Czech Technical University in Prague
Faculty of Nuclear Sciences and Physical Engineering
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Programme: Mathematical Engineering
Branch of Study: Mathematical Physics

High pT jets in RunII of the ATLAS Experiment

MASTER'S DEGREE PROJECT

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Submitted in: May 2015

Zadani prace

Statement

Prohlasuji. . .

V Praze dne ....

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Acknowledgment

Dekuji. . .

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Název práce:

Jety s vysokou příčnou hybností v RunII experimentu ATLAS

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Obor: Matematické Inženýrství

Druh práce: Diplomová práce

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CERN

Abstrakt: Abstrakt CZ

Klíčová slova: Klicova slova

Title:

High pT jets in RunII of the ATLAS Experiment

Author: Jan Lochman

Abstract: Abstrakt EN

Key words: Key words

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Introduction

Chapter 1

QCD

Is the purpose of theoretical physics to be no more than a cataloging of all the things that can happen when particles interact with each other and separate? Or is it to be an understanding at a deeper level in which there are things that are not directly observable (as the underlying quantized fields are) but in terms of which we shall have a more fundamental understanding?

Julian Schwinger

The theoretical framework of particle physics is called Standard Model (SM). The SM describes the way how the fundamental components of matter interact with each other through strong, weak and electromagnetic interactions. Mathematically the SM is gauge quantum field theory with local internal symmetries of the direct product group $SU(3)\times SU(2)\times U(1)$. Gauge bosons are assigned to generators of this symmetry - there are 8 massless gluons from $SU(3)$ intermediating strong interaction between quarks and 3 massive $W^{\pm}, Z$ bosons with 1 massless boson γ for electroweak $SU(2)\times U(1)$ sector. Higgs mechanism has to be introduced in electroweak sector to assign $W^{\pm}, Z$ bosons mass and as consequence the new particle - Higgs boson - emerges in the SM theory. All bosons have integer spin.

In addition to the bosons the SM introduces spin-1/2 fermions which are divided into three quark and three lepton families. Fermions are assumed to be point-like because there is no evidence for their internal structure to date. All fermions interact weakly, if they have electrical charge, they interact electromagnetically as well. Quarks are the only fundamental fermions which do interact strongly. System of fundamental particles of the SM is shown in figure 1.1.

Quarks bind together to form hadrons and there are hundreds (?source?) of known hadrons up to date. Theory describing the interaction between quarks is called Quantum Chromodynamics (QCD) which key features will be discussed in this chapter. The reasoning for quark existence and for the description their strong interaction as $SU(3)$

