A Short Note on Rationalizing Overreactions

Julio Ortiz *

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Abstract

Survey data on expectations show that errors and revisions tend to covary negatively which implies that forecasters exhibit overreactive behavior. While this empirical finding is used to motivate models of non-rational expectations, I argue that error predictability is not sufficient to reject rationality. I prove that a (rational) model of strategic interaction can deliver and identical OLS coefficient from an errors-on-revisions regression as (non-rational) models of overconfidence and diagnostic expectations. In light of this observational equivalence, I propose focusing on the persistence of revisions as a more robust reduced form exercise. Using data from the Survey of Professional Forecasters, I find evidence against linear rational expectations.

Keywords: Rational expectations. Noisy information. overreactions.

JEL Codes: D83, D84

*Boston University, Department of Economics, 270 Bay State Road, Boston, MA 02215; Phone: 201-230-1960; Email: jlortiz@bu.edu.

1 Introduction

Forecast error predictability is not sufficient to reject rationality. Recently, much of the literature on expectations formation has documented that ex-post errors and and ex-ante revisions are negatively related. The interpretation is that forecasters overreact to news. To make sense of this, several theories of non-rational expectations have been proposed. In this note, I argue that rational and non-rational models can deliver identical relationships between errors and revisions. Specifically, I show that the same OLS coefficient obtained from a reduced form regression of errors on revisions can arise in a rational and non-rational model alike. I consider two models of non-rational expectations and derive analytical results mapping their respective parameters to those in a rational model of strategic substitutability. In light of this result I recommend an alternate test of linear rational expectations that makes use of the persistence of revisions. Using this well known alternative test, I document evidence against *linear* rational expectations.

Related Literature

Several studies have used survey data to test for rationality.[1] A popular test of rationality that is used in the literature projects errors on revisions at the forecaster-level. Suppose that x_t is the target variable and $x^i_{t+h|t}$ is forecaster i's forecast devised at time t for horizon h. Then the empirical test is defined as

$$\underbrace{x_t - x^i_{t|t}}_{\text{Error}} = \beta_0 + \beta_1 [\underbrace{x^i_{t|t} - x^i_{t|t-1}}_{\text{Revision}}] + \epsilon^i_t \tag{1}$$

The coefficient in front of revisions is found to be negative for a number of macroeconomic variables ($\beta_1 < 0$). To make sense of such "overreactions" several theories of non-rational expectations have been proposed. A prominent model that can make sense of this fact is a model of overconfidence (Daniel (1993)). More recently, Bordalo et. al. (2019) develop a theory of diagnostic expectations. However, linear rational models can also deliver $\beta_1 < 0$. To do so, one must simply adjust the forecasters (symmetric) objective. In this note, I focus on relating overconfidence and diagnostic expectations models to a model of strategic interaction as in Woodford (2001).[2]

[1] Examples include Bordalo et. al. (2019), Fuhrer (2018), Dovern et. al. (2014), Crowe (2010), Paloviita and Viren (2013), Burgi (2016), Andrade and Le Bihan (2013).

[2] The focus here is on symmetric loss functions. Whereas asymmetric loss can generate $\beta_1 < 0$, these models also imply counterfactually biased consensus forecasts (see Bordalo et al. (2019)).

Set Up

I consider a noisy information environment. Optimal forecasts in this context are consistent with the mathematical expectations operator, $\mathbb{E}$. This will be the benchmark from which all other expectations will deviate in some manner. In keeping with Coibion and Gorodnichenko (2012, 2015) and Bordalo et. al. (2019) consider the following

$$\begin{aligned} \text{Exogenous State:} \quad & x_t = \rho x_{t-1} + w_t, \quad w_t \sim \mathcal{N}(0, \sigma_w^2) \\ \text{Private Signal:} \quad & y_t^i = x_t + v_t^i, \qquad v_t^i \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_v^2) \end{aligned}$$

The exogenous fundamental follows a simple AR(1) process. The dynamics of the state are innocuous for the results of the paper. Moreover, agents have access only to private information in the form of a noisy private signal y_t^i observed each period. I abstract away from more complex signal structures for simplicity, however, the assumptions on how agents receive information are unimportant for the results presented in subsequent sections.

From the Kalman filter, the optimal nowcast of x_t is $\mathbb{E}(x_t|\mathcal{I}_t^i) = x_{t|t}^i = (1-\kappa)x_{t|t-1}^i + \kappa y_t^i$ where $\mathcal{I}_t^i$ denotes individual i's information set at time t, and κ refers to the steady-state Kalman gain, $\kappa = \frac{\text{Var}(x_t - x_{t|t-1}^i)}{\text{Var}(x_t - x_{t|t-1}^i) + \sigma_v^2}$ which is the optimal (in the mean-square sense) weight placed on new information.[3]

2 Non-rational Expectations

Overconfidence Model

Daniel (1993) presents a theory of overconfidence in which individuals perceive their private signals to be more precise than they truly are. Specifically, forecasters perceive

$$y_t^i = x_t + v_t^i \qquad v_t^i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \breve{\sigma}_v^2)$$

where $\breve{\sigma}_v = \alpha \sigma_v$ such that $\alpha < 1$.

