$-\lvert x\rvert \le x \le \lvert x\rvert$	*Triangle inequality*: $\lvert x+y\rvert \le \lvert x\rvert + \lvert y\rvert$
$\sqrt{x^2} = \lvert x\rvert$	$\lvert -x\rvert$
If $\lim_{x\to c} g(x) = 0$, $\lim_{x\to c}\frac{\sin g(x)}{g(x)} = \lim_{x\to c}\frac{g(x)}{\sin g(x)} = 1$	$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{x}{\sin x} = 1$
If $\lim_{x\to c} g(x) = 0$, $\lim_{x\to c}\frac{\tan g(x)}{g(x)} = \lim_{x\to c}\frac{g(x)}{\tan g(x)} = 1$	$\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{x}{\tan x} = 1$
Derivative by first principle: $f'(x) = \lim_{h\to 0}\frac{f(x-h)-f(x)}{h}$ $f'(x) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$	Continuity: (i) $\lim_{x\to c} f(x)$ *exists* (ii) $\lim_{x\to c} f(x) = f(c)$

x^n	nx^{n-1}	$\ln x$	$\frac{1}{x}$
$\cos x$	$-\sin x$	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\sin x$	$\cos x$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan x$	$\sec^2 x$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec x$	$\sec x\tan x$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\csc x$	$-\csc x\cot x$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
$\cot x$	$-\csc^2 x$	$\csc^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
e^x	e^x	$\ln x$	$\frac{1}{x}$

Sum rule	$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
Product rule	$\frac{d}{dx}(uv) = \frac{du}{dx}v + \frac{dv}{dx}u$
Quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{v^2}$
Derivative of inverse function	$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$
$f(x)^{g(x)}$	$f(x)^{g(x)}\left(g'(x)\ln f(x) + \frac{f'(x)}{f(x)}g(x)\right)$
$a^{g(x)}$	$a^{g(x)}g'(x)\ln a$
Intermediate Value Theorem	If f be continuous on $[a,b]$ and k is a number between $f(a)$ and f(b), Then $f(c) = k$ for some $c \in [a,b]$.

Tangent equation	$y - f(x_0) = m(x - x_0)$
Normal equation	$y - f(x_0) = -\frac{1}{m}(x - x_0)$
LHopital's Rule	$\lim_{n\to c}\frac{f(x)}{g(x)} = \lim_{n\to c}\frac{f'(x)}{g'(x)}$ (*only when* $\lim_{n\to c} f(x) = \lim_{n\to c} g(x) = 0$ *or* ∞ *s.t.* $\frac{f(x)}{g(x)} = \frac{0}{0}$ *or* $\frac{\infty}{\infty}$)
$0^0/\infty^0/1^\infty$	$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^{\lim_{x\to 0^+} x\ln x}$
Rolle's Theorem	Let f be continuous on $[a,b]$ and differentiable on (a,b). If $f(a) = f(b)$, then there is at least one number c such that $f'(c) = 0$.
Mean Value Theorem	Let f be continuous on $[a,b]$ and differentiable on (a,b). Then, there is at least one number c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$
Critical points:	*i)* $f'(x) = 0$, *ii)* $f'(x)$ *does not exist*

$\sin^2 x + \cos^2 x = 1$	$\sin A\cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$
$\sin 2A = 2\sin A\cos A$	$\cos A\sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$
$\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A$ $= 2\cos^2 A - 1$	$\cos A\cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$
	$\sin A\sin B = -\frac{1}{2}(\cos(A+B) - \cos(A-B))$

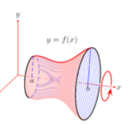
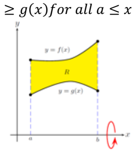
$\sin(\pi-\theta) = \sin\theta$ $\cos(\pi-\theta) = -\cos\theta$ $\tan(\pi-\theta) = -\tan\theta$	$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$ $\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$	$\sin(\pi+\theta) = -\sin\theta$ $\cos(\pi+\theta) = -\cos\theta$ $\tan(\pi+\theta) = \tan\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

