OUTPUTS:
24
    % plot of the desired simulation case setup
25
26
    % Xing Jin and Javier Lobato, created on 2018/04/03
27
    % Javier Lobato, last modified on 2018/04/15
29
    % Initialize position and velocity of the flock
30
     [boid,pred] = initialization(dim,n,prd);
31
32
    % Create a new figure to avoid superposition
33
    if plop
34
        figure(figNo)
36
    end
37
    % Test visionMode vector to analyze its possibilities:
38
           [0 0 0] -> not valid configuration
39
            [1 0 0] -> zor (zone of repulsion)
40
            [0 1 0] -> fov (field of vision)
41
           [0 \ 0 \ 1] \rightarrow kn \ (k-nearest boids)
           [1 1 0] -> zf (zone of repulsion + field of vision)
43
            [1 0 1] -> not valid configuration (additive configuration)
44
           [0 1 1] \rightarrow fk (field of vision + k-nearest)
45
46
            [1 1 1] -> not valid configuration (additive configuration)
47
    % The invalid configurations will raise an error and exit the function
48
    if sum(visionMode) == 3
49
        error('Not valid visionMode vector')
50
        % To force the system to leave the function, let's limit ts as zero
51
        ts = 0;
52
     elseif sum(visionMode) == 0
53
```

```
error('Not valid visionMode vector')
54
          ts = 0;
55
     elseif sum(visionMode) == 2
56
          if visionMode(3) == 0
57
              vM = 'zf';
58
          elseif visionMode(1) == 0
59
              vM = 'fk';
60
          else
61
              error('Not valid visionMode vector')
62
63
64
          end
     elseif sum(visionMode) == 1
65
          if visionMode(1) == 1
66
              vM = 'zor';
67
          elseif visionMode(2) == 1
68
              vM = 'fov';
69
          else
70
              vM = 'kn';
71
72
          end
     end
73
74
     % Preallocation of space for the maximum distance array
75
     \max_{dist} = zeros([(ts+1),1]);
76
77
     % Calculation of the maxmimum distance for the initialization
78
     max_dist(1) = maxDist(boid(:,1:2));
79
     % If the pltFov flag is True, get the index for the boid to track
81
82
     if pltFov
          ind = randsample(length(boid),1);
83
          boid(ind,1:2) = dim/2;
84
85
     else
          ind = 1; % Otherwise define it as 1 although it won't be used
86
87
     end
88
     % Time loop from 1 until the specified ts
89
     for i=1:ts
90
91
         % Update the position of the boids
          boid = updateboid(dim,n,boid,prd,pred,coe,zor,zop,fov,k,vLim,vCorr,vM);
92
              % (If there is a predator, also update it) though it will never be
93
              % a predator in the current experiment setup
              if prd
95
                   pred = updatepred(dim,n,boid,pred,prdSpd);
96
              end
97
          % Plot current position of boids and predator if desired
98
          if plop
99
              pltdistribution(dim,boid,prd,pred,arr,pltFov, zor, fov, k, ind, visionMode)
100
101
          end
          % Pause to see the figure
102
          pause(0.001)
103
          % Calculations of the largest distance between birds
104
105
          \max_{dist(i+1)} = \max_{dist(boid(:,1:2))};
     end
106
107
108
     end
```

#### maxDist.m file