As a matter of notation, let the forecaster's current period forecast as $\breve{x}_{t|t}^i$ and his one step ahead forecast as $\breve{x}_{t|t-1}^i$. Forecasters invoke the Kalman filter for the model in order for formulate their expectations. As a

[3] Note that the imperfect information enviornment is a more general formulation than a full-information environment. The model collapses to full-information rational expectations when $\sigma_v = 0$ so that $\kappa = 1$.

result, expectations are determined according to the following predict-update procedure

$$\breve{x}^i_{t|t-1} = \rho \breve{x}^i_{t-1|t-1} \qquad \text{(Predict)}$$

$$\breve{x}^i_{t|t} = \breve{x}^i_{t|t-1} + \breve{\kappa}(y^i_t - \breve{x}^i_{t|t-1}) \qquad \text{(Update)}$$

Proposition 1. *The OLS coefficient arising from an errors-on-revisions regression in the overconfidence model is*

$$\beta_1^{OC} = \frac{\mathbb{C}(x_t - \breve{x}^i_{t|t}, \breve{x}^i_{t|t} - \breve{x}^i_{t|t-1})}{\mathbb{V}(\breve{x}^i_{t|t} - \breve{x}^i_{t|t-1})} = \frac{(\alpha^2 - 1)\sigma_v^2}{\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2} < 0$$

Proof. See Appendix A. □

The coefficient β_1^{OC} is always negative since $\alpha < 1$.[4] Hence, the underlying overconfidence on the part of forecasters generates overreactions. In essence, forecasters believe their signals to be more precise than they truly are thereby leading them to place more weight on new information.

Diagnostic Expectations

Under diagnostic expectations, agents also overreact to new information so that the reported update is

$$x^{i,\theta}_{t|t} = x^i_{t|t} + \theta(x^i_{t|t} - x^i_{t|t-1})$$

while the one step ahead forecast is

$$x^{i,\theta}_{t|t-1} = \rho x^{i,\theta}_{t-1|t-1}$$

where $x^{i,\theta}_{t|t}$ denotes the individual's forecast for x_t made at time t, given the realization of y^i_t. In this model, $\theta > 0$ is a belief distortion due to the representativeness heuristic described in Kahneman and Tversky (1974).

The diagnostic expectations model can also deliver overreactions [5]

$$\beta_1^{DE} = \frac{\mathbb{C}(x_t - x^{i,\theta}_{t|t}, x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1})}{\mathbb{V}(x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1})} = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2\rho^2} < 0$$

[4] In the absence of overconfidence, $\alpha = 1$ and $\beta_1 = 0$.

[5] See Bordalo et al. (2019) for proof. When $\theta = 0$, diagnosticity disappears and $\beta_1 = 0$ as implied by error orthogonality.

3 Strategic Interaction

Overreactions, however, can also arise in a model of noisy information rational expectations, I next consider a model of strategic interaction. The model presented in this section draws from Morris and Shin (2002) and Woodford (2001). To obtain overreactions, I assume strategic substitutability. Intuitively, forecasters have the dual objective to minimize their squared errors and also distinguish themselves from the average forecast. More specifically, each forecaster wishes the minimize the following loss function

$$\min_{\{\tilde{x}^i_{t|t}\}} \mathbb{E}\left[(x_t - \tilde{x}^i_{t|t})^2 + R(\tilde{x}^i_{t|t} - F_t)^2 | \mathcal{I}^i_t\right] \tag{2}$$

where x_t is the realized fundamental, $\tilde{x}^i_{t|t}$ is forecaster i's *reported* current-period forecast, $\mathcal{I}^i_t$ is forecaster i's information set at time t, F is the consensus forecast, and $R < 0$ is the degree of strategic substitutability.[6] There are a number of possible microfoundations for strategic substitutability. Most prominently, see Ottaviano and Sørensen (2006). When $R = 0$, the loss function collapses to the familiar mean-squared loss.

The presence of strategic incentives makes higher order beliefs crucial to this model. In particular, the consensus nowcast at time t is denoted by F_t and it is defined as

$$F_t = \frac{1}{1+R} \sum_{k=0}^{\infty} \left(\frac{R}{1+R}\right)^k \mathbb{E}^{(k)}(x_t) = \frac{1}{1+R} x_{t|t} + \frac{R}{1+R} F_{t|t}$$

where $\mathbb{E}^{(k)}$ is the k^{th}-order expectation of x_t.

Taking the first order conditions of (2), it follows that the optimal reported prediction is

$$\tilde{x}^i_{t|t} = \frac{1}{1+R} x^i_{t|t} + \frac{R}{1+R} F^i_{t|t}$$

where $x^i_{t|t}$ is the optimal nowcast for the state and $F^i_{t|t}$ is forecaster i's prediction about what the consensus nowcast is at time t.

It follows that the forecast error in this model is [7]

$$x_t - \tilde{x}^i_{t|t} = (1-\lambda)\left[x_t - \frac{1}{1+R} x^i_{t|t-1} - \frac{R}{1+R} \rho F^i_{t-1|t-1}\right] - \lambda v^i_t$$

[6] For $R > 0$, forecasters exhibit strategic complementarities. That is, forecasters have an incentive to stay close to the consensus forecast.

[7] See Appendix A for details.

Model	Update rule
Overconfidence	$\breve{x}^i_{t\|t} = x^i_{t\|t} + (1-\breve{\kappa})\breve{x}^i_{t\|t-1} - (1-\kappa)x^i_{t\|t-1} + (\breve{\kappa}-\kappa)y^i_t$
Diagnostic Expectations	$x^{i,\theta}_{t\|t} = x^i_{t\|t} + \theta(x^i_{t\|t} - x^i_{t\|t-1})$
Strategic Substitutes	$\widetilde{x}^i_{t\|t} = x^i_{t\|t} - \frac{R}{1+R}(x^i_{t\|t} - F^i_{t\|t})$

Table 1: Update Rules Across Models

and the forecast revision is defined as

$$\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1} = \lambda(x_t - x^i_{t|t-1} + v^i_t)$$

where $\lambda = \frac{\kappa_1 + R\kappa_2}{1+R}$ and κ_1 and κ_2 are the elements of the $2\times$ 1 Kalman gain vector, κ.

