A New Fact to Discipline Models of Beliefs*

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September 2020

Abstract

Changes made to existing forecasts that are systematically correlated with subsequent forecast errors yield "error predictability," a salient feature of survey data. Full information rational expectations (FIRE) models, by contrast, produce unpredictable errors, a fact motivating many non-FIRE theories. Within this non-FIRE group, I demonstrate that empirical error predictability is not typically enough to distinguish across alternative belief systems. Instead, I highlight an additional empirical fact that, paired with error predictability, can serve to further disentangle competing models of beliefs. In particular, I emphasize that any model featuring Bayesian updating requires that forecast revisions be serially uncorrelated. Applying this logic to two widely studied theories and taking it to data from the Survey of Professional Forecasters favors a model of diagnostic expectations over a model of beliefs driven by strategic interaction.

Keywords: Rational expectations. Noisy information. Overreactions.
JEL Codes: D83, D84

*I would like to thank Stephen Terry and Constantin Burgi for their valuable feeback and helpful conversations
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1 Introduction

Forecast error predictability is a salient feature of survey data. Specifically, ex-ante forecast revisions can predict ex-post forecast errors. In their seminal paper, Coibion and Gorodnichenko (2015) document this fact which motivates departures from full information rational expectations (FIRE). At the forecaster-level, there is ample evidence of apparent overreactions (Bordalo et al., 2020).

Within the set of non-FIRE models, however, there are several theories consistent with such overreactions. I show that across specific non-FIRE models (diagnostic expectations and strategic interactions), one can recover an identical relationship between errors and revisions. In light of this, I offer an additional fact that can be used to further distinguish between models of expectations formation: the persistence of overlapping revisions.

At the forecaster-level, there tends to be a negative relationship between revisions made across two adjacent periods for a specific horizon. Some models are unable accommodate this fact. Intuitively, there is a subset of non-FIRE models that feature Bayesian updating. Such models require information efficiency that precludes the serial correlation of revisions. Based on the Survey of Professional Forecasters, I show that the data favor a model of diagnostic expectations over a model of strategic interaction.

Several studies have used survey data to test for FIRE.[1] In particular, a popular test often implemented in the literature projects errors on revisions. Suppose that x_t is the target variable and $x^i_{t+h|t}$ is forecaster i's forecast devised at time t for horizon h. Then the empirical test is defined as

$$\underbrace{x_t - x^i_{t|t}}_{\text{Error}} = \beta_0 + \beta_1[\underbrace{x^i_{t|t} - x^i_{t|t-1}}_{\text{Revision}}] + \epsilon^i_t \tag{1}$$

At the forecaster-level, the coefficient in front of revisions is found to be negative for a number of macroeconomic variables ($\beta_1 < 0$). To make sense of these apparent overreactions several non-FIRE theories have been proposed. Some of these theories feature information frictions while

[1] Examples include Bordalo et al. (2020), Fuhrer (2018), Dovern et al. (2015), Crowe (2010), Paloviita and Viren (2013), Burgi (2016), Andrade and Bihan (2013)

others incorporate behavioral frictions. I argue that both types of models are able to generate these apparent overreactions. A model with information frictions can generate this with an adjusted (symmetric) objective. In this note, I focus on relating diagnostic expectations models to a model of strategic interaction as in Woodford (2001).[2] [3]

I consider a noisy information environment. Optimal forecasts in this context are consistent with the mathematical expectations operator, $\mathbb{E}$. This will be the benchmark from which all other expectations will deviate in some manner. In keeping with (Coibion and Gorodnichenko, 2012, 2015) and Bordalo et al. (2020) consider the following

$$\begin{aligned} &\text{Exogenous State:} \quad x_t = \rho x_{t-1} + w_t, \quad w_t \sim \mathcal{N}(0, \sigma_w^2) \\ &\text{Private Signal:} \quad y_t^i = x_t + v_t^i, \quad v_t^i \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_v^2) \end{aligned}$$

The exogenous fundamental follows a simple AR(1) process.[4] Moreover, agents have access only to private information in the form of a noisy private signal y_t^i observed each period. I abstract away from more complex signal structures for simplicity, however, the assumptions on how agents receive information are unimportant for the results presented in subsequent sections.