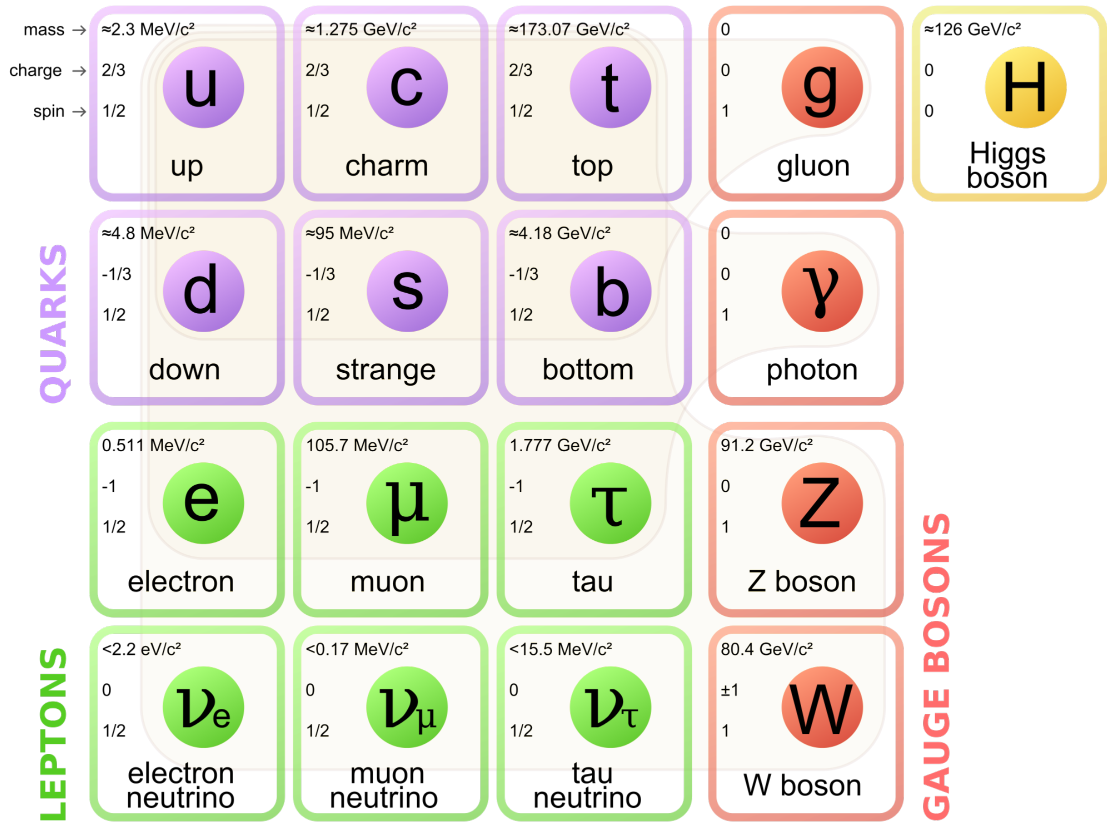

Figure 1.1: The system of fundamental particles of the SM. Figure from [1]

gauge theory will be presented. Running coupling constant will be discussed to split QCD into perturbative and non-perturbative regions - two regions, where QCD has to use different mathematical approach for description of strong interaction. Most of ideas presented here is overtaken from the following textbook [2].

1.1 Theoretical Ansatz

In 1950s there have already been discovered tens of new hadrons thanks to new particle accelerators and a lot of effort was exerted to categorize them. To each particle there was assigned a series of quantum numbers including isospin T with its third component T_3, hypercharge Y, electrical charge Q, strangeness S, baryon number B and others. Soon it was recognized, that there are some symmetries between these quantum numbers, like Gell-Mann–Nishijima relation [3, 4]

$$Q = T_3 + 1/2Y \quad , \quad Y = B + S + \ldots , \tag{1.1}$$

	S	Y	T	T_3	Q
p	0	1	1/2	1/2	1
n				-1/2	0
Σ^+	-1	0	1	1	1
Σ^0				0	0
Σ^-				-1	-1
Λ			0	0	0
Ξ^0	-2	-1	1/2	1/2	0
Ξ^-				-1/2	-1

Table 1.1: Quantum numbers of selected baryons known in 1950s. S strangeness, Y hypercharge, T isospin, T_3 third component of isospin, Q electrical charge.

where dots denotes charm, bottomness and topness. Some of the hadrons known by then are shown in table 1.1. In 1960s the known particles were successfully categorized with the so called Eightfold Way, which was published independently by Murray Gell-Mann (?citace?) and George Zweig (?citace?) in 1964. The Eightfold Way successfully predicted the existence of new particle Ω^{--} including its mass. Basic ideas of Eightfold way will be discussed in this section.

The key feature of Eightfold Way is to understand hadrons as the part of different representations of infinitesimal generators of $SU(3)$ flavor symmetry group. Lie algebra of $SU(3)$ is real eight-dimensional Lie algebra $\mathfrak{su}(3)$ which fundamental representation is usually derived from Gell-Mann matrices

$$
\begin{aligned}
&\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \\
&\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned}
\tag{1.2}
$$

The generators are usually chosen $g_a = \frac{1}{2}\lambda_a$ and obey the commutation relation $[g_a, g_b] = i f_{abc} g_c$ with f_{abc} being structure constants. Cartan subalgebra of fundamental representation of $\mathfrak{su}(3)$ is generated by $H_1 = g_3$ and $H_2 = g_8$. The eigenstates of three-dimensional representation of $\mathfrak{su}(3)$ can be chosen

$$
u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \leftrightarrow \left(\frac{1}{2}, \frac{\sqrt{3}}{6} \right), \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \leftrightarrow \left(-\frac{1}{2}, \frac{\sqrt{3}}{6} \right), \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \leftrightarrow \left(0, -\frac{\sqrt{3}}{3} \right),
\tag{1.3}
$$