Proposition 2. *The errors-on-revisions regression coefficient in the strategic interaction model is*

$$\beta_1^{SI} = \frac{R(\kappa_1 - \kappa_2)}{\kappa_1 + R\kappa_2}$$

Proof. See Appendix A. □

As expected, when $R = 0$, the coefficient $\beta_1^{SI} = 0$, consistent with rational expectations under standard mean squared loss. Given the assumption placed on R, this model can generate overreactions.[8]

4 An Equivalence Result for β_1

All three models discussed above are able to generate forecaster-level overreactions. Table 1 summarizes the updating rules for each of the three models. Note that all three models can be expressed as a (positive) deviation from the conditional expectation, $x^i_{t|t}$. Hence, forecast updates exceed what is called for by the optimal minimum mean square estimate.

[8] One might wonder whether forecasters can exhibit overreactions under strategic complementarities ($R > 0$). This can only occur if the weight placed private information when predicting the consensus forecast exceeds the weight placed on private information when predicting the state ($\kappa_2 > \kappa_1$). Given that the signal is more informative about x_t than F_t, it is never optimal for the forecaster to set $\kappa_2 > \kappa_1$.

Matching Errors-on-Revisions

Furthermore, all three models are capable to generating the same β_1 coefficient in (1).[9]

Consider first the mapping between the diagnostic expectations model and the strategic interaction model. Given $\beta_1^{DE} = \beta_1(\rho, \sigma_v, \sigma_w, \theta)$ in the diagnostic expectations model, we can match β_1 in the strategic interaction model by setting the degree of strategic substitution to be

$$R = \frac{\beta_1^{DE}\kappa_1}{\kappa_1 - \kappa_2(1 + \beta_1^{DE})}$$

Similarly, given $\beta_1^{SI} = \beta_1(\rho, \sigma_v^2, \sigma_w^2, R)$ in the strategic interaction model, one could construct a model diagnostic expectations that delivers an equivalent β_1 by setting the degree of diagnosticity, θ, to be equal to the largest root of the following quadratic

$$[(1 + \rho^2)\beta_1^{SI} + 1]\lambda^2 + (1 + 2\beta_1^{SI})\lambda + \beta_1^{SI} = 0$$

Finally, a model of overconfidence can deliver the same β_1 coefficient as a model of diagnostic expectations. Since the variance of the error depends nonlinearly on α, this is done by finding the α parameter such that

$$\alpha^2 - \frac{\beta_1^{DE}}{\sigma_v^2}\breve{\Psi}^i_{t|t-1}(\alpha) = \beta^{DE} + 1$$

where $\breve{\Psi}^i_{t|t-1}(\alpha)$ is the forecast error variance in the overconfidence model. Note that this is itself a function of α. Hence, by simply assessing the regression coefficient in an errors-on-revisions regression, one cannot necessarily distinguish across noisy information models of rational and non-rational expectations.

Simulation Results

The panels in Figure 1 plot the relationship between the three parameters $\{\alpha, \theta, R\}$ that are key in delivering identical β_1 coefficients.

[9] See the Appendix B for details on this result.

