Question 1

Consider two scenarios for a consumer planning consumption with income process $y_t = y_0\delta^t, \quad \forall t \geq 0$ and $F_0 = 0$.

Scenario 1 for Consumer The consumer maximizes the following welfare

$$U \equiv \max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{1}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{2}$$

$$\text{(transversality condition)} \tag{3}$$

Scenario 2 for Consumer The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of y_0 is potentially different (define it as y_0^{NB})

Define the PDV of utility (i.e., the welfare) of this as U^{NB}.

(a) Let $\beta = 0.95, R = 1.04, \delta = 1.02, y_0 = 1$, and $y_0^{NB} = 1$. Calculate U and U_{NB}.

(b) Let $y_0 = 1$. Now find a y_0^{NB} such that $U = U^{NB}$. The difference between y_0 and y_0^{NB} is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

(c) Maintain $y_0 = 1$. Now, let $\beta = .99, R = 1.04$, and $\delta = 1.01$. What is c_0 and F_1 here under Scenario 1? Repeat part (b) to find y_0^{NB} such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

Question 2

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$\max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{5}$$

$$F_{t+1} \geq -B \tag{6}$$

$$F_0 = 0 \tag{7}$$

$$\text{(transversality condition)} \tag{8}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1.[1] Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[1]Note that this equation deliberately is avoiding any t subscripts! This makes it a truly recursive expression.

Question 3

Consider a markov chain with two states: U for unemployment and E for employment.

- With probability $\lambda \in (0, 1)$, a person unemployed today becomes employed tomorrow.
- With probability $\alpha \in (0, 1)$, a person employed today becomes unemployed tomorrow.

(a) Let $N \geq 1$ be the number of periods until a currently unemployed person becomes employed. Calculate $\mathbb{E}[N]$.

(b) Let $M \geq 1$ be the number of periods until a currently employed person becomes unemployed. Calculate $\mathbb{E}[M]$.

(c) Please compute the fraction of time an infinitely lived person can expect to be unemployed and the fraction of time they can expect to be employed.

Question 4

An economy has 3 states for workers:

- U: unemployment.
- V: if they have found a potential employer and are being verified to see if they are a good fit.
- E: if a worker has been verified and is employed.

The probabilities that they jump between these states each period is:

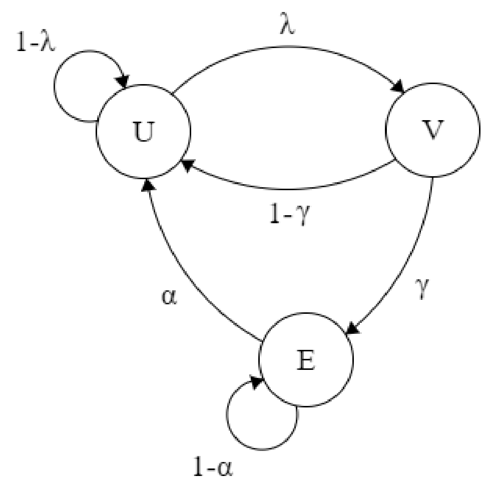

Figure 1: Markov Chain

i.e. probability γ they are a good fit, and the verification takes 1 period.

(a) Write a Markov transition matrix for this process, P.

(b) Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of P).

(c) If a worker is U today, write an expression for the probability they will be employed exactly j periods in the future (considering any possible transitions which end in employment at j periods).[2].

(d) Assume that $\alpha = 0, \lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist and the intuition from the perspective of the Markov chain.

[2]Note: This is only looking at j periods into the future. i.e. this is **not** the probability that they become at employed at least once during the j periods, which is a much more difficult calculation.