**Question 1**

Consider a markov chain with two states: $U$ for unemployment and $E$ for employment.

- With probability $\lambda \in (0,1)$, a person unemployed today becomes employed tomorrow.
- With probability $\alpha \in (0,1)$, a person employed today becomes unemployed tomorrow.

(a) Let $N \geq 1$ be the number of periods until a currently unemployed person becomes employed. Calculate $\mathbb{E}[N]$.

(b) Let $M \geq 1$ be the number of periods until a currently employed person becomes unemployed. Calculate $\mathbb{E}[M]$.

(c) Please compute the fraction of time an infinitely lived person can expect to be unemployed and the fraction of time they can expect to be employed.

**Question 2**

An economy has 3 states for workers:

- $U$: unemployment.
- $V$: if they have found a potential employer and are being verified to see if they are a good fit.
- $E$: if a worker has been verified and is employed.

The probabilities that they jump between these states each period is:

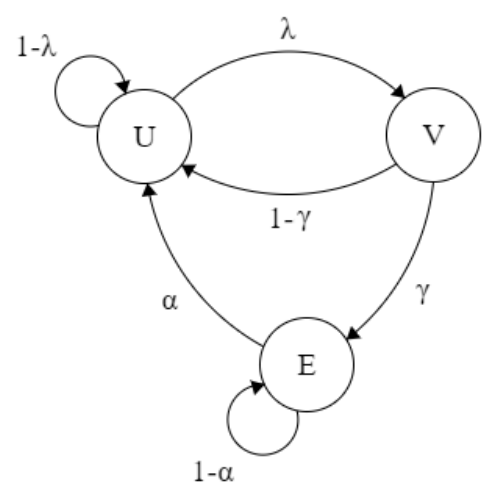


Figure 1: Markov Chain

i.e. probability $\gamma$ they are a good fit, and the verification takes 1 period.

(a) Write a Markov transition matrix for this process, $P$.

(b) Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of $P$).

(c) If a worker is $U$ today, write an expression for the probability they will be employed exactly $j$ periods in the future (considering any possible transitions which end in employment at $j$ periods).[1].

(d) Assume that $\alpha = 0, \lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist and the intuition from the perspective of the Markov chain.

[1]Note: This is only looking at $j$ periods into the future. i.e. this is **not** the probability that they become at employed at least once during the $j$ periods, which is a much more difficult calculation.

**Question 3**

Let $y_t \in \mathbb{R}$ be a sequence of dividends such that

$$y_{t+1} = y_t + w_{t+1}$$

where $w_{t+1} \sim N(0, \sigma^2)$ for some $\sigma > 0$. i.e. $\mathbb{E}_t [w_{t+1}] = 0$ and $\mathbb{E}_t \left[w_{t+1}^2\right] = \sigma^2$. An agent prices an asset as the expected PDV of dividends,

$$p_t = \mathbb{E}_t \left[ \sum_{j=0}^{\infty} \beta^j y_{t+j} \right]$$

Define the forecast error as the deviation in the expected price tomorrow calculated at time $t$ from the actual price tomorrow. i.e.,

$$FE_{t+1|t} \equiv p_{t+1} - \mathbb{E}_t [p_{t+1}]$$

(a) Setup in our canonical Linear Gaussian State Space model.

(b) Solve for $p_t$ in terms of $y_t$ and model intrinsics.

(c) Find the <u>expected forecast error</u>: $\mathbb{E}_t \left[FE_{t+1|t}\right] = \mathbb{E}_t [p_{t+1} - \mathbb{E}_t [p_{t+1}]]$

(d) Find the <u>variance of forecast errors</u>:

$$\begin{aligned} \mathbb{V}_t \left(FE_{t+1|t}\right) &\equiv \mathbb{E}_t \left[FE_{t+1|t}^2\right] - \left(\mathbb{E}_t \left[FE_{t+1|t}\right]\right)^2 \\ &= \mathbb{E}_t \left[(p_{t+1} - \mathbb{E}_t [p_{t+1}])^2\right] - (\mathbb{E}_t [p_{t+1} - \mathbb{E}_t [p_{t+1}]])^2 \end{aligned}$$

(e) Setup the problem recursively as $p_t$ define in terms of $p_{t+1}$. Test, using your solution from early in the question, if the stochastic process $p_t$ a martingale (i.e., just a random walk in this case)? Recall that a martingale is a process such that $\mathbb{E}_t [p_{t+1}] = p_t$. Give any intuition you can on this result.