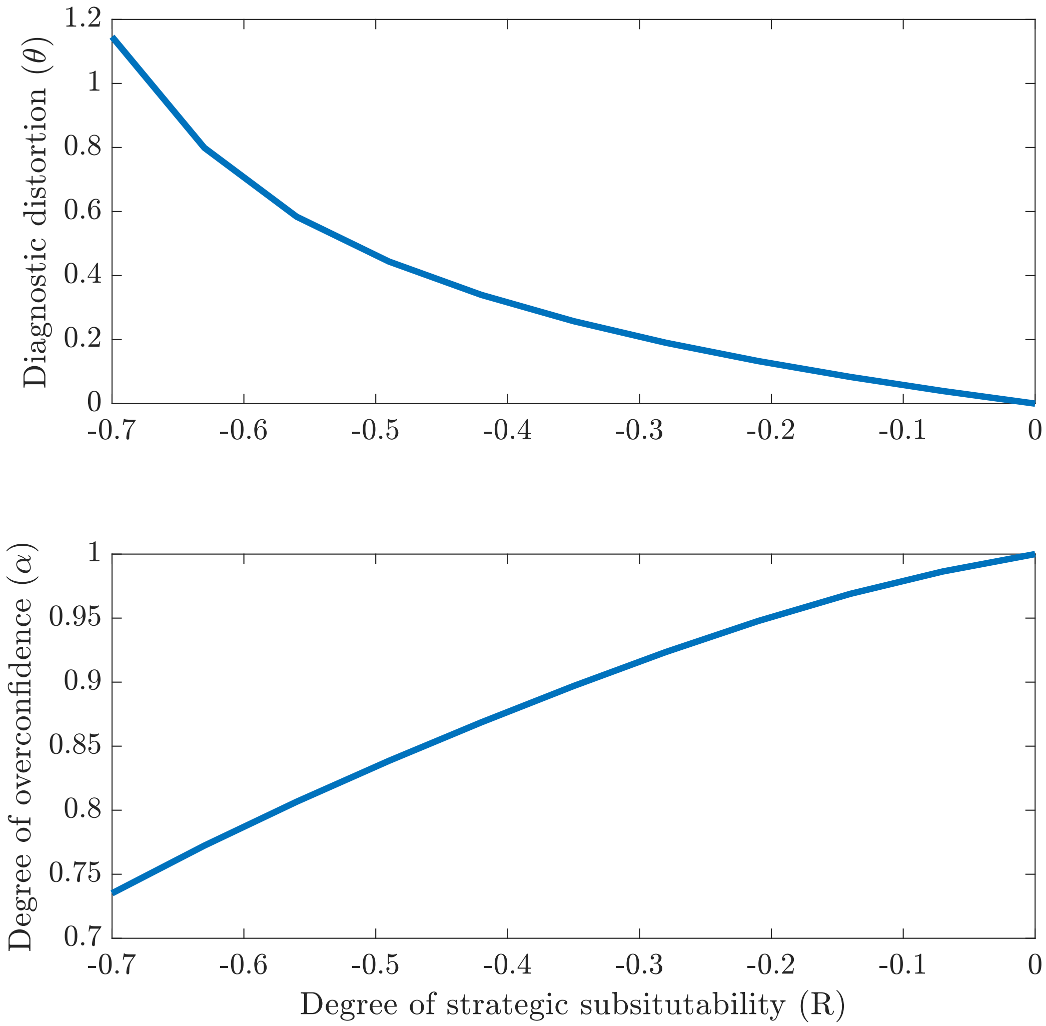

Figure 1: Mapping Reduced Form β_1 Across Models of Overreactions

Note: The figure plots the degree of strategic substitutability, R, that generates the same β_1 that is obtained by non-rational models of overreactions denoted by the degree of overreaction, θ (for diagnostic expectations) and the parameter governing perceived information precision, α (for overconfidence).

5 Alternate Test: Persistence of Revisions

While all three models can deliver identical β_1 coefficients, it is evident that they are not entirely equivalent. Hence, with enough data, one can discern across all three models. With that said, the focus of this note is to distinguish between two classes of linear noisy information models: rational and non-rational. This can be done without requiring several different data moments and instead simply considering the persistence of revisions.

Beyond forecast error orthogonality, Norhaus (1987) notes that revisions must be "informationally efficient". This requires the following condition to hold

$$\mathbb{E}(x^i_{t|t} - x^i_{t|t-1} | \mathcal{I}^i_t) = 0$$

In words, forecast revisions must be orthogonal to any variable residing in the forecasters information set.

$$\mathbb{E}[(x^i_{t|t} - x^i_{t|t-1})\mu] = 0 \quad \text{for } \mu \in \mathcal{I}^i_t$$

This is akin to the error orthogonality condition which has been the focus of conventional efficiency tests. However, whereas error orthogonality can be violated for some linear noisy information rational expectations models, revision orthogonality cannot. This is an artifact of Bayesian updating in a linear setting. In such models, the forecast revision is equal to the innovation error observed when the signal is received, scaled by the optimal Kalman gain. These innovation errors are unpredictable by definition. With this insight, we can run the following regression to test for rational expectations

$$\underbrace{x^i_{t|t} - x^i_{t|t-1}}_{\text{Revision}_t} = \gamma_0 + \gamma_1 [\underbrace{x^i_{t|t-1} - x^i_{t|t-2}}_{\text{Revision}_{t-1}}] + \varepsilon^i_t \tag{3}$$

From a practical standpoint, this testable implication also has the benefit of not requiring the econometrician to take a stand on which type of realized data to use (real time or revised).

Proposition 3. *The strategic interaction model delivers* $\gamma_1 = 0$.

Proof. See Appendix A. □

Table 2 reports a set of simulations results from all three models. I first fix the parameters of the strategic interaction model and I find the $\{\alpha, \theta\}$ that replicate β_1^{SI}. I then compute the simulated revision persistence coefficient, γ_1 across all models.[10] The table verifies that while the model can deliver identical β_1 coefficients, it is unable to deliver the same revision persistence. In particular, the strategic interaction model requires lagged revisions to have no predictive power over current revisions.

	β_1	γ_1
Overconfidence	-0.378	-0.104
	[0.058]	[0.059]
Diagnostic Expectations	-0.378	-0.378
	[0.058]	[0.023]
Strategic Interaction	-0.378	-0.015
	[0.058]	[0.062]

Table 2: β_1 and γ_1 Across Models

Note: The table displays the simulated β_1 and γ_1 coefficients across models. The parameterization is for the strategic interaction model $R = -0.7$, $\rho = 0.9$, $\sigma_v = 2.0$ and $\sigma_w = 1.5$. From here, I find the θ and α parameters such that β_1 is identical across models. With these parameters, I then simulate γ_1 for each model. I compute 10,000 simulations, each with 100 quarters (with additional 100 burn-in period) and 40 forecasters. Standard deviations reported in brackets.

Empirical Evidence

Data from the Survey of Professional Forecasters (SPF) suggests that forecast revisions are negatively related over time. Table 3 reports the γ_1 coefficient arising from (1) pooling across 15 variables in the SPF.[11] The empirical results suggest that models of overconfidence and diagnostic expectations are consistent with survey expectations whereas a rational model of strategic substitutability is not. Note that a non-zero γ_1 coefficient implies a rejection of *linear* rational expectations. Importantly, this need not extend to nonlinear environments.[12]

[10] Appendix C plots the simulated densities of the different γ_1 coefficients.
[11] See Appendix B for variable-by-variable results.
[12] See Ortiz (2020).

