

Search and the Labor Market

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1 Quick Review

1.1 Markov Chain Model of Unemployment

Recall model of unemployment:

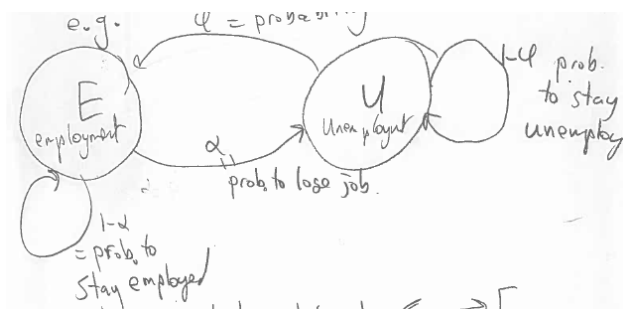


Figure 1: Lake Model with Markov Chains

where ϕ probability of U to E transition should come from consumer decisions.

- This model will endogenize based on dynamic decisions of consumers looking for, and potentially rejecting, jobs.
- This, in turn, endogenizes unemployment rate.

1.2 Random Variables Review

- p is a non-negative random variable with pdf (probability density function) $f(p)$ and cdf (cumulative distribution function):

$$F(p) \equiv \int_0^p f(s)ds \quad (1)$$

Assume $F(0) = 0$, $F(B) = 1$, where B is the upper bound.

- If p has continuous values on $[0, B]$, then:

$$\underbrace{\mathbb{E}[p]}_{\text{mean of } p} = \int_0^B p f(p) dp \quad (2)$$

An alternative formula using the CDF,¹

$$\mathbb{E}[p] = \int_0^B (1 - F(p)) dp \quad (3)$$

2 McCall Search Model

2.1 Setup

- Background:

- Infinitely lived risk-neutral worker wants to maximize the expected P.D.V:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t y_t \right], \text{ with } y_t = \begin{cases} w_t & \text{if employed} \\ c & \text{if unemployed} \end{cases} \quad (8)$$

where $\beta \in (0, 1)$

- Each period, an unemployment worker draws one offer to work at wage w forever,

¹To derive, Recall integration by parts,

$$\int_a^b u dv = \underbrace{uv \Big|_a^b}_{\substack{\text{evaluated at } b - \\ \text{evaluated at } a}} - \int_a^b v du \quad (4)$$

Using our definition of expectation:

$$\int_0^B \underbrace{p}_{\substack{u=p \\ \downarrow \\ du=dp}} \underbrace{f(p)dp}_{\substack{dv=f(p)dp \\ \downarrow \\ v=F(p)=\int_0^p f(s)ds}} = \int_0^B u dv \quad (5)$$

By formula,

$$= pF(p) \Big|_0^B - \int_0^B F(p) dp \quad (6)$$

$$= \underbrace{BF(B)}_B - 0 - \int_0^B F(p) dp = \underbrace{\int_0^B (1 - F(p)) dp}_{\text{Alternative formula for expectation}} = \mathbb{E}[p] \quad (7)$$

the wage is drawn from the distribution of wages in the economy $F(w)$, where $F(0) = 0$, $F(B) = 1$

- There is recall of previous wages each period
- Each period, can accept current wage draw from $F(w)$, or reject it and collect $c > 0$ (unemployment benefits) and draw again next period.
- If accept, the worker will work forever at w and can neither be fired nor quit.

- Problem:

- Find the worker's optimal strategy for accepting or rejecting draws

- Solution:

- Let Q = optimal value of the problem for a worker about to draw a wage offer

$$Q \equiv \underbrace{\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t y_t \right]}_{\text{expectation before wage is drawn}} \quad (9)$$

- Let $v(w)$ be the optimal value of the problem for a previously unemployed worker who has just drawn w and is about to decide what to do,

$$\underbrace{Q}_{\substack{\text{value} \\ \text{about to} \\ \text{draw}}} = \underbrace{\int_0^B v(w) f(w) dw}_{\text{expected value over draws}} \quad (10)$$

- Key observation: write recursively as value today in terms of tomorrow

$$\underbrace{v(w)}_{\substack{\text{value if} \\ \text{draw } w}} = \max_{\{\text{accept, reject}\}} \left\{ \underbrace{w}_{\text{Wage today}} + \underbrace{\beta v(w)}_{\substack{\text{Discounted value} \\ \text{keeping wage}}}, \underbrace{c}_{\substack{\text{Unemployment} \\ \text{Benefits today}}} + \underbrace{\beta \cdot Q}_{\substack{\text{Discounted value} \\ \text{drawing wage}}} \right\} \quad (11)$$