```
function [ max_dist ] = maxDist(boidPos)
    % MAXDIST: function that computes the maximum distance between boids in the
2
    % current flock distribution. Computing the distance between one bird to
    % every other bird, taking the maximum, repeating that for all birds and
    % taking the maximum of the maximums is an inefficient way of computing the
    % maximum distance between two points. Following the advice desrcibed:
                 https://stackoverflow.com/a/8006849
    % the maximum distance between the boids can be computed without the need
    % of looping twice the array
9
10
    % INPUTS:
11
    % boidPos = position of the boids in the flock
12
    % OUTPUTS:
14
    % max_dist = value of the maximum distance
15
16
    % Javier Lobato, created on 2018/04/15
17
18
    % Let's add the components x+y and store them
19
    adding = boidPos(:,1) + boidPos(:,2);
20
    % Let's substract the components x-y and store them
21
    substracting = boidPos(:,1) - boidPos(:,2);
23
^{24}
    % Lower left point is the min(x+y)
    [~, ill] = min(adding);
    % Upper left point is the min(x-y)
26
27
     [~, iul] = min(substracting);
    % Upper right point is the max(x+y)
28
     [~, iur] = max(adding);
    % Lower right point is the max(x-y)
30
     [~, ilr] = max(substracting);
31
32
    % Let's compute the distance between the most lower left and most upper
33
    % right point
34
    d1 = sqrt((boidPos(ill,1)-boidPos(iur,1))^2 + (boidPos(ill,2)-boidPos(iur,2))^2);
35
    % Let's compute the distance between the most upper left and most upper
37
    d2 = sqrt((boidPos(iul,1)-boidPos(ilr,1))^2 + (boidPos(iul,2)-boidPos(ilr,2))^2);
38
39
    % Take the biggest of these two measures
40
    \max_{dist} = \max(d1,d2);
41
42
    end
43
```

#### HW4b\_driver.m file

```
1
    HOMEWORK #4.B
2
    3
    % Javier Lobato, created on 2018/04/15
    close all; clear all; clc
    % Mode of vision for the flock of birds
    % The elements of the vector are:
         - proximity (circle of repulsion)
9
         - field of view (angle of the bird vision)
10
11
         - k-nearest birds (taking into account only the closest birds)
12
    %% Mode of vision: zone of repulsion
13
    % Case variables
14
    n = 200; dim = 200; ts = 1;
15
    % Bird configuration
16
    zor = 40; zop = 20; fov = 0.4*pi; k = 6; vLim = 20; vCorr = 5;
17
    coe = [1/100, 1, 1/8, 1];
    % Predator configuration
19
    prd = false; prdSpd = 0;
    % Plotting configuration
21
    plop = true; arrows = false; pltFov = true; figNo = 1;
    % Mode of vision
23
    visionMode = [true, false, false];
^{24}
    % Function calling
    swarmModel(n,dim,ts,zor,zop,fov,k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
26
    title('Zone of repulsion');
27
28
    %% Mode of vision: field of view
    % Case variables
30
    n = 200; dim = 200; ts = 1;
31
    % Bird configuration
    zor = 2; zop = 20; fov = 0.6*pi; k = 6; vLim = 20; vCorr = 5;
33
    coe = [1/100, 1, 1/8, 1];
    % Predator configuration
35
    prd = false; prdSpd = 0;
    % Plotting configuration
37
    plop = true; arrows = false; pltFov = true; figNo = 2;
38
    % Mode of vision
    visionMode = [false, true, false];
40
    % Function calling
41
42
    swarmModel(n,dim,ts,zor,zop,fov,k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
    title('Field of view');
44
    %% Mode of vision: k-nearest neighbors
45
    % Case variables
    n = 200; dim = 200; ts = 1;
47
    % Bird configuration
    zor = 2; zop = 20; fov = 0.4*pi; k = 15; vLim = 20; vCorr = 5;
49
    coe = [1/100, 1, 1/8, 1];
    % Predator configuration
51
    prd = false; prdSpd = 0;
52
    % Plotting configurationc
53
    plop = true; arrows = false; pltFov = true; figNo = 3;
```

