Question 1

A person who has just graduated from high school can enter the workforce now (at time t) and earn the future income process

$$w_{t+j}^h = \delta_h^j w_t^h$$

for $j = 0 \dots T$, where $\delta_h > 1$. Alternatively, if the person goes to college and graduate school, they start working in period $t + k$. They earn nothing while in school for the k periods, but have the following income process after they begin working

$$w_{t+j}^c = \delta_c^j w_t^c$$

for $j = k, \dots T$ and for $\delta_c > 1$. In either case, the agent retires at time $t + T + 1$. The discounts income at a rate $\beta \in (0, 1)$.

(a) Find a formula for the present discounted value of lifetime earnings <u>at time t</u> if they begin working in high school (i.e., PV_t^h) or if the go to college and begin working afterwords (i.e.i, PV_t^c). These formula should be in terms of $\beta, T, w_t^c, w_t^h, \delta_h, \delta_c$, and k.

(b) Assume that the consumer has period utility $u'(c) > 0, u''(c) < 0$, maximizes the present discounted value of consumption (as we did in class), and can borrow or save at an interest rate $R = 1/\beta$. Write an equation for starting college wages w_t^c that makes the consumer indifferent between working now or going to college.[1]

(c) Would this indifference equation hold if the consumer could not borrow?

[1] Don't get caught up in reducing and simplifying this expression if it is difficult. I want to make sure you have set it up correctly as an implicit equation of model parameters. Another hint: you can only compare present discounted values if they reflect discounting from the same starting point (e.g. both at time t).

Question 2

(**Sequential and Recursive**)

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{2}$$

$$F_{t+1} \geq -B \tag{3}$$

$$F_0 = 0 \tag{4}$$

$$\text{(transversality condition)} \tag{5}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1.[2] Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[2]Note that this equation deliberately is avoiding any t subscripts! This makes it a truly recursive expression.