Question 1

Take the search model we did in class with an endogenous choice of accepting a job, but add in the following elements:

- If you are unemployed and reject a wage, there is only a $\theta \in (0,1)$ probability to get a wage offer. Otherwise, you can recall a wage you previously rejected last period. (hint: they would never choose to recall a wage in equilibrium, but it may help you write down the Bellman equation cleanly.)
- There is an exogenous probability $\alpha \in (0,1)$ of being fired at the end of any period you are working. You then draw a new wage as an unemployed agent entering the next period with certainty (i.e., bypass the θ probability going into the next period.

To summarize the timing here: As in our example in class, let $v(w)$ be the value of coming a period unemployed with wage offer w and when they are about to choose to accept or reject.

If they reject an offer, they gain unemployment insurance c, and have the probability θ to gain the draw with expected value

$$Q = \int_0^B v(\hat{w}) f(\hat{w}) \mathrm{d}\hat{w}$$

If they accept they gain the wage w that period, and then have the α chance of being fired as they come into the next period—at which point they get the wage offer draw with certainty, as discussed. (Hint: if they are not fired, the value next period is $v(w)$, the same as if they were first offered w.)

(a) Draw a Markov chain with two states E and U. Let the probability of staying unemployed be λ which will end up endogenous. You will also need the α transition probability

(b) Write the value of a worker with wage offer w who chooses to reject the offer (hint: if they reject they don't necessarily gain a new draw, but could have $v(w)$ as their value next period since they can recall the rejected w).

(c) Write the value of a worker accepting the offer of w. (hint: may need to be recursive now, unlike what we did in class)

(d) Combine the values in the previous two parts to form a Bellman equation with $v(w)$ and the max for the choice.

(e) Write the equation for an indifference point $\bar{w}$, where they are at the threshold of accepting (or rejecting) the wage.[1]

(f) Assuming that you could numerically solve the previous equation to find a $\bar{w}$, what is the expression for the stationary proportion of unemployed workers as a function of $\alpha, \theta, f(\cdot)$, and $\bar{w}$. (Hint: derive the λ, then use older notes on unemployment. However, recall that the $U \to E$ transition only occurs if both a wage offer and endogenous acceptance occur).

[1] As in the case of class, the Bellman equation could be reorganized to eliminate the $v(\cdot)$ function to give an implicit equation in $\bar{w}$ and parameters. This is significantly trickier than what we did, so only try to simplify the indifference equation if you wish.

Question 2

Each period, a previously unemployed worker draws two offers to work forever at wage w from the cumulative distribution function ('cdf')

$$F(w) = \Big(w/B\Big)^{\frac{1}{2}}, \quad 0 \leq w \leq B$$

where $F(w) = \text{Prob}(\tilde{w} \leq w)$ where $\tilde{w}$ is a particular wage offer. Successive draws within a period and across periods are identically and independently distributed. The unemployed worker is free to inspect both offers in a period and, if he or she wants, accept the highest among offers he or she has drawn that period. Offers from past periods cannot be recalled. The offers are to work at the accepted wage forever. There is no option to quit after an offer has been accepted, and there is no prospect of being fired. The worker wants to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t y_t\right], \quad 0 < \beta < 1,$$

where

$$y_t = \begin{cases} w & \text{if employed at wage } w, \\ c & \text{if unemployed} \end{cases}$$

where w is the wage, c is unemployment compensation, and $\mathbb{E}[\ldots]$ is an expected value before the offers are drawn.

(a) Verify that $F(w) = \Big(w/B\Big)^{\frac{1}{2}}$ is a legitimate cdf. Find the cdf for the maximum of 2 draws (i.e., find the cdf of $z \equiv \max\{z_1, z_2\} \in \tilde{F}(z)$) and verify it is a cdf.

(b) Find the worker's optimal search strategy and show that it has a 'reservation wage' form.[2] Draw the value function for a worker given a particular w.

(c) Let $\bar{w}$ be the reservation wage. Find a formula for the reservation wage as a function of B, β, c.[3]

(d) Given this $\bar{w}$, find a formula for ψ = probability that an unemployed worker leaves unemployment this period as a function of $\bar{w}, B, \beta, c$ (eliminating $\bar{w}$ if you found a closed form solution in part c).