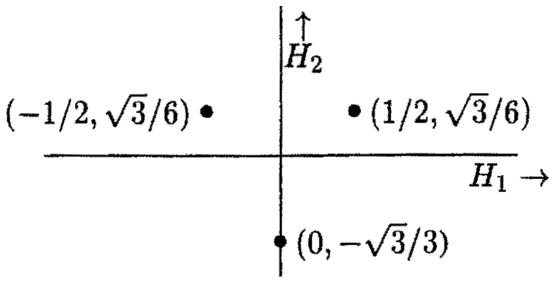

Figure 1.2: Eigenvalues of 3-dimensional representation of $\mathfrak{su}(3)$ Lie algebra. Figure from [5].

where the eigenvalues to generators of the Cartan subalgebra was assigned $H_1 u = \frac{1}{2}u$, $H_2 u = \frac{\sqrt{3}}{6}u$ and similarly for d and s eigenstates. These eigenvalues are shown in figure 1.2. Other important representation of $\mathfrak{su}(3)$ is eight-dimensional adjoint representation. This representation has the following eigenstates and corresponding eigenvalues

$$\begin{aligned}
\frac{1}{\sqrt{2}}\left(g_1 \pm i g_2\right) &\leftrightarrow (\pm 1, 0)\,, \\
\frac{1}{\sqrt{2}}\left(g_4 \pm i g_5\right) &\leftrightarrow \left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right), \\
\frac{1}{\sqrt{2}}\left(g_6 \pm i g_7\right) &\leftrightarrow \left(\mp\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right),
\end{aligned} \tag{1.4}$$

where again when denoting $A = \frac{1}{\sqrt{2}}(g_1 + i g_2)$ then the upper sign of the first expression reads $[H_1, A] = A$ and $[H_2, A] = 0$ and similarly for remaining 5 eigenstates. Defining

$$H_1 = T_3 \quad \text{and} \quad H_2 = \frac{\sqrt{3}}{2} Y \tag{1.5}$$

one can easily assign hadrons from table 1.1 to corresponding eigenvalues of adjoint representation in (1.4) according to its third component of isospin T_3 and its hypercharge Y. This is depicted in figure 1.3.

When the same redefinition is done to the eigenstates of three-dimensional representation in (1.3), one can assign to eigenstates the hypercharge Y and strangeness S as well. The concrete values for states u, d, s are shown in table 1.2.

It is possible to find another representations of Lie algebra, to which the observed hadrons can be assigned. The simplest way seems to be through highest weight defining representation. From eigenvalues of adjoint representation (1.4) one can find simple roots $\alpha^1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\alpha^2 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, which are defining the highest weights $\mu^1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$, $\mu^2 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{6}\right)$. New representation of Lie algebra can be constructed from highest weight. This procedure is described in [5] in detail.

	S	Y	T	T_3	Q
u	0	1/3	1/2	1/2	2/3
d				-1/2	-1/3
s	-1	-2/3	0	0	

Table 1.2: Quantum numbers of three quarks which existence was predicted by Gell-Mann and Zweig in 1964.

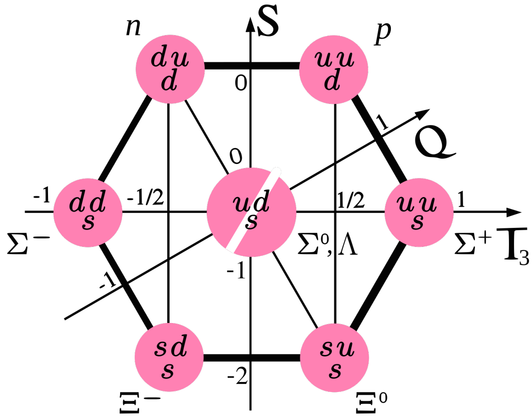

Figure 1.3: Baryonic octuplet encapsulating baryons from table 1.1. For baryons in this diagram, the relation $Y = S + 1$ holds. Figure from [6].

Representations defined by highest weight μ^1 or μ^2 respectively are called fundamental. Fundamental representation defined by μ^1 is usually denoted $\mathbf{3}$ and its weight diagram is shown at figure 1.2, corresponding to three different quark states. The second fundamental representation corresponds to three anti-quark states and is usually denoted $\bar{\mathbf{3}}$. Representation depicted in figure 1.3 is defined by the highest weight $\mu^1 + \mu^2$.