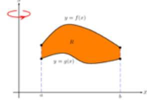
$\int (ax+b)^n\,dx \Rightarrow \frac{(ax+b)^{n+1}}{(n+1)a} + C$	$\int \sec(ax+b)\tan(ax+b)\,dx \Rightarrow \frac{1}{a}\sec(ax+b) + C$
$\int \frac{1}{ax+b}dx \Rightarrow \frac{1}{a}\ln\lvert ax+b\rvert + C$	$\int \csc(ax+b)\cot(ax+b)\,dx \Rightarrow -\frac{1}{a}\csc(ax+b) + C$
$\int e^{ax+b}\,dx \Rightarrow \frac{1}{a}e^{ax+b} + C$	$\int \frac{1}{a^2+(x+b)^2}dx \Rightarrow \frac{1}{a}\arctan(ax+b) + C$
$\int \sin(ax+b) \Rightarrow -\frac{1}{a}\cos(ax+b) + C$	$\int \frac{1}{\sqrt{a^2-(x+b)^2}} \Rightarrow \frac{1}{a}\arcsin(ax+b) + C$
$\int \cos(ax+b)\,dx \Rightarrow \frac{1}{a}\sin(ax+b) + C$	$\int \frac{-1}{\sqrt{a^2-(x+b)^2}}dx \Rightarrow \frac{1}{a}\arccos(ax+b) + C$
$\int \tan(ax+b)\,dx \Rightarrow \frac{1}{a}\ln\lvert\sec(ax+b)\rvert + C$	$\int \frac{1}{a^2-(x+b)^2}dx \Rightarrow \frac{1}{2a}\ln\left\lvert\frac{x+b+a}{x+b-a}\right\rvert + C$
$\int \sec(ax+b)\,dx \Rightarrow \frac{1}{a}\ln\lvert\sec(ax+b) + \tan(ax+b)\rvert + C$	$\int \frac{1}{(x+b)^2-a^2}dx \Rightarrow \frac{1}{2a}\ln\left\lvert\frac{x+b-a}{x+b+a}\right\rvert + C$
$\int \csc(ax+b)\,dx \Rightarrow -\frac{1}{a}\ln\lvert\csc(ax+b) + \cot(ax+b)\rvert + C$	$\int \frac{1}{\sqrt{(x+b)^2+a^2}}dx \Rightarrow \ln\left\lvert(x+b) + \sqrt{(x+b)^2+a^2}\right\rvert + C$
$\int \cot(ax+b)\,dx \Rightarrow -\frac{1}{a}\ln\lvert\csc(ax+b)\rvert + C$	$\int \frac{1}{\sqrt{(x+b)^2-a^2}}dx \Rightarrow \ln\left\lvert(x+b) + \sqrt{(x+b)^2-a^2}\right\rvert + C$
$\int \sec^2(ax+b)\,dx \Rightarrow \frac{1}{a}\tan(ax+b) + C$	$\int \sqrt{a^2-x^2}\,dx \Rightarrow \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$
$\int \csc^2(ax+b)\,dx \Rightarrow -\frac{1}{a}\cot(ax+b) + C$	$\int \sqrt{x^2-a^2}\,dx \Rightarrow \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left\lvert x + \sqrt{x^2-a^2}\right\rvert + C$

Integration by substitution:

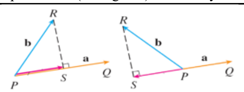
Expression	Substitution	Identity involved
$\sqrt{a^2-(x+b)^2}$	*Let* $x+b = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2+(x+b)^2}$	*Let* $x+b = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{(x+b)^2-a^2}$	*Let* $x+b = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