[2]Hints: (1) setup the model recursively in a way isomorphic to the model from class, (2) For any n and independent draws, $z_1, \ldots, z_n$ from cdf $F(z)$, the cdf of the maximum of these is $z \equiv \max\{z_1, \ldots z_n\} \sim F(z)^n$

[3]Hint: While you could solve for $\bar{w}$ directly, feel free to leave it in an *implicit* form if you are finding the algebra difficult. However, you should be able to eliminate any recursive value functions

Question 3

A household supplies 1 unit of labor inelastically (i.e., doesn't value leisure) and consumes two types of goods in quantities: c_1 (e.g., apples) and c_2 (e.g., oranges).[4] The preferences are

$$u(c_1, c_2) = a c_1^{\alpha} c_2^{1-\alpha}$$

with $\alpha \in (0,1)$ and $a > 0$.

The production technology uses labor (ℓ) as its only input and can produce good 1 or good 2 with separate constant returns to scale production functions: $y_1 = z_1 \ell_1$ and $y_2 = z_2 \ell_2$. with productivities $z_1 > 0$ and $z_2 > 0$.

The total labor allocated to producing each good cannot add to more than the total labor endowment: $\ell_1 + \ell_2 = 1$ (i.e., holds at equality due to inelastic supply).

(a) Carefully define a feasible allocation in this economy

(b) Formulate the planner's problem. (hint: how many constraints and choice variables are there compared to the examples in class?)

(c) Provide a system of equations which would solve the planning problem, and solve for the allocation.

Now, assume that firms operate the production technology, and that both firms and consumers are price takers for the market prices of labor and goods. Let q_1 and q_2 be the prices of the 2 goods. Let w be the wage of the consumer per unit of labor.

(d) What is a price system for this economy?

(e) What is the consumer's budget constraint? What are the choice variables of the consumer? (Hint: careful on what they are not allowed to choose). Write down the consumer's problem.

(f) What are the profits of firm 1 with output y_1? What are the choice variables of the firm? (Hint: again, careful on what they are not allowed to choose). Write down the full profit maximization problem of firm type 1. Repeat for firm type 2.

(g) Carefully define a competitive equilibrium for this economy.

(h) Solve for the competitive equilibrium. Does this decentralize the planner's problem?

[4] Hint: Equations combining "apples" and "oranges" directly without any prices may be correct, but they may not be useful as they are physically district objects.

Question 4

A price taking consumer has an exogenous endowment $\{y_t\}_{t=0}^{\infty}$. They choose consumption to maximize their welfare given a discount rate $\beta \in (0,1)$, and a concave $u(\cdot)$.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$\text{s.t.} \sum_{t=0}^{\infty} q_t^0 c_t \leq \sum_{t=0}^{\infty} q_t^0 y_t \tag{2}$$

where $\{q_t\}_{t=0}^{\infty}$ are the price of one unit of consumption good delivered at time t measured in units of time 0 consumption (i.e., use a $q_0^0 = 1$ normalization of the price level here).

Assume the utility has the form,

$$u(c) = \begin{cases} \frac{1}{1-\gamma} c^{1-\gamma} & \text{if } \gamma > 0, \gamma \neq 1 \\ \log c & \text{if } \gamma = 1 \end{cases}$$

Note: you will note that the marginal utility, $u'(c) = c^{-\gamma}$ holds for all $\gamma > 0$, including the special log case. This means you will not need to treat it separately.

Assume there is a large number of identical agents in the economy, all with identical processes $y_t = y_0 \delta^t$ for $0 < \delta < 1/\beta$.

Finally, recall that if $q_0^0 = 1$, then r_{0t} is the "yield to maturity on a t-period zero-coupon bond purchased at time 0" through,

$$\frac{q_t^0}{q_0^0} \equiv \frac{1}{(1 + r_{0t})^t}$$

(a) What is the feasibility condition in the economy (i.e. relate c_t and y_t)? (hint: can use a representative agent with a large number of price taking agents).

(b) Solve for q_t^0 in this model, explaining why q_0^0 can be chosen for convenience. Then use this to find r_{0t} from the definition above.

(c) In the special case of $\gamma = 1$, compute q_t^0 and r_{0t}. Compute the special case of $\gamma = 1$ and $\delta = 1$.

(d) Interpret r_{01} if $\gamma = 1$ for the $\delta > 1$ and $\delta < 1$ cases

(e) Interpret r_{01} for $\gamma > 0$ and $\delta = 1$. In particular, discuss any reliance on γ.