Special interest is in representations with dimensions 10, 8 and 1. These multiplets are present in decompositions $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$, which correspond to the baryons composed of three quarks, and $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ corresponding to mesons from quark and anti-quark.

Important feature of quark model just presented is its capability to predict hadron masses. This is done using Gell-Mann–Okubo mass formula (?citace?)

$$M = a_0 + a_1 S + a_2 \left(T(T+1) - \frac{1}{4} S^2 \right), \tag{1.6}$$

where a_0, a_1 and a_2 are free parameters which are common for all hadrons in one multiplet.

In less than a year after publication of Gell-Mann–Zweig quark model, Sheldon Lee Glashow and James Bjorken proposed (?citace?) an extension which predicted existence of fourth flavor of quark - charm quark.

In 1973 the Makoto Kobayashi and Toshihide Moskawa proposed (?citace?) that the existence of 6 different quark flavors could explain the experimental observation of CP violation.

1.2 Experimental Ground

In the previous section it was shown the hadrons can be categorized using representations of $\mathfrak{su}(3)$ Lie algebra. This lead to the model, where baryons were composed of three quarks. In this section, some experimental evidences will be presented to support quark model. First the scattering reactions will be discussed. It will be shown, that the lepton scattering on nucleons can be explained by assumption, that nucleons are composed of point-likes pin-1/2 particles. Next discussion will address the fact, that there are three color charges - this will address the question, why the group $SU(3)$ is connected to the theory of strong interaction.

1.2.1 Scattering Reactions

One of the possibilities, how to find out, if there is some inner structure in nucleon N, are the scattering reactions

$$e^- \, (E \gg 1\,\mathrm{GeV}) + N \to e^- + N, \tag{1.7}$$
$$\nu_e \, (E \gg 1\,\mathrm{GeV}) + N \to \nu_e + N, \tag{1.8}$$

where the condition $E \gg 1\,\mathrm{GeV}$ is explicitly written to ensure the wavelength of lepton being $< 0.2\,\mathrm{fm}$. By the first scattering reaction, the information about electric charge distribution in nucleon can be extracted, whereas the second scattering reaction informs us about weak charge distribution. Further only (1.7) will be discussed. Feynmann diagram of this process is depicted with kinematics variables and vertex algebraic structures in figure 1.4.

Because of Lorentz-invariance of QED, the matrix element of the nucleon vertex $\bar{u}(P', S')\Gamma_\mu u(P, S)$ has to be Lorentz-vector. This restricts the possible form of Γ_μ to the following algebraic structure

$$\Gamma_\mu = A\gamma_\mu + BP'_\mu + CP_\mu + iDP'^\nu\sigma_{\mu\nu} + iEP^\nu\sigma_{\mu\nu}, \tag{1.9}$$

where A,...,E depend only on Lorentz-invariant quantities. Next condition which has to be taken into account, is gauge invariance of matrix element, which can be written in the form

$$q^\mu \bar{u}(P', S')\Gamma_\mu u(P, S). \tag{1.10}$$

The further computation of cross section is straightforward and the result can be easily generalized to nonelastic scattering by which the nucleon in final state decays. The result is usually written using inelasticity parameter $y = \frac{E-E'}{E}$, $0 \leq y \leq 1$ and $y = 0$

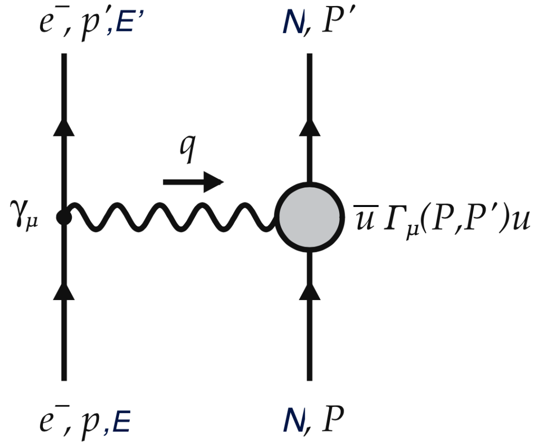

Figure 1.4: Scattering reaction $e^-N \to e^-N$. Figure from [2]

is in the case of elastic scattering, Bjorken variable $x = \frac{Q^2}{2P\cdot q}$, $0 < x \leq 1$, $x = 1$ denoting elastic scattering and finally instead of negative value q^2 the $Q^2 = -q^2$ is used. Final result can be than written in the form

$$\left.\frac{d^2\sigma}{dxdy}\right|_{eN} = \frac{8\pi M_N E\alpha^2}{Q^4}\left[xy^2F_1^{eN}(Q^2,x) + (1-y)F_2^{eN}(Q^2,x)\right]. \tag{1.11}$$