From the Kalman filter, the optimal nowcast of x_t is

$$\mathbb{E}(x_t|\mathcal{I}_t^i) = x_{t|t}^i = (1-\kappa)x_{t|t-1}^i + \kappa y_t^i$$

where $\mathcal{I}_t^i$ denotes individual i's information set at time t, and κ refers to the steady-state Kalman gain, $\kappa = \frac{\text{Var}(x_t - x_{t|t-1}^i)}{\text{Var}(x_t - x_{t|t-1}^i) + \sigma_v^2}$ which is the optimal (in the mean-square sense) weight placed on new information.[5]

In Sections 2 and 3, I describe diagnostic expectations and strategic interaction, and analytically

[2] The focus here is on symmetric loss functions. Whereas asymmetric loss can generate $\beta_1 < 0$, these models also imply counterfactually biased consensus forecasts (see Bordalo et al. (2020).

[3] I also consider a model of overconfidence in Appendix B.

[4] The dynamics of the state are innocuous for the results of the paper.

[5] Note that the imperfect information enviornment is a more general formulation than a full-information environment. The model collapses to full-information rational expectations when $\sigma_v = 0$ so that $\kappa = 1$.

document the OLS coefficients from an errors-on-revisions regression that each theory delivers. In Section 4, I show that by varying the extent of diagnosticity and the degree of strategic substitutability, both theories can match the same level of observed overreaction. I then present a new fact in Section 5 and empirical evidence favoring diagnostic expectations. Section 6 concludes.

2 Diagnostic Expectations

Under diagnostic expectations, agents do not update their expectations in a rational matter, but instead overreact as follows:

$$x^{i,\theta}_{t|t} = x^i_{t|t} + \underbrace{\theta(x^i_{t|t} - x^i_{t|t-1})}_{\text{Overreaction}}$$

The one step ahead forecaster under diagnostic expectations is

$$x^{i,\theta}_{t|t-1} = \rho x^{i,\theta}_{t-1|t-1}$$

where $x^{i,\theta}_{t|t}$ denotes the individual's forecast for x_t made at time t, given the realization of y^i_t.

In this model, $\theta > 0$ is a belief distortion due to the representativeness heuristic described in Tversky and Kahneman (1974). Intuitively, more recent information is more easily recalled and therefore overweighted when formulating beliefs. Diagnostic expectations are forward looking and are therefore robust to the Lucas critique.

The diagnostic expectations model, therefore, delivers overreactions [6]

$$\beta_1^{DE} = \frac{\mathbb{C}(x_t - x^{i,\theta}_{t|t}, x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1})}{\mathbb{V}(x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1})} = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2\rho^2} < 0$$

When $\theta = 0$, diagnosticity disappears and $\beta_1 = 0$ as implied by error orthogonality.

[6] See **?** for proof.

3 Strategic Interaction

Overreactions, however, can also arise in a noisy information rational expectations environment. I next consider a model of strategic interaction. The model presented in this section draws from **?** and Woodford (2001). To obtain overreactions, I assume strategic substitutability. Intuitively, forecasters have the dual objective to minimize their squared errors and also distinguish themselves from the average forecast. More specifically, each forecaster wishes the minimize the following loss function

$$\min_{\{\widetilde{x}^i_{t|t}\}} \mathbb{E}\left[(x_t - \widetilde{x}^i_{t|t})^2 + R(\widetilde{x}^i_{t|t} - F_t)^2 | \mathcal{I}^i_t\right] \tag{2}$$

where x_t is the realized fundamental, $\widetilde{x}^i_{t|t}$ is forecaster i's *reported* current-period forecast, $\mathcal{I}^i_t$ is forecaster i's information set at time t, F is the consensus forecast, and $R < 0$ is the degree of strategic substitutability.[7] There are a number of possible microfoundations for strategic substitutability. Most prominently, see **?**. When $R = 0$, the loss function collapses to the familiar mean-squared loss.

