

Question 1

Let $y_t \in \mathbb{R}$ be a sequence of dividends such that

$$y_{t+1} = y_t + w_{t+1}$$

where $w_{t+1} \sim N(0, \sigma^2)$ for some $\sigma > 0$. i.e. $\mathbb{E}_t[w_{t+1}] = 0$ and $\mathbb{E}_t[w_{t+1}^2] = \sigma^2$. An agent prices an asset as the expected PDV of dividends,

$$p_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j y_{t+j} \right]$$

Define the forecast error as the deviation in the expected price tomorrow calculated at time t from the actual price tomorrow. i.e.,

$$FE_{t+1|t} \equiv p_{t+1} - \mathbb{E}_t[p_{t+1}]$$

- (a) Setup in our canonical Linear Gaussian State Space model.
- (b) Solve for p_t in terms of y_t and model intrinsics.
- (c) Find the expected forecast error: $\mathbb{E}_t[FE_{t+1|t}] = \mathbb{E}_t[p_{t+1} - \mathbb{E}_t[p_{t+1}]]$
- (d) Find the variance of forecast errors:

$$\begin{aligned} \mathbb{V}_t(FE_{t+1|t}) &\equiv \mathbb{E}_t[FE_{t+1|t}^2] - (\mathbb{E}_t[FE_{t+1|t}])^2 \\ &= \mathbb{E}_t[(p_{t+1} - \mathbb{E}_t[p_{t+1}])^2] - (\mathbb{E}_t[p_{t+1} - \mathbb{E}_t[p_{t+1}]])^2 \end{aligned}$$

- (e) Setup the problem recursively as p_t define in terms of p_{t+1} . Test, using your solution from early in the question, if the stochastic process p_t a martingale (i.e., just a random walk in this case)? Recall that a martingale is a process such that $\mathbb{E}_t[p_{t+1}] = p_t$. Give any intuition you can on this result.

Question 2

Consider a variation of the example in class where the government makes a surprise announces that it will borrow money, give the money to the consumers as a “stimulus”, and eventually pay it back through taxing consumers (i.e., no chance of a government default).

The agent’s income is $y_t = \delta^t$ for $\delta > 1$, and assume that: $\beta R = 1$, $u'(c) > 0$, and $u''(c) < 0$. Furthermore, assume that the consumer’s and government face the same interest rate $R > 0$. Let $F_0 = 0$.

The government makes there announcement between time 0 and time 1 (i.e., after the consumer has already chosen c_0 and F_1 thinking that their income will follow y_t). The precise announcement is that at time 1, the consumer’s are given $\alpha > 0$ as *extra* income as a stimulus (thereby increasing their y_1 from what they had previously anticipated). That is income is now,

$$y_1 = \delta + \alpha$$

Instead of paying back deterministically, the government will pay back the loan at period k (which will be stochastic). To pay the loan, the government taxes the total value of the loan + interest. For example, if they paid it off in period k , then the labor income of a consumer at period k would be

$$y_k = \delta^k - \alpha R^{k-1}$$

Otherwise, the consumer’s income follows the same y_t process. While the consumer doesn’t know exactly when the loan will be repaid, they know the correct distribution of payment dates upon the announcement 1:

$$\mathbb{P}(\text{pay at } k) = p(k) \geq 0$$

where $\sum_{k=2}^{\infty} p(k) = 1$.

- (a) First, assuming the standard permanent income model, calculate the optimal sequence $\{c_t\}_{t=0}^{\infty}$ at $t = 0$, before the government announces the policy. Note that at this point, the consumer believes they have a deterministic income stream and that the government is not going to borrow or tax.
- (b) After the surprise announcement, what is the new optimal path of consumption chosen?¹ What is $c_1 - c_0$? Interpret the effect of the stimulus on consumption.
- (c) Does it matter if the consumer knows the true distribution of payment dates, or the timing of the taxes to pay for the loan?

¹Hint: at time 1 the consumer’s income is now stochastic, but it is very linear and simple.

Question 3

A consumer's optimal decision rule for consumption satisfies

$$c_t = (1 - \beta) \left[F_t + E_t \sum_{j=0}^{\infty} \beta^j y_{t+j} \right] \quad (1)$$

where c_t is consumption, β^{-1} is the gross one-period interest rate (i.e., $\beta R = 1$), which is constant over time, y_t is the consumer's income at time t , F_t is the consumer's financial assets at the beginning of t , and $E_t(\cdot)$ means the best forecast of (\cdot) (whatever (\cdot) is), conditional on information that the consumer knows at t . At time t , assume that the consumer knows current and past values of y_t 's, but not future values. The consumer's labor income follows the random process

$$y_{t+1} = \delta_0 + \delta_1 y_t + \delta_2 y_{t-1} + \sigma \epsilon_{t+1}$$

where $\{\epsilon_{t+1}\}_{t=0}^{\infty}$ is an independently and identically distributed (iid) sequence of scalar normally distributed scalar random variables, each with mean 0 and variance 1. (Please make whatever assumptions you want about δ_1 and δ_2 in order to make the subsequent questions meaningful.)

- (a) Given available information at time t , give an expression for the consumer's expected income j periods into the future, $E_t y_{t+j}$
- (b) Find an expression for the consumer's decision rule of the form

$$c_t = (1 - \beta) \left[F_t + \alpha_0 + \alpha_1 y_t + \alpha_2 y_{t-1} \right].$$

Please describe how to find formulas for $\alpha_0, \alpha_1, \alpha_2$.

- (c) Measured in constant 2015 dollars, the changes in consumption for this consumer over the last year (which started out better than it ended) were as follows:

quarter	$c_t - c_{t-1}$
2018I	1000
2018II	0
2018III	0
2018IV	-4000

What can you infer from these consumption change numbers, if anything, about the consumer's past, present, and future labor income? Can you interpret these in the context of "surprise"?