```
% Mode of vision
55
    visionMode = [false, false, true];
56
    % Function calling
57
    swarmModel (n, dim, ts, zor, zop, fov, k, coe, prd, prdSpd, vLim, vCorr, plop, pltFov, arrows, figNo, visionMode); \\
    title('k-nearest neighbors');
59
60
    %% Mode of vision: zone of repulsion + field of view
61
    % Case variables
62
    n = 200; dim = 200; ts = 1;
63
    % Bird configuration
64
    zor = 40; zop = 20; fov = 0.6*pi; k = 6; vLim = 20; vCorr = 5;
    coe = [1/100, 1, 1/8, 1];
66
    % Predator configuration
    prd = false; prdSpd = 0;
68
    % Plotting configurations
69
    plop = true; arrows = false; pltFov = true; figNo = 4;
70
    % Mode of vision
71
    visionMode = [true, true, false];
72
    % Function calling
73
    swarmModel(n,dim,ts,zor,zop,fov,k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
74
    title('Zone of repulsion + Field of view');
75
76
    %% Mode of vision: field of view + k-nearest neighbors
77
    % Case variables
78
    n = 200; dim = 200; ts = 1;
79
    % Bird configuration
80
    zor = 2; zop = 20; fov = 0.4*pi; k = 10; vLim = 20; vCorr = 5;
    coe = [1/100, 1, 1/8, 1];
82
    % Predator configuration
83
    prd = false; prdSpd = 0;
84
    % Plotting configurationc
85
    plop = true; arrows = false; pltFov = true; figNo = 5;
87
    % Mode of vision
    visionMode = [false, true, true];
    % Function calling
89
    swarmModel(n,dim,ts,zor,zop,fov,k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
90
    title('Field of view + k-nearest neighbors');
91
```

### dataAnalysis.m file

```
HOMEWORK #4.B
2
    3
    % Javier Lobato, created on 2018/04/15
    close all; clear all; clc;
    %% Parameter sweep of the k-nearest neighbors
    % Case variables
    n = 50; dim = 200; ts = 100;
9
    % Bird configuration
10
    zor = 2; zop = 20; fov = 0.8*pi; vLim = 30; vCorr = 3;
    coe = [1/100, 1, 1/8, 1];
12
    % Predator configuration
13
    prd = false; prdSpd = 0;
14
    % Plotting configurationc
15
    plop = true; arrows = false; pltFov = false; figNo = 1;
16
    % Mode of vision
17
    visionMode = [false, true, true];
    % Parameter sweep vector
19
    k = [1,5,10,15,20];
    % Number of runs
21
    runs = 10;
    % Maximum diameter for this parameter sweep
23
    diam = zeros([length(k),runs,ts+1]);
24
    % Function calling
25
    for j=1:length(k)
26
27
        for i=1:runs
            diam(j,i,:) = swarmModel(n,dim,ts,zor,zop,fov,...
28
            k(j),coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
29
        end
30
31
    end
32
33
    figure(2)
    % Plot the mean and standard deviation for k-neearest parameter sweep
34
    c = (winter(length(k)));
35
    lw = ['-o', '-v', '-h', '-h', '-^', '-v'];
    hold on
37
    for j=1:length(k)
38
        errorbar(0:1:100,reshape(mean(diam(j,:,:)/sqrt(2*dim^2),2),[1,101]),...
39
        reshape(std(diam(j,:,:)/sqrt(2*dim^2)),[1,101]),lw(j),'Color',c(j,:),'MarkerFaceColor',c(j,:))
40
        legendList{j} = ['k = ',num2str(k(j))];
41
    end
42
    hold off
43
    ylim([0,1])
44
    ley = legend(legendList);
45
    set(ley, 'FontSize',14);
46
    xlabel('Timesteps')
47
    ylabel('Normalized maximum size of the flock')
    set(gca,'FontSize',14)
49
    title(['k variation'], 'FontSize', 20)
51
    %% Parameter sweep of the field of vision
52
    % Case variables
53
    n = 50; dim = 200; ts = 100;
54
```