The presence of strategic incentives makes higher order beliefs crucial to this model. In particular, the consensus nowcast at time t is denoted by F_t and it is defined as

$$F_t = \frac{1}{1+R}\sum_{k=0}^{\infty}\left(\frac{R}{1+R}\right)^k \mathbb{E}^{(k)}(x_t) = \frac{1}{1+R}x_{t|t} + \frac{R}{1+R}F_{t|t}$$

where $\mathbb{E}^{(k)}$ is the k^{th}-order expectation of x_t.

Taking the first order conditions of (2), it follows that the optimal reported prediction is

$$\widetilde{x}^i_{t|t} = \frac{1}{1+R}x^i_{t|t} + \frac{R}{1+R}F^i_{t|t}$$

where $x^i_{t|t}$ is the optimal nowcast for the state and $F^i_{t|t}$ is forecaster i's prediction about what the consensus nowcast is at time t.

[7] For $R > 0$, forecasters exhibit strategic complementarities. That is, forecasters have an incentive to stay close to the consensus forecast.

It follows that the forecast error in this model is [8]

$$x_t - \widetilde{x}^i_{t|t} = (1-\lambda)\left[x_t - \frac{1}{1+R}x^i_{t|t-1} - \frac{R}{1+R}\rho F^i_{t-1|t-1}\right] - \lambda v^i_t$$

and the forecast revision is defined as

$$\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1} = \lambda(x_t - x^i_{t|t-1} + v^i_t)$$

where $\lambda = \frac{\kappa_1 + R\kappa_2}{1+R}$ and κ_1 and κ_2 are the elements of the $2\times$ 1 Kalman gain vector, κ.

Proposition 1. *The errors-on-revisions regression coefficient in the strategic interaction model is*

$$\beta^{SI}_1 = \frac{R(\kappa_1 - \kappa_2)}{\kappa_1 + R\kappa_2}$$

Proof. See Appendix A. □

As expected, when $R = 0$, the coefficient $\beta^{SI}_1 = 0$, consistent with rational expectations under standard mean squared loss. Given the assumption placed on R, this model can generate overreactions.[9]

4 Matching Errors-on-Revisions

Both models discussed above are able to generate forecaster-level overreactions. Table 1 summarizes the updating rules for each of the three models. Note that these models can be expressed as a (positive) deviation from the conditional expectation, $x^i_{t|t}$. Hence, forecast updates exceed what is called for by the optimal minimum mean square estimate.

Both models are capable to generating the same β_1 coefficient in (1).[10] Consider first the

[8] See Appendix A for details.

[9] One might wonder whether forecasters can exhibit overreactions under strategic complementarities ($R > 0$). This can only occur if the weight placed private information when predicting the consensus forecast exceeds the weight placed on private information when predicting the state ($\kappa_2 > \kappa_1$). Given that the signal is more informative about x_t than F_t, it is never optimal for the forecaster to set $\kappa_2 > \kappa_1$.

[10] See the Appendix A for details on this result.

Table 1: Update Rules Across Models

Model	Update rule
Diagnostic Expectations	$x_{t\|t}^{i,\theta} = x_{t\|t}^{i} + \theta(x_{t\|t}^{i} - x_{t\|t-1}^{i})$
Strategic Interaction	$\widetilde{x}_{t\|t}^{i} = x_{t\|t}^{i} - \frac{R}{1+R}(x_{t\|t}^{i} - F_{t\|t}^{i})$

Note: The table reports the updating rules for a model of diagnostic expectations and a model of strategic interaction.

mapping between the diagnostic expectations model and the strategic interaction model. Given $\beta_1^{DE} = \beta_1(\rho, \sigma_v, \sigma_w, \theta)$ in the diagnostic expectations model, we can match β_1 in the strategic interaction model by setting the degree of strategic substitution to be