The eN sub(super)script stresses the fact, we are dealing with scattering (1.7). F_1^{eN} and F_2^{eN} are the so called structure functions, which are not determinable by the theory just presented - they have to be measured experimentally.

Structure constants were first measured by eP scattering at SLAC in 1968 (?citacce?) and shown the following results

1. for $Q^2 \geq 1\,\mathrm{GeV}$, there is no significant dependence of structure functions on Q^2 and
2. for $Q^2 \geq 1\,\mathrm{GeV}$, $F_2 \approx 2xF_1$.

These results can be also seen from figure (?figure?) and can be explained by assumption nucleon being composed of point-like spin-1/2 constituents, for which R. P. Feynmann used term partons. To ith parton, it is possible to assign momentum $P_{i,\xi}$

$$P_{i,\mu} = \xi_i P_\mu + \Delta P_{i,\mu} \quad , \quad \max_\mu(\Delta P_\mu) \ll \max_\mu P_\mu, \tag{1.12}$$

where $\xi_i \in \langle 0, 1\rangle$ and $\Delta P_{i,\mu}$ comes from the interaction between partons and it is assumed, the momentum coming from this interaction is much smaller than the total nucleon momentum P_μ. In addition, probabilities $f_i(\xi_i)$ that ith parton will carry ξ_i fraction of total momentum fulfilling

$$\int d\xi_i f_i(\xi_i) = 1 \tag{1.13}$$

must be defined. Then for scattering reaction (1.7) it can be derived the total cross section formula

$$\left.\frac{d^2\sigma}{dxdy}\right|_{eN} = \frac{4\pi M_N E\alpha^2}{Q^4}\left[y^2 + 2(1-y)\right]\sum_i f_i(x) q_i^2 x. \tag{1.14}$$

where in for ith parton its electrical charge q_i was introduced. The last expression and (1.11) can be compared as polynomials in y resulting in

$$F_1^{eN}(x) = \frac{1}{2}\sum_i f_i(x) q_i^2 \quad , \quad F_2^{eN}(x) = \sum_i f_i(x) q_i^2 x. \tag{1.15}$$

It can be easily checked, that $F_2^{eN}(x) = 2xF_1^{eN}(x)$. Functions $f_i(x)$ just introduced are called Parton Density Functions (PDF) and their important role in QCD will be discussed in (?somewhere?) in more details.

Important conclusion from analyzing of scattering reactions is, that the experimental results can be explained by assumption nucleons being consisted of spin-1/2 point-like partons, now called quarks.

1.2.2 Number of Colors

Despite the strong confidence in parton model, theory which would describe the interaction between partons was still missing. There was no direct evidence on how the theory would look like in (?year?). The theory of electroweak unification successfully suggested, that the gauge theories are the right theories for description of world at subatomic level, but to construct gauge theory of strong interaction the number of colors first had to be known.

Number of colors N_C is the number of different kinds of quarks of the same flavor with respect to the new interaction. In this part, three arguments will be presented to demonstrate, that $N_C = 3$.

The first argument is the analysis of the electron-positron anihilation into the pair of fermion and anti-fermion

$$e^+e^- \to f\bar{f}. \tag{1.16}$$

Feynmann diagram of this reaction is shown in figure 1.5, where constants sitting in two vertices are denoted. α stands for fine structure constants and Q_f for charge of fermion f in units of positron charge. Total cross section has to be proportional to

$$\sigma(e^-e^+ \to f\bar{f}) \sim Q_f^2\alpha^2. \tag{1.17}$$

In the case fermion f being quark, there is new degeneracy in final state coming from different colors of quarks in final state - the total cross section has to be multiplied by factor N_C. Experimentally, the so called R-factor is measured

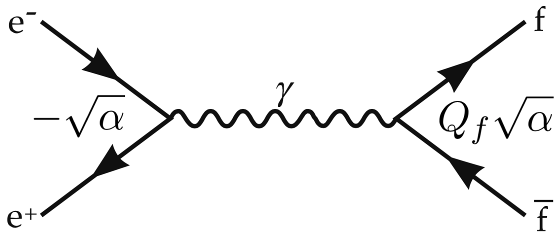

Figure 1.5: $e^+e^- \to f\bar{f}$ annihilation with vertex constants. α stands for fine structure constant, Q_f for charge of fermion f in units of positron charge.