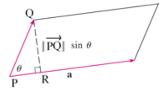
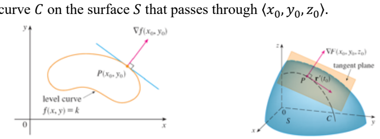
Integration by parts	$\int uv\,dx = u\cdot\left(\int v\,dx\right) - \int\left(u'\cdot\int v\,dx\right)dx$
Riemann's sum	$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left\{\sum_{k=1}^{n}\left(\frac{b-a}{n}\right)f\left(a + k\frac{(b-a)}{n}\right)\right\}$ $\int_a^b f(x)\,dx \approx \sum_{k=1}^{n}\left(\frac{b-a}{n}\right)f\left(a + k\frac{(b-a)}{n}\right)$, *when n is large*
Fundamental Theorem of Calculus	$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\cdot u'(x)$
Improper integrals (Type 1) $[a,\infty]$ $[-\infty,b]$ $[-\infty,\infty]$	$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$ $\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$ $\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$
Improper integrals (Type 2) $(a,b]$ $[a,b)$ (a,b)	$\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx$ $\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Area under curve bounded by the curve, lines $y=0$, $x=a$ and $x=b$	$A = \int_a^b f(x)\,dx$ $A = \int_c^d f(y)\,dy$
Area between curves $f(x) \ge g(x)$ *for all* $a \le x \le b$	$A = \int_a^b (f(x) - g(x))\,dx$
Area between curves	$A = \int_a^b \lvert f(x) - g(x)\rvert\,dx$
Solid of Revolution by Disk Method (Rotated around x-axis)  $V = \pi\int_a^b f(x)^2\,dx$	Volume bounded by two curves $f(x) \ge g(x)$ *for all* $a \le x \le b$  $V = \pi\int_a^b f(x)^2\,dx - \pi\int_a^b g(x)^2\,dx$

Cylindrical Shell Method (Rotated around y-axis)  $V = 2\pi\int_a^b x\lvert f(x)\rvert\,dx$	Cylindrical Shell Method of two curves  $V = 2\pi\int_a^b x\lvert f(x) - g(x)\rvert\,dx$
Length of curve	*Arc length* $= \int_a^b \sqrt{1 + f'(x)^2}\,dx$

Theorem 6.1.	Let f be a function, and $\{a_n\}$ be a sequence s.t. $f(n) = a_n$ for all n. If $\lim_{x\to\infty} f(x) = L$, then $\lim_{x\to\infty} a_n = L$.
	If $\{a_n\}$ and $\{a_n\}$ are convergent sequences and c is a constant, then: • $\lim_{x\to\infty} ca_n = c\lim_{x\to\infty} a_n$ • $\lim_{x\to\infty}(a_n \pm b_n) = \lim_{x\to\infty} a_n \pm \lim_{x\to\infty} b_n$ • $\lim_{x\to\infty} a_nb_n = \lim_{x\to\infty} a_n \lim_{x\to\infty} b_n$ • $\lim_{x\to\infty}\frac{a_n}{b_n} = \frac{\lim_{x\to\infty} a_n}{\lim_{x\to\infty} b_n}$, if $\lim_{x\to\infty} b_n \ne 0$
Theorem 6.2. Squeeze Theorem for Sequence	If $a_n \le b_n \le c_n$ for all n and $\lim_{x\to\infty} a_n = \lim_{x\to\infty} c_n = L$, then $\lim_{x\to\infty} b_n = L$.
Theorem 6.3	If $\sum_{n=1}^\infty a_n$ and $\sum_{n=1}^\infty b_n$ are convergent series, so are the series $\sum_{n=1}^\infty ca_n$ and $\sum_{n=1}^\infty(a_n + b_n)$. Moreover, $\sum_{n=1}^\infty ca_n = c\sum_{n=1}^\infty a_n$ *and* $\sum_{n=1}^\infty(a_n + b_n) = \sum_{n=1}^\infty a_n + \sum_{n=1}^\infty b_n$
Lemma 6.4	*If* $\sum_{n=1}^\infty a_n$ *is convergent, then* $\lim_{x\to\infty} a_n = 0$.
Theorem of 6.5 Test for Divergent	*If* $\lim_{x\to\infty} a_n$ *does not exist or if* $\lim_{x\to\infty} a_n \ne 0$, *then the series* $\sum_{n=1}^\infty a_n$ *is divergent*
Theorem 6.6	*A series* $\sum_{n=1}^\infty a_n$ *of nonnegative terms converges iff its partial sums are bounded from above (i.e.* $\exists$ *a constant K s.t.* $S_n < K$ *for all n)*
Theorem 6.7 Integral test	*Suppose* $a_n = f(n)$, *where f is a continuous, positive, decreasing function of x for all* $x \ge 1$. *Then,* *If* $\int_c^\infty f(x)\,dx$ *is convergent (that is equals to a constant), then* $\sum_{n=c}^\infty a_n$ *is converg* *If* $\int_c^\infty f(x)\,dx$ *is divergent, then* $\sum_{n=c}^\infty a_n$ *is divergent.*
Theorem 6.8 p-series	*The p = series* $\sum_{n=1}^\infty \frac{1}{n^p}$ *is convergent iff* $p > 1$.
Theorem 6.9 Comparison test	*Suppose* $\sum_{n=1}^\infty a_n, \sum_{n=1}^\infty b_n$ *are series with positive terms s.t.* $a_n \le b_n$ *If* $\sum_{n=1}^\infty b_n$ *is convergent, then* $\sum_{n=1}^\infty a_n$ *is convergent.* *If* $\sum_{n=1}^\infty a_n$ *is divergent, then* $\sum_{n=1}^\infty b_n$ *is divergent*
Theorem 6.10 Ratio test Theorem 6.11 Root test	6.10 − *Suppose* $\sum_{n=1}^\infty a_n$ *is a series s.t.* $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L$, 6.11 − *Suppose* $\sum_{n=1}^\infty a_n$ *is a series s.t.* $\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert} = L$, *(L is a finite number or* ∞*)* *If* $0 \le L < 1$, $\sum_{n=1}^\infty a_n$ *is absolutely convergent.* $(\sum_{n=1}^\infty \lvert a_n\rvert$ *is convergent)* *If* $L > 1$, *then* $\sum_{n=1}^\infty a_n$ *is divergent* *If* $L = 1$, *ratio test is inconclusive*
Theorem 6.12 Alternating series test	*If* b_n *is a sequence of positive numbers s.t. (i)* b_n *is decreasing, and (ii)* $\lim_{n\to\infty} b_n = 0$, *then the alternating series* $\sum_{n=1}^\infty(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + \cdots$ *is convergent.*
Absolute convergence	*If* $\sum_{n=1}^\infty \lvert a_n\rvert$ *is convergent, then* $\sum_{n=1}^\infty a_n$ *is convergent*