Recognizing if you accepting w once, you would keep accepting it, $v(w|\text{accept}) = w + \beta v(w|\text{accept})$ for accepted wage. So $v(w|\text{accept}) = \frac{w}{1-\beta}$, and

$$\underbrace{v(w)}_{\substack{\text{value if} \\ \text{draw } w}} = \max_{\{\text{accept, reject}\}} \left\{ \underbrace{\frac{w}{1-\beta}}_{\substack{\text{PDV of} \\ \text{wage forever}}}, \underbrace{c}_{\substack{\text{unemployment} \\ \text{benefits today}}} + \underbrace{\beta \cdot Q}_{\substack{\text{discounted} \\ \text{value tomorrow}}} \right\} \quad (12)$$

Note: $c + \beta \cdot Q$ is independent of w , since $Q = \int v(w')f(w')dw'$

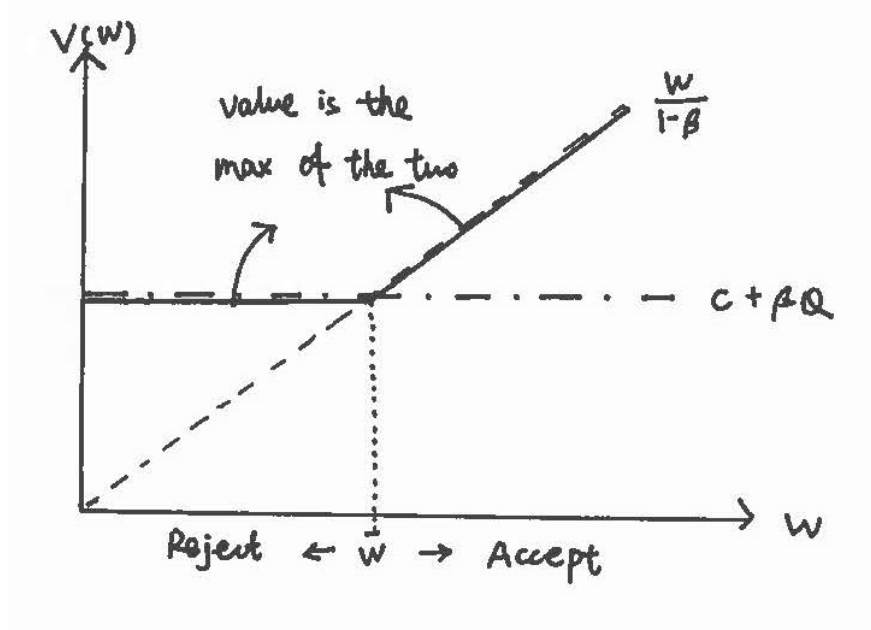


Figure 2: McCall model

– Plug in for Q to find a functional equation in $v(w)$,

$$v(w) = \max \left\{ \frac{w}{1-\beta}, c + \beta \int_0^B v(w')f(w')dw' \right\} \quad (13)$$

2.2 Solve the Bellman Equation

The solution to this problem is a $v(w)$ and an accept or reject policy which fulfills (13) for the “deep parameters” β , $f(\cdot)$, c .²

Using Guess-and-verify Start with the functional equation from (13). Guess there is a \bar{w} where agent is indifferent between accept and reject

²A numerical method is to iterate the following fixed-point map

$$v_{j+1}(w) = \max \left\{ \frac{w}{1-\beta}, c + \beta \int_0^B v_j(w')f(w')dw' \right\} \quad (14)$$

Let $v_0(w) = 0$, evaluate $v_1(w)$, etc. Guaranteed to converge to a stationary $v(w)$

$$v(\bar{w}) = \frac{\bar{w}}{1-\beta} = c + \beta \int_0^B v(w')f(w')dw', \text{ where } \begin{cases} \text{reject if } w' < \bar{w} \\ \text{accept if } w' > \bar{w} \end{cases} \quad (15)$$

$$= c + \beta \int_0^{\bar{w}} v(w')f(w')dw' + \beta \int_{\bar{w}}^B v(w')f(w')dw' \quad (16)$$

$$= c + \beta \underbrace{\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw'}_{\text{reject value by definition of } \bar{w}} + \beta \underbrace{\int_{\bar{w}}^B \frac{w'}{1-\beta} f(w')dw'}_{\text{accept value}} \quad (17)$$

Use $\bar{w} = \int_0^{\bar{w}} \bar{w}f(w')dw' + \int_{\bar{w}}^B \bar{w}f(w')dw'$ on the LFS

$$\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw' + \int_{\bar{w}}^B \frac{\bar{w}}{1-\beta} f(w')dw' = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw' + \beta \int_{\bar{w}}^B \frac{w'}{1-\beta} f(w')dw' \quad (18)$$