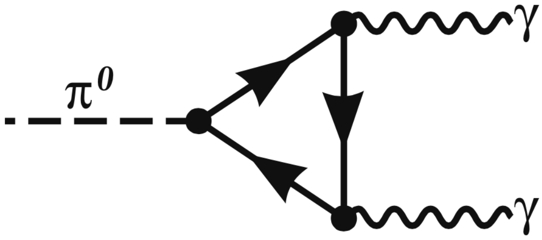

Figure 1.6: $\pi^0 \to 2\gamma$ decay with closed fermion loop.

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \left(\sum_q Q_q^2\right) N_C, \tag{1.18}$$

where sum on the left hand side is over all possible quark states. When the quark model proposed by Gell-Mann a Zweig is used, then

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] N_C = \frac{2}{3} N_C. \tag{1.19}$$

Experimental results for R-ratio have shown (?citace?, ?concrete value?), that $N_C = 3$.

The second argument is the measurement of decay width of π_0 meson. Decay is depicted in figure 1.6. For decay width Γ it can be derived

$$\Gamma = 7.63 \left(\frac{N_C}{3}\right)^2 \text{ eV}, \tag{1.20}$$

which, compared to the experimental value (?citace?, ?concrete value?) leads again to $N_C = 3$.

The third argument is purely theoretical and states, that the SM is internally consistent only if there are three colors (?citace?). This indicates that there is some linking between electroweak and strong sector of SM and motivates the search for Grand Unified Theories.

1.3 QCD as Gauge Theory

Putting arguments of previous section all together, there are strong experimental evidences, that nucleons consist of point-like spin-1/2 particles called quarks and that quarks bring into the theory new degeneracy factor $N_C = 3$, which can be understood as three different strong charges called colors.

Nowadays the quark-quark strong interaction is understood as an $SU(3)$ gauge theory in a degree of freedom called color. Gell-Mann matrices (1.2) can be chosen as generators of $SU(3)$. These matrices act on quark color triplets wave functions

$$\psi(x) = \begin{pmatrix} \psi_r(x) \\ \psi_g(x) \\ \psi_b(x) \end{pmatrix} . \tag{1.21}$$

Following the Yang-Mills theory (?citace?), to each generator $\frac{\lambda^a}{2}$ gluon field $A^a_\mu(x)$ and gluon file strength tensor

$$F^a_{\mu\nu} = \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\right) \tag{1.22}$$

is assigned where g denotes the coupling constant of strong interaction and f^{abc} are structure constant defined in section 1.1. QCD Lagrangian

$$\mathscr{L}_{\text{QCD}} = \bar{\psi}\left(-i\partial_\mu + g\frac{\lambda}{2}A^a_\mu(x)\right)\gamma^\mu\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \tag{1.23}$$

is invariant under local transformation

$$\psi(x) \ \ \rightarrow \psi'(x) = \mathrm{e}^{ig\Theta(x)}\psi(x), \tag{1.24}$$

$$A_\mu(x) \rightarrow \mathrm{e}^{ig\Theta(x)}\left(A_\mu(x) + \frac{i}{g}\partial_\mu\right)\mathrm{e}^{-ig\Theta(x)}, \tag{1.25}$$

where

$$\Theta(x) = \frac{1}{2}\lambda^a\Theta^a(x) \quad , \quad A_\mu(x) = \frac{1}{2}\lambda^a A^a_\mu(x). \tag{1.26}$$

There is no mass term in Lagrangian (1.23) because mass term $m\bar{\psi}\psi$ vary under gauge transformation (1.24). Origin of mass term lies in Higgs mechanism (?citace?) which will not be discussed here.