Theorem 6.14	*For a given power series* $\sum_{n=0}^{\infty} c_n(x-a)^n$, *only one of the following possiblities hold*: (i) The series converges at $x=a$ only. (ii) The series converges for all x. (iii) There is a positive number R such that the series converges if $\lvert x-a\rvert<R$ and diverges if $\lvert x-a\rvert>R$.
Radius of converges (R)	If $\lim_{n\to\infty}\left\lvert\frac{c_{n+1}}{c_n}\right\rvert=L$ or $\lim_{n\to\infty}\sqrt[n]{\lvert c_n\rvert}=L$, where L is a real number or ∞ Then $R=\frac{1}{L}$
Theorem 6.15	*If the power series* $\sum_{n=0}^{\infty} c_n(x-a)^n$ *has* $R>0$, *then the function* f $f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n$ *is differentiable on interval* $\lvert x-a\rvert<R$, *and* i. $f'(x)=\sum_{n=0}^{\infty} nc_n(x-a)^{n-1}$, *for* $\lvert x-a\rvert<R$ ii. $\int f(x)\,dx=\sum_{n=0}^{\infty} c_n\frac{(x-a)^{n+1}}{n+1}+C$, *for* $\lvert x-a\rvert<R$
Theorem 6.16 Taylor series and Maclaurin series	*Taylor series*: $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\cdots$ *Maclaurin series*: (*Taylor series when* $a=0$) $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}(x)^n=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots$
Power series for special functions (Maclaurin series)	$\frac{1}{1-x}=1+x+x^2+x^3+\cdots+x^n+\cdots$ $\left(\sum_{n=0}^{\infty}x^n,\ \text{geometric series}\right), R=1$ $e^x=1+x+\frac{x^2}{2!}+\cdots$ $\left(\sum_{n=0}^{\infty}\frac{1}{n!}x^n\right), R=\infty$ $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$ $\left(\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{n}\cdot x^n\right), R=1$ $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$ $\left(\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{(2n+1)!}\cdot x^{2n+1}\right), R=\infty$ $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$ $\left(\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{(2n)!}\cdot x^{2n}\right), R=\infty$ $\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots$ $\left(\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{2n+1}\cdot x^{2n+1}\right), R=1$
Theorem 7.1: Distance	*Distance* $\lvert P_1P_2\rvert$ *between points* $P_1(x_1,y_1,z_1)$ *and* $P_2(x_2,y_2,z_2)$: $\lvert P_1P_2\rvert=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
Theorem 7.2: Sphere	*Equation of sphere with center* $C(h,k,l)$ *and radius* r: $(x-h)^2+(y-k)^2+(z-l)^2=r^2$
Properties of Vectors	$\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{b}+\boldsymbol{a}$; $c(\boldsymbol{a}+\boldsymbol{b})=c\boldsymbol{a}+c\boldsymbol{b}$ $\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$; $(c+d)\boldsymbol{a}=c\boldsymbol{a}+d\boldsymbol{a}$ $\boldsymbol{a}+0=\boldsymbol{a}$; $(cd)\boldsymbol{a}=c(d\boldsymbol{a})$ $\boldsymbol{a}+(-\boldsymbol{a})=0$; $1\boldsymbol{a}=\boldsymbol{a}$
Length	*Length* $\lVert\boldsymbol{a}\rVert=\sqrt{x^2+y^2+z^2}$, where $\boldsymbol{a}=\langle x,y,z\rangle$
Unit vector	*If* $\boldsymbol{a}\neq\boldsymbol{0}$, *then a unit vector in the same direction as* $\boldsymbol{a}$ *is*: $\boldsymbol{u}=\frac{\boldsymbol{a}}{\lVert\boldsymbol{a}\rVert}$
Dot product	*Given* $\boldsymbol{a}=\langle a_1,a_2,a_3\rangle,\boldsymbol{b}=\langle b_1,b_2,b_3\rangle$ $\boldsymbol{a}\cdot\boldsymbol{b}=a_1b_1+a_2b_2+a_3b_3$ $\boldsymbol{0}\cdot\boldsymbol{a}=0$ $\boldsymbol{a}\cdot\boldsymbol{a}=\lVert\boldsymbol{a}\rVert^2$
Theorem 7.7	*Let* θ *be the angle between nonzero vectors* $\boldsymbol{a}$ *and* $\boldsymbol{b}$, $\boldsymbol{a}\cdot\boldsymbol{b}=\lVert\boldsymbol{a}\rVert\lVert\boldsymbol{b}\rVert\cos\theta$
Theorem 7.8	Two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular (orthogonal) if and only if $\boldsymbol{a}\cdot\boldsymbol{b}=0$
Projections	 $\vec{PS}$ *is the vector projection of* $\boldsymbol{b}$ *onto* $\boldsymbol{a}$, *denoted as* $proj_{\boldsymbol{a}}\boldsymbol{b}$ *Scalar projection of* $\boldsymbol{b}$ *onto* $\boldsymbol{a}$, *denoted as* $comp_{\boldsymbol{a}}\boldsymbol{b}=\lVert\boldsymbol{b}\rVert\cos\theta=\frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\lVert\boldsymbol{a}\rVert}$ *Then,* $proj_{\boldsymbol{a}}\boldsymbol{b}=comp_{\boldsymbol{a}}\boldsymbol{b}\times\frac{\boldsymbol{a}}{\lVert\boldsymbol{a}\rVert}$
Theorem 7.9 Distance – Point to plane	The shortest distance from a point $P(x_0,y_0,z_0)$ to plane $ax+by+cz=d$ is given by: $\frac{\lvert ax_0+by_0+cz_0-d\rvert}{\sqrt{a^2+b^2+c^2}}$
Cross product	*Given* $\boldsymbol{a}=\langle a_1,a_2,a_3\rangle,\boldsymbol{b}=\langle b_1,b_2,b_3\rangle$ $\boldsymbol{a}\times\boldsymbol{b}=(a_2b_3-a_3b_2)\boldsymbol{i}+(a_3b_1-a_1b_3)\boldsymbol{j}+(a_1b_2-a_2b_1)\boldsymbol{k}$ The vector $\boldsymbol{a}\times\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$