	Nowcast	One-Quarter Ahead	Two-Quarters Ahead
Estimate	-0.174***	-0.212**	-0.299***
	(0.045)	(0.034)	(0.027)
Observations	57,417	57,180	55,151

* p<0.10, ** p<0.05, *** p<0.01

Table 3: Pooled OLS Forecast Revision Persistence Regressions

Note: The table reports the estimated coefficients of forecast revision persistence at the current, one-, and two-quarter ahead horizons. Columns differ in horizon considered. Standard errors are as in Driscoll and Kraay (1998). Data used for estimation come from SPF.

6 Conclusion

In this note, I show that the popular errors-on-revisions coefficient used in the expectations formation literature is insufficient to motivate a departure from rationality. By way of example, I show that two popular models of non-rational expectations can deliver the same errors-on-revisions coefficient as in a rational strategic interactions model. Given this, I propose testing for rationality by instead projecting revisions on their past values. This testable implication is robust to general quadratic loss functions that might otherwise deliver a non-zero covariance between errors and revisions under rational expectations. Using survey from the Survey of Professional Forecasters, I find evidence against linear rational expectations.

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Appendix: A Note on Rationalizing Over-Reactions

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Contents

Appendix A Derivations and Proofs

A.1 Proof of Proposition 1

$$
\begin{aligned}
\beta_1^{OC} &= \frac{\mathbb{C}(x_t - \breve{x}^i_{t|t}, \breve{x}^i_{t|t} - \breve{x}^i_{t|t-1})}{\mathbb{V}(\breve{x}^i_{t|t} - \breve{x}^i_{t|t-1})} \\
&= \frac{(1-\breve{\kappa})\breve{\kappa}\mathbb{V}(x_t - x^i_{t|t-1}) - \breve{\kappa}^2\sigma_v^2}{\breve{\kappa}^2[\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2]} \\
&= \frac{(1-\breve{\kappa})\mathbb{V}(x_t - x^i_{t|t-1}) - \breve{\kappa}\sigma_v^2}{\breve{\kappa}[\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2]} \\
&= \frac{\frac{\breve{\sigma}_v^2\mathbb{V}(x_t - \breve{x}^i_{t|t-1})}{\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \breve{\sigma}_v^2} - \frac{\mathbb{V}(x_t - \breve{x}^i_{t|t-1})\sigma_v^2}{\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \breve{\sigma}_v^2}}{\breve{\kappa}[\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2]} \\
&= \frac{(\alpha^2 - 1)\sigma_v^2\breve{\kappa}}{\breve{\kappa}[\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2]} \\
&= \frac{(\alpha^2 - 1)\sigma_v^2}{\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2}
\end{aligned}
$$

A.2 Deriving Errors and Revisions for SI Model

In state space form, we have

$$
\begin{aligned}
\begin{bmatrix} x_t \\ F_t \end{bmatrix} &= \begin{bmatrix} \rho & 0 \\ \lambda\rho & (1-\lambda)\rho \end{bmatrix} \begin{bmatrix} x_{t-1} \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda \end{bmatrix} w_t \\
y_t^i &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ F_t \end{bmatrix} + v_t^i
\end{aligned}
$$

Or more compactly,

$$
\begin{aligned}
\mathbf{s_t} &= \mathbf{A}\mathbf{s_{t-1}} + \mathbf{B}w_t \\
y_t^i &= \mathbf{C}\mathbf{s_t} + v_t^i
\end{aligned}
$$

Invoking the Kalman filter

$$
\begin{aligned}
\mathbf{s^i_{t|t}} &= (\mathbf{I} - \kappa\mathbf{C})\mathbf{A}\mathbf{s^i_{t-1|t-1}} + \kappa\mathbf{C}\mathbf{A}\mathbf{s_{t-1}} + \kappa\mathbf{C}\mathbf{B}w_t + \kappa v^i_t \\
\mathbf{s^i_{t|t-1}} &= \mathbf{A}\mathbf{s^i_{t-1|t-1}}
\end{aligned}
$$

which implies that

$$
\begin{aligned}
x^i_{t|t} &= (1-\kappa_1)\rho x^i_{t-1|t-1} + \kappa_1 y^i_t \\
F^i_{t|t} &= (\lambda-\kappa_2)\rho x^i_{t-1|t-1} + (1-\lambda)\rho F^i_{t-1|t-1} + \kappa_2 y^i_t
\end{aligned}
$$

and

$$
\begin{aligned}
x^i_{t|t-1} &= \rho x^i_{t-1|t-1} \\
F^i_{t|t-1} &= \lambda\rho x^i_{t-1|t-1} + (1-\lambda)\rho F^i_{t-1|t-1}
\end{aligned}
$$

where $\lambda = \frac{\kappa_1 + R\kappa_2}{1+R}$ and κ_1, κ_2 are the Kalman gains belonging to the two-dimensional column vector κ. Letting $\xi = \begin{bmatrix} \frac{1}{1+R} & \frac{R}{1+R} \end{bmatrix}$, we have

$$
\begin{aligned}
\widetilde{x}^i_{t|t} &= \xi\mathbf{s^i_{t|t}} \\
&= \frac{1}{1+R}x^i_{t|t} + \frac{R}{1+R}F^i_{t|t} \\
&= \frac{1}{1+R}\Big[(1-\kappa_1)\rho x^i_{t-1|t-1} + \kappa_1 y_{it}\Big] + \frac{R}{1+R}\Big[(\lambda-\kappa_2)\rho x^i_{t-1|t-1} + (1-\lambda)\rho F^i_{t-1|t-1} + \kappa_2 y_{it}\Big] \\
&= \frac{1-\kappa_1 + R(\lambda-\kappa_2)}{1+R}x^i_{t|t-1} + \lambda y_{it} + \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} \\
&= \frac{1-\lambda}{1+R}x^i_{t|t-1} + \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} + \lambda y_{it}
\end{aligned}
$$

and

$$\begin{aligned}
\widetilde{x}^i_{t|t-1} &= \xi \mathbf{s^i_{t|t-1}} \\
&= \frac{1}{1+R}\rho x^i_{t-1|t-1} + \frac{R}{1+R}\Big[\lambda\rho x^i_{t-1|t-1} + (1-\lambda)\rho F^i_{t-1|t-1}\Big] \\
&= \frac{1+R\lambda}{1+R}x^i_{t|t-1} + \frac{(1-\lambda)\rho R}{1+R}F^i_{t-1|t-1}
\end{aligned}$$