```
% Bird configuration
55
     zor = 2; zop = 20; k = 6; vLim = 30; vCorr = 3;
56
     coe = [1/100, 1, 1/8, 1];
57
     % Predator configuration
     prd = false; prdSpd = 0;
59
     % Plotting configurations
     plop = true; arrows = false; pltFov = false; figNo = 1;
61
     % Mode of vision
62
     visionMode = [false, true, true];
63
     % Parameter sweep vector
64
     fov = [0.2*pi, 0.4*pi, 0.6*pi, 0.8*pi, 1.0*pi, 1.2*pi];
     % Number of runs
66
     runs = 10;
     % Maximum diameter for this parameter sweep
68
     diam = zeros([length(k),runs,ts+1]);
69
     % Function calling
70
     for j=1:length(fov)
71
         for i=1:runs
72
              diam(j,i,:) = swarmModel(n,dim,ts,zor,zop,fov(j),...
73
              k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
74
         end
75
     end
76
77
     figure(3)
78
     % Plot the mean and standard deviation for field of view parameter sweep
79
     c = (winter(length(fov)));
80
     lw = ['-o', '-v', '-h', '-h', '-^', '-v'];
     hold on
82
83
     for j=1:length(fov)
         errorbar(0:1:100,reshape(mean(diam(j,:,:)/sqrt(2*dim^2),2),[1,101]),...
84
         reshape(std(diam(j,:,:)/sqrt(2*dim^2)),[1,101]),lw(j),'Color',c(j,:),'MarkerFaceColor',c(j,:))
85
         legendList{j} = ['fov = ',num2str(fov(j)/pi),'? rad'];
86
87
     hold off
     ylim([0,1])
89
     ley = legend(legendList);
     set(ley,'FontSize',14);
91
92
     xlabel('Timesteps')
     ylabel('Normalized maximum size of the flock')
93
     set(gca, 'FontSize',14)
94
     title(['Field of view variation'], 'FontSize', 20)
96
     %% Parameter sweep of the size of the zone of repulsion
     % Case variables
98
     n = 50; dim = 200; ts = 100;
99
     % Bird configuration
100
     zop = 20; fov = 0.8*pi; k = 6; vLim = 30; vCorr = 3;
101
     coe = [1/100, 1, 1/8, 1];
102
103
     % Predator configuration
     prd = false; prdSpd = 0;
104
     % Plotting configurationc
105
106
     plop = true; arrows = false; pltFov = false; figNo = 1;
     % Mode of vision
107
     visionMode = [false, false, true];
108
     % Parameter sweep vector
109
     zor = [1,2,3,4,5];
110
111
     % Number of runs
```

```
112
     runs = 10;
     % Maximum diameter for this parameter sweep
113
     diam = zeros([length(k),runs,ts+1]);
114
     % Function calling
115
     for j=1:length(zor)
116
         for i=1:runs
117
              diam(j,i,:) = swarmModel(n,dim,ts,zor(j),zop,fov,...
118
              k,coe,prd,prdSpd,vLim,vCorr,plop,pltFov,arrows,figNo,visionMode);
119
          end
120
121
     end
123
     figure(4)
     % Plot the mean and standard deviation for zone of repulsion parameter sweep
     c = (winter(length(zor)));
125
     lw = ['-o','-v','-h','-h','-^'];
126
     hold on
127
     for j=1:length(zor)
128
          errorbar(0:1:100,reshape(mean(diam(j,:,:)/sqrt(2*dim^2),2),[1,101]), ...
          reshape(std(diam(j,:,:)/sqrt(2*dim^22)),[1,101]),lw(j),'Color',c(j,:),'MarkerFaceColor',c(j,:))\\
130
131
          legendList{j} = ['zor = ',num2str(zor(j))];
     end
132
     hold off
133
     ylim([0,1])
134
     ley = legend(legendList);
135
     set(ley,'FontSize',14);
136
     xlabel('Timesteps')
137
     ylabel('Normalized maximum size of the flock')
138
     set(gca, 'FontSize',14)
139
     title(['Zone of repulsion variation'], 'FontSize', 20)
140
```