Question 1

A person who has just graduated from high school can enter the workforce now (at time t) and earn the future income process

$$w_{t+j}^h = \delta_h^j w_t^h$$

for $j = 0 \dots T$, where $\delta_h > 1$. Alternatively, if the person goes to college and graduate school, they start working in period $t + k$. They earn nothing while in school for the k periods, but have the following income process after they begin working

$$w_{t+j}^c = \delta_c^j w_t^c$$

for $j = k, \dots T$ and for $\delta_c > 1$. In either case, the agent retires at time $t + T + 1$. The discounts income at a rate $\beta \in (0, 1)$.

(a) Find a formula for the present discounted value of lifetime earnings <u>at time t</u> if they begin working in high school (i.e., PV_t^h) or if the go to college and begin working afterwords (i.e.i, PV_t^c). These formula should be in terms of $\beta, T, w_t^c, w_t^h, \delta_h, \delta_c$, and k.

(b) Assume that the consumer has period utility $u'(c) > 0, u''(c) < 0$, maximizes the present discounted value of consumption (as we did in class), and can borrow or save at an interest rate $R = 1/\beta$. Write an equation for starting college wages w_t^c that makes the consumer indifferent between working now or going to college.[1]

(c) Would this indifference equation hold if the consumer could not borrow?

[1] Don't get caught up in reducing and simplifying this expression if it is difficult. I want to make sure you have set it up correctly as an implicit equation of model parameters. Another hint: you can only compare present discounted values if they reflect discounting from the same starting point (e.g. both at time t).

Question 2

Let $y_t \in \mathbb{R}$ be a sequence of dividends such that

$$y_{t+1} = y_t + w_{t+1}$$

where $w_{t+1} \sim N(\gamma, \sigma^2)$ for some $\sigma > 0$ and $\gamma \in \mathbb{R}$. i.e. $\mathbb{E}_t\left[w_{t+1}\right] = \gamma$ and $\mathbb{E}_t\left[(w_{t+1} - \mathbb{E}_t\left[w_{t+1}\right])^2\right] = \sigma^2$. An agent prices an asset as the expected PDV of dividends,

$$p_t = \mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j y_{t+j}\right]$$

Define the forecast error as the deviation in the expected value calculated at time t from the actual value the next period. e.g. $FE^y_{t+1|t} \equiv y_{t+1} - \mathbb{E}_t\left[y_{t+1}\right]$.

(a) Convert the dividend process to one with a normalized Gaussian term, i.e. replace w_{t+1} with a $\epsilon_{t+1} \sim N(0, 1)$

(b) Setup in our canonical Linear Gaussian State Space model, defining the appropriate state as x_t.

(c) Solve for p_t in terms of x_t and model intrinsics.[2]

(d) Find the expected forecast error of x_{t+1}: $\mathbb{E}_t\left[FE^x_{t+1|t}\right] = \mathbb{E}_t\left[x_{t+1} - \mathbb{E}_t\left[x_{t+1}\right]\right]$

(e) Find the expected forecast error of y_{t+1}:[3] $\mathbb{E}_t\left[FE^y_{t+1|t}\right] = \mathbb{E}_t\left[y_{t+1} - \mathbb{E}_t\left[y_{t+1}\right]\right]$

(f) Find the variance of forecast errors:

$$\begin{aligned}
\mathbb{V}_t\left(FE^y_{t+1|t}\right) &\equiv \mathbb{E}_t\left[\left(FE^y_{t+1|t}\right)^2\right] - \left(\mathbb{E}_t\left[FE^y_{t+1|t}\right]\right)^2 \\
&= \mathbb{E}_t\left[\left(y_{t+1} - \mathbb{E}_t\left[y_{t+1}\right]\right)^2\right] - \left(y_{t+1} - \mathbb{E}_t\left[y_{t+1}\right]\right)^2
\end{aligned}$$

Interpret any dependence of the forecast error on the drift parameter, γ.

(g) Find the expected forecast error of p_{t+1}:[4] $\mathbb{E}_t\left[FE^p_{t+1|t}\right] = \mathbb{E}_t\left[p_{t+1} - \mathbb{E}_t\left[p_{t+1}\right]\right]$

(h) Setup the problem recursively as p_t define in terms of p_{t+1}. Solve the recursive problem with guess-and-verify, using your previous solution as a guide, and exploiting your Linear Gaussian state space setup. Feel free to leave things as matrices where appropriate.

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[2] Hint: You can use the appropriate formulas and leave it in terms of matrices if you have correctly put it into the state space.

[3] Hint: Very similar to the previous one, but you will need to use the G matrix. I expect you to do the (simple) matrix algebra here.

[4] Hint: leave this in matrix form until the end, and then simplify.