Theorem 7.11	*If* θ *is the angle between* $\boldsymbol{a}$ *and* $\boldsymbol{b}$, $\lVert\boldsymbol{a}\times\boldsymbol{b}\rVert=\lVert\boldsymbol{a}\rVert\lVert\boldsymbol{b}\rVert\sin\theta$ Use cross product to: i. To find area of a parallelogram ii. To find distance from a point to a line in $\mathbb{R}^3$ 
Line parametric equation	$x(t)=x_0+at,\quad y(t)=y_0+bt,\quad z(t)=z_0+ct$ $\boldsymbol{r}(t)=\langle x_0,y_0,z_0\rangle+t\langle a,b,c\rangle$
Equation of plane	$\boldsymbol{n}\cdot(\boldsymbol{r}-\boldsymbol{r_0})=0$, *where* $\boldsymbol{n}$ *is the normal vector* $\boldsymbol{n}\cdot\boldsymbol{r}=\boldsymbol{n}\cdot\boldsymbol{r_0}$ *Linear equation*: $ax+by+cz+d=0$
Derivative of Vector-valued function	Let $\boldsymbol{r}(t)=\langle f(t),g(t),h(t)\rangle$, where f,g,h are differentiable at $t=a$, then r is differentiable at $t=a$: $\boldsymbol{r}'(a)=\langle f'(a),g'(a),h'(a)\rangle$
Arc length	Let C curve be $\boldsymbol{r}(t)=\langle f(t),g(t),h(t)\rangle$, *Length from a to b* $=\int_a^b\sqrt{f'(t)^2+g'(t)^2+h'(t)^2}\,dt$
Level curve	Two-dimensional graph of equation $f(x,y)=k$, for some constant k
Contour plot	Graph of numerous level curves $f(x,y)=k$, for representative value of k that are equally spaced
Clairaut's Theorem	*Suppose* f *is defined ona disk* D *that contains* (a,b). *If functions* f_{xy} *and* f_{yx} *are both continuous on* D, *then* $f_{xy}(a,b)=f_{yx}(a,b)$
Equation of Tangent Plane	*A normal vector to the tangent plane is*: $\boldsymbol{n}=\langle f_x(a,b),f_y(a,b),-1\rangle$ Hence, the equation of tangent plane is given: $\boldsymbol{n}\cdot\langle x-a,y-b,z-c\rangle=0$, *where* $c=f(a,b)$ $f_x(a,b)(x-a)+f_y(a,b)(y-b)-(z-f(a,b))=0$
Chain rule	Suppose $z=f(x,y)$, where $x=g(t)$ and $y=h(t)$ are differentiable functions of f, then $\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$ Suppose $z=f(x,y)$, where $x=g(s,t)$ and $y=h(s,t)$, then $\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$ *and* $\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$
Implicit differentiation 2 – IV	Suppose equation $F(x,y,z)=0$, $\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)}$ *and* $\frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$
	Suppose f is differentiable at (a,b). Let Δx and Δy be small increments in x and y from (a,b): $\Delta z\approx dz=f_x(a,b)dx+f_y(a,b)dy=f_x(a,b)\Delta x+f_y(a,b)\Delta y$
Directional derivatives (2-D and 3-D)	If $f(x,y)$ is a differentiable function, then f has a directional derivative in the direction of any unit vector $\boldsymbol{u}=\langle a,b\rangle$ $D_{\boldsymbol{u}}f(x,y)=f_x(x,y)a+f_y(x,y)b=\langle f_x,f_y\rangle\cdot\langle a,b\rangle=\langle f_x,f_y\rangle\cdot\boldsymbol{u}$ For functions of three variables $f(x,y,z)$ $D_{\boldsymbol{u}}f(x,y,z)=\nabla f(x,y,z)\cdot\boldsymbol{u}$
Gradient	$\nabla f(x,y)=\langle f_x,f_y\rangle=\frac{\partial f}{\partial x}\boldsymbol{i}+\frac{\partial f}{\partial y}\boldsymbol{j}$ $\nabla f(x,y,z)=\frac{\partial f}{\partial x}\boldsymbol{i}+\frac{\partial f}{\partial y}\boldsymbol{j}+\frac{\partial f}{\partial z}\boldsymbol{k}$
Level curve, level surface vs ∇f	$\nabla f(x_0,y_0)$ is perpendicular/normal to the level curve $f(x,y)=k$ at the point (x_0,y_0) $\nabla F(x_0,y_0,z_0)\cdot\boldsymbol{r}'(t_0)=0$, that is $\nabla F(x_0,y_0,z_0)$ is perpendicular/normal to tangent vector $\boldsymbol{r}'(t_0)$ to any curve C on the surface S that passes through $\langle x_0,y_0,z_0\rangle$. 
Tangent plane to level surface	$\nabla F(x_0,y_0,z_0)\cdot\langle x-x_0,y-y_0,z-z_0\rangle=0$ $F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$
Maximising rate	Let θ be angle between vectors $\boldsymbol{u}$ and ∇f $D_{\boldsymbol{u}}f(P)=\lVert\nabla f(P)\rVert\cos\theta$ In direction of $\nabla f(P)$, $\lVert\nabla f(P)\rVert$ is the maximum rate of increase of f at P In direction of $-\nabla f(P)$, $-\lVert\nabla f(P)\rVert$ is the maximum rate of decrease of f at P
Theorem 8.17	If f has a local maximum/minimum at (a,b) and the first-order derivatives of f exists there, $f_x(a,b)=f_y(a,b)=0$
Critical points	$f_x(a,b)=0$ *and* $f_y(a,b)=0$, *OR one of the partial derivatives does not exist*

