Question 1

(GE with 2 goods)

A household supplies 1 unit of labor inelastically (i.e., doesn't value leisure) and consumes two types of goods in quantities: c_1 (e.g., apples) and c_2 (e.g., oranges).[1] The preferences are

$$u(c_1, c_2) = ac_1^{\alpha} c_2^{1-\alpha}$$

with $\alpha \in (0, 1)$ and $a > 0$.

The production technology uses labor (ℓ) as its only input and can produce good 1 or good 2 with separate constant returns to scale production functions: $y_1 = z_1\ell_1$ and $y_2 = z_2\ell_2$. with productivities $z_1 > 0$ and $z_2 > 0$.

The total labor allocated to producing each good cannot add to more than the total labor endowment: $\ell_1 + \ell_2 = 1$ (i.e., holds at equality due to inelastic supply).

(a) Carefully define a feasible allocation in this economy

(b) Formulate the planner's problem. (hint: how many constraints and choice variables are there compared to the examples in class?)

(c) Provide a system of equations which would solve the planning problem, and solve for the allocation.

Now, assume that firms operate the production technology, and that both firms and consumers are price takers for the market prices of labor and goods. Let q_1 and q_2 be the prices of the 2 goods. Let w be the wage of the consumer per unit of labor.

(d) What is a price system for this economy?

(e) What is the consumer's budget constraint? What are the choice variables of the consumer? (Hint: careful on what they are not allowed to choose). Write down the consumer's problem.

(f) What are the profits of firm 1 with output y_1? What are the choice variables of the firm? (Hint: again, careful on what they are not allowed to choose). Write down the full profit maximization problem of firm type 1. Repeat for firm type 2.

(g) Carefully define a competitive equilibrium for this economy.

(h) Solve for the competitive equilibrium. Does this decentralize the planner's problem?

[1] Hint: Equations combining "apples" and "oranges" directly without any prices may be correct, but they may not be useful as they are physically district objects.

Question 2

A price taking consumer has an exogenous endowment $\{y_t\}_{t=0}^{\infty}$. They choose consumption to maximize their welfare given a discount rate $\beta \in (0,1)$, and a concave $u(\cdot)$.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$\text{s.t.} \sum_{t=0}^{\infty} q_t^0 c_t \leq \sum_{t=0}^{\infty} q_t^0 y_t \tag{2}$$

where $\{q_t\}_{t=0}^{\infty}$ are the price of one unit of consumption good delivered at time t measured in units of time 0 consumption (i.e., use a $q_0^0 = 1$ normalization of the price level here).

Assume the utility has the form,

$$u(c) = \begin{cases} \frac{1}{1-\gamma} c^{1-\gamma} & \text{if } \gamma > 0, \gamma \neq 1 \\ \log c & \text{if } \gamma = 1 \end{cases}$$

Note: you will note that the marginal utility, $u'(c) = c^{-\gamma}$ holds for all $\gamma > 0$, including the special log case. This means you will not need to treat it separately.

Assume there is a large number of identical agents in the economy, all with identical processes $y_t = y_0 \delta^t$ for $0 < \delta < 1/\beta$.

Finally, recall that if $q_0^0 = 1$, then r_{0t} is the "yield to maturity on a t-period zero-coupon bond purchased at time 0" through,

$$\frac{q_t^0}{q_0^0} \equiv \frac{1}{(1 + r_{0t})^t}$$

(a) What is the feasibility condition in the economy (i.e. relate c_t and y_t)? (hint: can use a representative agent with a large number of price taking agents).

(b) Solve for q_t^0 in this model, explaining why q_0^0 can be chosen for convenience. Then use this to find r_{0t} from the definition above.

(c) In the special case of $\gamma = 1$, compute q_t^0 and r_{0t}. Compute the special case of $\gamma = 1$ and $\delta = 1$.

(d) Interpret r_{01} if $\gamma = 1$ for the $\delta > 1$ and $\delta < 1$ cases

(e) Interpret r_{01} for $\gamma > 0$ and $\delta = 1$. In particular, discuss any reliance on γ.