Question 1

A dividend process follows the deterministic process:

$$x_{t+1} = A \cdot x_t \tag{1}$$

where x_t is an $n \times 1$ vector and A is an $n \times n$ matrix, and

$$y_t = G \cdot x_t \tag{2}$$

where y_t is the dividend(a scalar), and G is an $1 \times n$ vector. Assume future profits are discounted by $\beta \in (0, 1)$, and that $I - \beta A$ is invertible.

(a) What is the stock price of the firm p_t, in terms of A and G if there was no bubble?

(b) What is the stock price of the firm today, p_t, in terms of the price tomorrow, p_{t+1}, and the state today, x_t?

(c) A friend guesses that the stock price should be

$$p_t = H \cdot x_t + c \cdot \lambda^t \tag{3}$$

for some vector $H \in \mathbb{R}^n$ and scalars c, λ.
Get as far as you can in finding formulas for H, c, λ. (**Hint**: use the guess and verify to find the undetermined constants, with the recursive definition of the price from (b)).

(d) Is H unique? How about c and λ?

Question 2

A dividend obeys:

$$y_{t+1} = \lambda_0 + \lambda_1 y_t + \lambda_2 y_{t-1} \tag{4}$$

where y_t is scalar

The stock price obeys:

$$p_t = \sum_{j=0}^{\infty} \beta^j y_{t+j} \tag{5}$$

(a) Find a solution for the price p_t of the form:

$$p_t = a_0 + a_1 y_t + a_2 y_{t-1} \tag{6}$$

for some a_0, a_1, and a_2 in terms of model parameters.[1] (No need to actually invert matrices, etc. to find the solution to the particular a_0, a_1, a_2)

[1] **Hint:** Set it up as a linear state space.

Question 3

Take an asset which owns claims a single claim on two streams of dividends (both paying out to the owner of the asset at time t):

- $d_{At+1} = (1+\delta_A)d_{At}$ for $\delta_A \geq 0$
- $d_{Bt+1} = (1+\delta_B)d_{Bt}$ for $\delta_B \geq 0$ where $d_{A0} = d_{B0} = 1$. Let the price of this asset, using discount **rate** $\rho > 0$ (i.e. $\frac{1}{1+\rho}$ is the discount factor) be p_t^{AB}.

That is, if I own 1 unit of the asset at time t, I get $y_t = d_{At} + d_{Bt}$ in payoffs.

(a) Write this problem in our linear state space model.

(b) Find an expression for the price, p_0^{AB}, of the underlying asset at time 0 using the tools from our linear state space models.[2]

(c) Roughly describe the conditions on δ_1, δ_2 and ρ required for this to be a well defined problem.

(d) Now assume that instead of a joint asset, consider an asset, priced at p_t^A which only has claims to the d_{At} sequence, and another p_t^B with claims to the d_{Bt} sequence. Calculate p_0^A and p_0^B.

(e) Describe the intuition for how p_0^{AB}, p_0^A, p_0^B relate, and how agents would behave different if the relationship was broken.

[2]Hint: to take the inverse of a diagonal matrix, just take the reciprocal along the diagonals. i.e. $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix}$