Question 1

Consider a markov chain with two states: U for unemployment and E for employment.

- With probability $\lambda \in (0,1)$, a person unemployed today becomes employed tomorrow.
- With probability $\alpha \in (0,1)$, a person employed today becomes unemployed tomorrow.

(a) Let $N \geq 1$ be the number of periods until a currently unemployed person becomes employed. Calculate $\mathbb{E}[N]$.

(b) Let $M \geq 1$ be the number of periods until a currently employed person becomes unemployed. Calculate $\mathbb{E}[M]$.

(c) Please compute the fraction of time an infinitely lived person can expect to be unemployed and the fraction of time they can expect to be employed.

Question 2

An economy has 3 states for workers:

- U: unemployment.
- V: if they have found a potential employer and are being verified to see if they are a good fit.
- E: if a worker has been verified and is employed.

The probabilities that they jump between these states each period is:

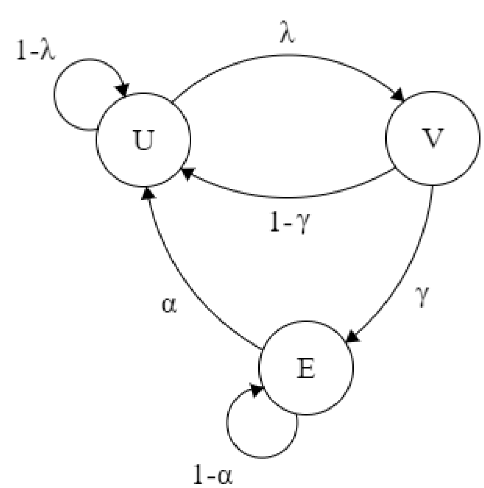

Figure 1: Markov Chain

i.e. probability γ they are a good fit, and the verification takes 1 period.

(a) Write a Markov transition matrix for this process, P.

(b) Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of P).

(c) If a worker is U today, write an expression for the probability they will be employed exactly j periods in the future (considering any possible transitions which end in employment at j periods).[1].

(d) Assume that $\alpha = 0, \lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist and the intuition from the perspective of the Markov chain.

[1]Note: This is only looking at j periods into the future. i.e. this is **not** the probability that they become at employed at least once during the j periods, which is a much more difficult calculation.