$$R = \frac{\beta_1^{DE}\kappa_1}{\kappa_1 - \kappa_2(1 + \beta_1^{DE})}$$

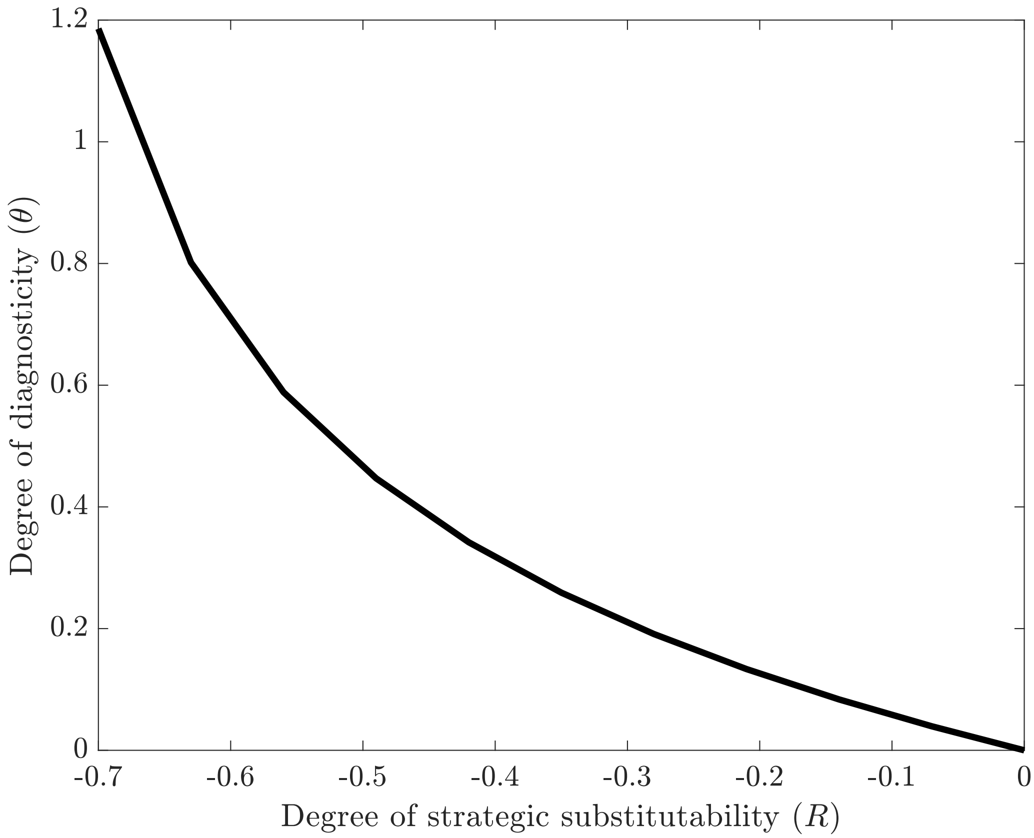
Similarly, given $\beta_1^{SI} = \beta_1(\rho, \sigma_v^2, \sigma_w^2, R)$ in the strategic interaction model, one could construct a model diagnostic expectations that delivers an equivalent β_1 by setting the degree of diagnosticity, θ, to be equal to the largest root of the following quadratic

$$[(1 + \rho^2)\beta_1^{SI} + 1]\lambda^2 + (1 + 2\beta_1^{SI})\lambda + \beta_1^{SI} = 0$$

Hence, by simply assessing the regression coefficient in an errors-on-revisions regression, one cannot necessarily distinguish across noisy information models of rational and non-rational expectations.

The panels in Figure 1 plot the relationship between the two parameters $\{\theta, R\}$ that are key in delivering identical β_1 coefficients.

Figure 1: Mapping β_1 Across Models of Overreactions



Note: Each point on the line corresponds to a specific value of β_1. The figure plots the degree of strategic substitutability, R, that generates the same β_1 that is obtained by a model of diagnostic expectations.

5 A New Fact: Persistence of Revisions

While both models can deliver identical β_1 coefficients, they make different predictions about forecaster behavior. Hence, with enough data, one can discern between the two. More broadly, with enough data, we can distinguish between two subsets of non-FIRE models. I show that we can make progress on this front by merely focusing on an additional fact: the persistence of overlapping revisions.