QCD Lagrangian (1.23) together with gauge transformations (1.24) are sufficient for determination of Feynman rules - key ingredient in perturbative QCD which will be discussed in next section.

By derivation of gluon propagator, one has to add to the QCD Lagrangain the so called gauge-fixing term

$$\mathscr{L}^{\text{gauge-fixing}}_{\text{QCD}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu_a\right)^2, \tag{1.27}$$

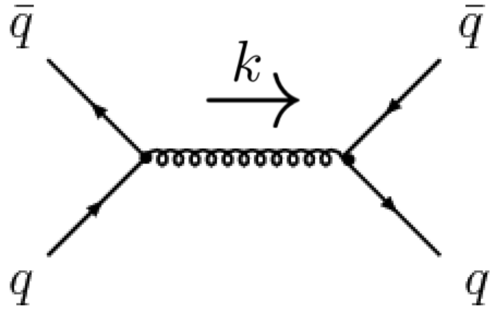

Figure 1.7: Leading order Feynmann diagrams in scattering reaction $q\bar{q} \to q\bar{q}$ with denoted transfered momentum k.

which confines the possible gauges to one class parametrized by real parameter ξ. In non-Abelian gauge theories this term must be supplemented by the so called ghost term which brings into the theory new unphysical scalar particle obeying fermionic statistics. More details on so called Faddev-Popov ghost filed can be found in (?source?).

1.4 Perturbative QCD

Quantum Electrodynamics (QED) and QCD are both quantum filed gauge theories, but they differ in one key feature - the former is Abelian whereas the latter is not. The non-Abelian character of QCD leads to new phenomenons which have the origin in QCD Lagrangian (1.23) which directly leads to triple and quartic gluonic interactions. In this section one remarkable consequence will be discussed - the running coupling constant.

Assume scattering process

$$q\bar{q} \to q\bar{q}, \tag{1.28}$$

which is depicted in the lowest order of perturbation theory by the Feynman graph 1.7. Except contribution of this graph to the scattering amplitude (which is the only contribution $\sim g^2$) there are 12 other Feynman diagrams with contributions $\sim g^4$. These are depicted in figure 1.8.

The contributions from new Feynman diagrams are calculated in [2] in detail. There is shown, that all this contributions together are logarithmically divergent. This divergence can be removed, when from the scattering amplitude for arbitrary momentum transfer k^2 scattering amplitude for fixed momentum transfer $k^2 = -M^2$ is subtracted. This is how the renormalized coupling constant g_R is obtained and here is its final expression

$$g_R = g_0 - \frac{g_0^3}{16\pi^2}\left(\frac{11}{2} - \frac{1}{3}N_F\right)\ln\left(\frac{-k^2}{M^2}\right) + \mathscr{O}(g_0^5). \tag{1.29}$$

g_0 stands for the coupling constant measured at the renormalization scale $k^2 = -M^2$ and N_F is the number of different quark flavors with mass $m^2 \ll |k^2|$. Dependence

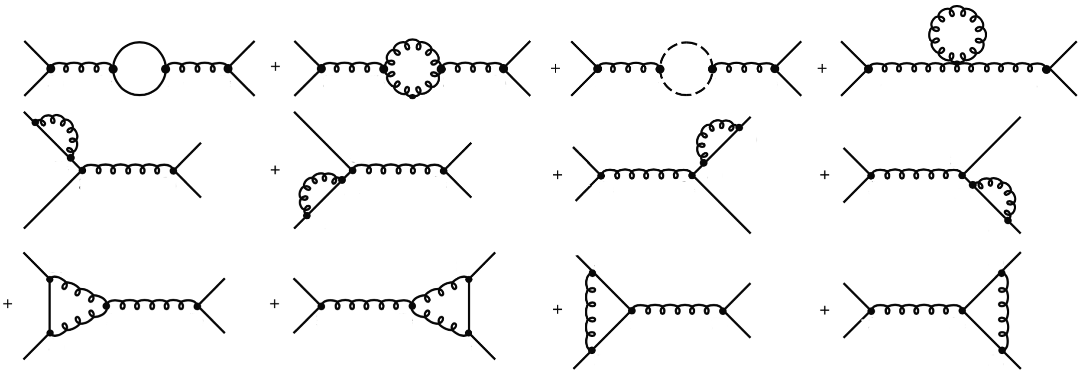

Figure 1.8: Next to the leading order Feynmann diagrams in scattering reactions $q\bar{q} \to q\bar{q}$. Dashed line represents scalar ghost particle.

