Infinite Sums

$$\sum_{j=0}^{\infty} a^j = \frac{1}{1-a}, \quad \sum_{j=0}^{T} a^j = \frac{1-a^{T+1}}{1-a}$$

$$\sum_{j=1}^{\infty} j\, a^{j-1} = \frac{1}{(1-a)^2}$$

Linear State Space Model (LSS)

$$x_{t+1} = A\, x_t$$

$$y_t = G\, x_t$$

LSS PDV

$$\sum_{j=0}^{\infty} \beta^j y_{t+j} = G\,(I - \beta A)^{-1} x_t$$

Permanent Income Model (PIM)

$$\max_{\{c_t,\, F_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t u(c_t),$$

$$\text{s.t.} \quad F_{t+1} = R\Big(F_t + y_t - c_t\Big), \quad t = 0, \ldots$$

PIM Lifetime Budget Constraint

$$\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t+j} = F_t + \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j y_{t+j}$$

PIM Euler Equation

$$u'(c_t) = \beta R\, u'(c_{t+1})$$

PIM Solution with $\beta R = 1$

$$\bar{c} = (1-\beta)\Big[\sum_{j=0}^{\infty} \beta^j y_{t+j} + F_t\Big]$$

PIM with $F_{t+1} \geq 0$ Euler Equation

$$\begin{aligned} &u'(c_t) = \beta R\, u'(c_{t+1}) \quad \text{or} \\ &u'(c_t) > \beta R\, u'(c_{t+1}) \quad \text{and } c_t = F_t + y_t \end{aligned}$$

Expected Value for Discrete RV

$$\mathbb{E}\left[Y\right] = \sum_{n=1}^{N} \mathbb{P}\Big(Y = y_n\Big) y_n = \sum_{n=1}^{N} \pi_n y_n = \pi \cdot y$$

Evolution of PMF for Markov Chain

$$\pi_{t+j} = \pi_t\, P^j, \quad \text{with } P_{ij} = \mathbb{P}\left[Y_{t+1} = j \mid Y_t = i\right]$$

Asset Pricing with Markov Chain

$$p_t(x_t) = \mathbb{E}_t\Big[\sum_{j=0}^{\infty} \beta^j Y_{t+j}\Big] = G\Big(I - \beta P^{\top}\Big)^{-1} x_t,$$

$$\text{where } x_t = \pi_t^{\top}, \quad G = \begin{bmatrix} y_1 & \cdots & y_N \end{bmatrix}.$$

Normal Random Variable Linearity

If $z \sim N(\mu, \sigma^2)$, then $z \sim \mu + \sigma w, \quad w \sim N(0,1)$.

Linear Gaussian State Space (LGSS)

$$\begin{aligned} x_{t+1} &= A\, x_t + C\, w_{t+1}, \quad w_{t+1} \sim N(0, I), \\ y_t &= G\, x_t. \end{aligned}$$

LGSS Forecasting

$$\mathbb{E}_t\left[x_{t+j}\right] = A^j x_t,$$

$$\mathbb{E}_t\left[y_{t+j}\right] = G\, A^j x_t,$$

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j y_{t+j}\right] = G\,(I - \beta A)^{-1} x_t.$$

Stochastic PIM (SPIM) Euler Equation

$$u'(c_t) = \beta R\, \mathbb{E}_t\Big[u'(c_{t+1})\Big]$$

SPIM with $\beta R = 1$

$$c_t \cong (1-\beta)\Big(F_t + \mathbb{E}_t\Big[\sum_{j=0}^{\infty} \beta^j y_{t+j}\Big]\Big)$$

SPIM with $\beta R = 1$

$$c_{t+1} - c_t \cong (1-\beta) \sum_{j=0}^{\infty} \beta^j \Big(\mathbb{E}_{t+1}\left[y_{t+j+1}\right] - \mathbb{E}_t\left[y_{t+j+1}\right]\Big)$$

SPIM with $\beta R = 1$ and LGSS

$$c_{t+1} - c_t \cong (1-\beta)\, G\,(I - \beta A)^{-1} C\, w_{t+1}.$$