Second derivative test	Define discriminant D for the point (a,b): $D=D(a,b)=f_{xx}(a,b)f_{yy}(a,b)-[f_{xy}(a,b)]^2$ If $D>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local maximum; If $D>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local minimum; If $D<0$, then $f(a,b)$ is a saddle point of f; If $D=0$, then no conclusion can be drawn
Double integral	If $f(x,y)\geq0$, the volume V of the solid above rectangle R and below surface $z=f(x,y)$ is *Volume* $=\iint_R f(x,y)\,dA$
Iterated integral	$\int_a^b\int_c^d f(x,y)\,dydx$ First integrate with respect to y from c to d (keeping x fixed), then with respect to x from a to b
Fubini's Theorem	If f is continuous on the rectangle $R=[a,b]\times[c,d]$, $\iint_R f(x,y)\,dA=\int_a^b\int_c^d f(x,y)\,dydx=\int_c^d\int_a^b f(x,y)\,dxdy$
Area of plane region	Let $f(x,y)=1$ over a given region D. Then the area of D is $A(D)=\iint_D 1\,dA$
Conversion to Polar Coordinates	If f is continuous on a polar rectangle R: $R=\{(r,\theta):0\leq a\leq r\leq b,\alpha\leq\theta\leq\beta\}$ then, $\iint_R f(x,y)\,dA=\int_\alpha^\beta\int_a^b f(r\cos\theta,r\sin\theta)\,r\,drd\theta$
Surface area	$\iint_D dS=\iint_D\sqrt{f_x^2+f_y^2+1}\,dA$

