Question 1

A price taking consumer has an exogenous endowment $\{y_t\}_{t=0}^{\infty}$. They choose consumption to maximize their welfare given a discount rate $\beta \in (0,1)$, and a concave $u(\cdot)$.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$\text{s.t.} \sum_{t=0}^{\infty} q_t^0 c_t \leq \sum_{t=0}^{\infty} q_t^0 y_t \tag{2}$$

where $\{q_t\}_{t=0}^{\infty}$ are the price of one unit of consumption good delivered at time t measured in units of time 0 consumption (i.e., use a $q_0^0 = 1$ normalization of the price level here).

Assume the utility has the form,

$$u(c) = \begin{cases} \frac{1}{1-\gamma} c^{1-\gamma} & \text{if } \gamma > 0, \gamma \neq 1 \\ \log c & \text{if } \gamma = 1 \end{cases}$$

Note: you will note that the marginal utility, $u'(c) = c^{-\gamma}$ holds for all $\gamma > 0$, including the special log case. This means you will not need to treat it separately.

Assume there is a large number of identical agents in the economy, all with identical processes $y_t = y_0 \delta^t$ for $0 < \delta < 1/\beta$.

Finally, recall that if $q_0^0 = 1$, then r_{0t} is the "yield to maturity on a t-period zero-coupon bond purchased at time 0" through,

$$\frac{q_t^0}{q_0^0} \equiv \frac{1}{(1 + r_{0t})^t}$$

(a) What is the feasibility condition in the economy (i.e. relate c_t and y_t)? (hint: can use a representative agent with a large number of price taking agents).

(b) Solve for q_t^0 in this model, explaining why q_0^0 can be chosen for convenience. Then use this to find r_{0t} from the definition above.

(c) In the special case of $\gamma = 1$, compute q_t^0 and r_{0t}. Compute the special case of $\gamma = 1$ and $\delta = 1$.

(d) Interpret r_{01} if $\gamma = 1$ for the $\delta > 1$ and $\delta < 1$ cases

(e) Interpret r_{01} for $\gamma > 0$ and $\delta = 1$. In particular, discuss any reliance on γ.

Question 2

Consider a standard setup of the neoclassical growth model in a competitive equilibrium: A representative consumer orders its welfare by[1]

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

where $0 < \beta < 1$. Or $\beta \equiv \frac{1}{1+\rho}$ for $\rho > 0$. The technology in the economy is,

$$y_t = f(k_t) = z k_t^{\alpha}$$

for $0 < \alpha < 1$ and $z > 0$. Labor of mass 1 is supplied inelastically.

Given exogenous government expenditures of real goods, g_t, the feasibility condition is

$$c_t + k_{t+1} + g_t \leq y_t + (1 - \delta) k_t$$

In a competitive equilibrium, the government will finance g_t through taxes on capital or lump-sump taxes, $\{\tau_{kt}, \tau_{ht}\}$. Negative taxes are subsidies.

(a) Find the steady state level of capital and consumption $\left\{\bar{k}, \bar{c}\right\}$ if $g_t = \tau_{kt} = \tau_{ht} = 0$.

(b) Now, assume that while the government will still have $g_t = 0$, they can choose a constant tax ($\bar{\tau}_k > 0$) or subsidize ($\bar{\tau}_k < 0$) the return to capital faced by the consumer. Since they have no need for expenditures, then if $\bar{\tau}_k > 0$ the government simply rebates the revenues to consumers as a lump-sum subsidy ($\bar{\tau}_h < 0$). Similarly, to pay for a capital subsidy the government sets a lump-sum tax. Find the steady state $\left\{\bar{k}, \bar{c}\right\}$ for a given $\bar{\tau}_k$ tax (or subsidy).[2]

(c) The objective of government (A) is to maximize steady state consumption per capita by choosing the $\bar{\tau}_k$. Formulate this as an optimal problem for the government, and solve for its optimal $\bar{\tau}_k$ policy and the corresponding steady state $\left\{\bar{c}, \bar{k}\right\}$. What is the sign of $\bar{\tau}_k$, and why?

(d) Now, a new government (B) comes to power with the objective of maximizing consumer welfare (i.e. our usual objective) by choosing a constant $\bar{\tau}_k$. Find the optimal $\bar{\tau}_k$ policy and the corresponding steady state $\left\{\bar{c}, \bar{k}\right\}$. What is the sign of $\bar{\tau}_k$, and why?

(e) Assuming that government (A) was in power for a long-time and the economy was in a steady state. The new government (B) is elected with no anticipation, and associated new tax policy is immediately changed to the optimal value forever. Draw the dynamics of $\{k_t, c_t\}_{t=0}^{\infty}$ as the economy evolves from the initial steady state of government (A) to the new steady state of government (B).

(f) Compare the steady states of the two governments to discuss whether $\bar{\tau}_k$ was set too high or too low in government (A).[3]

[1] Let c_t, k_t, y_t, and g_t be in per-capita terms.
[2] Hint: $\bar{\tau}_h$ adjusts to balance the government's budget and is non-distorting.
[3] Be explicit on what criteria one should use to make this judgment.