Question 1

Stochastic Interest Rates

Consider an economy where aggregate output is either Y_L or Y_H, where $0 < Y_L < Y_H$. This output follows where the probability that $Y = Y_L$ is $0 < \pi < 1$ and the probability that $Y = Y_H$ is $1-\pi$, independently over time. Simplifying to only consider 2 consecutive periods, t and $t+1$, a representative consumer has preferences over consumption streams ordered by

$$u(c_t) + \beta\mathbb{E}\left[u(c_{t+1})\right], \quad 0 < \beta < 1, \text{ and standard } u'(c) > 0, u''(c) < 0, u'(0) = \infty$$

where $\beta \equiv \frac{1}{1+\rho}$. As this is a representative agent, the resource constraint is $c_{t+1} = Y_L$ or $c_{t+1} = Y_H$, depending on the realization of output. c_t could be either Y_L or Y_H, but is known at time t. The representative agent is a *price taker* with access to *complete markets* for trading state and time contingent assets,

- Let the price of an asset delivering 1 unit of consumption at time t consumption to be q (i.e., this would be q_t^t in the infinite horizon notation)
- Let the price of a state contingent asset that delivers 1 unit of consumption at time $t+1$ in state L as q_L. This delivers nothing if the state is H. (In infinite horizon notation, this might be $q_{t+1}^t(Y_{t+1} = L)$).
- Similarly, define q_H to be the price of a state contingent asset that delivers 1 unit of consumption at time $t+1$ in state H, and nothing in state L.

(a) Precisely define a feasible allocation and a price system for this economy.

(b) Given the representative agent has a known income of $Y_t \in \{Y_L, Y_H\}$ and a potential income at time $t+1$ of either Y_L or Y_H, define the consumer's budget constraint for trading the complete set of assets at time t to finance consumption c_t, $c_{t+1}(L)$, and $c_{t+1}(H)$.

(c) Write down the consumer's problem, and a precisely define a competitive equilibrium for this economy with the representative consumer.

(d) Solve the consumer's problem directly (i.e. take first order conditions given some Lagrange multiplier on the budget constraint) for the optimal decision rules, and use the feasibility constraint to find the prices q, q_L, and q_H. Normalize the price level to $q = 1$. Do the other prices depend on whether Y_t is currently Y_L or Y_H? Why?

(e) What is the risk free price of an asset which delivers a unit of consumption at time $t+1$ with certainty (i.e., regardless of whether Y_L or Y_H is realized)? Does this price depend on whether Y_t is currently Y_L or Y_H? Interpret the equation in terms of income smoothing incentives.[1]

[1]Hint: compare the current marginal utility, $u'(Y_t)$ to the expected marginal utility tomorrow, $\mathbb{E}\left[u'(Y_{t+1})\right]$ and consider how that interacts with smoothing incentives inherent implied by the $u(\cdot)$ function.

Question 2

(Transitions and Government Objective Functions)

Consider a standard setup of the neoclassical growth model in a competitive equilibrium: A representative consumer orders its welfare by[2]

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

where $0 < \beta < 1$. Or $\beta \equiv \frac{1}{1+\rho}$ for $\rho > 0$. The technology in the economy is,

$$y_t = f(k_t) = z k_t^{\alpha}$$

for $0 < \alpha < 1$ and $z > 0$. Labor of mass 1 is supplied inelastically.

Given exogenous government expenditures of real goods, g_t, the feasibility condition is

$$c_t + k_{t+1} + g_t \leq y_t + (1 - \delta) k_t$$

In a competitive equilibrium, the government will finance g_t through taxes on capital or lump-sump taxes, $\{\tau_{kt}, \tau_{ht}\}$. Negative taxes are subsidies.

(a) Find the steady state level of capital and consumption $\left\{\bar{k}, \bar{c}\right\}$ if $g_t = \tau_{kt} = \tau_{ht} = 0$.

(b) Now, assume that while the government will still have $g_t = 0$, they can choose a constant tax ($\bar{\tau}_k > 0$) or subsidize ($\bar{\tau}_k < 0$) the return to capital faced by the consumer. Since they have no need for expenditures, then if $\bar{\tau}_k > 0$ the government simply rebates the revenues to consumers as a lump-sum subsidy ($\bar{\tau}_h < 0$). Similarly, to pay for a capital subsidy the government sets a lump-sum tax. Find the steady state $\left\{\bar{k}, \bar{c}\right\}$ for a given $\bar{\tau}_k$ tax (or subsidy).[3]

(c) The objective of government (A) is to maximize steady state consumption per capita by choosing the $\bar{\tau}_k$. Formulate this as an optimal problem for the government, and solve for its optimal $\bar{\tau}_k$ policy and the corresponding steady state $\left\{\bar{c}, \bar{k}\right\}$. What is the sign of $\bar{\tau}_k$, and why?

(d) Now, a new government (B) comes to power with the objective of maximizing consumer welfare (i.e. our usual objective) by choosing a constant $\bar{\tau}_k$. Find the optimal $\bar{\tau}_k$ policy and the corresponding steady state $\left\{\bar{c}, \bar{k}\right\}$. What is the sign of $\bar{\tau}_k$, and why?

(e) Assuming that government (A) was in power for a long-time and the economy was in a steady state. The new government (B) is elected with no anticipation, and associated new tax policy is immediately changed to the optimal value forever. Draw the dynamics of $\{k_t, c_t\}_{t=0}^{\infty}$ as the economy evolves from the initial steady state of government (A) to the new steady state of government (B).

(f) Compare the steady states of the two governments to discuss whether $\bar{\tau}_k$ was set too high or too low in government (A).[4]

---

[2] Let c_t, k_t, y_t, and g_t be in per-capita terms.

[3] Hint: $\bar{\tau}_h$ adjusts to balance the government's budget and is non-distorting.

[4] Be explicit on what criteria one should use to make this judgment.