Hence,

$$\begin{aligned}
\widetilde{x}^i_{t|t} &= \frac{1-\lambda}{1+R}x^i_{t|t-1} + \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} + \lambda y^i_t \\
\widetilde{x}^i_{t|t-1} &= \frac{1+R\lambda}{1+R}x^i_{t|t-1} + \frac{(1-\lambda)\rho R}{1+R}F^i_{t-1|t-1}
\end{aligned}$$

Furthermore,

$$F_t = \widetilde{x}_{t|t} = (1-\lambda)\rho F_{t-1} + \lambda x_t$$

$$\widetilde{x}_{t|t-1} = \lambda\rho x_{t-1|t-1} + (1-\lambda)\rho F_{t-1}$$

From here, it follows that

$$\begin{aligned}
x_t - \widetilde{x}^i_{t|t} &= x_t - \frac{1-\kappa_1+(\lambda-\kappa_2)R}{1+R}x^i_{t|t-1} - \lambda y^i_t - \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} \\
&= (1-\lambda)x_t - \frac{1-\kappa_1+(\lambda-\kappa_2)R}{1+R}x^i_{t|t-1} - \lambda v^i_t - \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} \\
&= (1-\lambda)x_t - \frac{1-\lambda}{1+R}x^i_{t|t-1} - \frac{(1-\lambda)R\rho}{1+R}F^i_{t-1|t-1} - \lambda v^i_t \\
&= (1-\lambda)\Big[x_t - \frac{1}{1+R}x^i_{t|t-1} - \frac{R}{1+R}\rho F^i_{t-1|t-1}\Big] - \lambda v^i_t
\end{aligned}$$

and the forecast revision is

$$\begin{aligned}
\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1} &= \frac{(1-\kappa_1)+(\lambda-\kappa_2)R-(1+R\lambda)}{1+R}x^i_{t|t-1} + \lambda x_t + \lambda v^i_t \\
&= -\frac{\kappa_1-\kappa_2 R}{1+R}x^i_{t|t-1} + \lambda x_t + \lambda v^i_t \\
&= \lambda(x_t - x^i_{t|t-1} + v^i_t)
\end{aligned}$$

A.3 Proof of Proposition 2

$$\begin{aligned}
\beta_1^{SI} &= \frac{\mathrm{Cov}(x_t - \widetilde{x}^i_{t|t}, \widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1})}{\mathrm{Var}(\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1})} \\
&= \frac{(1-\lambda)\lambda \mathrm{Cov}(x_t, x_t - x^i_{t|t-1}) - \lambda^2\sigma_v^2}{\lambda^2\left[\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2\right]} \\
&= \frac{1-\lambda}{\lambda}\frac{\mathrm{Var}(x_t) - \mathrm{Cov}(x_t, x^i_{t|t-1})}{\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2} - \frac{\sigma_v^2}{\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2} \\
&= \frac{1-\lambda}{\lambda}\frac{\frac{\sigma_w^2}{1-\rho^2} - \frac{\rho^2\kappa_1\cdot\frac{\sigma_w^2}{1-\rho^2}}{1-(1-\kappa_1)\rho^2}}{\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2} - (1-\kappa_1) \\
&= \frac{1-\lambda}{\lambda}\frac{\frac{\sigma_w^2}{1-\rho^2}\frac{1-\rho^2}{1-(1-G)\rho^2}}{\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2} - (1-\kappa_1) \\
&= \frac{1-\lambda}{\lambda}\frac{\mathrm{Var}(x_t - x^i_{t|t-1})}{\mathrm{Var}(x_t - x^i_{t|t-1}) + \sigma_v^2} - (1-\kappa_1) \\
&= \frac{1-\lambda}{\lambda}\kappa_1 - (1-\kappa_1) \\
&= \frac{1-\kappa_1 + R(1-\kappa_2)}{\kappa_1 + R\kappa_2}\kappa_1 - (1-\kappa_1) \\
&= \frac{R(\kappa_1-\kappa_2)}{\kappa_1 + R\kappa_2}
\end{aligned}$$

A.4 Equating β_1 Across Models

DE → SI

$\beta_1^{DE} = \frac{-\theta(1+\theta)}{(1+\theta)^2+\rho^2\theta^2}$ and $\beta_1^{SI} = \frac{R(\kappa_1-\kappa_2)}{\kappa_1+R\kappa_2}$.