of g_R on transfered momentum k^2 is evident, but there are another two intertwined dependences - on normalization scale M and on coupling constant at renormalization scale $g_0 = g_R|_{k^2=-M^2}$. For next purpose, it is convenient use the dependence schema

$$g_R = g_R(-k^2, g_0(M)) \tag{1.30}$$

which allows us to use advantages of β-function and using equation 1.29 leads to following differential equation for g_0

$$\beta(g_0) \equiv M \left(\frac{\partial g_R}{\partial M}\right)_{-k^2=M^2} = M \left(\frac{dg_0}{dM}\right)_{-k^2=M^2} \tag{1.31}$$

$$= -b_0 g_0^3 + \mathscr{O}(g_0^5), \quad b_0 = \frac{1}{16\pi^2}\left(11 - \frac{2N_F}{3}\right), \tag{1.32}$$

which can be solved directly to obtain coupling constant g_0 for arbitrary scale $-k^2$

$$\int_{g_0(M^2)}^{g_0(-k^2)} \frac{dg_0}{g_0^3} = -b_0 \int_{M^2}^{-k^2} \frac{dM}{M} \tag{1.33}$$

with solution

$$\alpha_S(-k^2) = \frac{\alpha_S(M^2)}{1 + \frac{\alpha_S(M^2)}{4\pi}\left(11 - \frac{2N_F}{3}\right)\ln\left(\frac{-k^2}{M^2}\right)}, \quad g_0^2(-k^2) = 4\pi\alpha_S(-k^2), \tag{1.34}$$

which is the final expression for running coupling constant up to one-loop order. This dependence corresponds to experimental data which are depicted in figure 1.9. Coupling constant decreases with increasing momentum transfer allowing the use of the perturbation theory.

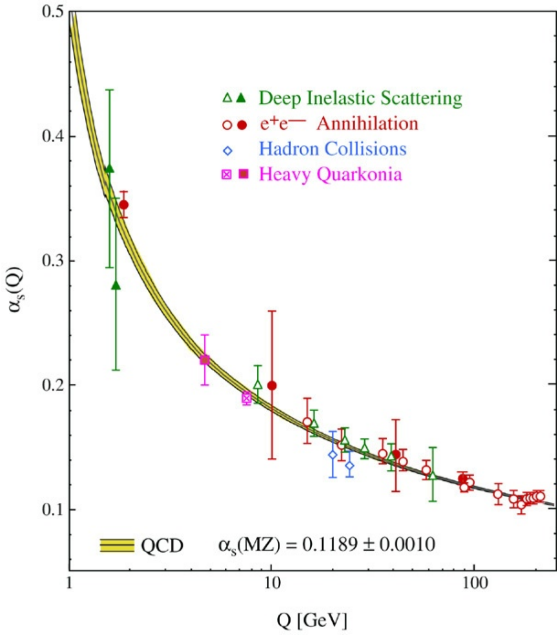

Figure 1.9: Experimental measurements of running coupling constant. $Q = \sqrt{|k^2|}$ in comparison to (1.34). (?source?) (?maybe better picture?)

But when the momentum transfer decreases, there is special value $-k^2 = \Lambda^2$ for which the last expression diverges

$$-1 = \frac{\alpha_S(M^2)}{4\pi}\left(11 - \frac{2N_F}{3}\right)\ln\left(\frac{\Lambda^2}{M^2}\right). \tag{1.35}$$

Experimental value is $\Lambda = \ldots$ (?source?) and demonstrates, that perturbative QCD cannot be used at low energy transfers. In fact, the running coupling constant $\alpha_S(-k^2)$ reaches value ~ 1 on momenta transfers $\sqrt{|k^2|} \sim 500\,\mathrm{MeV}$.

To understand e.g. structure of proton with rest mass $< 1\,\mathrm{GeV}$ it is clear non-perturbative QCD has to be used. The ideas of non-perturbative QCD will be introduced in next section. (?maybe?)

1.5 Non-Perturbative QCD

1.6 Basic principles of QCD

Chapter 2

QCD on ATLAS

Chapter 3

ATLAS Detector

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