**Question 1**

Consider a variation of the example in class where the government makes a surprise announces that it will borrow money, give the money to the consumers as a "stimulus", and eventually pay it back through taxing consumers (i.e., no chance of a government default).

The agent's income is $y_t = \delta^t$ for $\delta > 1$, and assume that: $\beta R = 1$, $u'(c) > 0$, and $u''(c) < 0$. Furthermore, assume that the consumer's and government face the same interest rate $R > 0$.

The government makes there announcement between time 0 and time 1 (i.e., after the consumer has already chosen $c_0$ and $F_1$ thinking that their income will follow $y_t$). The precise announcement is that at time 1, the consumer's are given $\alpha > 0$ as *extra* income as a stimulus (thereby increasing their $y_1$ from what they had previously anticipated). That is income is now,

$$y_1 = \delta + \alpha$$

Instead of paying back deterministically, the government will pay back the loan at period $k$ (which will be stochastic). To pay the loan, the government taxes the total value of the loan + interest. For example, if they paid it off in period $k$, then the labor income of a consumer at period $k$ would be

$$y_k = \delta^k - \alpha R^{k-1}$$

Otherwise, the consumer's income follows the same $y_t$ process. While the consumer doesn't know exactly when the loan will be repaid, they know the correct distribution of payment dates upon the announcement 1:

$$\mathbb{P}\,(\text{pay at k}) = p(k) \geq 0$$

where $\sum_{k=2}^{\infty} p(k) = 1$.

(a) First, assuming the standard permanent income model, calculate the optimal sequence $\{c_t\}_{t=0}^{\infty}$ at $t = 0$, before the government announces the policy. Note that at this point, the consumer believes they have a deterministic income stream and that the government is not going to borrow or tax.

(b) After the surprise announcement, what is the new optimal path of consumption chosen?[1] What is $c_1 - c_0$? Interpret the effect of the stimulus on consumption.

(c) Does it matter if the consumer knows the true distribution of payment dates, or the timing of the taxes to pay for the loan?

[1] Hint: at time 1 the consumer's income is now stochastic, but it is very linear and simple.

**Question 2**

A consumer's optimal decision rule for consumption satisfies

$$c_t = (1-\beta)\Big[F_t + E_t \sum_{j=0}^{\infty} \beta^j y_{t+j}\Big] \tag{1}$$

where $c_t$ is consumption, $\beta^{-1}$ is the gross one-period interest rate (i.e., $\beta R = 1$), which is constant over time, $y_t$ is the consumer's income at time $t$, $F_t$ is the consumer's financial assets at the beginning of $t$, and $E_t(\cdot)$ means the best forecast of $(\cdot)$ (whatever $(\cdot)$ is), conditional on information that the consumer knows at $t$. At time $t$, assume that the consumer knows current and past values of $y_t$'s, but not future values. The consumer's labor income follows the random process

$$y_{t+1} = \delta_0 + \delta_1 y_t + \delta_2 y_{t-1} + \sigma \epsilon_{t+1}$$

where $\{\epsilon_{t+1}\}_{t=0}^{\infty}$ is an independently and identically distributed (iid) sequence of scalar normally distributed scalar random variables, each with mean 0 and variance 1. (Please make whatever assumptions you want about $\delta_1$ and $\delta_2$ in order to make the subsequent questions meaningful.)

(a) Given available information at time $t$, give an expression for the consumer's expected income $j$ periods into the future, $E_t y_{t+j}$

(b) Find an expression for the consumer's decision rule of the form

$$c_t = (1-\beta)\Big[F_t + \alpha_0 + \alpha_1 y_t + \alpha_2 y_{t-1}\Big].$$

Please describe how to find formulas for $\alpha_0, \alpha_1, \alpha_2$.

(c) Measured in constant 2015 dollars, the changes in consumption for this consumer over the last year (which started out better than it ended) were as follows:

| quarter | $c_t - c_{t-1}$ |
|---|---|
| 2018I | 1000 |
| 2018II | 0 |
| 2018III | 0 |
| 2018IV | -4000 |

What can you infer from these consumption change numbers, if anything, about the consumer's past, present, and future labor income? Can you interpret these in the context of "surprise"?

**Question 3**

Consider two scenarios for a consumer planning consumption with income process $y_t = y_0\delta^t, \quad \forall t \geq 0$ and $F_0 = 0$.

**Scenario 1 for Consumer** The consumer maximizes the following welfare

$$U \equiv \max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{2}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{3}$$

$$\text{(transversality condition)} \tag{4}$$

**Scenario 2 for Consumer** The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of $y_0$ is potentially different (define it as $y_0^{NB}$)

Define the PDV of utility (i.e., the welfare) of this as $U^{NB}$.

(a) Let $\beta = 0.95, R = 1.04, \delta = 1.02, y_0 = 1$, and $y_0^{NB} = 1$. Calculate $U$ and $U_{NB}$.

(b) Let $y_0 = 1$. Now find a $y_0^{NB}$ such that $U = U^{NB}$. The difference between $y_0$ and $y_0^{NB}$ is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

(c) Maintain $y_0 = 1$. Now, let $\beta = .99, R = 1.04$, and $\delta = 1.01$. What is $c_0$ and $F_1$ here under Scenario 1? Repeat part (b) to find $y_0^{NB}$ such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

**Question 4**

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$\max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{5}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{6}$$

$$F_{t+1} \geq -B \tag{7}$$

$$F_0 = 0 \tag{8}$$

$$\text{(transversality condition)} \tag{9}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined $k_0$ and $k_1$.[2] Solve for $k_0$ and $k_1$ and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[2] Note that this equation deliberately is avoiding any $t$ subscripts! This makes it a truly recursive expression.