Given $\{\rho, \sigma_w, \sigma_v, \theta\}$ we solve for R by setting $\beta_1^{DE} = \beta_1^{SI}$

$$\begin{aligned}
\frac{R(\kappa_1-\kappa_2)}{\kappa_1 + R\kappa_2} &= \beta_1^{DE} \\
R(\kappa_1-\kappa_2) &= \beta_1^{DE}(\kappa_1 + R\kappa_2) \\
R(\kappa_1-\kappa_2) &= \beta_1^{DE}\kappa_1 + \beta_1^{DE}R\kappa_2 \\
R(\kappa_1-\kappa_2) - \beta_1^{DE}R\kappa_2 &= \beta_1^{DE}\kappa_1 \\
R &= \frac{\beta_1^{DE}\kappa_1}{\kappa_1 - (1+\beta_1^{DE})\kappa_2}
\end{aligned}$$

SI→DE

Given $\{\rho, \sigma_w, \sigma_v, R\}$ we solve for θ by setting $\beta_1^{DE} = \beta_1^{SI}$

$$
\begin{aligned}
\frac{-\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2} &= \beta_1^{SI} \\
-\theta(1+\theta) &= \beta_1^{SI}(1+\theta)^2 + \beta_1^{SI}\rho^2\theta^2 \\
-\theta - \theta^2 &= \beta_1^{SI}(1 + 2\theta + \theta^2) + \beta_1^{SI}\rho^2\theta^2 \\
0 &= \beta_1^{SI} + 2\beta_1^{SI}\theta + \theta + \beta_1^{SI}\theta^2 + \theta^2 + \beta_1^{SI}\rho^2\theta^2 \\
0 &= \beta_1^{SI} + (2\beta_1^{SI} + 1)\theta + [1 + (1+\rho^2)\beta_1^{SS}]\theta^2
\end{aligned}
$$

OC→DE

Given $\{\rho, \sigma_w, \sigma_v, \theta\}$, we solve for α by setting $\beta_1^{DE} = \beta_1^{OC}$

$$
\begin{aligned}
\frac{(\alpha^2 - 1)\sigma_v^2}{\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2} &= \beta_1^{DE} \\
\alpha^2 &= \frac{\beta_1^{DE}}{\sigma_v^2}[\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) + \sigma_v^2] + 1 \\
\alpha^2 - \frac{\beta_1^{DE}}{\sigma_v^2}\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) &= \beta_1^{DE} + 1
\end{aligned}
$$

and solving for the α parameter for which the above equality holds. Note that due to the recursive nature of the OC model, $\mathbb{V}(x_t - \breve{x}^i_{t|t-1}) \equiv \breve{\Psi}^i_{t|t-1}(\alpha)$ is itself a function of α.

A.5 Proof of Proposition 3

It suffices to show that $\text{Cov}(\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1}, \widetilde{x}^i_{t|t-1} - \widetilde{x}^i_{t|t-2}) = 0$. The current period revision is

$$
\begin{aligned}
\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1} &= \frac{1 - \lambda - 1 - R\lambda}{1+R} x^i_{t|t-1} + \lambda y_{it} \\
&= \lambda(y_{it} - x^i_{t|t-1})
\end{aligned}
$$

and the previous period revision can be expressed as

$$
\begin{aligned}
\widetilde{x}^i_{t|t-1} - \widetilde{x}^i_{t|t-2} &= \frac{1}{1+R}(x^i_{t|t-1} - x^i_{t|t-2}) + \frac{R}{1+R}(F^i_{t|t-1} - F^i_{t|t-2}) \\
&= \frac{\rho}{1+R}(x^i_{t-1|t-1} - x^i_{t-1|t-2}) + \frac{R}{1+R}\big[\lambda\rho x^i_{t-1|t-1} + (1-\lambda)\rho F^i_{t-1|t-1} - \lambda\rho x^i_{t-1|t-2} \\
&- (1-\lambda)\rho F^i_{t-1|t-2}\big] \\
&= \frac{\rho}{1+R}\big[\kappa_1(y_{it-1} - x^i_{t-1|t-2})\big] + \frac{R}{1+R}\big[\lambda\rho(x^i_{t-1|t-1} - x^i_{t-1|t-2}) + (1-\lambda)\rho(F^i_{t-1|t-1} - F^i_{t-1|t-2})\big] \\
&= \frac{\rho\kappa_1}{1+R}\big[x_{t-1} - x^i_{t-1|t-2} + v_{it-1}\big] + \frac{R}{1+R}\big[\lambda\rho\kappa_1(x_{t-1} - x^i_{t-1|t-2} + v_{it-1}) \\
&+ (1-\lambda)\rho\kappa_2(x_{t-1} - x^i_{t-1|t-2} + v_{it-1})\big] \\
&= \left[\frac{\rho\kappa_1 + R\lambda\rho\kappa_1 + R(1-\lambda)\rho\kappa_2}{1+R}\right](x_{t-1} - x^i_{t-1|t-2} + v_{it-1}) \\
&= \frac{\rho(\kappa_1 + R\kappa_2) + \rho R\lambda(\kappa_1 - \kappa_2)}{1+R}(x_{t-1} - x^i_{t-1|t-2} + v_{it-1}) \\
&= \rho\lambda\left(1 + \frac{R(\kappa_1 - \kappa_2)}{1+R}\right)(x_{t-1} - x^i_{t-1|t-2} + v_{it-1})
\end{aligned}
$$