Beyond forecast error orthogonality, Nordhaus (1987) notes that revisions must be "informationally efficient." This requires the following condition to hold

$$\mathbb{E}(x^i_{t|t} - x^i_{t|t-1} | \mathcal{I}^i_t) = 0$$

In words, forecast revisions must be orthogonal to any variable residing in the forecasters information set.

$$\mathbb{E}[(x^i_{t|t} - x^i_{t|t-1})\mu] = 0 \quad \text{for } \mu \in \mathcal{I}^i_t$$

This is akin to the error orthogonality condition which has been the focus of conventional efficiency tests. However, whereas error orthogonality can be violated for some linear noisy information rational expectations models, revision orthogonality cannot. This is an artifact of Bayesian updating in a linear setting. In such models, the forecast revision is equal to the innovation error observed when the signal is received, scaled by the optimal Kalman gain. These innovation errors are unpredictable by definition. With this insight, we can run the following regression to test for rational expectations

$$\underbrace{x^i_{t|t} - x^i_{t|t-1}}_{\text{Revision}_t} = \gamma_0 + \gamma_1[\underbrace{x^i_{t|t-1} - x^i_{t|t-2}}_{\text{Revision}_{t-1}}] + \varepsilon^i_t \tag{3}$$

From a practical standpoint, this testable implication also has the benefit of not requiring the econometrician to take a stand on which type of realized data to use (real-time or revised).

Table 2: Revision Persistence Distinguishes the Models

	Errors-on-revisions (β_1)	Revision persistence (γ_1)
Diagnostic Expectations	-0.36	-0.38
	(0.06)	(0.02)
Strategic Interaction	-0.36	-0.02
	(0.06)	(0.06)

Note: The table displays the simulated β_1 and γ_1 coefficients across models. The parameterization is for the strategic interaction model $R = -0.7$, $\rho = 0.9$, $\sigma_v = 2.0$ and $\sigma_w = 1.5$. From here, I find the θ parameter such that β_1 is identical across models. With these parameters, I then simulate γ_1 for each model. I compute 10,000 simulations, each 100 quarters long (with additional 100 periods discarded) and 40 forecasters. Standard deviations reported in parentheses.

Proposition 2. *The strategic interaction model delivers* $\gamma_1 = 0$.

Proof. See Appendix A. □

Table 2 reports a set of simulations results from all three models. I first fix the parameters of the strategic interaction model and I find the $\{\alpha, \theta\}$ that replicate β_1^{SI}. I then compute the simulated revision persistence coefficient, γ_1 across all models.[11] The table verifies that while the model can deliver identical β_1 coefficients, it is unable to deliver the same revision persistence. In particular, the strategic interaction model requires lagged revisions to have no predictive power over current revisions.

Data from the Survey of Professional Forecasters (SPF) suggests that forecast revisions are negatively related over time. Table 3 reports the γ_1 coefficient arising from (1) pooling across 15 variables in the SPF.[12] The empirical results suggest that models of diagnostic expectations are consistent with survey expectations whereas a rational model of strategic substitutability is not.

[11] Appendix D plots the simulated densities of the different γ_1 coefficients.
[12] See Appendix C for variable-by-variable results.

Table 3: Pooled OLS Forecast Revision Persistence Regressions

	Nowcast	One-Quarter Ahead	Two-Quarters Ahead
Estimate	-0.174***	-0.212**	-0.299***
	(0.045)	(0.034)	(0.027)
Observations	57,417	57,180	55,151

Note: The table reports the estimated coefficients of forecast revision persistence at the current, one-, and two-quarter ahead horizons. Columns differ in horizon considered. Standard errors are as in Driscoll and Kraay (1998). Data used for estimation come from the Surey of Professional Forecasters. *** denotes 1% significance, ** denotes 5% significance, and * denotes 10% significance.

6 Conclusion

In this note, I show that the popular errors-on-revisions coefficient used in the expectations formation literature is insufficient to motivate a departure from rationality. By way of example, I show that a popular model of non-rational expectations can deliver the same errors-on-revisions coefficient as in a rational strategic interactions model. Given this I offer a new fact for further discerning across non-FIRE models which requires projecting revisions on their past values. Using survey from the Survey of Professional Forecasters, I find evidence favoring diagnostic expectations over strategic interactions.

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