

**Question 1**

Consider a variation of the example in class where the government makes a surprise announces that it will borrow money, give the money to the consumers as a “stimulus”, and eventually pay it back through taxing consumers (i.e., no chance of a government default).

The agent’s income is  $y_t = \delta^t$  for  $\delta > 1$ , and assume that:  $\beta R = 1$ ,  $u'(c) > 0$ , and  $u''(c) < 0$ . Furthermore, assume that the consumer’s and government face the same interest rate  $R > 0$ .

The government makes there announcement between time 0 and time 1 (i.e., after the consumer has already chosen  $c_0$  and  $F_1$  thinking that their income will follow  $y_t$ ). The precise announcement is that at time 1, the consumer’s are given  $\alpha > 0$  as *extra* income as a stimulus (thereby increasing their  $y_1$  from what they had previously anticipated). That is income is now,

$$y_1 = \delta + \alpha$$

Instead of paying back deterministically, the government will pay back the loan at period  $k$  (which will be stochastic). To pay the loan, the government taxes the total value of the loan + interest. For example, if they paid it off in period  $k$ , then the labor income of a consumer at period  $k$  would be

$$y_k = \delta^k - \alpha R^{k-1}$$

Otherwise, the consumer’s income follows the same  $y_t$  process. While the consumer doesn’t know exactly when the loan will be repaid, they know the correct distribution of payment dates upon the announcement 1:

$$\mathbb{P}(\text{pay at } k) = p(k) \geq 0$$

where  $\sum_{k=2}^{\infty} p(k) = 1$ .

- First, assuming the standard permanent income model, calculate the optimal sequence  $\{c_t\}_{t=0}^{\infty}$  at  $t = 0$ , before the government announces the policy. Note that at this point, the consumer believes they have a deterministic income stream and that the government is not going to borrow or tax.
- After the surprise announcement, what is the new optimal path of consumption chosen?<sup>1</sup> What is  $c_1 - c_0$ ? Interpret the effect of the stimulus on consumption.
- Does it matter if the consumer knows the true distribution of payment dates, or the timing of the taxes to pay for the loan?

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<sup>1</sup>Hint: at time 1 the consumer’s income is now stochastic, but it is very linear and simple.

**Question 2**

A consumer's optimal decision rule for consumption satisfies

$$c_t = (1 - \beta) \left[ F_t + E_t \sum_{j=0}^{\infty} \beta^j y_{t+j} \right] \quad (1)$$

where  $c_t$  is consumption,  $\beta^{-1}$  is the gross one-period interest rate (i.e.,  $\beta R = 1$ ), which is constant over time,  $y_t$  is the consumer's income at time  $t$ ,  $F_t$  is the consumer's financial assets at the beginning of  $t$ , and  $E_t(\cdot)$  means the best forecast of  $(\cdot)$  (whatever  $(\cdot)$  is), conditional on information that the consumer knows at  $t$ . At time  $t$ , assume that the consumer knows current and past values of  $y_t$ 's, but not future values. The consumer's labor income follows the random process

$$y_{t+1} = \delta_0 + \delta_1 y_t + \delta_2 y_{t-1} + \sigma \epsilon_{t+1}$$

where  $\{\epsilon_{t+1}\}_{t=0}^{\infty}$  is an independently and identically distributed (iid) sequence of scalar normally distributed scalar random variables, each with mean 0 and variance 1. (Please make whatever assumptions you want about  $\delta_1$  and  $\delta_2$  in order to make the subsequent questions meaningful.)

- (a) Given available information at time  $t$ , give an expression for the consumer's expected income  $j$  periods into the future,  $E_t y_{t+j}$
- (b) Find an expression for the consumer's decision rule of the form

$$c_t = (1 - \beta) \left[ F_t + \alpha_0 + \alpha_1 y_t + \alpha_2 y_{t-1} \right].$$

Please describe how to find formulas for  $\alpha_0, \alpha_1, \alpha_2$ .

- (c) Measured in constant dollars relative to a base year, the changes in consumption for this consumer over the last year (which started out better than it ended) were as follows:

quarter	$c_t - c_{t-1}$
I	1000
II	0
III	0
IV	-4000

What can you infer from these consumption change numbers, if anything, about the consumer's past, present, and future labor income? Can you interpret these in the context of "surprise"?