Then,

$$
\begin{aligned}
\mathrm{Cov}(\widetilde{x}^i_{t|t} - \widetilde{x}^i_{t|t-1}, \widetilde{x}^i_{t|t-1} - \widetilde{x}^i_{t|t-2}) &= \mathrm{Cov}\left[\lambda(y_{it} - x^i_{t|t-1}), \rho\lambda\left(1 + \frac{R(\kappa_1 - \kappa_2)}{1+R}\right)(x_{t-1} - x^i_{t-1|t-2} + v_{it-1})\right] \\
&= \mathrm{Cov}\Big[\lambda(1-\kappa_1)\rho(x_{t-1} - x^i_{t-1|t-2}) - \lambda\kappa_1\rho v_{it-1} + \lambda(v_{it} + w_t), \\
&\rho\lambda\left(1 + \frac{R(\kappa_1 - \kappa_2)}{1+R}\right)(x_{t-1} - x^i_{t-1|t-2} + v_{it-1})\Big] \\
&= (\lambda\rho)^2\left[1 + \frac{R(\kappa_1 - \kappa_2)}{1+R}\right]\left[(1-\kappa_1)\Psi - \kappa_1\sigma_v^2\right] \\
&= 0
\end{aligned}
$$

By definition, $\kappa_1 = \frac{\Psi}{\Psi + \sigma_v^2}$ where Ψ is the steady state forecast error variance (i.e. the variance that solves the Ricatti equation: $\Psi = (1-\kappa_1)\rho^2\Psi + \sigma_w^2$). From this it follows that the last term in hard brackets is zero.

Appendix B Empirics

Variable	Mnemonic	β_1
Consumer price index	CPI	-0.205***
		(0.067)
Employment	EMP	-0.083
		(0.197)
Housing starts	HOUSING	0.093**
		(0.041)
Industrial production	IP	-0.212***
		(0.045)
Nominal GDP	NGDP	0.113***
		(0.027)
GDP Deflator	PGDP	-0.302***
		(0.045)
Real consumption	RCONSUM	-0.303***
		(0.065)
Real federal government spending	RFEDGOV	-0.240***
		(0.060)
Real GDP	RGDP	-0.169***
		(0.038)
Real nonresidential investment	RNRESIN	-0.180***
		(0.056)
Real residential investment	RRESINV	-0.152**
		(0.074)
Real state/local government spending	RSLGOV	-0.236***
		(0.058)
3-month Treasury bill	TBILL	0.056
		(0.071)
10-year Treasury bond	TBOND	-0.004
		(0.045)
Unemployment rate	UNEMP	0.167**
		(0.068)

* p<0.10, ** p<0.05, *** p<0.01

Table B1: Pooled Revision Persistence Regressions at $h = 0$, by Variable

Note: The table reports the OLS coefficients from revision persistence regressions across 15 macroeconomic variables reported in the Survey of Professional Forecaters. Column (3) reports the coefficient in front of the revision at the forecaster-level. The revisions are for current period forecasts ($h = 0$).

Appendix C Simulation Results

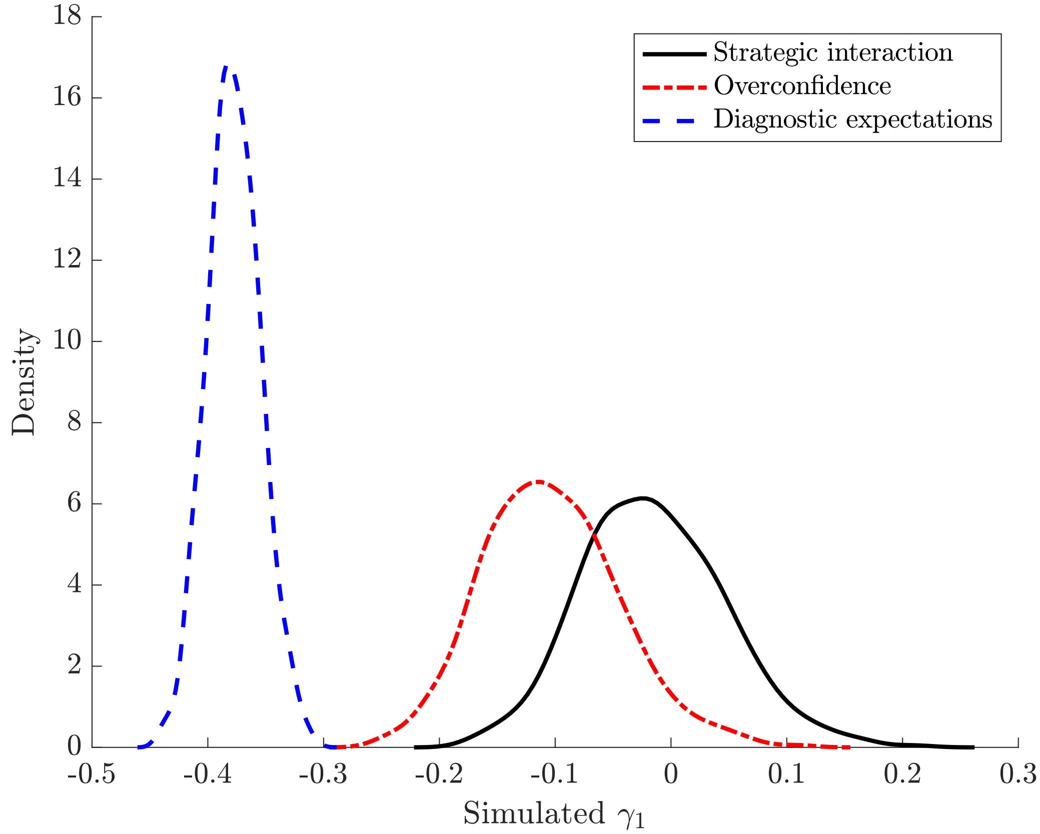

Figure C1: Mapping Over-reactions to Strategic Substitutability

Note: The figure plots the simulated densities of the various γ_1 coefficients across the three models described in the main text.