Ordinary Differential Equations

Separable ODE	$\frac{dy}{dx}=f(x)g(x)$ *Separating the variables*: $\frac{1}{g(y)}dy=f(x)\,dx$
Reduction to separable form	$y'=g\left(\frac{y}{x}\right)$ where g is any function of $\frac{y}{x}$. Let $v=\frac{y}{x}$, then $y=vx$ and $y'=v+xv'$. Then the equation can be written as $v+xv'=g(v)$, which is separable. A differential equation of the form $y'=f(ax+by+c)$ where f is continuous and $b\neq0$ (if $b=0$, the equation is separable) can be solved by setting $u=ax+by+c$
Linear first order ODE	$\frac{dy}{dx}+P(x)y=Q(x)$ *Let* $I(x)=e^{\int P(x)dx}$, then $y\cdot I(x)=\int I(x)\cdot Q(x)\,dx$
Bernoulli equation	$y'+p(x)y=q(x)y^n$ Then let $u=y^{1-n}$, substituting into the equation $u'+(1-n)p(x)u=(1-n)q(x)$ Solve in the form of linear first order ODE above.
Malthus Model of Population	Let $N(t)$ be the total population, B be the per capita birth rate, D be the per capita death rate, $\frac{dN}{dt}=(B-D)N=kN$ $N(t)=N_0e^{kt}$ D is an increasing function of N, such that $D=sN$ (s is a constant), $\frac{dN}{dt}=BN-sN^2,\quad N(0)=N_0$ Rewriting as Bernoulli equation and solving, $N=\frac{N_\infty}{1+\left(\frac{N_\infty}{N_0}-1\right)e^{-Bt}}$ where $N_\infty=\frac{B}{s}$, which is the carrying capacity (logistic equilibrium population)
Half-life	$t_{\frac{1}{2}}=\frac{\ln2}{k}$