

Modelo Linear Geral

$$Y = X\beta + \epsilon, \quad \epsilon \sim N(0, \sigma^2 I)$$

$$\text{EMV de } \beta \text{ é } \hat{\beta} = (X^T X)^{-1} X^T Y$$

$$\hat{Y} = X\hat{\beta} = \underbrace{X(X^T X)^{-1} X^T}_{H = \text{hat}} Y$$

$$\hat{\beta} = \underbrace{(X^T X)^{-1} X^T}_{\substack{\text{n\~ao} \\ \text{aleat\~orio}}} Y \sim N\left(\underbrace{(X^T X)^{-1} X^T X}_{I} \beta, (X^T X)^{-1} \sigma^2\right)$$

\uparrow
 $Y \sim N(X\beta, \sigma^2 I)$

$$E(\hat{\beta}) = E\left(\underbrace{(X^T X)^{-1} X^T}_{\text{}} Y\right) = (X^T X)^{-1} X^T E(Y) = \\ = (X^T X)^{-1} X^T X \beta = \beta$$

$$\text{Var}(\hat{\beta}) = (X^T X)^{-1} X^T \text{Var}(Y) X (X^T X)^{-1} = \\ = (X^T X)^{-1} X^T \sigma^2 I X (X^T X)^{-1} = \sigma^2 (X^T X)^{-1}.$$

$$\hat{\beta} \sim N(\beta, (X^T X)^{-1} \sigma^2).$$