Subtract $\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw'$ from both sides

$$(1-\beta) \int_{\bar{w}}^B \frac{\bar{w}}{1-\beta} f(w')dw' - c = \frac{1}{1-\beta} \int_{\bar{w}}^B (\beta w' - \bar{w}) f(w')dw' \quad (19)$$

Add $\bar{w} \int_{\bar{w}}^B f(w')dw'$ to both sides

$$\bar{w} \left[\int_0^{\bar{w}} f(w')dw' + \int_{\bar{w}}^B f(w')dw' \right] - c = \frac{\beta}{1-\beta} \int_{\bar{w}}^B (w' - \bar{w}) f(w')dw' \quad (20)$$

Simplify to get an **algebraic** equation in \bar{w}

$$\underbrace{\bar{w} - c}_{\text{cost of searching one more period}} = \underbrace{\frac{\beta}{1-\beta} \int_{\bar{w}}^B (w' - \bar{w}) f(w')dw'}_{\text{benefit of searching one more period}} \quad (21)$$

Given $\beta, c, f(\cdot)$, we can solve for \bar{w} , then use policy: $\begin{cases} \text{reject if } w < \bar{w} \\ \text{accept if } w > \bar{w} \end{cases}$

Cost and Benefits Let $h(w)$ be the benefit of search if at w

$$h(w) = \frac{\beta}{1-\beta} \int_w^B (w' - w) f(w')dw' \quad (22)$$

Then,

- $h(0) = \frac{\beta}{1-\beta} \mathbb{E}[w] > 0$
- $h(B) = 0$

- $h'(w) < 0, h'' > 0$

The cost of searching is the lost wages, w and the direct cost c

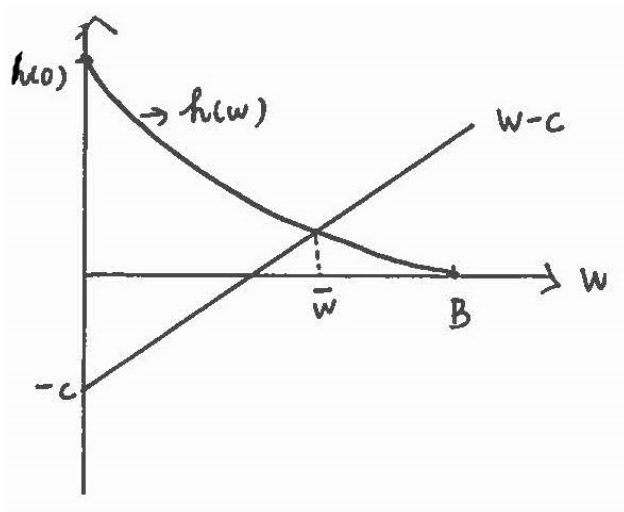


Figure 3: search model

2.3 The probability to accept

- $\text{Prob}(\text{accept} \mid \text{unemployed}) = \text{Prob}(w \geq \bar{w}) = 1 - F(\bar{w}) \equiv \pi$

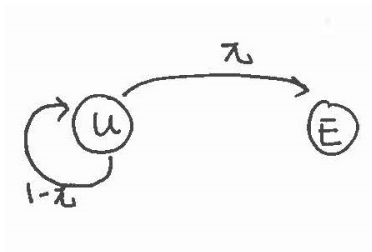


Figure 4: markov chain in search model

- In this basic setup, no firing or quitting, so E is absorbing. (Problem set includes firing)

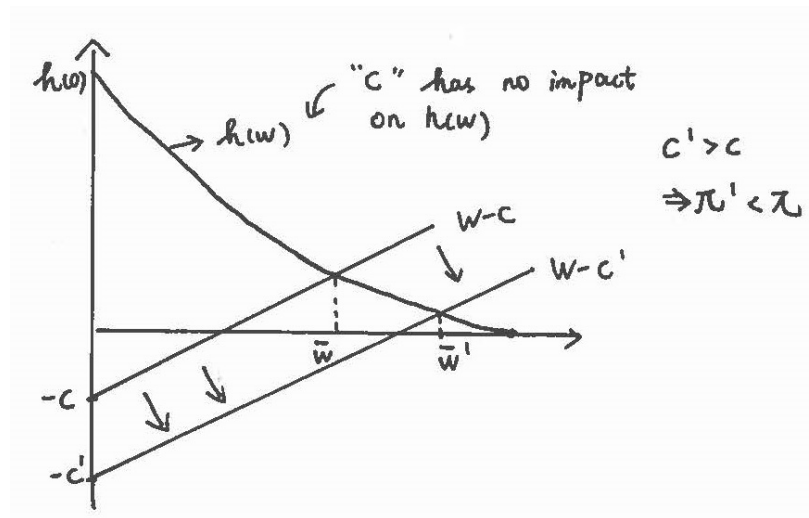


Figure 5: What is role of unemployment benefits?